3D Topological States of Matter and the Vortex Quantum Hall Effect

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coll. P. Sodano and C.A. Trugenberger; arXiv:1104.2485; arXiv:1105.5375 to appear in PRB

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- Conclusions

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- Topological order: new quantum order not based on SSB (Wen):
  - gapped in the bulk;
  - gapless edge excitations.
- Low energy effective field theories for such states involve topological field theories
  - background independent;
  - ground state degeneracy;
  - quasiparticles have fractional statistics.

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- Topological insulators are materials that are insulating in the bulk but support conducting edge excitations and can exist also in 3D (Fu, Kane, Mele, Moore, Balents)
- First experimental observation in 2008 (Hsieh et al.)

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- Which is the topological field theory that describes this new phase of matter? Topological BF action
- BF action was first proposed as a field theory description of topological phases of condensed matter systems in 1996 (Sodano, Trugenberger, MCD; recently reintroduced by Moore and Cho):
  - topological superconductors;
  - topological insulators;
  - topological confinement.

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- Model for Josephson Junction arrays (Sodano, Trugenberger, MCD); $\phi_\mu \propto \epsilon_{\mu\nu\alpha}\partial_\nu a_\alpha$ = vortex matter current

3D

- Natural generalization:

$$j_{\mu} \propto \epsilon_{\mu\nu\alpha\beta}\partial_{\nu}b_{\alpha\beta} \text{ , charge fluctuations}$$

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- In application to condensed matter we are interested in the case in which $p = 1 \Rightarrow a_1$ and b_{D-1}

BF Theory in (D+1)-dimensions

$$S_{TM} = \int_{M_{D+1}} \frac{k}{2\pi} a_1 \wedge db_{D-1} + \; (S_{BF})$$

$$\frac{-1}{2e^2} da_1 \wedge *da_1 + \frac{(-1)^{D-1}}{2g^2} db_{D-1} \wedge *db_{D-1}$$

where k is a dimensionless parameter.

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- Supports edges excitations (Momen, Balachandran; Cho and Moore)

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- The phase structure is determined by the condensation (lack of) of topological defects

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  $f^\mu \equiv \frac{1}{6} \epsilon^{\mu\nu\alpha\beta} f_{\nu\alpha\beta}$, $f_{\mu\nu\rho} = \partial_\mu b_{\nu\rho} + \partial_\nu b_{\rho\mu} + \partial_\rho b_{\mu\nu}$

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- $\epsilon = \frac{1}{\lambda}$ electric permittivity; $\mu = \eta$ magnetic permeability

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- It describes magnetoelectric polarizability

Phase structure analysis

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- Compactness imply that gauge variables become angular variables: periodicity
- To implement periodicity one has to introduce two integer forms Q_1 and M_2 in the Euclidean partition function

$$S \to S + \int_{M_{3+1}} a_1 \wedge *Q_1 + b_2 \wedge *M_2$$

  - Q_1 = closed electric loops;
  - M_2 = closed magnetic surfaces.

- Analysis based on:
  - free energy arguments, however an analytic expression is not known for the entropy of random surfaces $\Rightarrow$ qualitative argument;
  - expectation value of Wilson loop, L_W, for an electric charge q and 't Hooft surface, S_H, for a vortex line with flux ϕ.

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- e and $\alpha = ml \geq O(1)$ are fixed parameters, vary the dimensionless parameters k, $\lambda(1/\epsilon)$ and $\eta(\mu)$:
  - $k^2\lambda \ll \alpha^2/e^2$ and $\eta \ll \pi^2/e^2$ electric condensation phase;
  - $k^2\lambda \gg \alpha^2/e^2$ and $\eta \gg \pi^2/e^2$ magnetic condensation phase;
  - in between both types of topological excitations are dilute: topological insulating phase.

- Express L_W and S_H in term of external gauge potentials A_μ and $B_{\mu\nu}$ and compute the induced charge and vortex current:
  - $j_\mu^{\mathrm{ind}} \propto A_\mu$: London equation for induced charges current, perfect conductivity and photon mass in the charge condensation phase;
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- $T = 0$ quantum phase structure:
  - $\epsilon \gg \frac{k^2 e^2}{\alpha^2}, \mu \ll \frac{\pi^2}{e^2} \rightarrow$ top. superconductor
  - intermediate regime $\rightarrow$ top. insulator
  - $\epsilon \ll \frac{k^2 e^2}{\alpha^2}, \mu \gg \frac{\pi^2}{e^2} \rightarrow$ top. confinement

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- Julia-Toulouse mechanism (Quevedo and Trugenberger): the condensation of topological defects in solid state media generates new hydrodynamical modes for the low-energy effective theory

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  - What is the fate of the photon in the topological confinement phase?
- Need to compute the effective electromagnetic action induced by the condensation of the topological defects in these new phases of matter
- Julia-Toulouse mechanism (Quevedo and Trugenberger): the condensation of topological defects in solid state media generates new hydrodynamical modes for the low-energy effective theory
- These new modes are the long wavelength fluctuations of the continuous distribution of topological defects

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$$S \to S + \int d^4x \; ieA_\mu j_\mu + i\phi F_{\mu\nu}\phi_{\mu\nu} =$$

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- The last term produces the Axion electrodynamics

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- Mass arises as a consequence of quantum mechanical condensation of topological excitations: mechanism of topological superconductivity (Allen, Bowick, Lahiri)

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- If $\theta = 0$ no BF term is present in $S^{TC}_{\rm eff}$

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- Topological matter with a compact BF term can realize $U(1)$ confinement via the Stückelberg mechanism.

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- The BF term represents a vortex quantum Hall effect for dyonic strings (2D vortex QHE first studied by Horovitz)
- The two dual forms $\phi_{\mu\nu}$ and $\omega_{\mu\nu}$ become selfdual

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  - External electric field induces a magnetic vortex current perpendicular to both the applied electric field and the direction of the magnetic flux

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- The quantum phase structure is governed by three parameters that drive the condensation of topological defects: the BF coupling, the electric permittivity and the magnetic permeability of the material
- 3 possible phases: topological superconductor, topological insulator and charge confinement

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- Vortex quantum Hall effect might find an application for the dissipationless transport of information stored on dyonic vortices in oblique confinement phases