From Classical to Quantum Critical Phenomena in Condensed Matter Physics:

1) Recollection of some old features

2) a specific case: Superconductivity in Cuprates as an avoided quantum criticality, recent results

PRB 2011 and in preparation

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Criticality

Up to the sixties of the last century the entire world of condensed matter or every N-body system in a stable phase was considered to be reducible to a collection of quasi-particles.

Puzzling behavior arises in the proximity of criticality (the first example of what is now called "complexity") where **the collective phenomena do not arise as a simple superposition of single microscopic events and the laws of great numbers are modified (violation of $1/N^{1/2}$ law).** It is no more true that each sufficiently large portion has an average behavior independent from the rest.
The statistical aspects and new universal simplifying aspects are now dominating with respect to the previous approaches (many body theory) of approximately solving the dynamics to reduce each system to a gas of quasi-particles.

The most successful use of Renormalization Group, both in the field theoretic (1969) and the Wilson (1971) approaches, is of course in critical phenomena with the summation of infrared singular perturbation terms to give, for d<4, the critical power law behavior of physical response functions.

However perturbation theory can be singular even in stable phases with **finite** physical response functions, e.g.:

Interacting Fermions at d=1.

Interacting Bosons with condensation for for d≤3. (Pistolesi et al Phys. Rev. B **69**, 024513 (2004) and references therein)

Both cases deal with stable liquid phases.

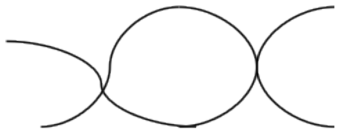
Stability therefore requires exact cancellation of singularities at all orders in perturbation theory in the response functions:

additional symmetries and related Ward Identities (relations between different skeleton structures), which, together with RG, allow for a closure of hierarchical equations and exact solution of the problems.

I only recall the interacting Fermi system at d=1, just to introduce to the non Fermi liquid problem relevant for the Cuprates.

Luttinger model was considered with linear spectrum ($\varepsilon_k = v_F(k - k_F)$) and forward scattering (g_2, g_4)

$-k_F$ $+k_F$



$\approx \ln k$ d=1, d+1-2=0 infrared ln singularity

susceptibility, specific heat, compressibility finite:

exact cancellation of singularities implemented by additional charge (spin) conservation at each Fermi point separately in addition to the total charge (spin) conservation and related Ward Identities

$$\omega\Lambda_0 - q\vec{\Lambda} = G^{-1}(\varepsilon + \omega, p + q) - G^{-1}(\varepsilon, \omega) \, , \qquad \vec{\Lambda} = v_F vers \vec{p} \Lambda_0$$

The first WI implements the total charge conservation and relates the current-vertex Λ and the charge-vertex Λ_0 to the one particle Green function. The last WI (separate left and right charge conservation) allows to eliminate Λ to give $\Lambda_0 = \Lambda_0[G]$

This closes the Dyson equation for G

The quasi-particle weight vanishes at the Fermi surface with an anomalous dimension η :

Non Fermi Liquid (FL), Charge and spin modes are separated

Single particle moves as a composite object due to the strong mixing with charge and spin modes via the RPA marginal effective interaction $D(q,\omega)$.

For a review see W.Metzner, C.Castellani and C.D C
Adv. in Phys. 47, 317-445 (1997).

2d metals? Can the same mechanism leading to Luttinger liquid in 1d lead to a non-FL in 2d as for the Cu-O planes of Cuprates?

Answer: No, with short range interaction.

In d dimension the generic integrals in k are proportional to $(\sin\theta)^{d-2}$ and for $1 \leq d < 2$ are peaked for $\theta=0, \pi$ $\longrightarrow$

Almost all relevant k vectors are parallel or anti-parallel $\longrightarrow$

Tomographic Luttinger model for which the additional WI, valid in 1d, still holds asymptotically near the Fermi Surface

$$\vec{\Lambda} = v_F vers\vec{p}\Lambda_0$$

However $D(q,\omega)$ is now averaged over the over the transverse momenta and, being marginal in 1d, scales to zero in d>1.

We obtain therefore a normal Fermi Liquid (FL) as soon as d>1 unless a singular potential compensate this reduction of mixing with collective modes. This is the case when the interaction is mediated by critical modes near an instability.
This is one of the reasons which lead us to look for a quantum critical point (QCP) in the Cuprates separating two states underlying superconductivity.

In classical critical phenomena a knowledge of the fundamental symmetry inherent to each specific problem (in particular of the order parameter) is required to make the proper choice of the basic variables entering the Landau-Wilson functional on which the RGT acts.

In quantum criticality (QC) (Hertz 1976, Millis 1993,…) at T=0 not only the symmetry of the local order parameter enters. The dynamics is inextricably mixed in the generalized Landau-Wilson functional of the coarse grained fluctuating field of the order parameter.
QC behavior usually extends in a wide T-region above QCP

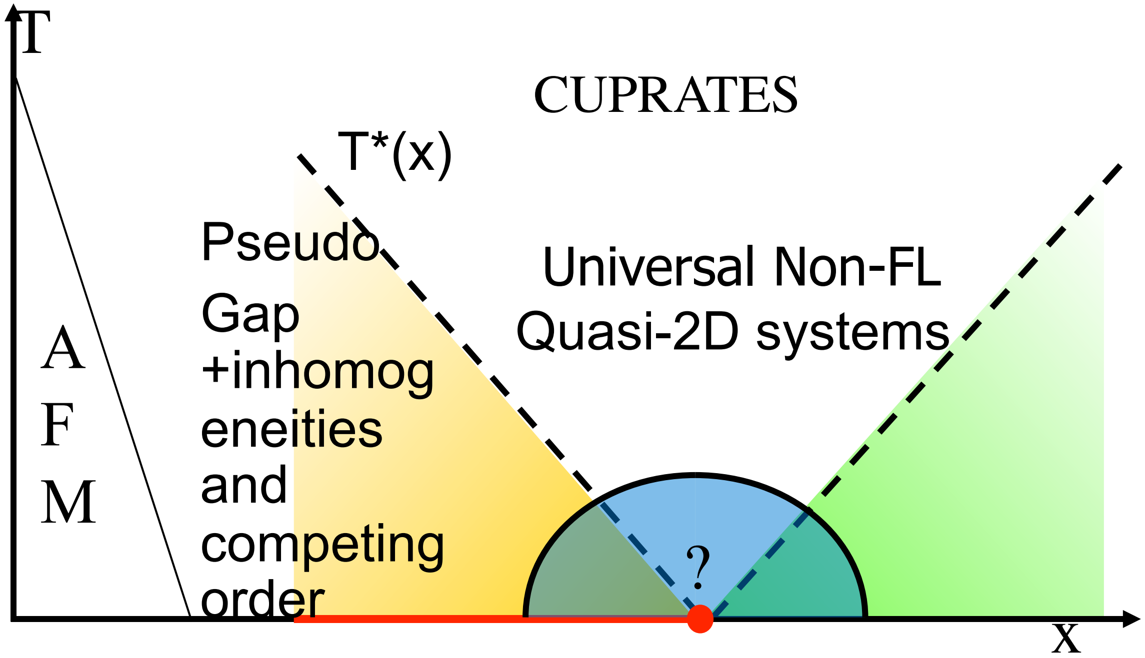
Long time correlation fluctuations are generated with divergent relaxation time $\tau_o \approx \xi^z$; the dynamical index z determines how the frequency relates to the wave vector k in the dynamical Landau-Wilson functional. z specifies if the modes of the order parameter are propagating (z=1) or damped and diffusive, z=2 (AFM).

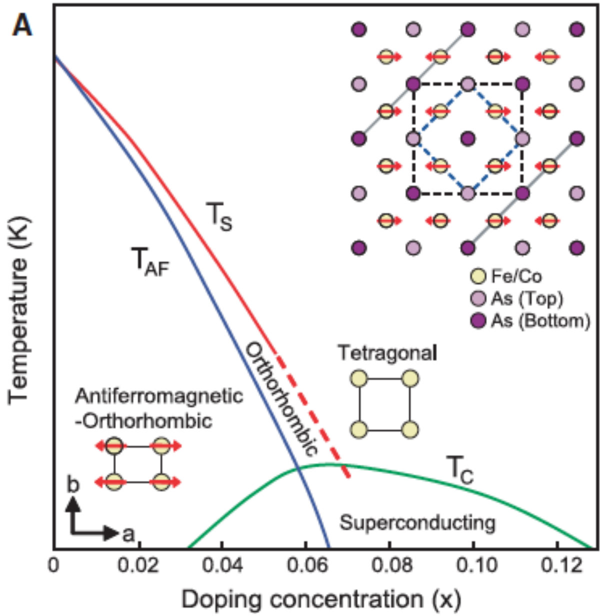
Unraveling the dynamics of specific QC and competitions eventually leading to unconventional criticality is one hot branch of research and the starting point for future more sophisticated field theory approaches. In particular in Cuprates disentangling the relevant modes (of the underlying state on which high temperature superconductivity (HTS) establishes) acting also as a glue for pairing, is a relevant current issue. It is also a starting point to unravel which phase is competing with superconductivity.

Examples of “hidden” *QCP* and schematic phase diagrams

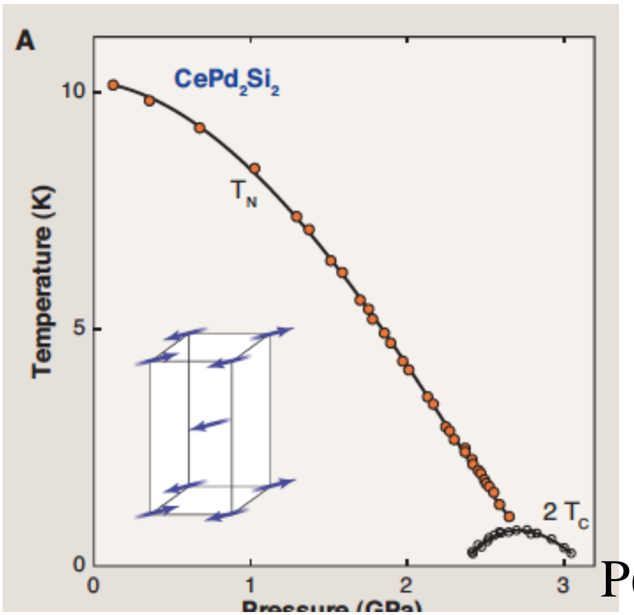
$Ca(Fe_{1-x}Co_x)_2As_2$

Chuang et al 2010





Heavy
Fermions



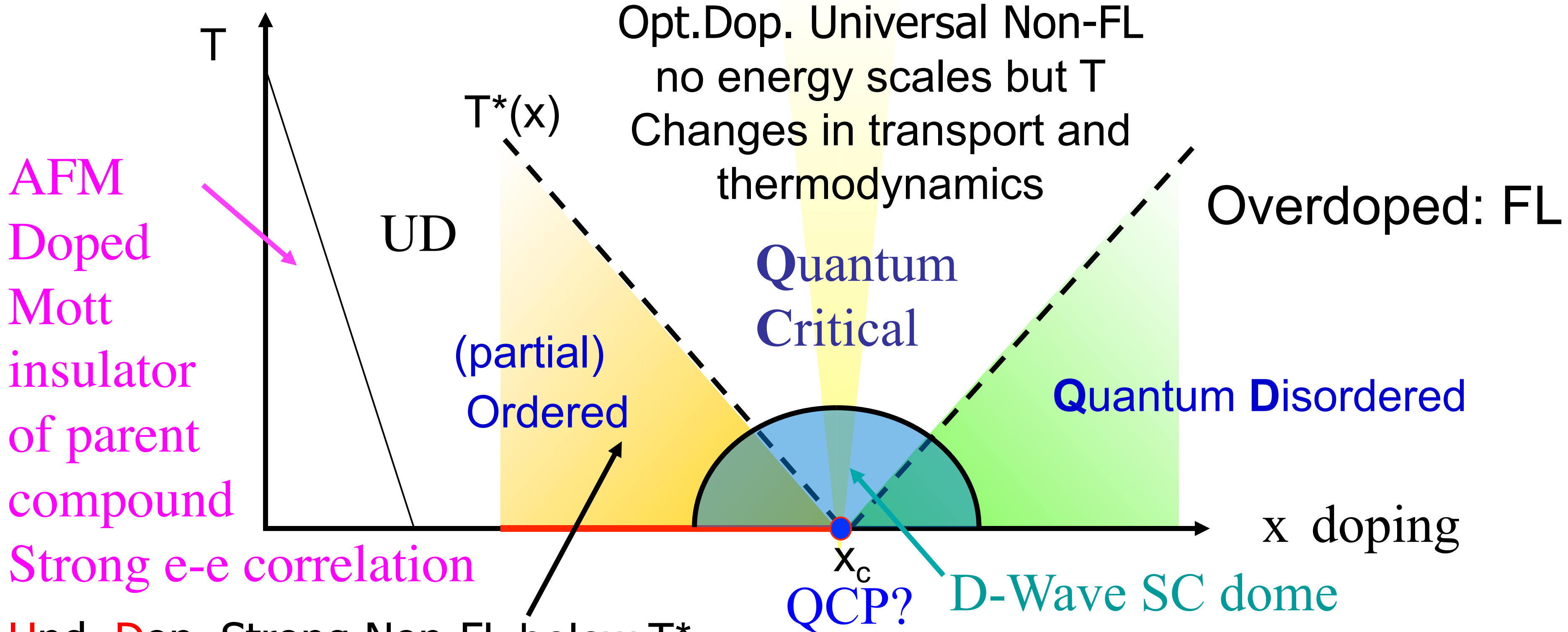
P(Gpa)

SDW fluctuations act as **glue** for d-wave SC
Electrons reorganize: new state to avoid QCP.

High Temperature Superconductors: the SC dome seems to hide a QCP due to competition between two different ground states.
What is the competing order and the related critical modes, mediators of pairing?

Generic phase diagram of Cuprates, e.g. $La_{2-x}Sr_xCuO_4$

Weakly coupled CuO_2 planar structure Quasi-2D



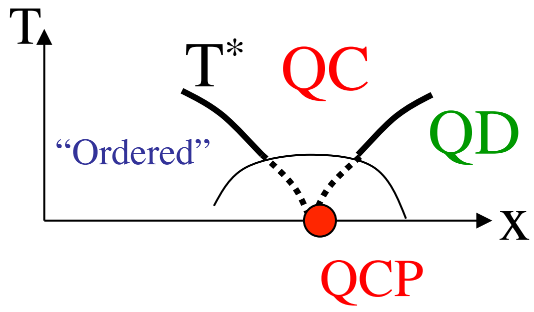
Und. Dop. Strong Non-FL below T*
Pseudo Gap for charge and spin in some regions of the Fermi surface
Variety of spin and charge ordered structures: stripe-like (dynamical with smectic order) (LSCO, YBCO), checkerboard or droplets (Bi2212)

Competing order? Inhomogeneous states in UD with a hidden QCP?
Are the fluctuations of the hidden order the glue for d-wave pairing and the cause of the non-FL behavior?

Cuprates: a (hidden) QCP around OpD ?

Experimentally: Physics changes qualitatively around OpD [e.g.:

i) transport when SC is suppressed by a strong magnetic field H or impurities
ii) thermodynamic quantities like heat capacity
(e.g. Tallon and Loram 2001)]



Theoretically: Several proposals of QCP

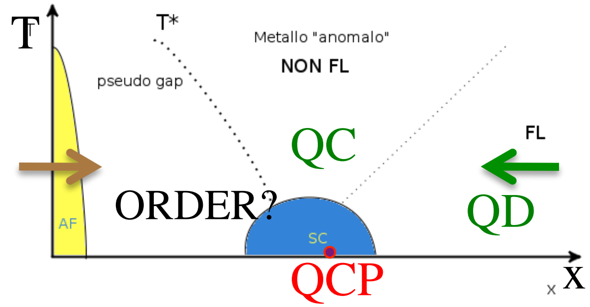
-Incommens. CDW-Stripes *(Castellani et al 1995)*, $q_c \approx (0,\pm\pi/2), (\pm\pi/2,0)$
-Circulating currents *(Varma 1994,…)*, $q_c=0$
-Pomeranchuk instability *(Metzner 2003)*, (nematicity) $q_c=0$
-spin waves *(Chubukov, Pines,…)*, $q_c \approx (\pi,\pi)$
-d-Density Wave (modulation of current)
(Benfatto et al 2000; Chakravarty et al 2001)

$T^* \approx T_{order}$?
Crucial role of critical modes for retarded effective interaction?
I will concentrate on our proposal

Instability of homogeneous systems
one-, three-band Hubbard +…

Strong correlation : **Phase separation is easy**

+long range Coulomb forces: **frustrated phase separation, stripes**



From low doping (Mott insulator): (Emery, Kivelson) expulsion of holes from AF background → Spin driven [anti-phase] Stripes

From high doping: instability of correlated Fermi Liquid (large FS) (Rome group) within the Hubbard-Holstein model in the mean field slave boson approach → ICDW with finite q_c modulation and a QCP (x_c=0.19) as end-point of a critical line $T_{co}(x) \approx T^*$ → Charge driven Stripes

Inhomogeneous State as a bridge between FL and doped AFM separated by a hidden QCP

Charge and spin interconnected with continuous evolution on doping acting as glue in the effective retarded interaction?

Due to the onset of heterogeneity, ($T_{co}(x) \approx T^*(x)$) dynamical charge and spin fluctuations mediate strong T, x and q retarded interaction among quasi particles

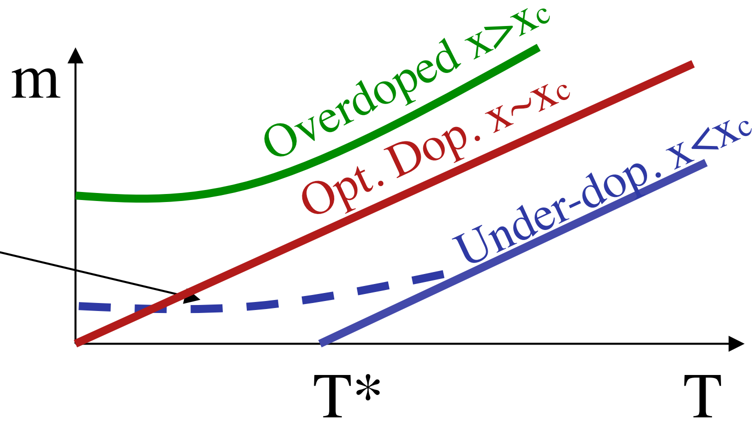
Specifying the glue and leading to a comprehensive scenario for the Cuprates

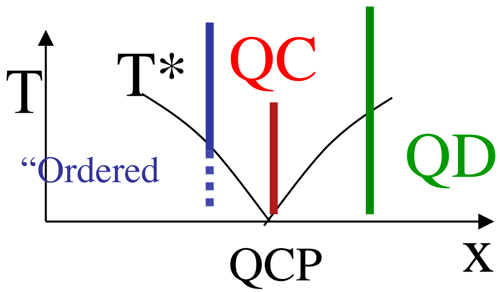
$$D(\vec{q},\omega) = -\frac{1}{m + \nu|\vec{q} - \vec{q}_c|^2 - i\omega - \frac{\omega^2}{\overline{\Omega}}}$$

$m \sim \xi^{-2}$ depends on proximity to the "missed instability"

OvD $m \approx x - x_c$ $x = x_c$ $m \approx T$ *UD* $m \approx T - T_{co}(x)$

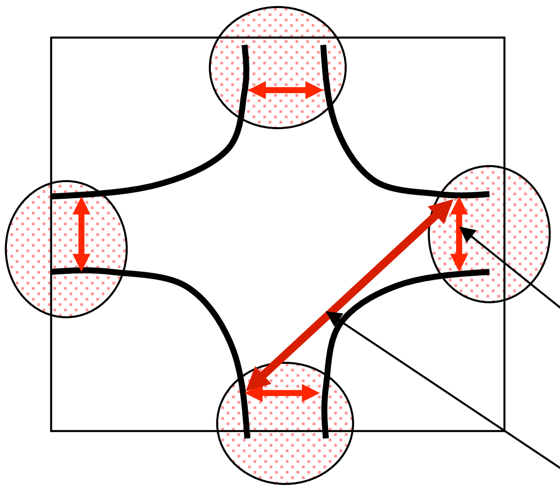
If order is not truly
static-long-range
m never vanishes





Can we identify these collective modes ?

Caprara et al
PRB 2011



$$q_c \approx \left(\pm\frac{\pi}{2},0\right),\left(0,\pm\frac{\pi}{2}\right)$$

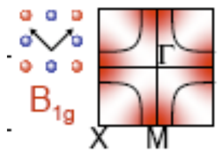
$$q_s \approx (\pm\pi,\pm\pi)$$

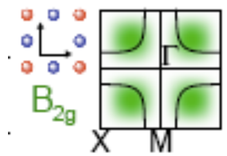
Nearly critical spin and charge modes strongly couple same regions but different branches of the Fermi Surface

In Raman spectroscopy the probed k-space is selected with specific form factors in the vertices, which assume different signs in the various parts of the FS:

B_{1g} $\quad \gamma_k = \cos(k_x) - \cos(k_y)$

B_{2g} $\quad \gamma_k = \sin(k_x)\sin(k_y)$

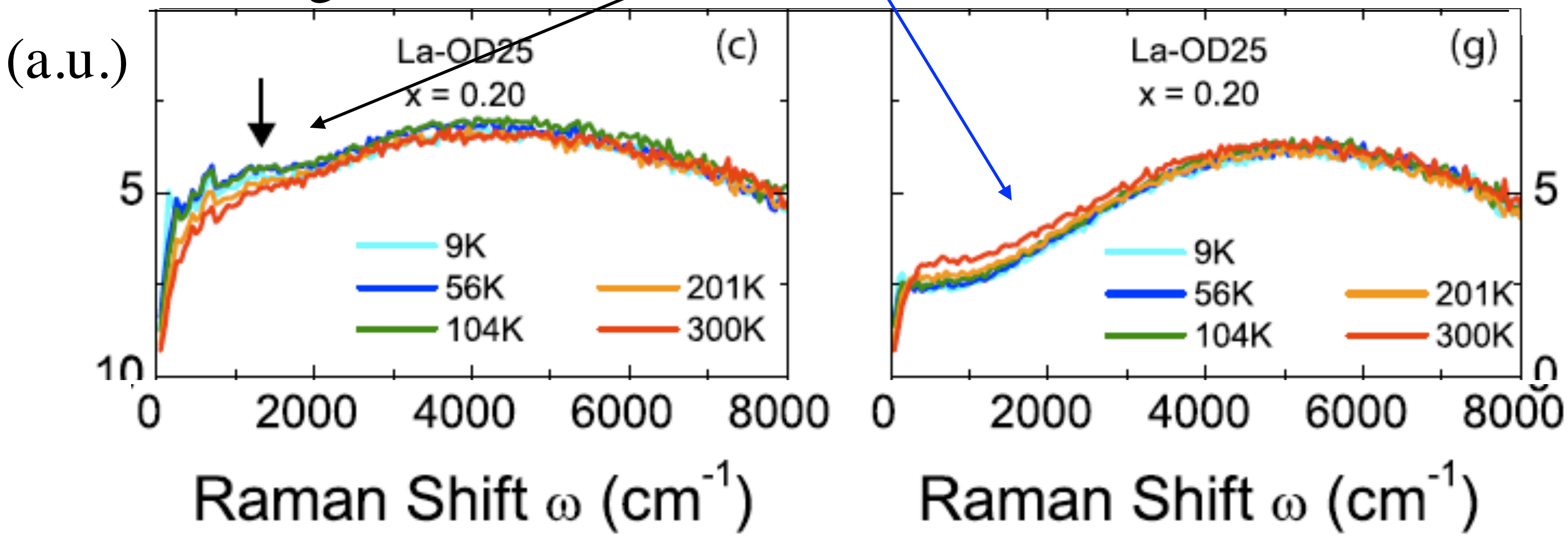




Spin and charge modes can then be unraveled.

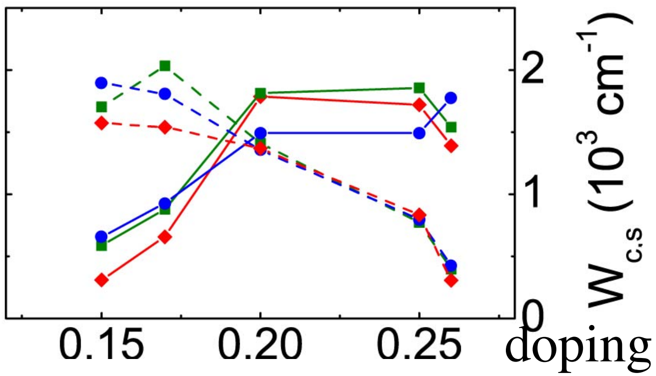
Experimentally indeed different shapes below ~4000 cm^{-1} e.g.

B1g   B2g



Theoretically: At leading order in the critical modes (one mode exchanged, low energy, linearized bands, vertices evaluated at E_F…), symmetry arguments based on the different q_c's and on the different sign of the form factors select **charge modes for B_{2g}** and **spin modes for B_{1g}**.

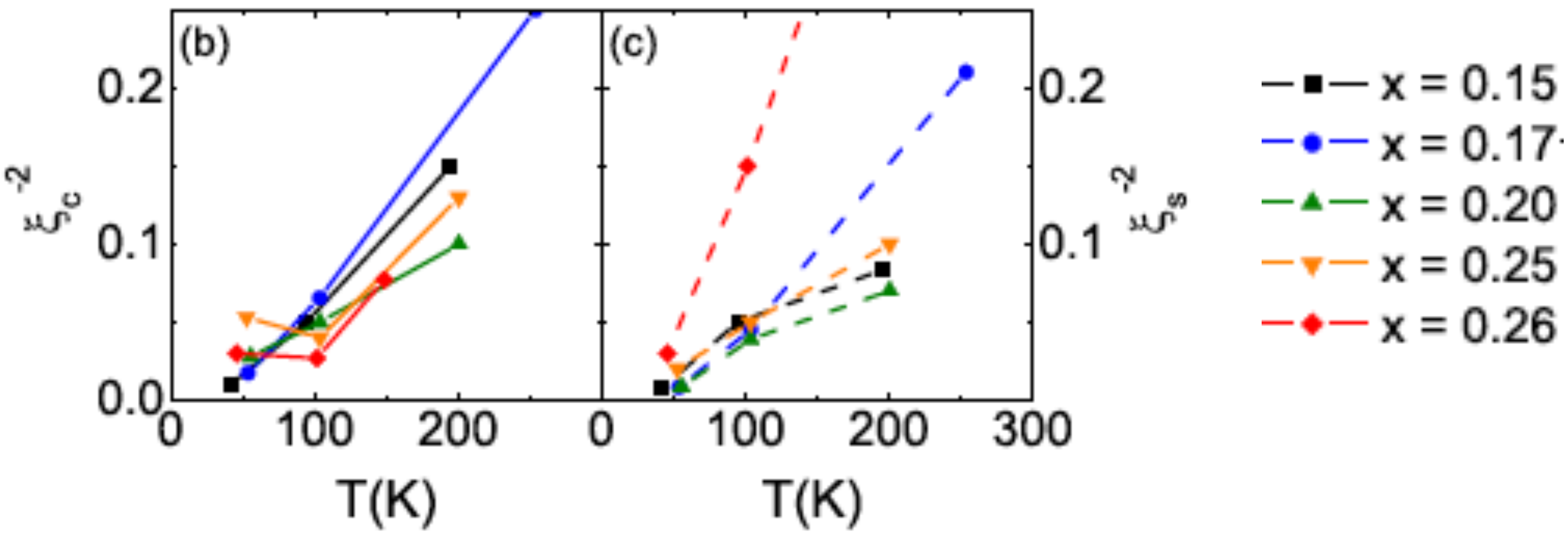
Fitting the data at various dopings ($0.15<x<0.26$) and temperatures we obtain the glue function α^2F *and the spectral weights of the modes*.



spin (dotted lines) and charge (full lines) spectral weight W_s and W_c as function of doping

W_s decreases while W_c increases with doping crossing at x=0.19 according to the proposed scenario

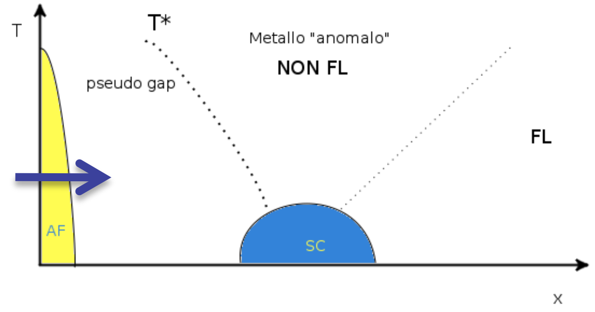
T-Dependence of charge and spin square inverse correlation length



Consistent with an underlying QC behaviour for charge ordering: $\xi_c^{-2} \rightarrow 0$ as T$\rightarrow$ 0 at x=0.20, at x>0.20 saturates, at x<0.20 goes to zero at a finite T*. QCP is usually reported at x=0.19.

I will now examine the low doping region.

From low doping

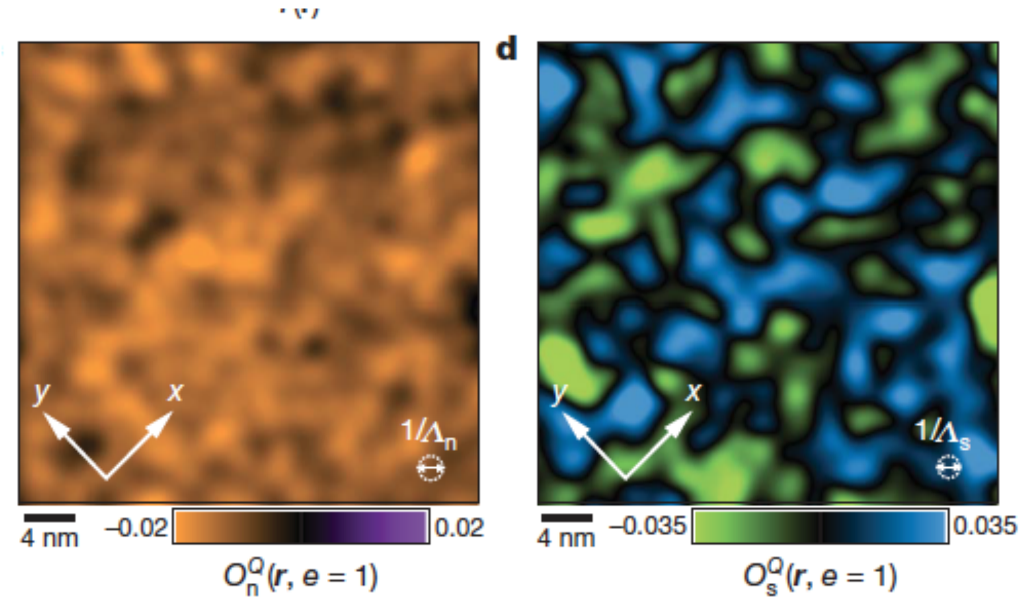


Heterogeneity e.g. in underdoped $Bi_2CaCu_2O_{8+y}$, Lawler et al 2010 [STM], Mesaros et al 2011)

Nematic order homogeneous at cell level (differences in the two O-sites of the cell in the Cu-O planes)

Smectic order heterogeneous

Nematic *Smectic*



e=1, at E= local pseudogap

Is nematicity a sign of fluctuating density wave order, (melted smectic stripes) or vice versa has independent formation?

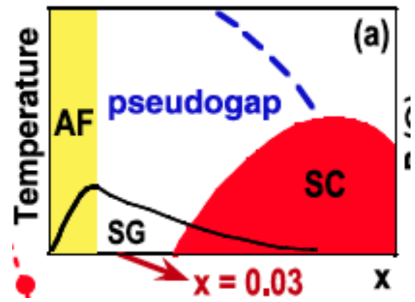
At low doping where spin and charge glassiness is present, vortex-antivortex (V-A) segments form the seeds of nematicity and of incommensurate smectic structures. (G. Seibold et al in preparation, *See also Berciu and John 2004, Timm and Bennemann 2000)*

Dilute limit; Spin glass phase

1(3)-band Hubbard model (U,t,t')
Gutzwiller approximation ⟶
Two holes form Vortex-Antivortex pairs which tend to arrange in 1D segments.

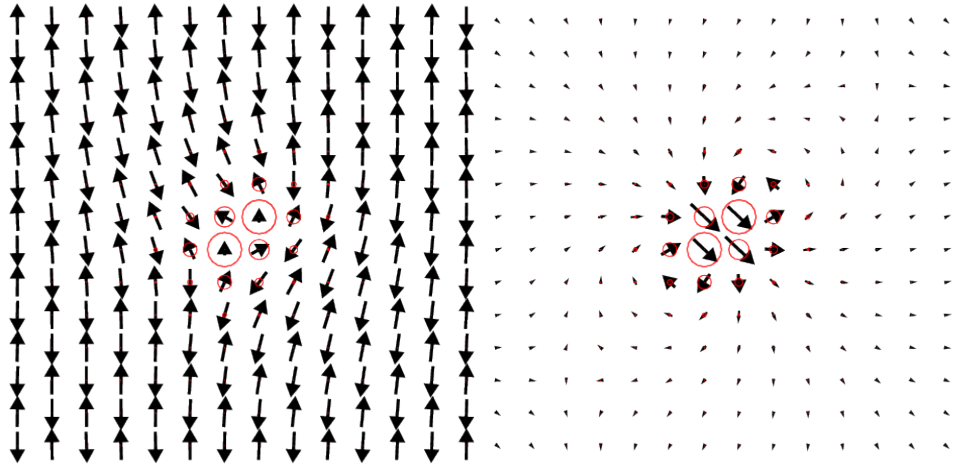
Dipole-dipole magnetic interaction between two pairs in 2D indeed favors nose to tail alignment.
Analogy with dipolar fluids where standard phase separation with isotropic aggregation is replaced by a defect induced (ends and junctions of chains) topological PS (Tlusty and Safran 2000).

For -0.3<t'/t<-0.2 crossover from diagonal to vertical-horizontal configuration.
Each segment of n pairs has 2n charges immersed in a compensating charged background ➔ Chain's length limited to a maximum $n_{c.}$=5-6 unit cells

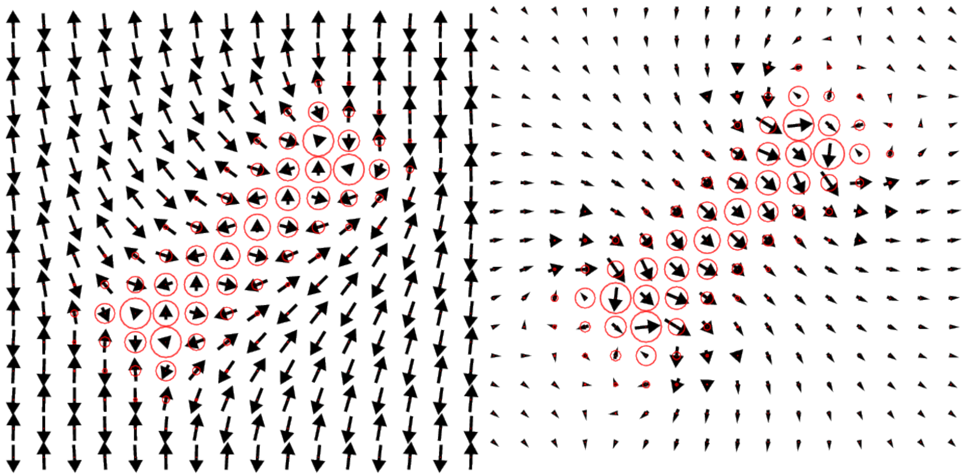


Diagonal V-A pair

spin/charge   spin currents



4 diagonally neighbored V-A pairs



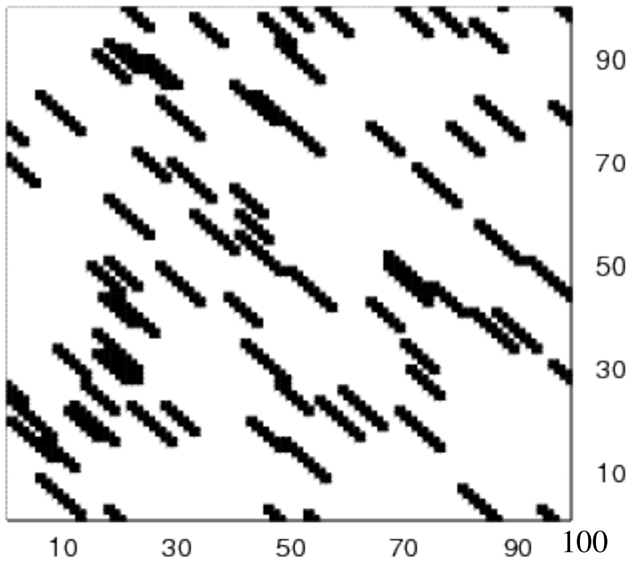
Spin and charge response from V-A chains with parallel configuration

Average over 20 random stripe segment configurations:

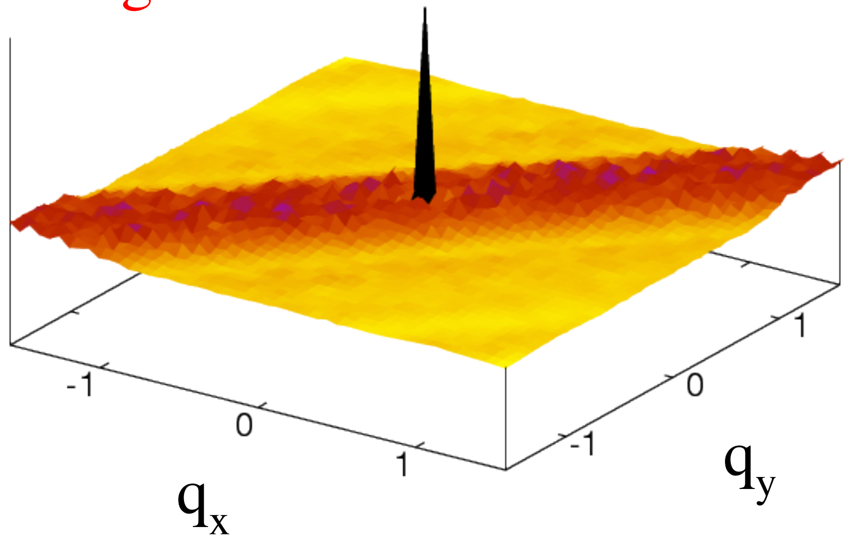
Doping: $x=0.05$

Segment length: $6\pm4\ a_{ortho}$

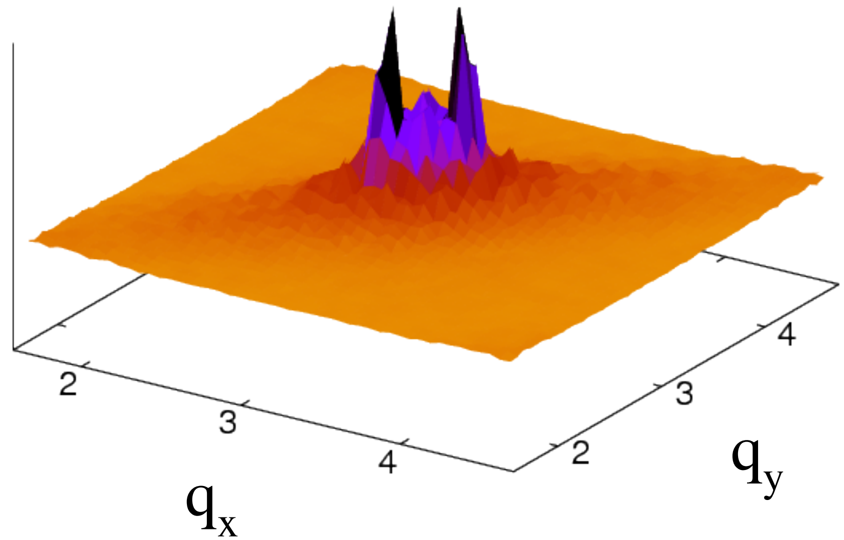
Distance: $10\pm4\ a_{ortho}$



Charge structure factor



Spin structure factor

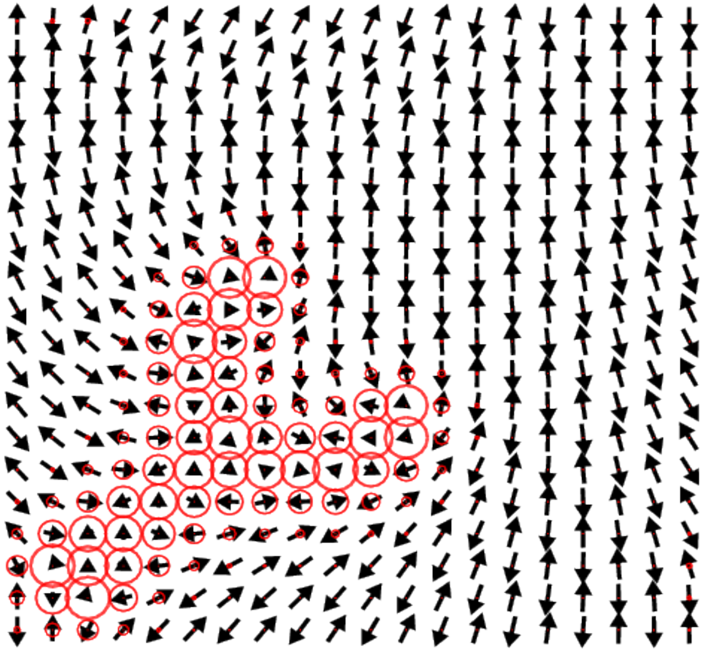


Glass of nematic V-A chains gives rise to smectic correlations at least in the spin sector at low doping

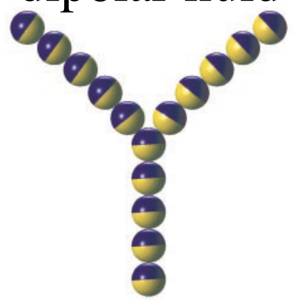
Increasing doping and temperature branching is favored and more complex structures (checkerboard, bubbles,…) should appear out of spin (and charge) nematic glass.

Diagonal (Vertical) junction: 20x20 lattice, 14 holes, U/t=8

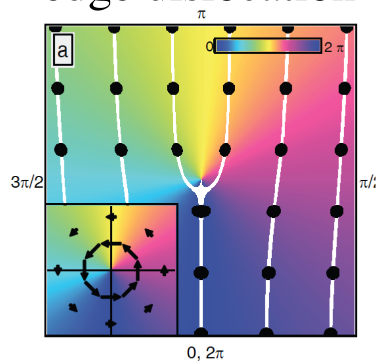
t'/t=-0.1



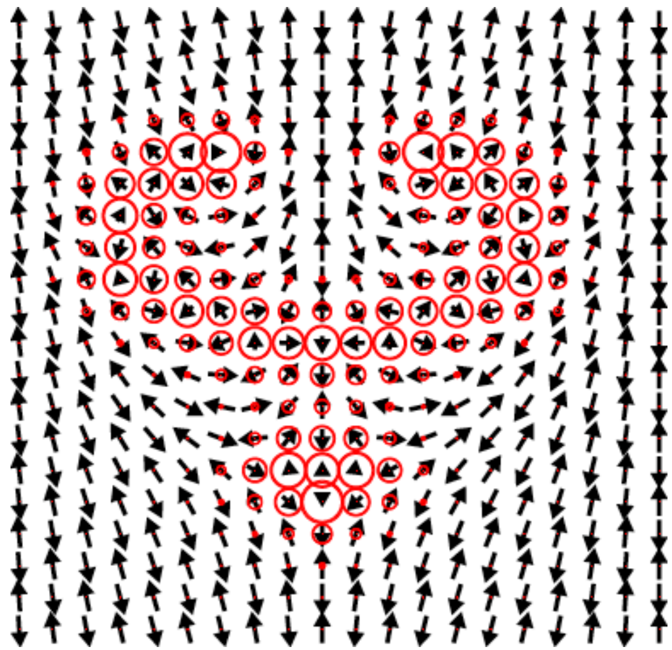
junction in dipolar fluid



edge dislocation



t'/t=-0.2



Both structures tend to favor closed structures.

New feature : gas of topological excitations, ends and junctions.

Charge and spin interconnected up to QCP and the scenario becomes complete.

Summary and conclusion

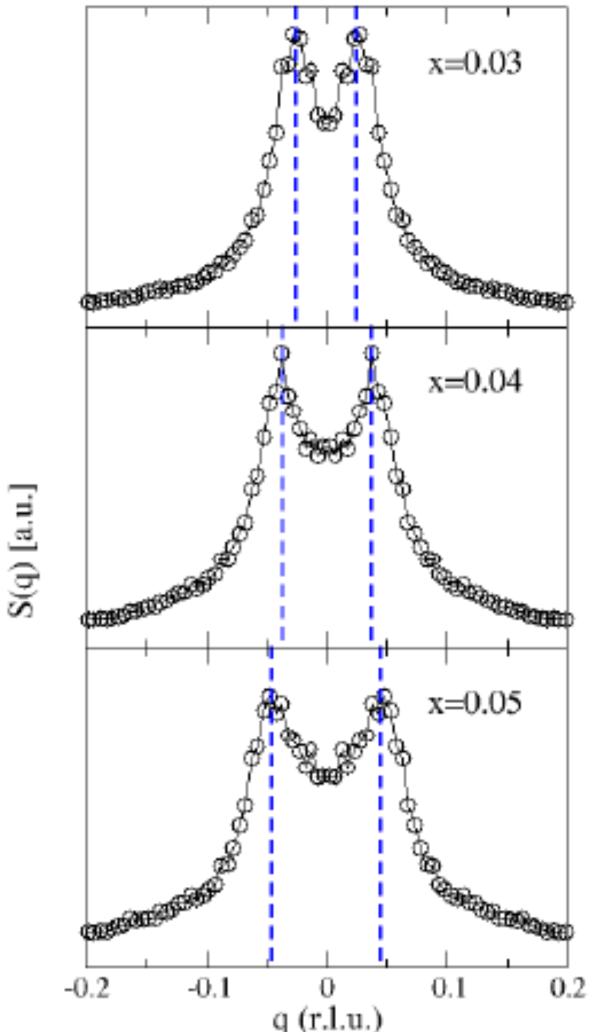
Within the framework of RG approach, I recalled the case of d=1 interacting electron system plagued by infrared divergences well inside the liquid stable phase. Additional symmetries and related Ward Identities implements cancellation of singularities to all orders in the response functions and allow for the asymptotic solution of the problem. A non Fermi liquid (Luttinger liquid) is obtained, which however reduces to a FL as soon as d>1, except in the presence of singular effective interaction, e. g. nearby an instability. This seems to be the case of Cupates.

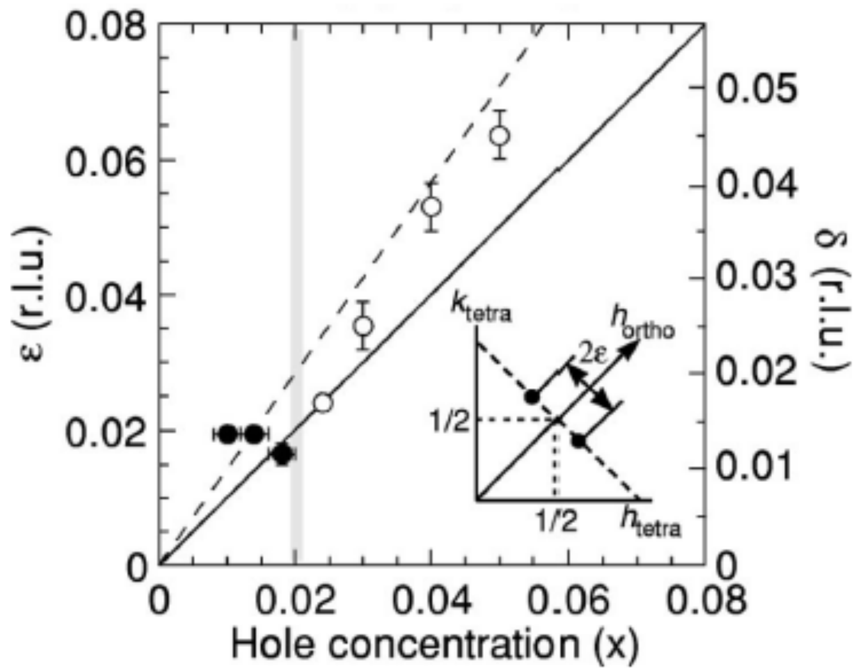
As glue mediators in Cuprates, Raman spectroscopy identifies two modes, spin and charge with different characteristic wave vectors . The relative importance of the two scattering mechanisms switches from spin to charge by increasing doping.
Simultaneous presence of spin and charge-density fluctuations suggests both charge and spin ordering as competing phase.

Confirmation of Quantum Criticality leading to inhomogeneous state formation with various morphologies (stripes, droplets,…) arising as Fermi Liquid instability of charge modulation at high doping (Rome proposal) evolving into spin dominated structures when the AFM region is approached at low doping (Emery and Kivelson proposal).

At low doping glass of nematic V-A chain segments give rise to smectic correlations like in stripe phase (at least in the spin sector) and are the seeds for more compact structures (checkerboard, bubbles…)

the segments are distributed at random.

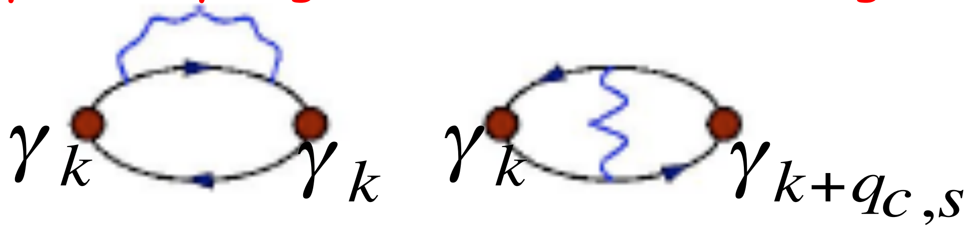




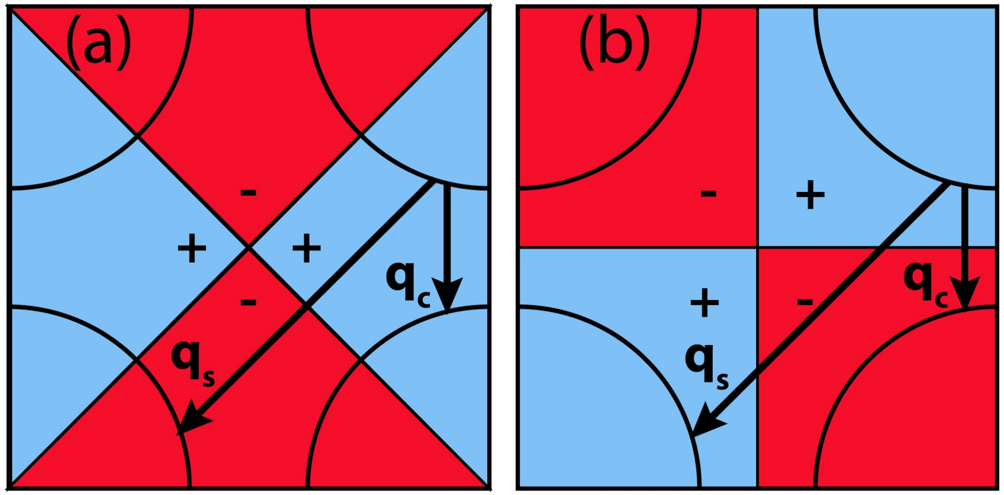
M. Matsuda et al., PRB (2002)

Cuts of spin structure factor computed for x=0.03, 0.04, 0.05 in the glass phase.Vertical dahsed lines are the experimental incommensurabilites (Matsuda) .

Symmetry arguments for the leading contribution to Raman response



The two diagrams cancel at leading order when the two vertices are equal they add when the two vertices are opposite



B_{2g}

B_{1g}

	SPIN	CHARGE
B_{1g}	ALLOWED	FORBIDDEN
B_{2g}	FORBIDDEN	ALLOWED

Morphologies in Cuprates :

"Dynamical Stripes": YBCO LSCO Neutron Tranquada, Mook, Abbamonte (RSXS)…
Charge period (when observed) ½ spin (incommensurate) period. ξ up to ≈20 periods
(Cu-O) oriented modulation; diagonal in LASCO x<0.05

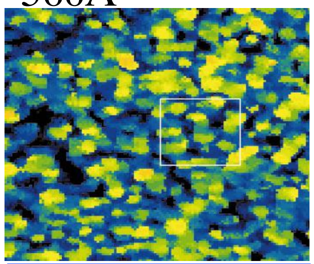
Or checkerboard (STM) :
-$Ca_{2-x}Na_xO_2Cl_2$ (x=0.08, 0.10, 0.12) Conductance map *4ax4a* and 4a/3x4a/3 modulation . ξ small slightly increasing with x — Hanaguri et al 2004
-$Ca_{1.88}Na_{0.12}O_2Cl_2$, $Bi_2Sr_2Dy_{0.2}Ca_{0.8}Cu_2O_{8+\delta}$ *4a-Unidirectional* domains — Kohsaka et al 2007

-$Bi_2Sr_2CaCu_2O_{8+\delta}$:
i) *4.5ax4.5a modulation* Antinodal decoherence coincident with *emergence of charge order* — Mc Elroy et al 2005
ii) *modulation (4.7±0.2)a*. ξ≈5 periods — Vershinin et al 2004

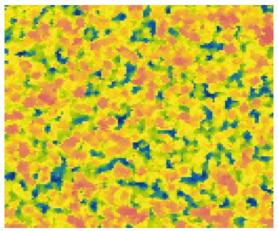
Or islands: Bi2212 *Gap Δ maps from STM* Lang et al 2002

Underdoped (x=0.14)
(Δ)=50meV
L=30~40 A°

560A



Insulating Like



Superconducting Like

Slightly OD (x=0.18)
(Δ)=35meV

see also Boyer et al 2007

B-drops in A-phase or vice versa