



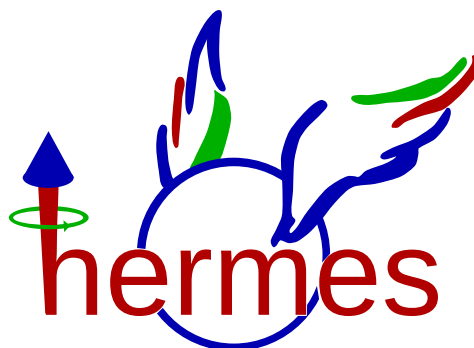
Transversity and Transverse Momentum Dependent Distribution and Fragmentation Functions

G. Schnell

Universiteit Gent

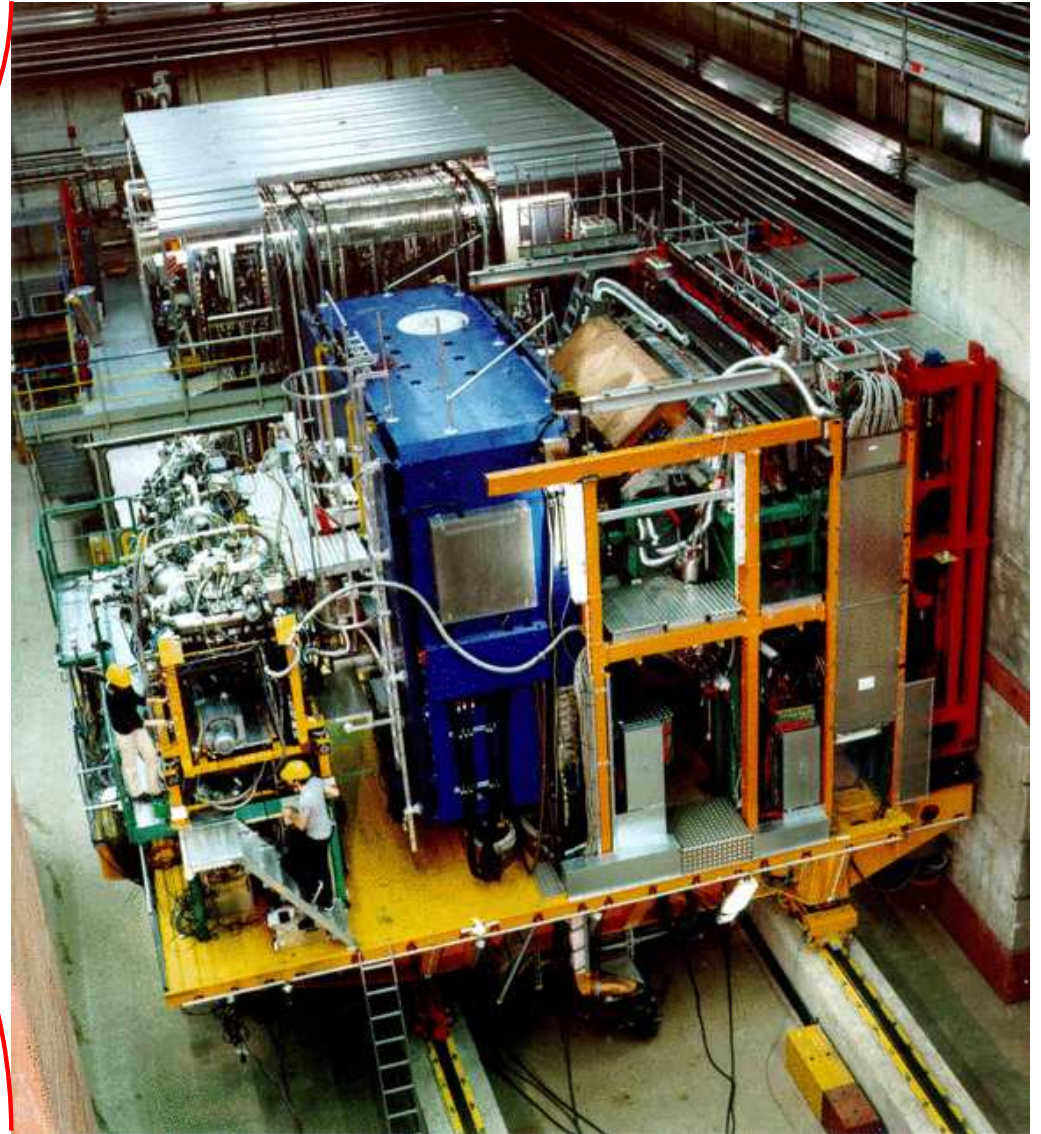
gunar.schnell@desy.de

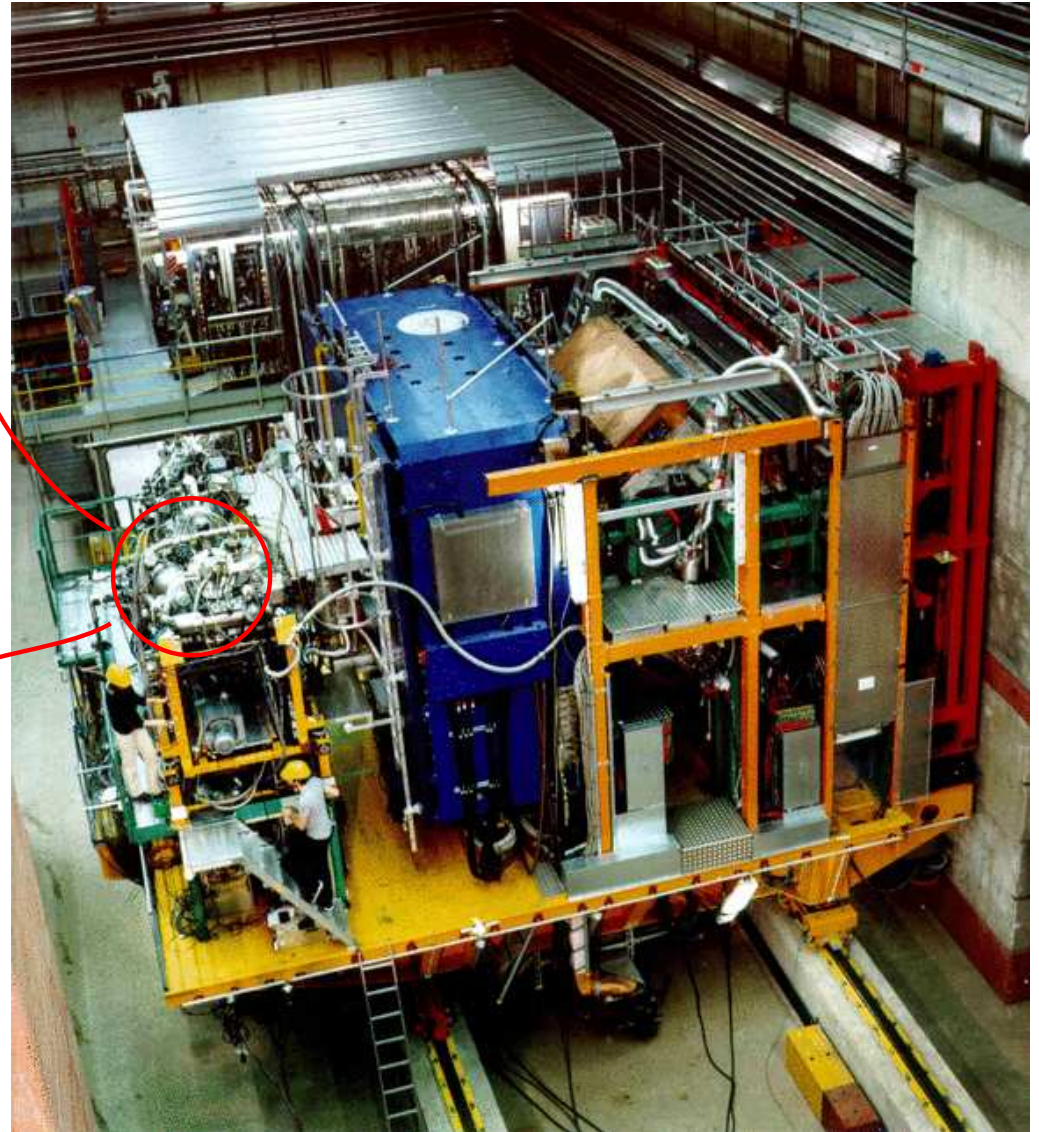
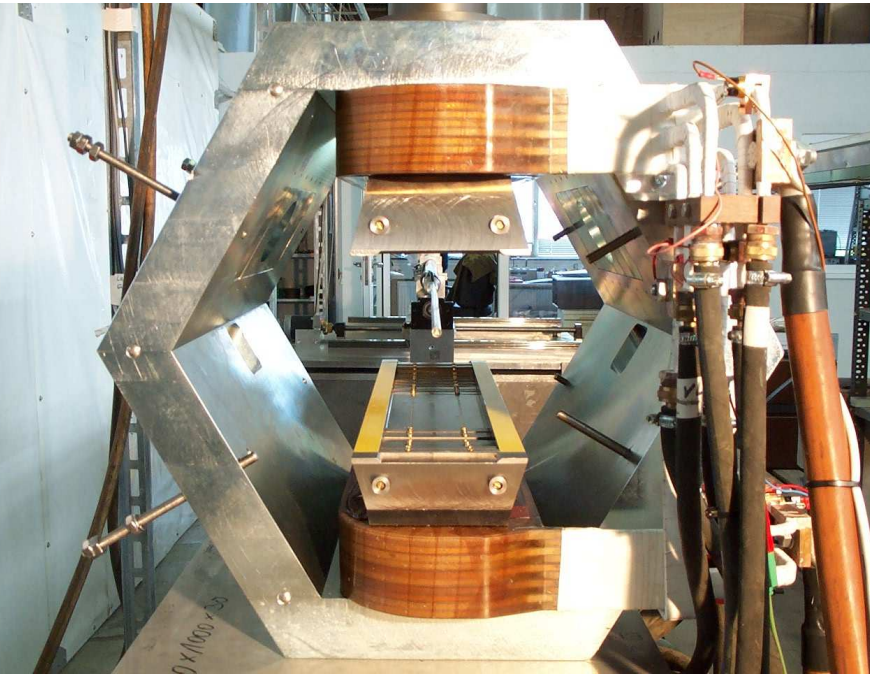
For the



Collaboration

27.5GeV positron beam of HERA





- atomic beam source
- ⇒ pure gas target
- transversely pol. hydrogen
polarization $\sim 75\%$
- other targets possible



Transversity Measurements

How can one measure the chiral-odd transversity?

Need another chiral-odd object!

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Need another chiral-odd object!

⇒ Semi-Inclusive DIS

$$\sigma^{ep \rightarrow ehX} = \sum_q h_1^q \otimes \sigma^{eq \rightarrow eq} \otimes FF^{q \rightarrow h}$$

⇓

chiral-odd

DF

⇓

chiral-odd

FF

⏟

CHIRAL EVEN

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chiral-odd

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⏟

CHIRAL EVEN

→ chiral-odd FF as a **polarimeter** of transv. quark polarization



Semi-Inclusive 2-Hadron Production

2-Hadron Fragmentation

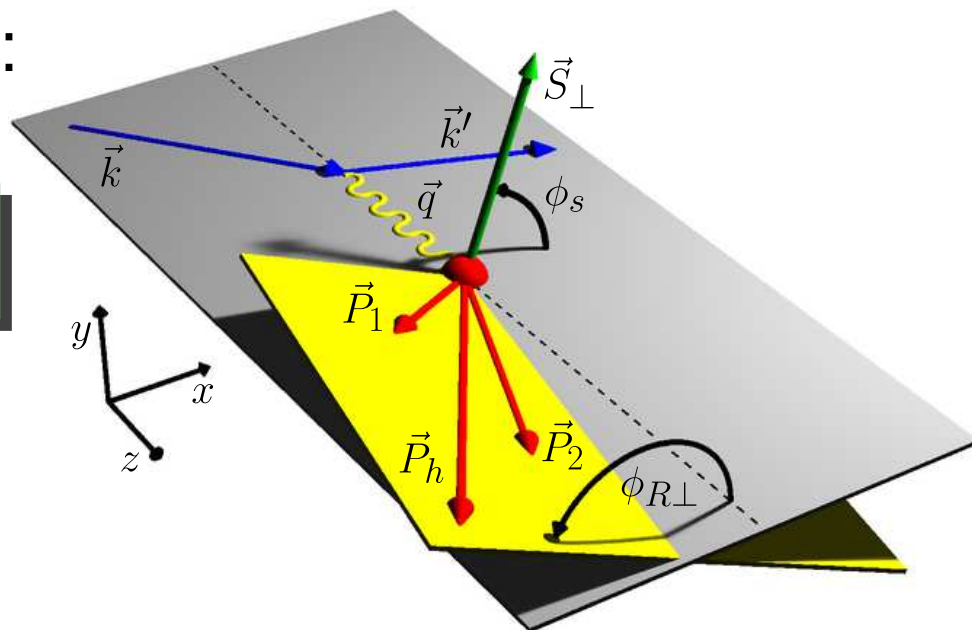
polarized 2-hadron cross section:

(**U**npolarized beam, **T**ransversely pol. target)

$$\sigma_{UT} \sim \sin(\phi_{R\perp} + \phi_S) \sum e_q^2 h_1^q H_1^{\triangleleft}$$

$$H_1^{\triangleleft} = H_1^{\triangleleft}(z, \zeta, M_{\pi\pi}^2)$$

$$(\zeta \sim z_1/(z_1 + z_2))$$



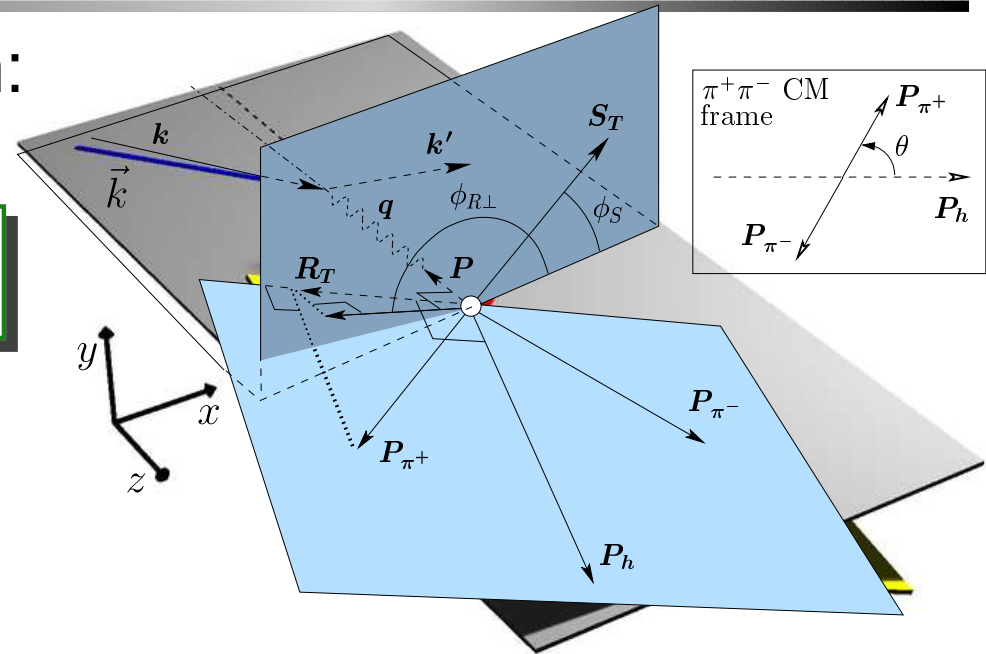
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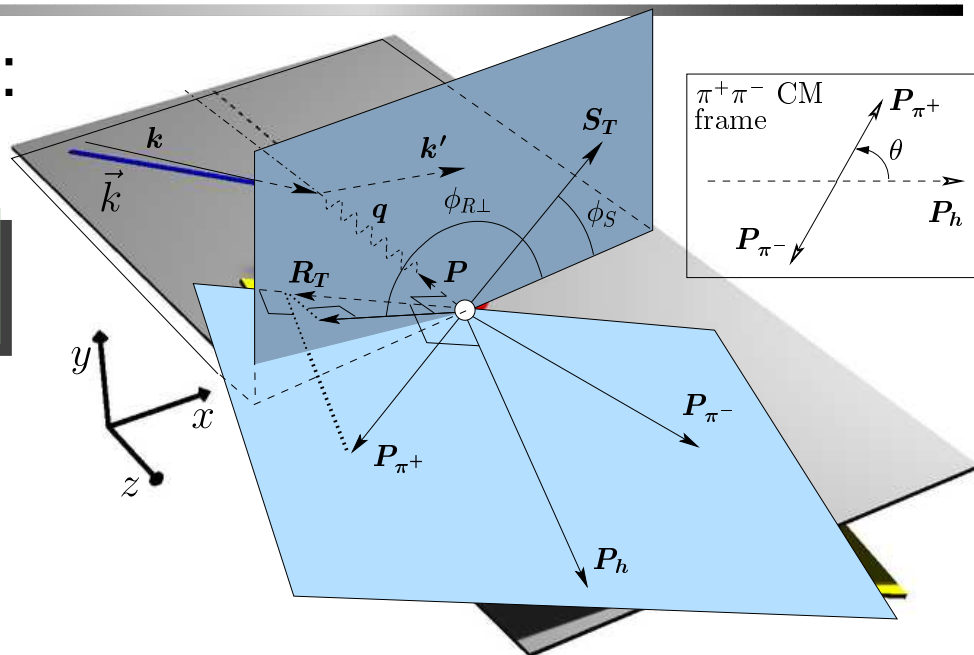
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$$(\zeta \sim z_1/(z_1 + z_2))$$



difficult to measure directly $\sigma_{UT} \equiv \sigma_{U\uparrow} - \sigma_{U\downarrow}$

$\Rightarrow$ measure **cross section asymmetry** A_{UT} :

$$A_{UT} \equiv \frac{1}{\langle |S_T| \rangle} \frac{N_{2\pi}^\uparrow(\phi_{R\perp}, \phi_S, \theta) - N_{2\pi}^\downarrow(\phi_{R\perp}, \phi_S, \theta)}{N_{2\pi}^\uparrow(\phi_{R\perp}, \phi_S, \theta) + N_{2\pi}^\downarrow(\phi_{R\perp}, \phi_S, \theta)}$$

$\uparrow\downarrow \dots$ target spin states
 $N_{2\pi} \dots$ (norm.) 2π yield
 $S_T \dots$ target polarization

2-Hadron Fragmentation

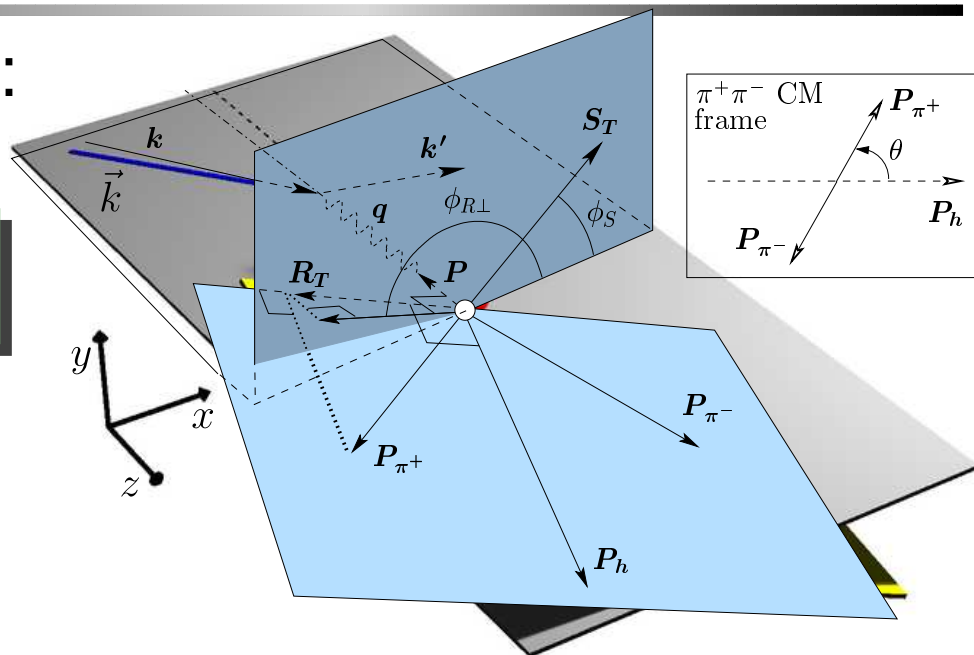
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$\uparrow\downarrow \dots$ target spin states
 $N_{2\pi} \dots$ (norm.) 2π yield
 $S_T \dots$ target polarization

But: asymmetry involves unknown unpolarized 2π cross section

Interference Fragmentation – Models

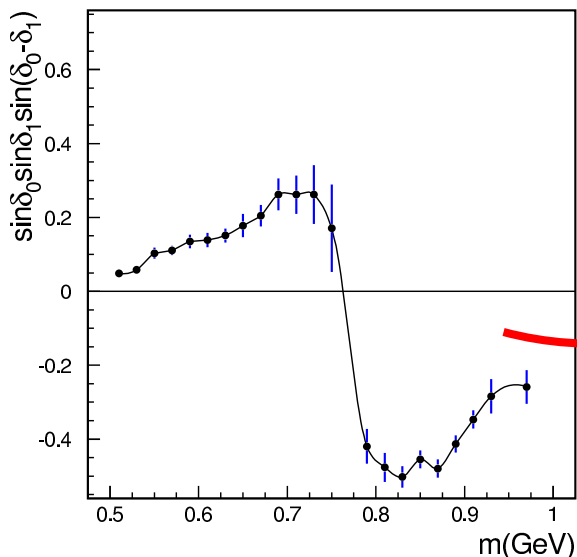
$$A_{UT} \sim \sin(\phi_{R\perp} + \phi_S) \sin \theta h_1 H_1^{\triangleleft}$$

Expansion of $H_1^{\triangleleft}$ in Legendre moments:

$$H_1^{\triangleleft}(z, \cos \theta, M_{\pi\pi}^2) = H_1^{\triangleleft,sp}(z, M_{\pi\pi}^2) + \cos \theta H_1^{\triangleleft,pp}(z, M_{\pi\pi}^2)$$

describe interference between 2 pion pairs coming from different production channels.

about $H_1^{\triangleleft,sp}$:



Jaffe et al. [[hep-ph/9709322](#)]:

$$H_1^{\triangleleft,sp}(z, M_{\pi\pi}^2) = \frac{\sin \delta_0 \sin \delta_1 \sin(\delta_0 - \delta_1)}{\delta_0 (\delta_1)} H_1^{\triangleleft,sp'}(z)$$

$\delta_0 (\delta_1) \rightarrow \text{S(P)-wave phase shifts}$

$$= \mathcal{P}(M_{\pi\pi}^2) H_1^{\triangleleft,sp'}(z)$$

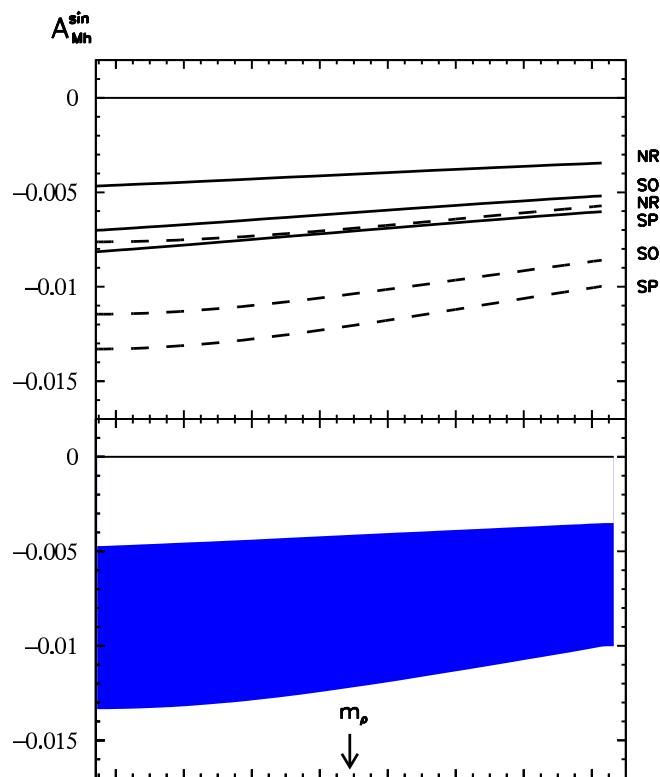
$\Rightarrow A_{UT}$ might depend strongly on $M_{\pi\pi}$

Interference Fragmentation – Models

$$A_{UT} \sim \sin(\phi_{R\perp} + \phi_S) \sin \theta h_1 H_1^{\triangleleft}$$

Expansion of $H_1^{\triangleleft}$ in Legendre moments:

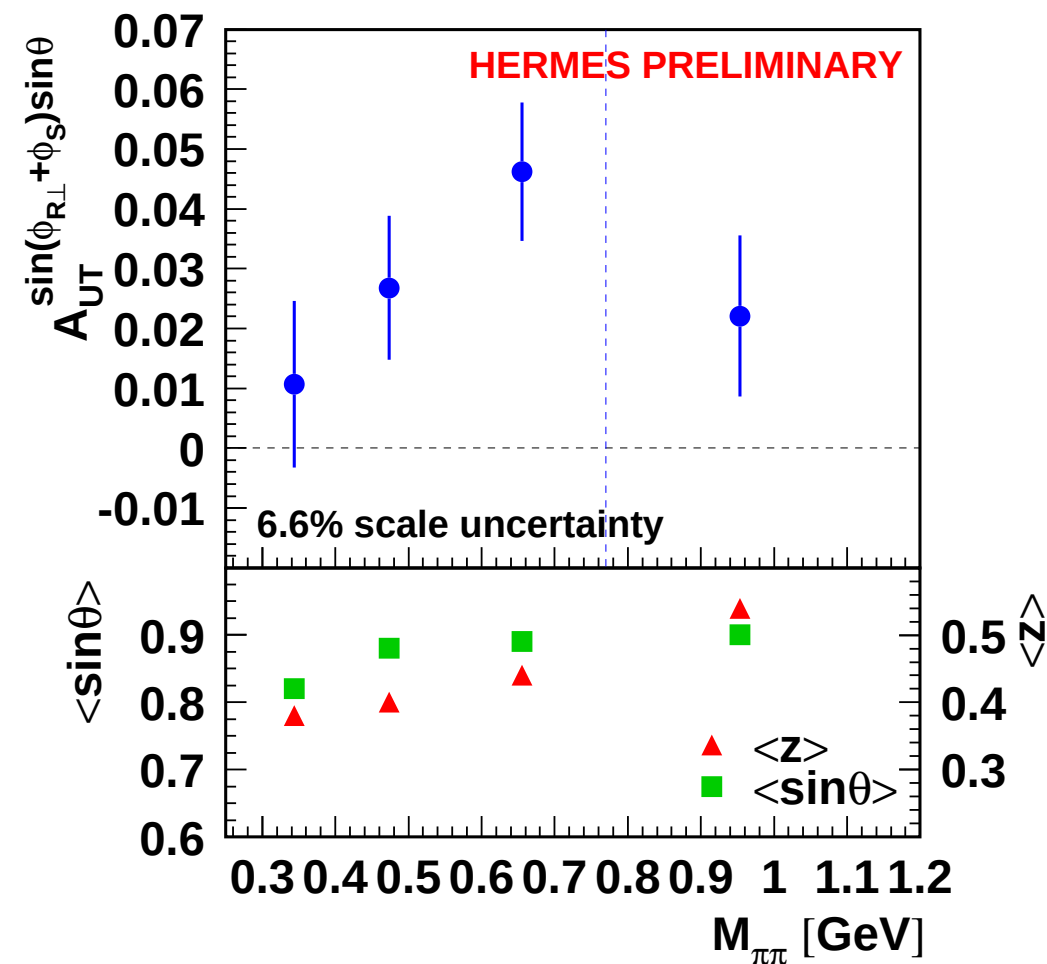
$$H_1^{\triangleleft}(z, \cos \theta, M_{\pi\pi}^2) = H_1^{\triangleleft,sp}(z, M_{\pi\pi}^2) + \cos \theta H_1^{\triangleleft,pp}(z, M_{\pi\pi}^2)$$



Radici et al. [hep-ph/0110252]:

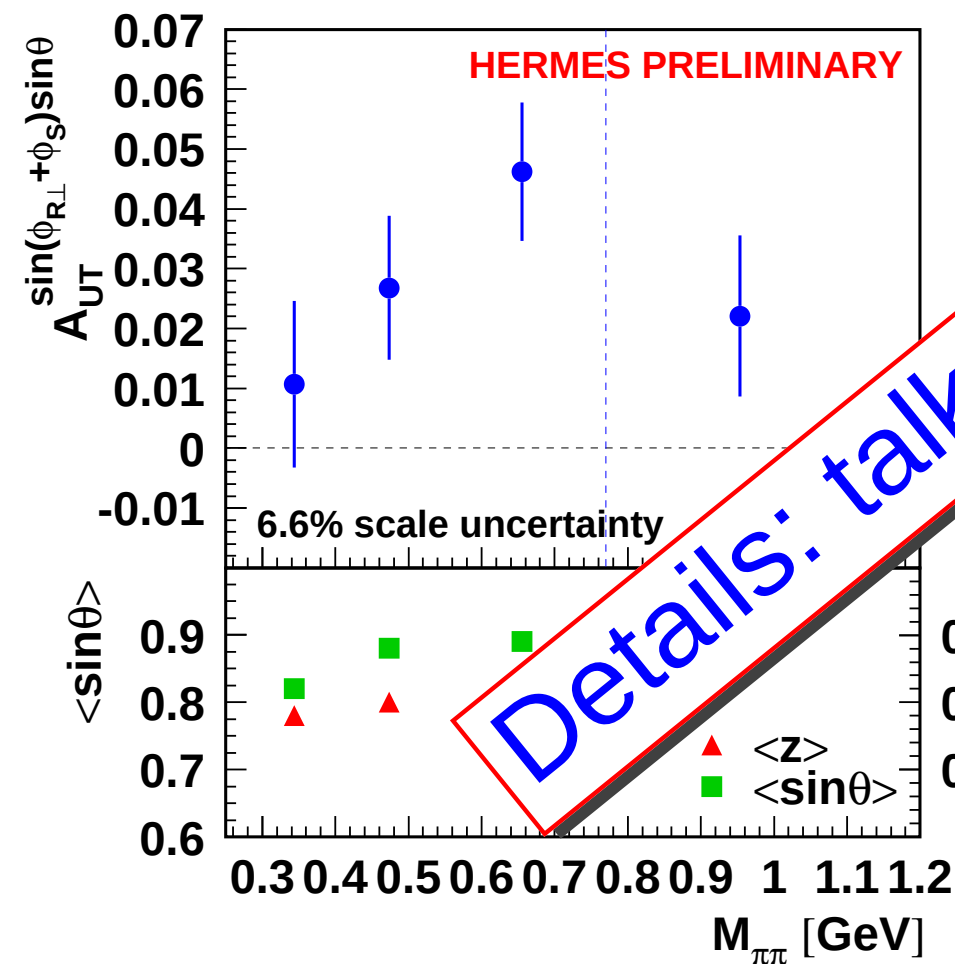
- completely different model, not predicting a sign change of the asymmetry

Mass Dependence of A_{UT}



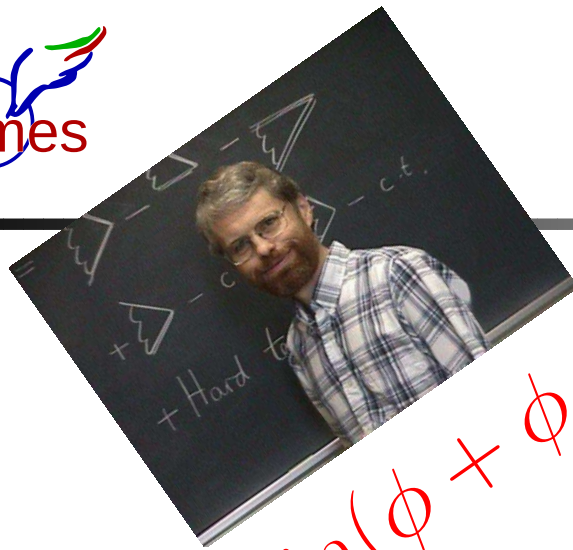
- 2-hadron (aka Interference) FF is not zero!
- asymmetry grows with $M_{\pi\pi}$ below ρ^0 mass
- positive asymmetries in all invariant mass bins
- rules out predicted sign change at ρ^0 mass (Jaffe et al.)
- to extract transversity (h_1) need Interference FF from Belle (or BaBar etc.)

Mass Dependence of A_{UT}

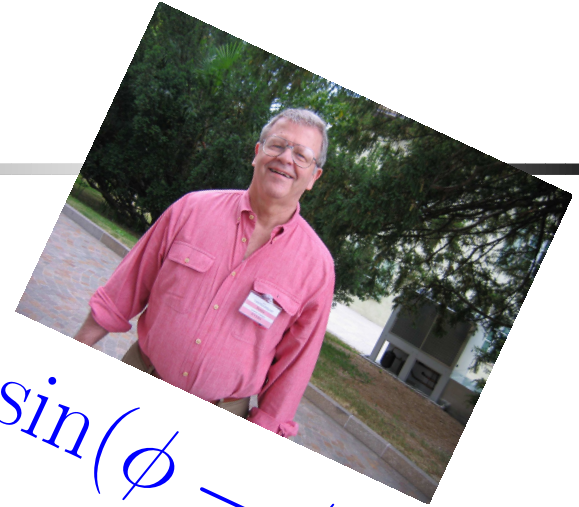


Details: talk by F. Giordani

- 2-body (interference) FF is not
- asymmetry grows with $M_{\pi\pi}$ below ρ^0 mass
- positive asymmetries in all invariant mass bins
- rules out predicted sign change at ρ^0 mass (Jaffe et al.)
- to extract transversity (h_1) need Interference FF from Belle (or BaBar etc.)



$$\sin(\phi + \phi_S)$$



$$\sin(\phi - \phi_S)$$

Semi-Inclusive 1-Hadron Production



$$\cos 2\phi$$

SSA & Unintegrated Distribution and Fragmentation Functions

Leading-Twist

Distribution Functions

$$f_1 = \text{yellow circle with blue center}$$

$$g_1 = \text{yellow circle with blue center and right arrow} - \text{yellow circle with blue center and left arrow}$$

$$h_1 = \text{yellow circle with blue center and up arrow} - \text{yellow circle with blue center and down arrow}$$

$$g_{1T} = \text{yellow circle with blue center and right arrow, up arrow} - \text{yellow circle with blue center and left arrow, up arrow}$$

$$f_{1T}^\perp = \text{yellow circle with blue center and up arrow} - \text{yellow circle with blue center and down arrow}$$

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Chiral-odd transversity h_1 must couple to chiral-odd FF

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Chiral-odd transversity h_1 must couple to chiral-odd FF
 $\Rightarrow H_1$ is the only k_T -integrated chiral-odd FF $\Rightarrow$ DSA
 (Example: transverse-spin transfer in Λ -production)

SSA & Unintegrated Distribution and Fragmentation Functions

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Chiral-odd transversity h_1 must couple to chiral-odd FF
 can use k_T -unintegrated chiral-odd FF $\Rightarrow$ T-odd Collins FF
 $\Rightarrow$ leads to Single-Spin Asymmetrie (SSA)

SSA & Unintegrated Distribution and Fragmentation Functions

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SSAs require one and only one T-odd function

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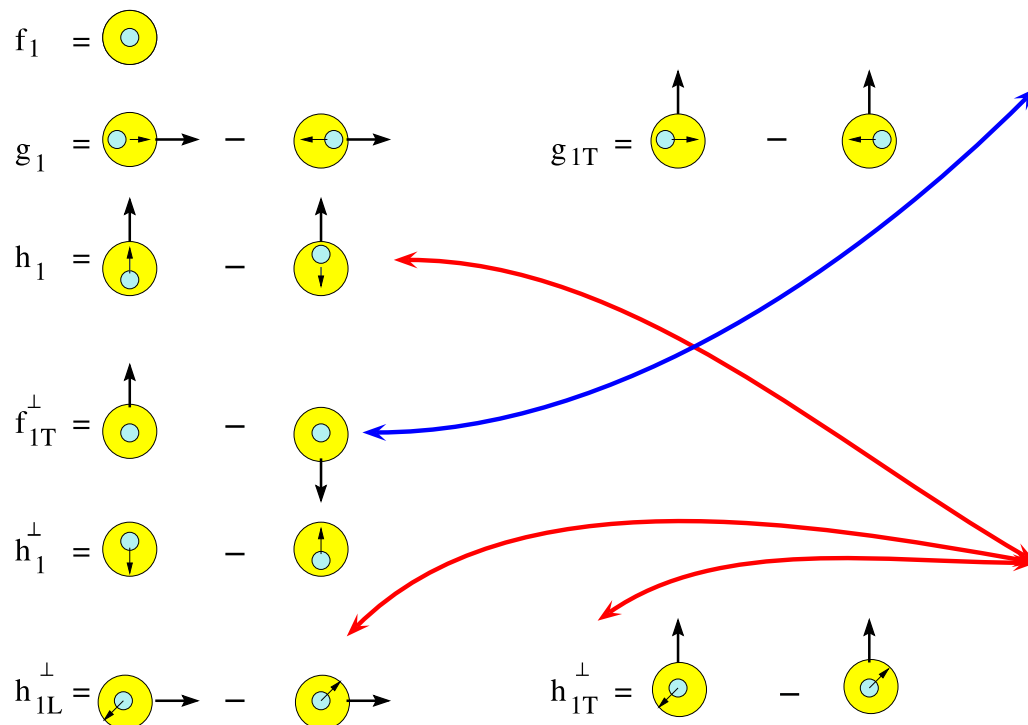
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SSAs require one and only one T-odd function
 $\Rightarrow$ SSAs through **Collins function**

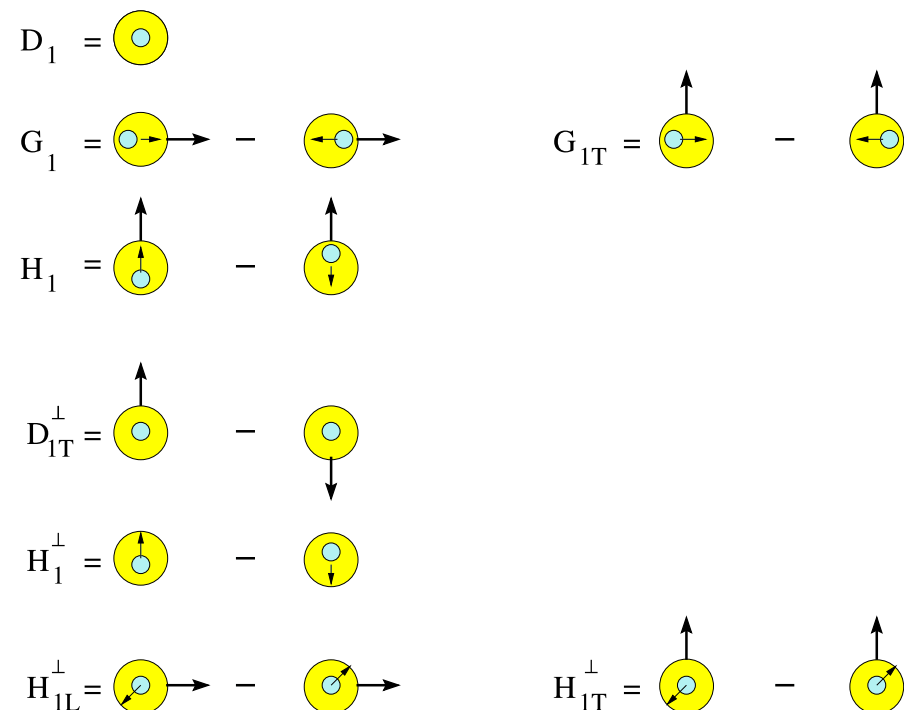
SSA & Unintegrated Distribution and Fragmentation Functions

Leading-Twist

Distribution Functions



Fragmentation Functions



SSAs require one and only one T-odd function

⇒ SSAs through **Collins function** or **Sivers function**

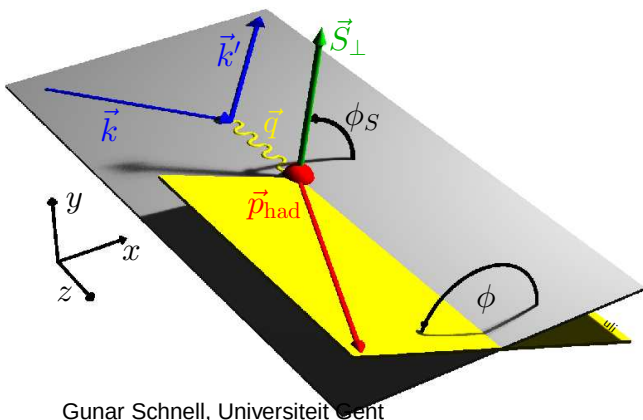
(Boer-Mulders DF couples to H_1 , but SSA requires polarization of final state!)

SIDIS Cross Section

(up to subleading order in $1/Q$)

$$\begin{aligned}
 d\sigma = & d\sigma_{UU}^0 + \cos 2\phi d\sigma_{UU}^1 + \frac{1}{Q} \cos \phi d\sigma_{UU}^2 + \lambda_e \frac{1}{Q} \sin \phi d\sigma_{LU}^3 \\
 & + S_L \left\{ \sin 2\phi d\sigma_{UL}^4 + \frac{1}{Q} \sin \phi d\sigma_{UL}^5 + \lambda_e \left[d\sigma_{LL}^6 + \frac{1}{Q} \cos \phi d\sigma_{LL}^7 \right] \right\} \\
 & + S_T \left\{ \sin(\phi - \phi_S) d\sigma_{UT}^8 + \sin(\phi + \phi_S) d\sigma_{UT}^9 + \sin(3\phi - \phi_S) d\sigma_{UT}^{10} \right. \\
 & \quad \left. + \frac{1}{Q} (\sin(2\phi - \phi_S) d\sigma_{UT}^{11} + \sin \phi_S d\sigma_{UT}^{12}) \right. \\
 & \quad \left. + \lambda_e \left[\cos(\phi - \phi_S) d\sigma_{LT}^{13} + \frac{1}{Q} (\cos \phi_S d\sigma_{LT}^{14} + \cos(2\phi - \phi_S) d\sigma_{LT}^{15}) \right] \right\}
 \end{aligned}$$

σ_{XY}
 Beam Target
 Polarization



Mulders and Tangemann, Nucl. Phys. B 461 (1996) 197

Boer and Mulders, Phys. Rev. D 57 (1998) 5780

Bacchetta et al., Phys. Lett. B 595 (2004) 309

“Trento Conventions”, Phys. Rev. D 70 (2004) 117504

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 & + S_T \left\{ \sin(\phi - \phi_S) d\sigma_{UT}^8 + \sin(\phi + \phi_S) d\sigma_{UT}^9 + \sin(3\phi - \phi_S) d\sigma_{UT}^{10} \right. \\
 & \quad \left. + \frac{1}{Q} (\sin(2\phi - \phi_S) d\sigma_{UT}^{11} + \sin \phi_S d\sigma_{UT}^{12}) \right. \\
 & \quad \left. + \lambda_e \left[\cos(\phi - \phi_S) d\sigma_{LT}^{13} + \frac{1}{Q} (\cos \phi_S d\sigma_{LT}^{14} + \cos(2\phi - \phi_S) d\sigma_{LT}^{15}) \right] \right\}
 \end{aligned}$$

σ_{XY}

Beam Target
Polarization

This talk:

$\sin \phi d\sigma_{LU}^3$	$\sin \phi d\sigma_{UL}^5$	...	Subleading Twist
$\sin(\phi - \phi_S) d\sigma_{UT}^8$		...	Sivers Effect
$\sin(\phi + \phi_S) d\sigma_{UT}^9$		...	Collins Effect

SIDIS Cross Section

(up to subleading order in $1/Q$)

$$\begin{aligned}
 d\sigma = & d\sigma_{UU}^0 + \cos 2\phi d\sigma_{UU}^1 + \frac{1}{Q} \cos \phi d\sigma_{UU}^2 + \lambda_e \frac{1}{Q} \sin \phi d\sigma_{LU}^3 \\
 & + S_L \left\{ \sin 2\phi d\sigma_{UL}^4 + \frac{1}{Q} \sin \phi d\sigma_{UL}^5 + \lambda_e \left[d\sigma_{LL}^6 + \frac{1}{Q} \cos \phi d\sigma_{LL}^7 \right] \right\} \\
 & + S_T \left\{ \sin(\phi - \phi_S) d\sigma_{UT}^8 + \sin(\phi + \phi_S) d\sigma_{UT}^9 + \sin(3\phi - \phi_S) d\sigma_{UT}^{10} \right. \\
 & \quad \left. + \frac{1}{Q} (\sin(2\phi - \phi_S) d\sigma_{UT}^{11} + \sin \phi_S d\sigma_{UT}^{12}) \right. \\
 & \quad \left. + \lambda_e \left[\cos(\phi - \phi_S) d\sigma_{LT}^{13} + \frac{1}{Q} (\cos \phi_S d\sigma_{LT}^{14} + \cos(2\phi - \phi_S) d\sigma_{LT}^{15}) \right] \right\}
 \end{aligned}$$

σ_{XY}

Beam Target

Polarization

Also Interesting: $\sin \phi_S d\sigma_{UT}^{12}, \cos \phi_S d\sigma_{LT}^{14} \dots \Rightarrow$ **Transversity, g_2**

(and under study!)

$\cos \phi d\sigma_{UU}^2$

$\dots$ **Cahn Effect**

$\cos 2\phi d\sigma_{UU}^1$

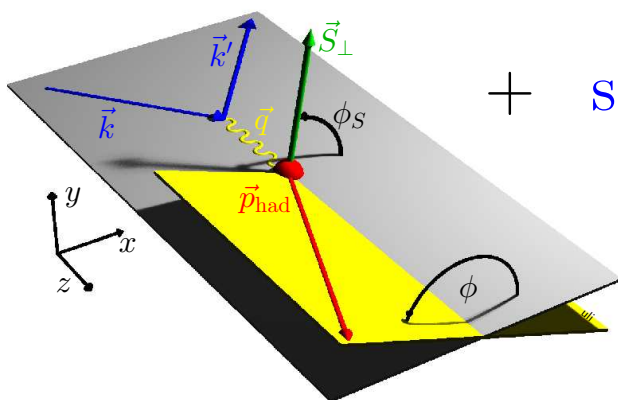
$\dots$ **Boer-Mulders Effect**

Azimuthal Single-Spin Asymmetries

$$A_{UT}(\phi, \phi_S) = \frac{1}{\langle |S_{\perp}| \rangle} \frac{N_h^{\uparrow}(\phi, \phi_S) - N_h^{\downarrow}(\phi, \phi_S)}{N_h^{\uparrow}(\phi, \phi_S) + N_h^{\downarrow}(\phi, \phi_S)}$$

$$\sim \sin(\phi + \phi_S) \sum_q e_q^2 \mathcal{I} \left[\frac{k_T \hat{P}_{h\perp}}{M_h} h_1^q(x, p_T^2) H_1^{\perp, q}(z, k_T^2) \right]$$

$$+ \sin(\phi - \phi_S) \sum_q e_q^2 \mathcal{I} \left[\frac{p_T \hat{P}_{h\perp}}{M} f_{1T}^{\perp, q}(x, p_T^2) D_1^q(z, k_T^2) \right]$$



+ ... $\mathcal{I}[\dots]$: convolution integral over initial (p_T) and final (k_T) quark transverse momenta

$\Rightarrow$ 2D-fit of A_{UT} to get Collins and Sivers asymmetries:

$$A_{UT}(\phi, \phi_S) = 2 \langle \sin(\phi - \phi_S) \rangle_{UT} \sin(\phi - \phi_S) + 2 \langle \sin(\phi + \phi_S) \rangle_{UT} \sin(\phi + \phi_S)$$

Resolving the Convolution Integral

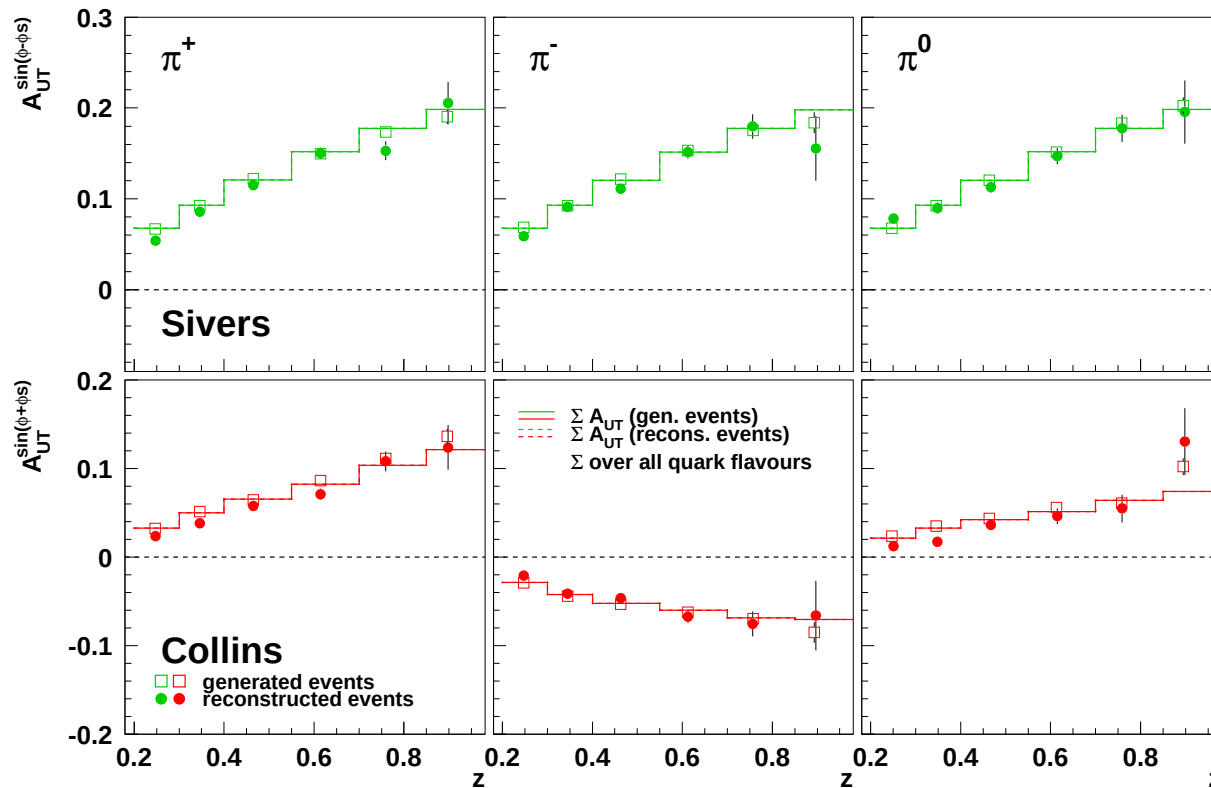
Weight with transverse hadron momentum $P_{h\perp}$ to resolve convolution:

$$\begin{aligned}\tilde{A}_{UT}(\phi, \phi_S) &= \frac{1}{\langle S_{\perp} \rangle} \frac{\sum_{i=1}^{N^+} P_{h\perp,i} - \sum_{i=1}^{N^-} P_{h\perp,i}}{N^+ + N^-} \\ &\sim \sin(\phi + \phi_C) \cdot \sum_q e_q^2 h_1^q(x) z H_1^{\perp(1),q}(z) \quad (1): \quad p_T^2/k_T^2\text{-moment of} \\ &\quad \text{distribution / fragmentation} \\ &\quad - \sin(\phi - \phi_S) \cdot \sum_q e_q^2 f_{1T}^{\perp(1),q}(x) z D_1^q(z) \quad \text{function} \\ &\quad + \dots\end{aligned}$$

$\Rightarrow$ 2D-fit of $\tilde{A}_{UT}$ to get Collins and Sivers asymmetries:

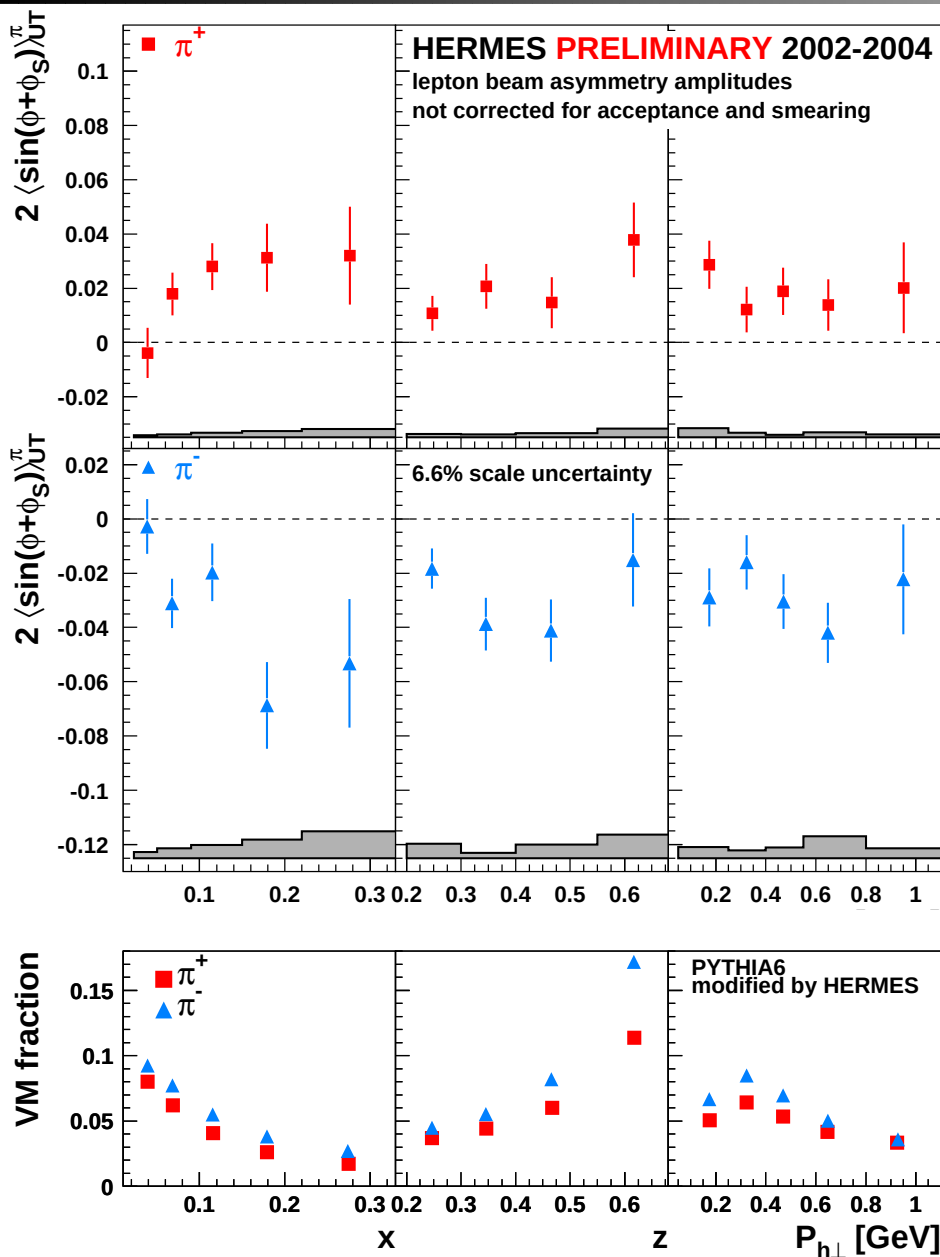
$$\begin{aligned}\tilde{A}_{UT}(\phi, \phi_S) &= M_{\pi} 2 \left\langle \frac{P_{h\perp}}{M_{\pi}} \sin(\phi + \phi_S) \right\rangle_{UT}(x, z) \sin(\phi + \phi_S) \\ &\quad + M_p 2 \left\langle \frac{P_{h\perp}}{M_p} \sin(\phi - \phi_S) \right\rangle_{UT}(x, z) \sin(\phi - \phi_S)\end{aligned}$$

- generate Collins and Sivers asymmetries (Gaussian Ansatz in p_T^2)
- analyze MC data like experimental data and extract asymmetries:



- Collins-Sivers cross contamination negligible
- insensitive to $\cos(2\phi)$ moments in unpolarized cross section
- insensitive to transverse target tracking corrections

Collins Asymmetries 2002-2004



published[†] results **confirmed** with much higher statistical precision

overall scale uncertainty of 6.6%

positive for π^+ and negative for π^- as maybe expected ($\delta u > 0$
 $\delta d < 0$)

unexpected **large π^- asymmetry**

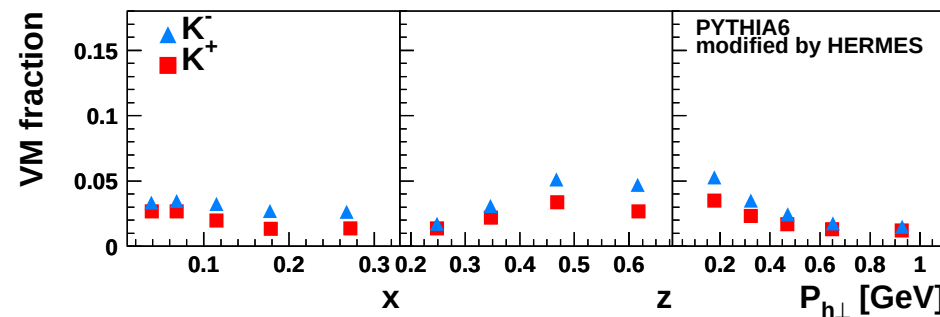
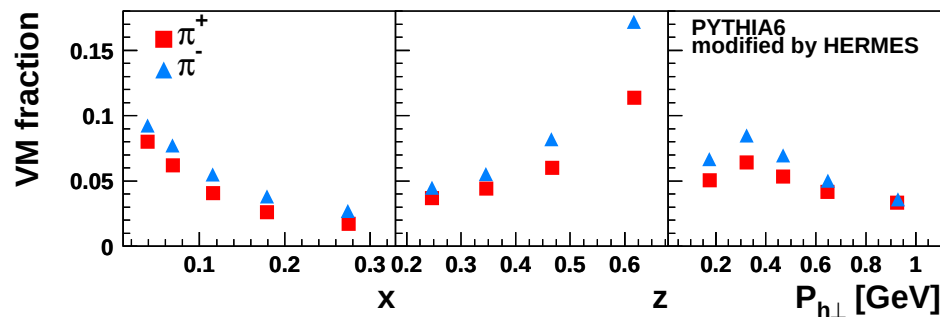
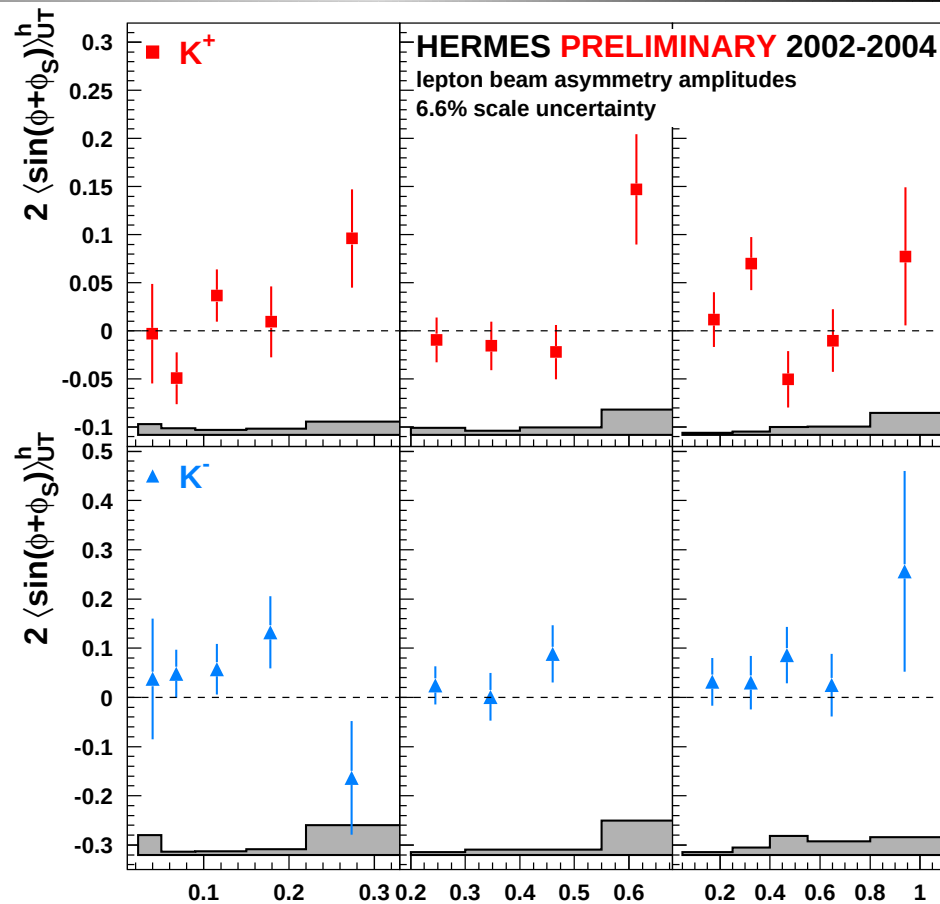
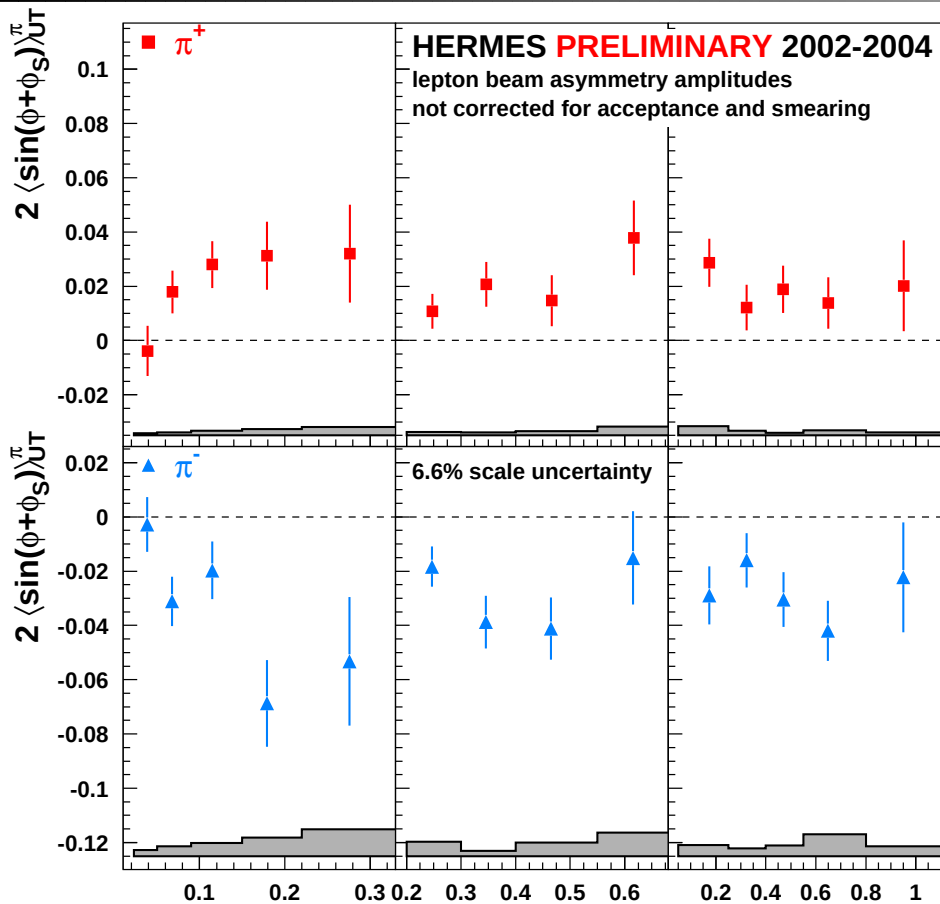
$\Rightarrow$ role of **disfavored** Collins FF

most likely: $H_1^{\perp, disf} \approx -H_1^{\perp, fav}$

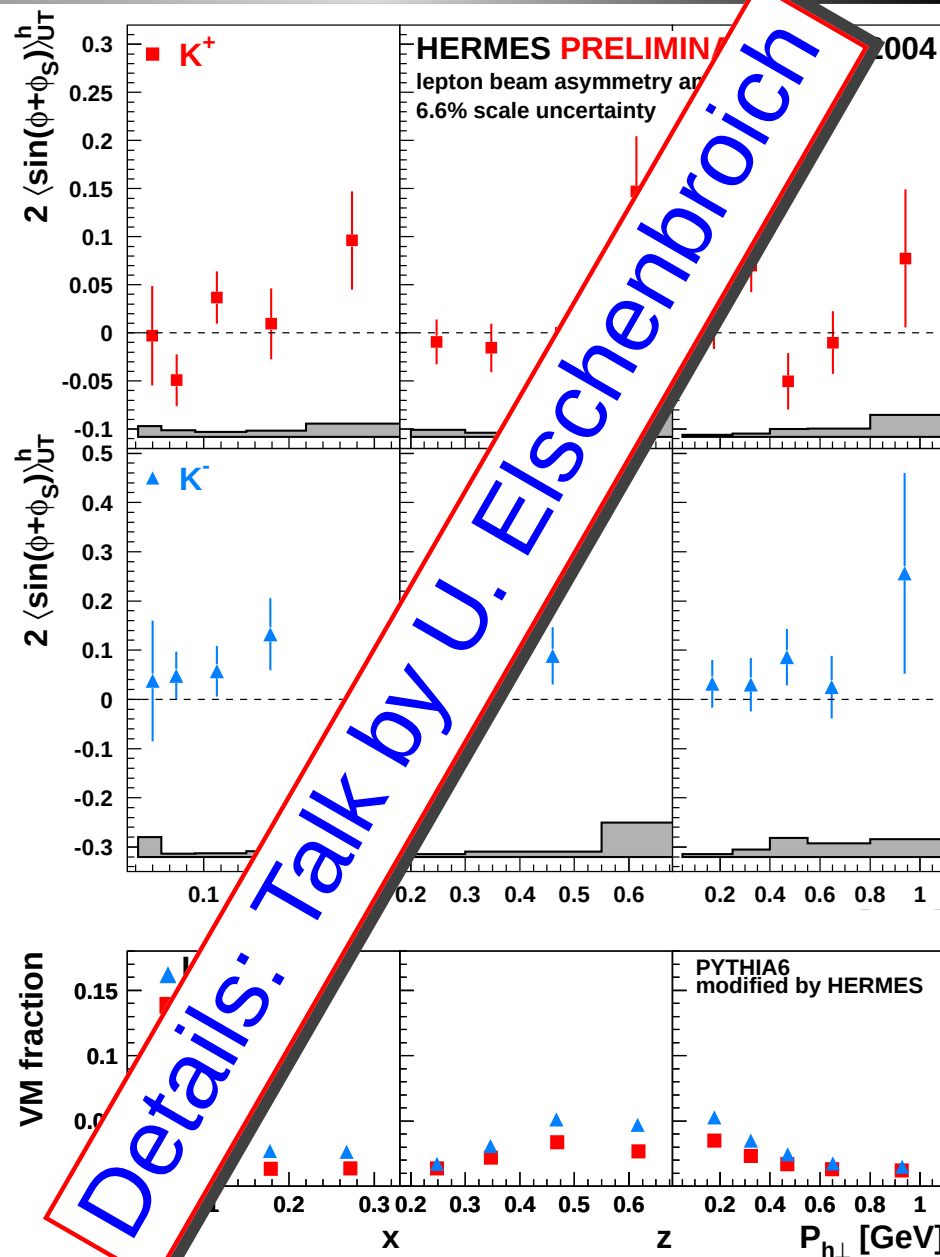
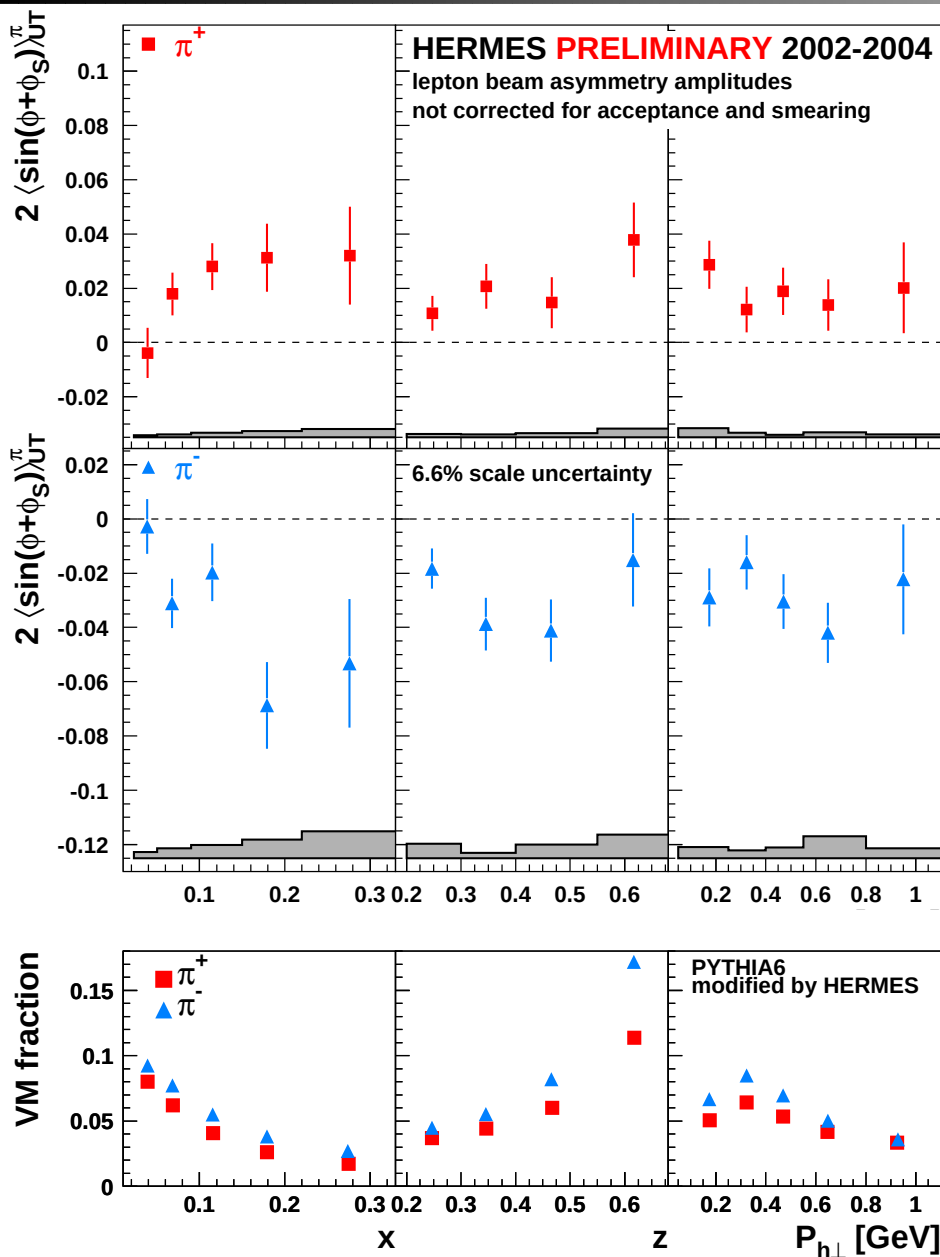
partially large contribution from decay of exclusively produced vector mesons

[†] [A. Airapetian et al, Phys. Rev. Lett. 94 (2005) 012002]

Collins Asymmetries 2002-2004



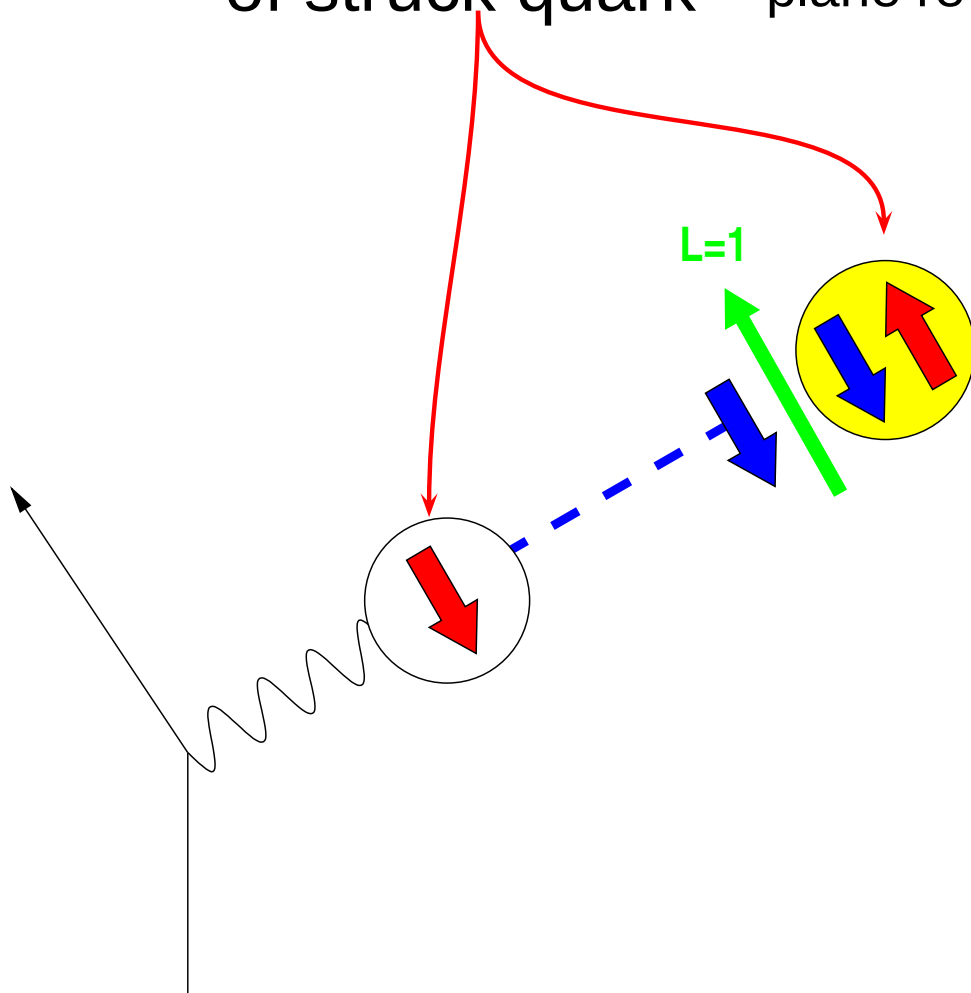
Collins Asymmetries 2002-2004



Details: Talk by U. Elschenbroich

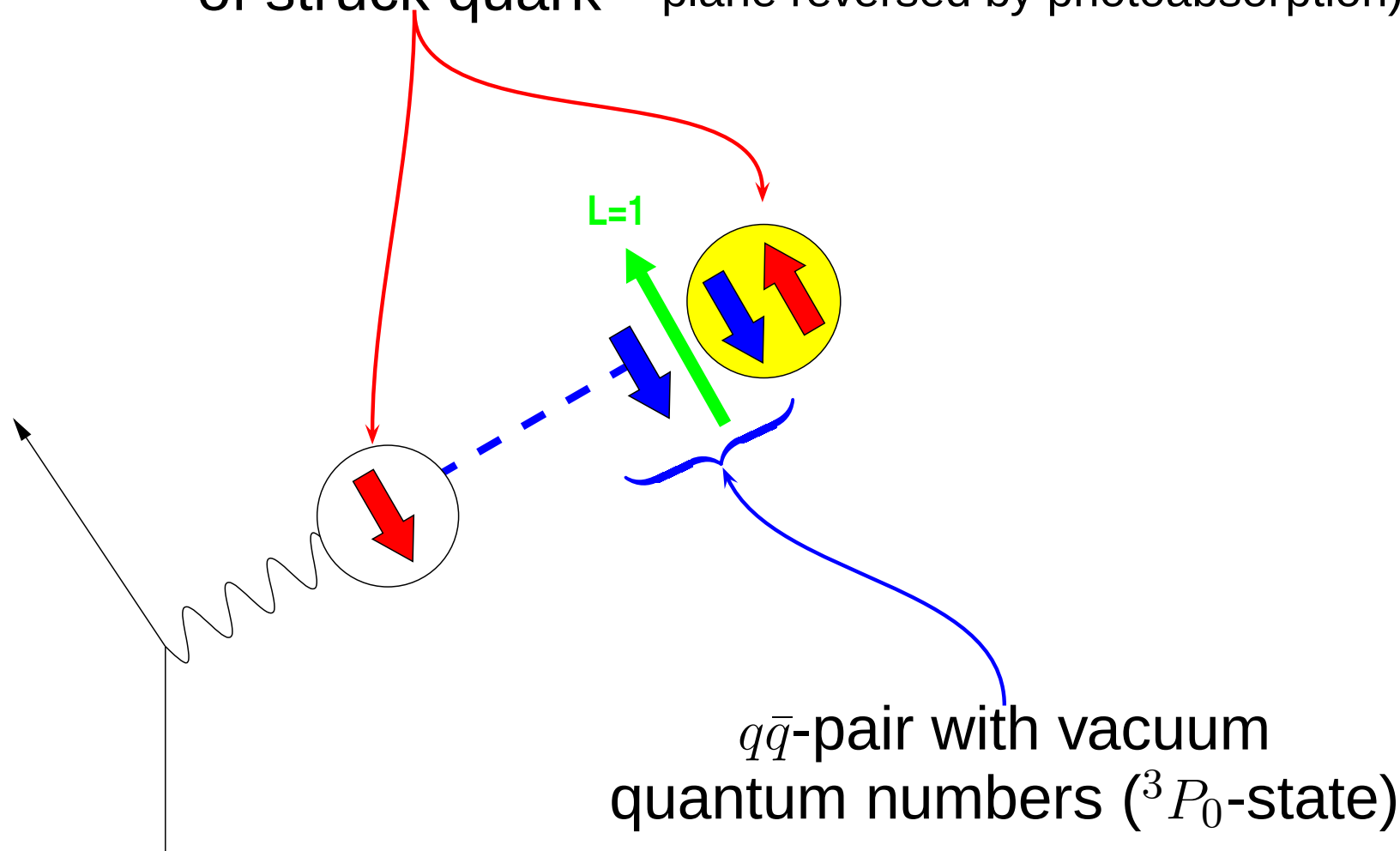
Understanding the Collins FF - String Model Interpretation (Artru)

transverse spin
of struck quark (polarization component in lepton scattering
plane reversed by photoabsorption)



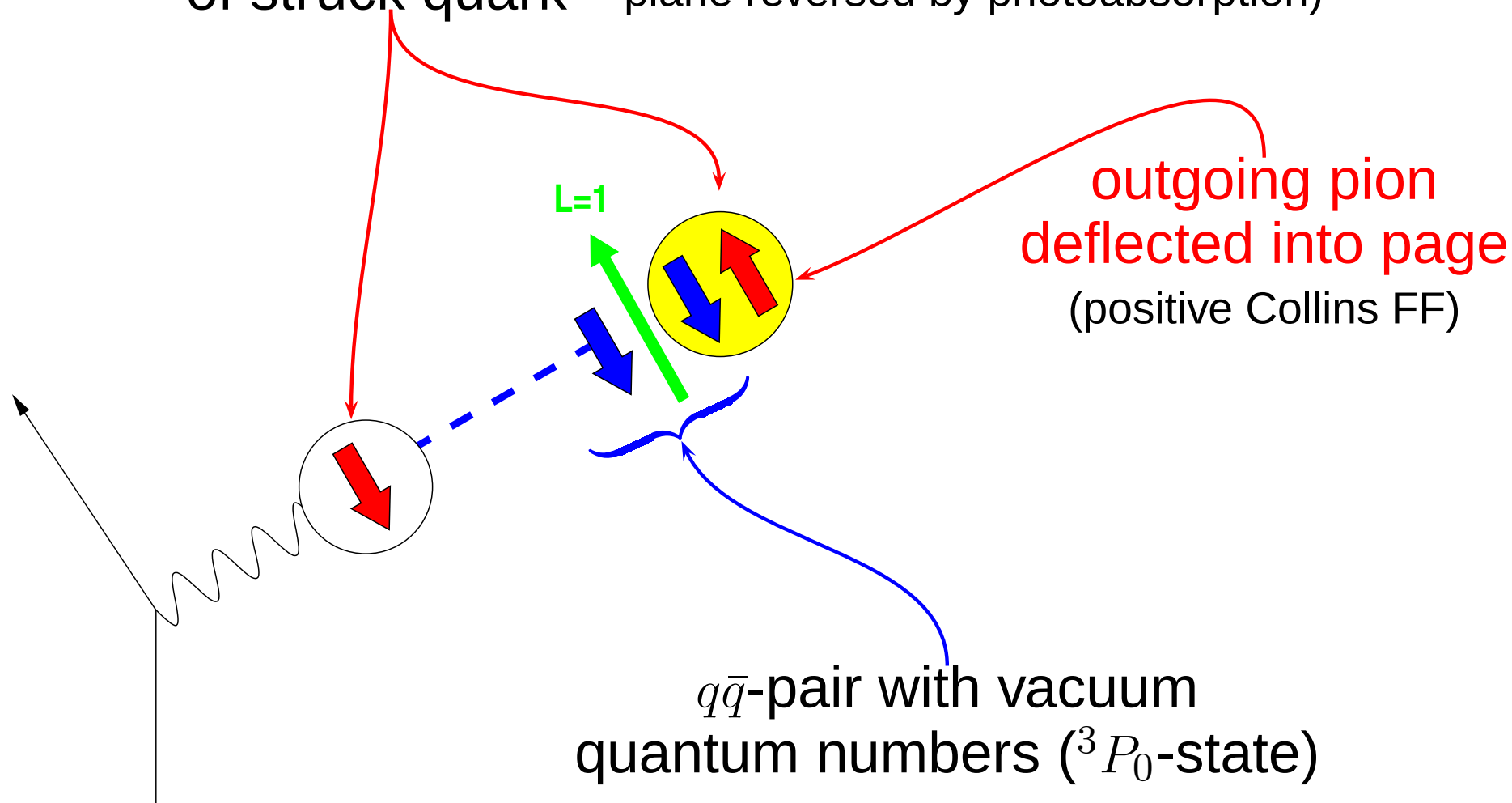
Understanding the Collins FF - String Model Interpretation (Artru)

transverse spin of struck quark (polarization component in lepton scattering plane reversed by photoabsorption)



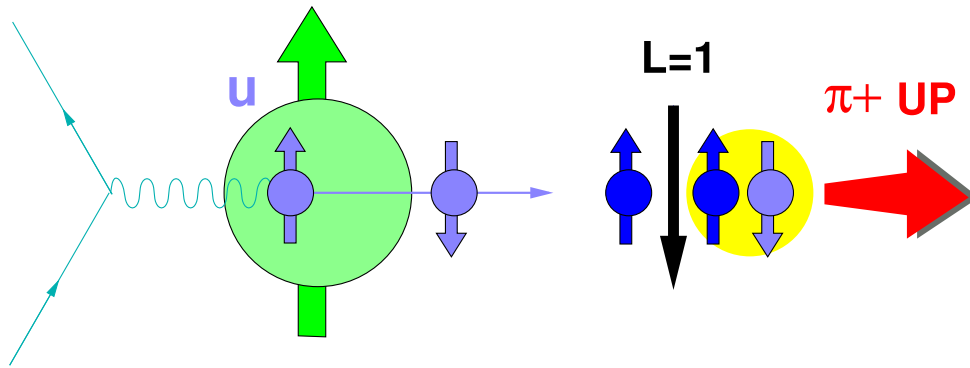
Understanding the Collins FF - String Model Interpretation (Artru)

transverse spin of struck quark (polarization component in lepton scattering plane reversed by photoabsorption)

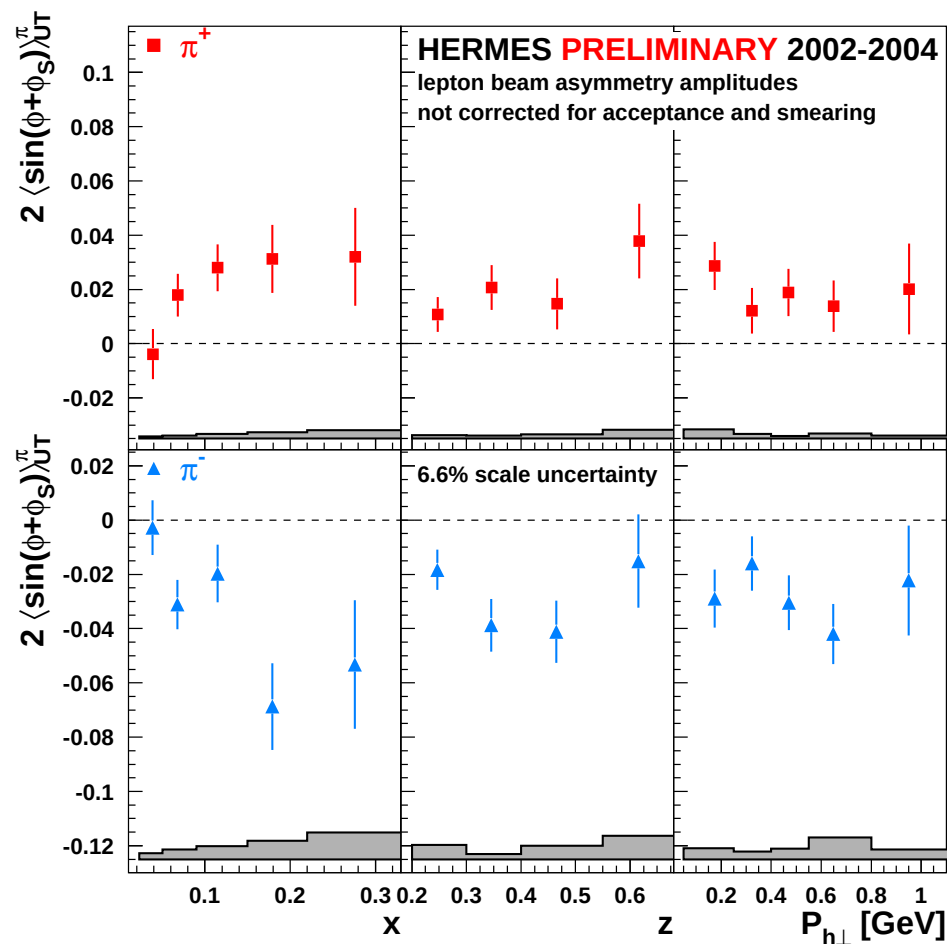


The Collins Effect

Artru Model vs. HERMES

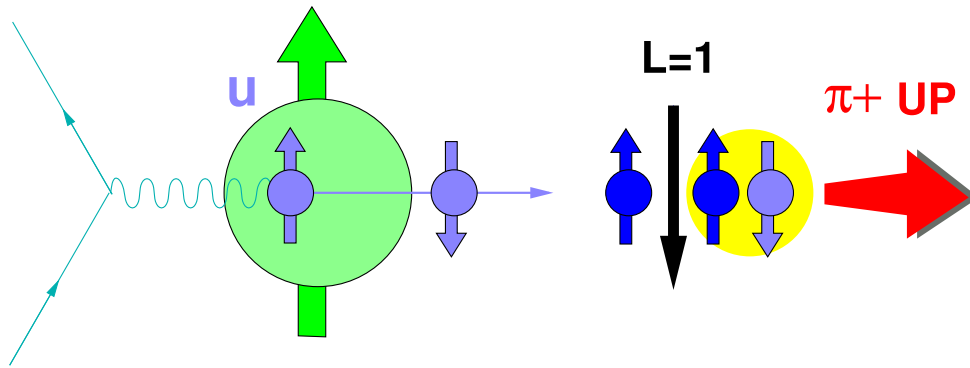


$$\left. \begin{array}{l} \phi_S = 0 \\ \phi = \pi/2 \end{array} \right\} \sin(\phi + \phi_S) > 0$$

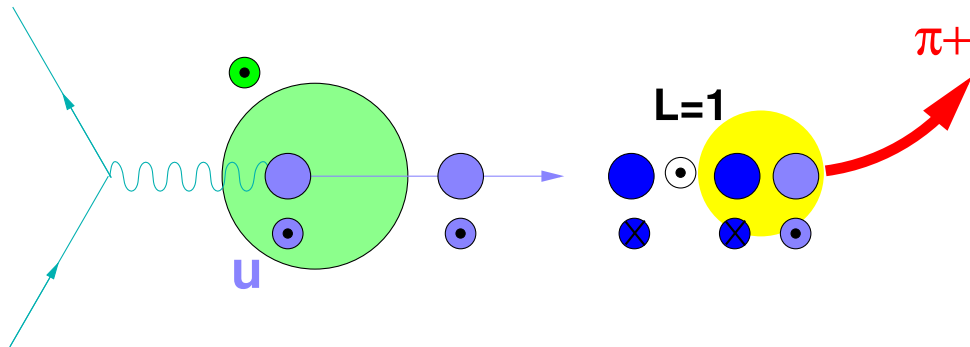


$$\left. \begin{array}{l} \phi_S = 0 \\ \phi = \pi/2 \end{array} \right\} \sin(\phi + \phi_S) > 0$$

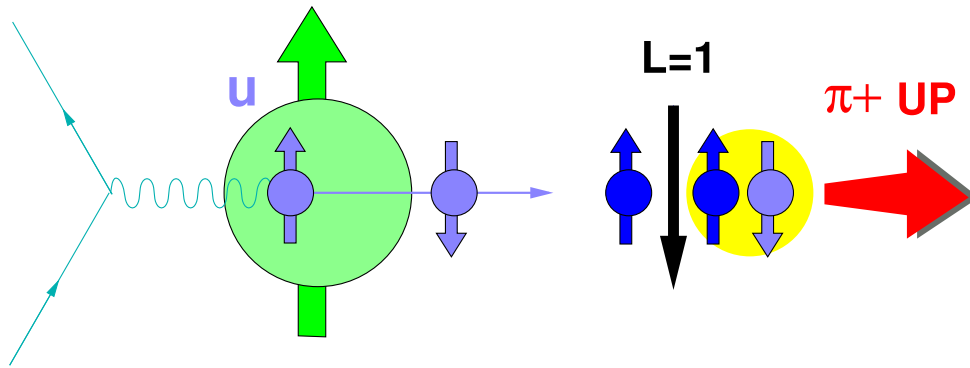




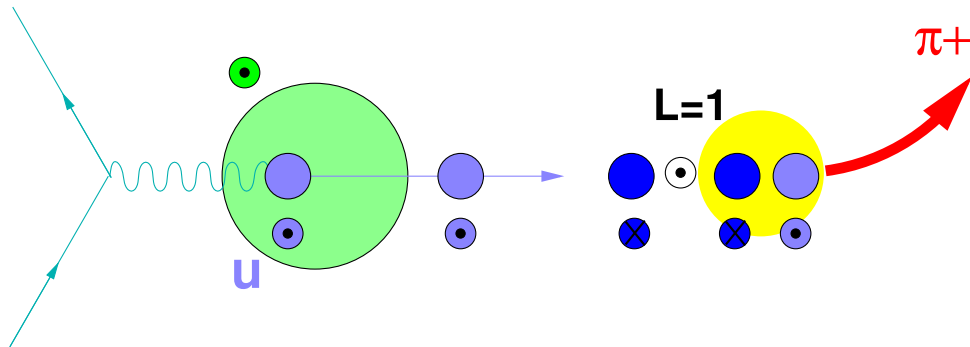
$$\left. \begin{array}{l} \phi_S = 0 \\ \phi = \pi/2 \end{array} \right\} \sin(\phi + \phi_S) > 0$$



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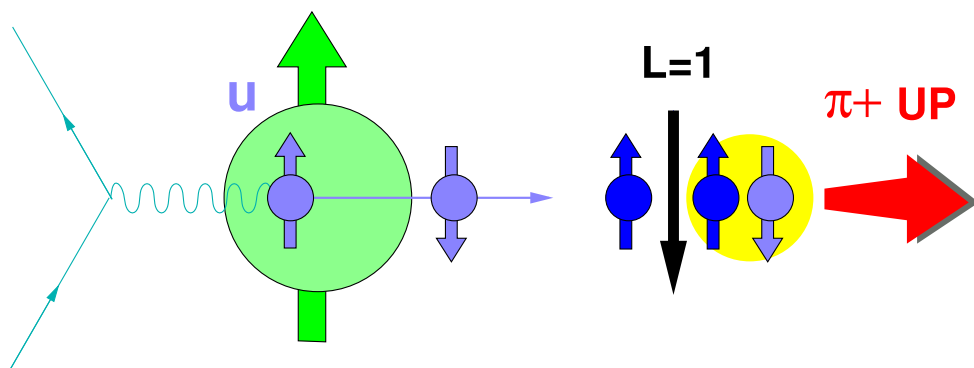


$$\left. \begin{array}{l} \phi_S = 0 \\ \phi = \pi/2 \end{array} \right\} \sin(\phi + \phi_S) > 0$$

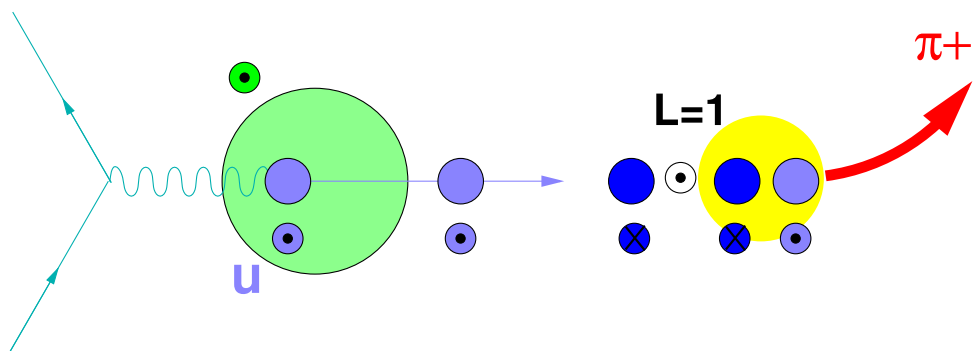


$$\left. \begin{array}{l} \phi_S = \pi/2 \\ \phi = 0 \end{array} \right\} \sin(\phi + \phi_S) > 0$$





$$\left. \begin{array}{l} \phi_S = 0 \\ \phi = \pi/2 \end{array} \right\} \sin(\phi + \phi_S) > 0$$



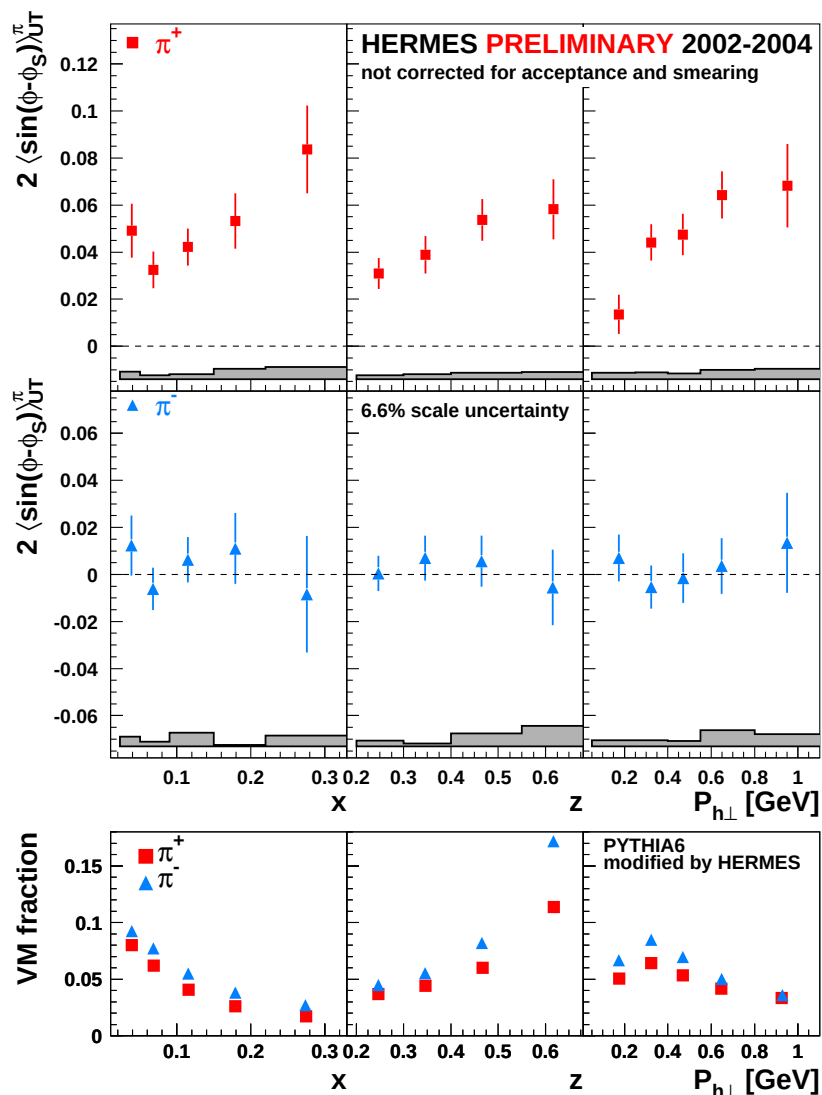
$$\left. \begin{array}{l} \phi_S = \pi/2 \\ \phi = 0 \end{array} \right\} \sin(\phi + \phi_S) > 0$$



Artru model and HERMES results in agreement!
(assuming u -quark transversity positive)

Results on Sivers Amplitudes from 2002-2004 data

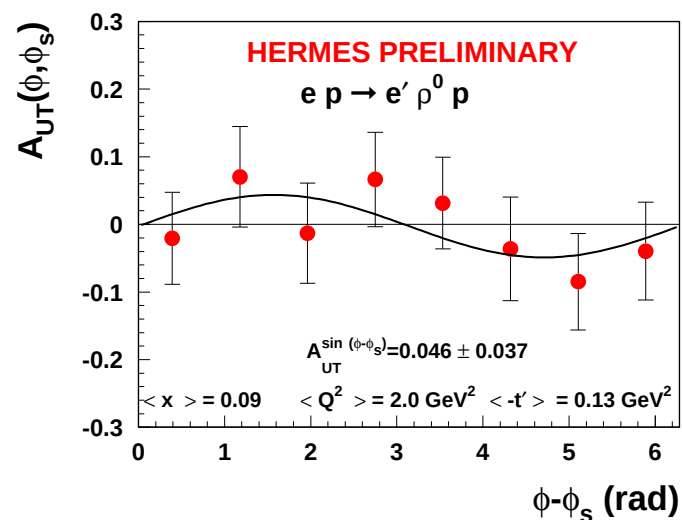
$$2 \langle \sin(\phi - \phi_S) \rangle_{UT} \propto - \sum_q e_q^2 \mathcal{I} \left[\frac{p_T \hat{P}_{h\perp}}{M} f_{1T}^{\perp,q}(x, p_T^2) D_1^q(z, K_T^2) \right]$$



π^+ : positive; π^- : consistent with zero

$\Rightarrow$ first evidence for non-zero Sivers fct.:
 $f_{1T}^{\perp,u} < 0$ (u -quark dominance)

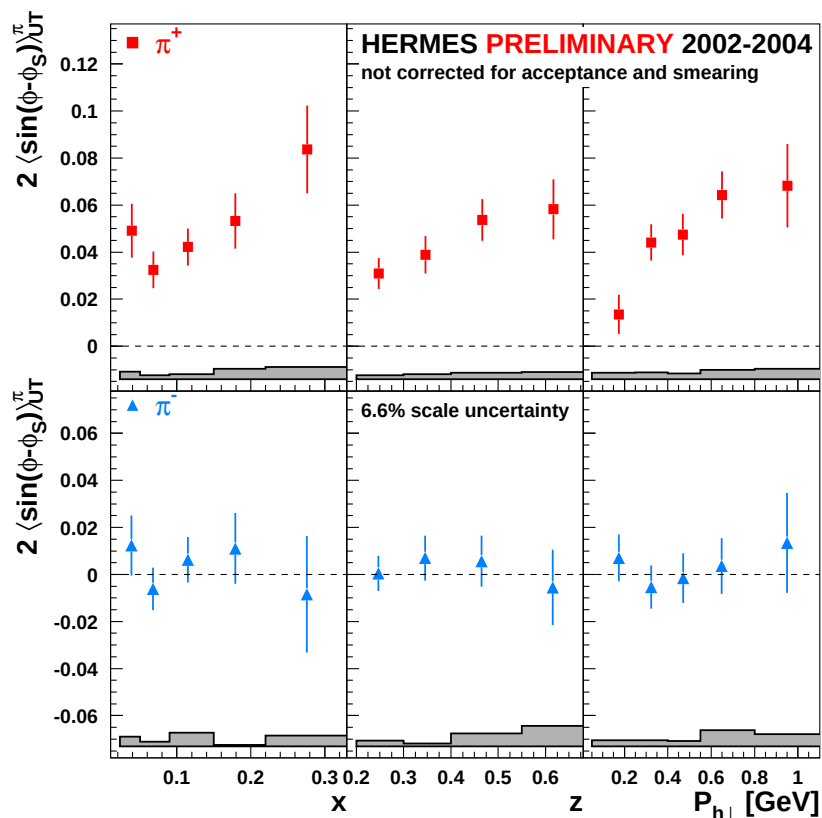
$\Rightarrow$ Exclusive ρ^0 asymmetry (2005 prel.):



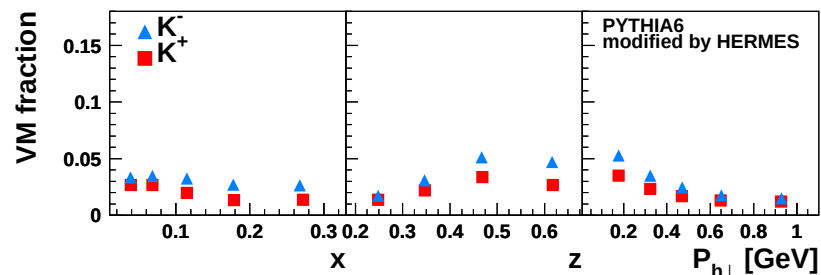
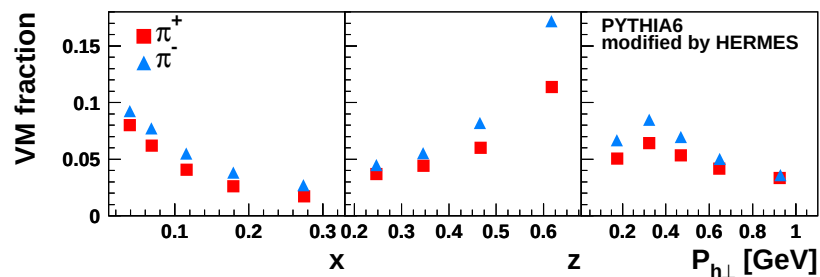
$\Rightarrow$ small syst. error from vector mesons

Results on Sivers Amplitudes from 2002-2004 data

$$2 \langle \sin(\phi - \phi_S) \rangle_{UT} \propto - \sum_q e_q^2 \mathcal{I} \left[\frac{p_T \hat{P}_{h\perp}}{M} f_{1T}^{\perp,q}(x, p_T^2) D_1^q(z, K_T^2) \right]$$

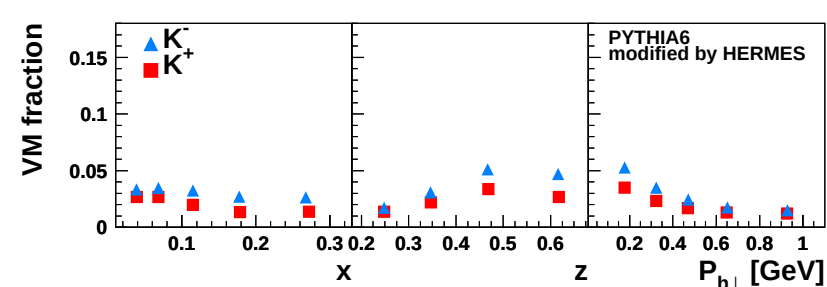
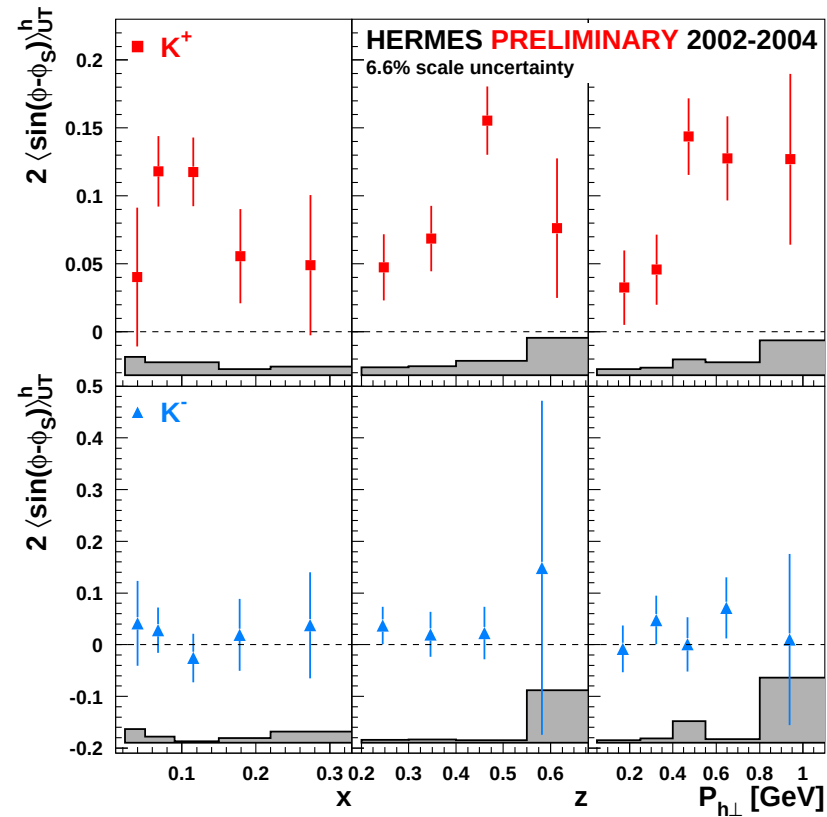
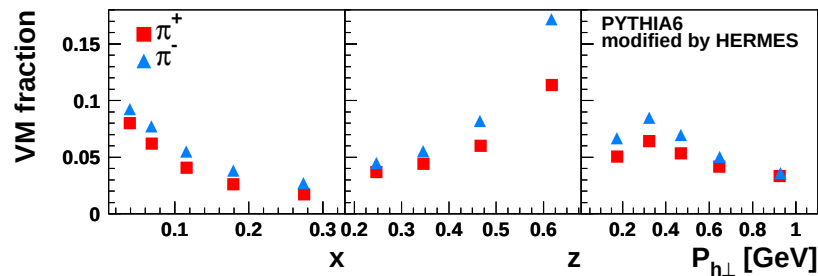
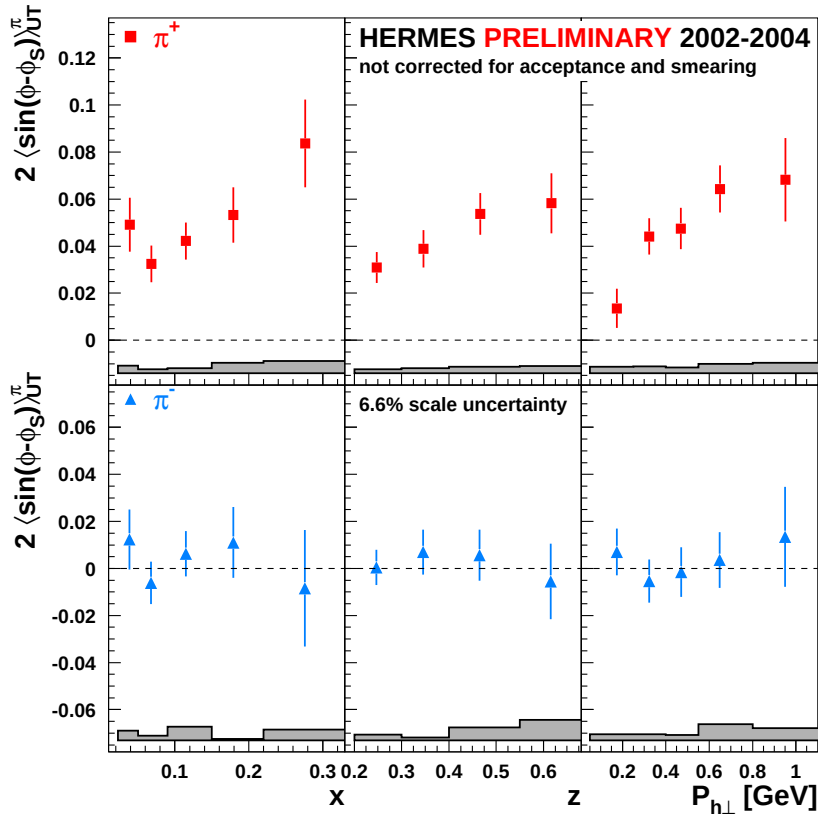


smaller VM contribution to kaon sample



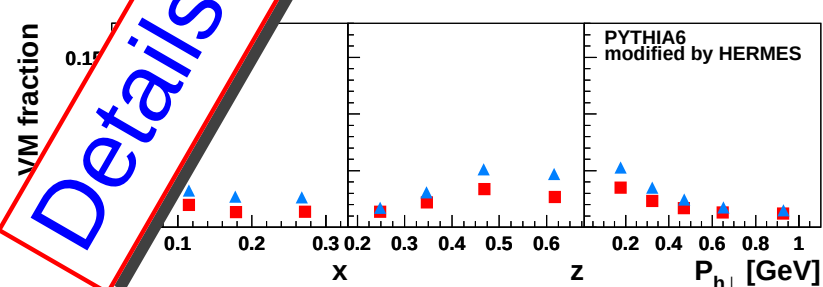
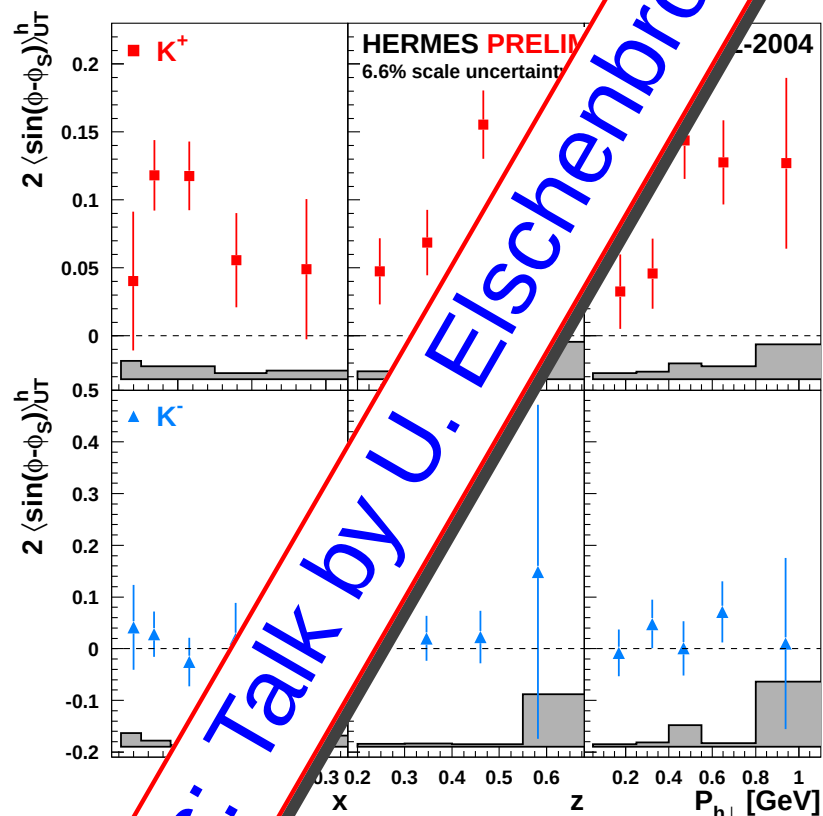
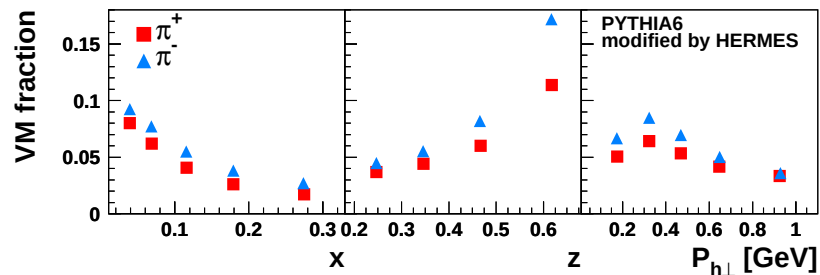
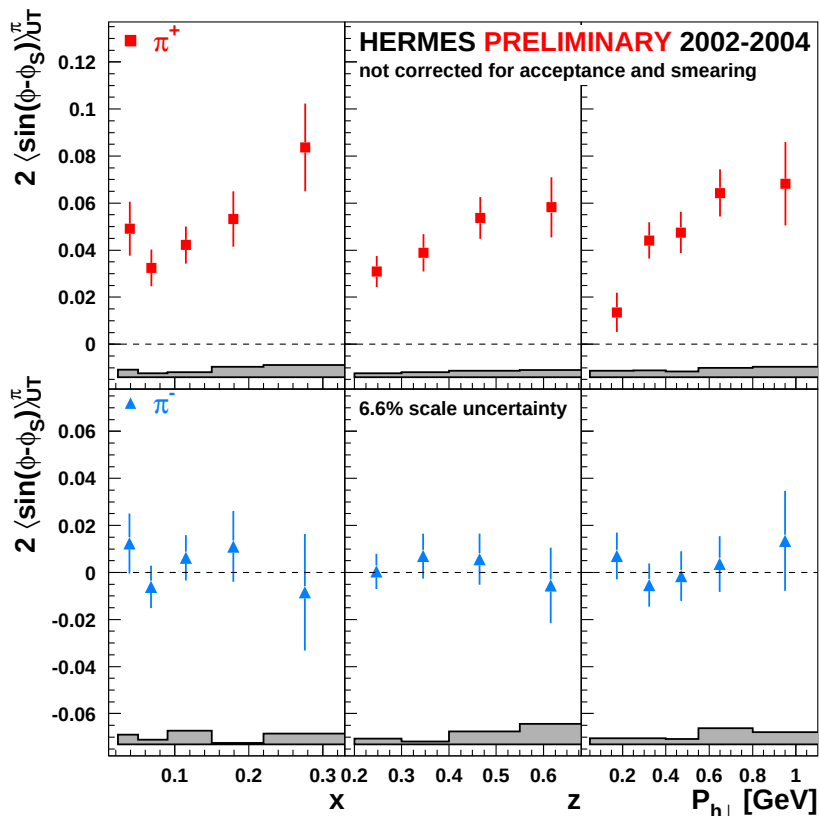
Results on Sivers Amplitudes from 2002-2004 data

$$2 \langle \sin(\phi - \phi_S) \rangle_{UT} \propto - \sum_q e_q^2 \mathcal{I} \left[\frac{p_T \hat{P}_{h\perp}}{M} f_{1T}^{\perp,q}(x, p_T^2) D_1^q(z, K_T^2) \right]$$



Results on Sivers Amplitudes from 2002-2004 data

$$2 \langle \sin(\phi - \phi_S) \rangle_{UT} \propto - \sum_q e_q^2 \mathcal{I} \left[\frac{p_T \hat{P}_{h\perp}}{M} f_{1T}^{\perp,q}(x, p_T^2) D_1^q(z) \right]$$



Details: Talk by U. Elschenbroich

Chromodynamic Lensing

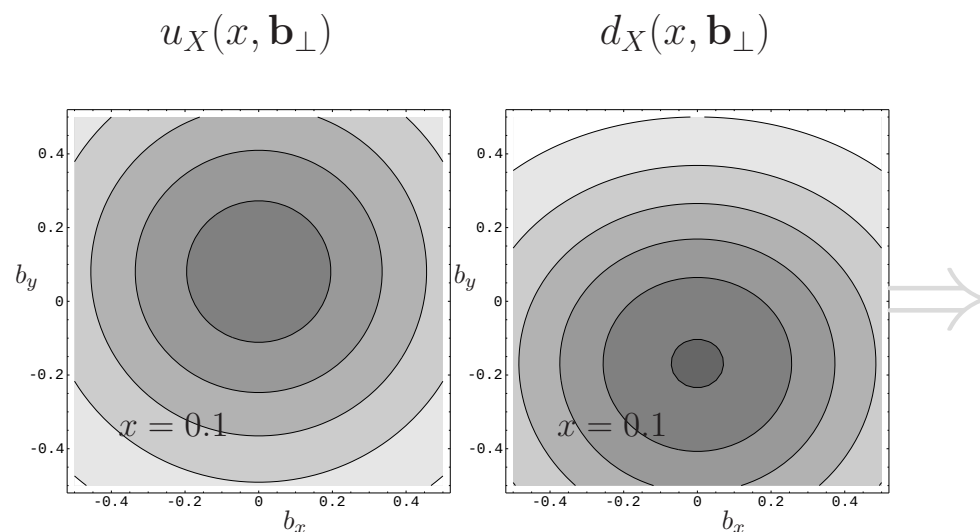
Understanding the Sivers Moments

approach by M. Burkardt:

[hep-ph/0309269]

spatial distortion of q -distribution

(obtained using anom. magn. moments
& impact parameter dependent PDFs)



approach by M. Burkardt:

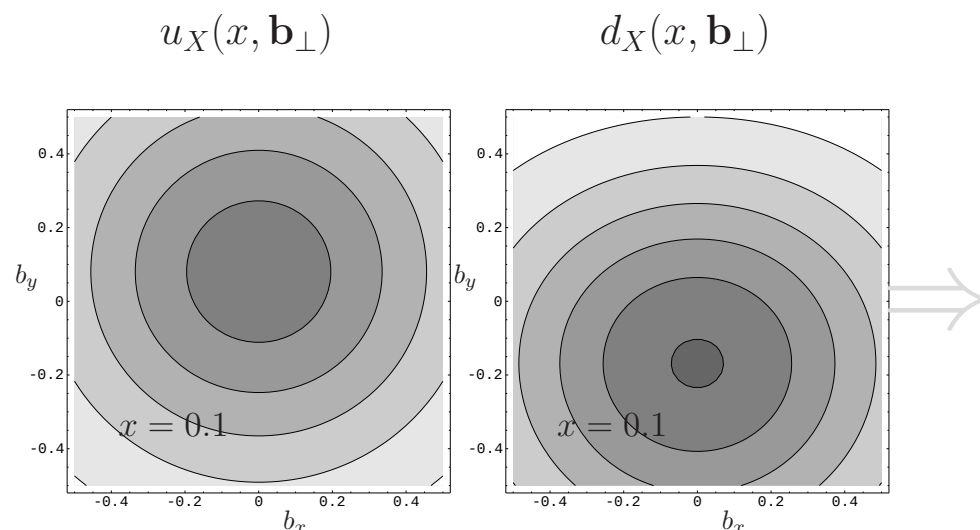
[hep-ph/0309269]

spatial distortion of q-distribution

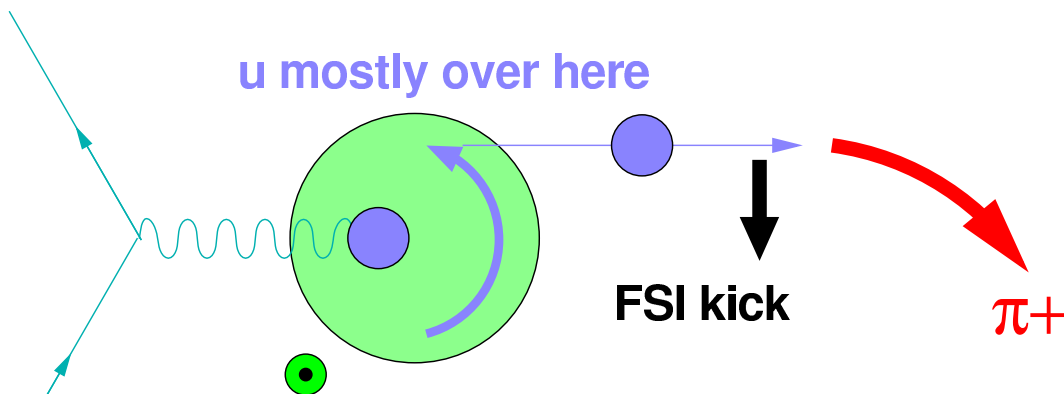
(obtained using anom. magn. moments
& impact parameter dependent PDFs)

+ attractive QCD potential
(gluon exchange)

⇒ transverse asymmetries



u mostly over here



$$\left. \begin{array}{l} \phi_S = \pi/2 \\ \phi = \pi \end{array} \right\} \sin(\phi - \phi_S) > 0$$

approach by M. Burkardt:

spatial distortion of q-distribution

(obtained using anom. magn. moments
& impact parameter dependent PDFs)

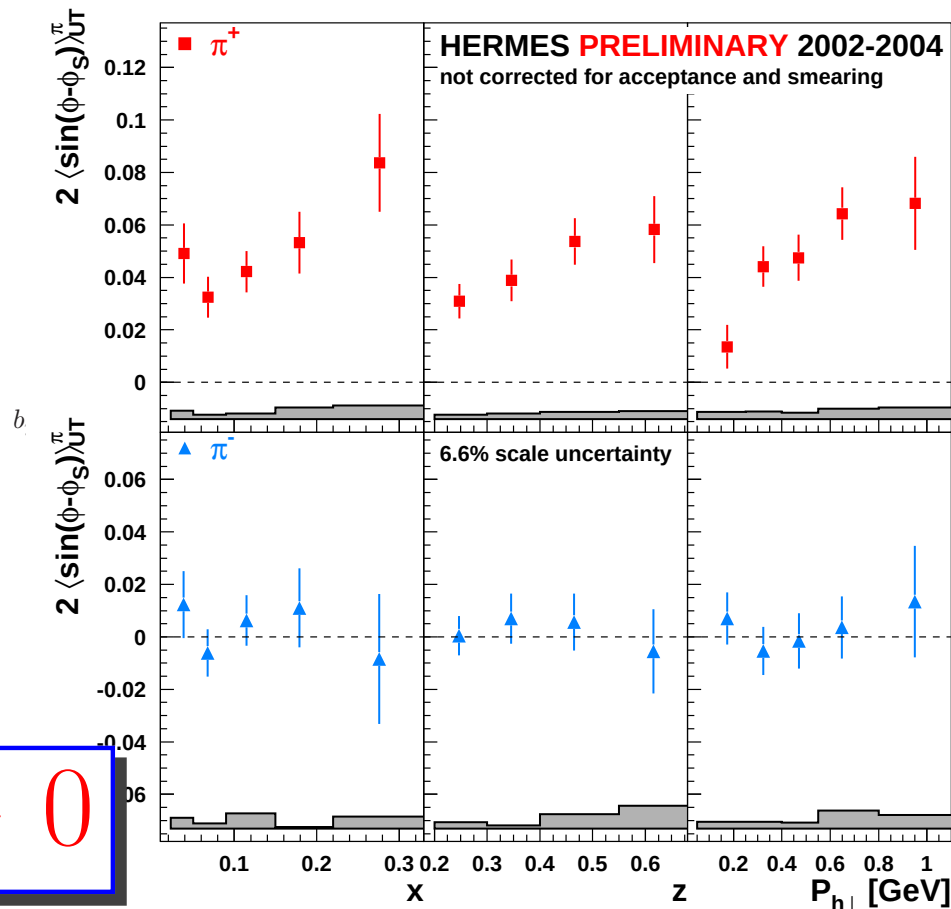
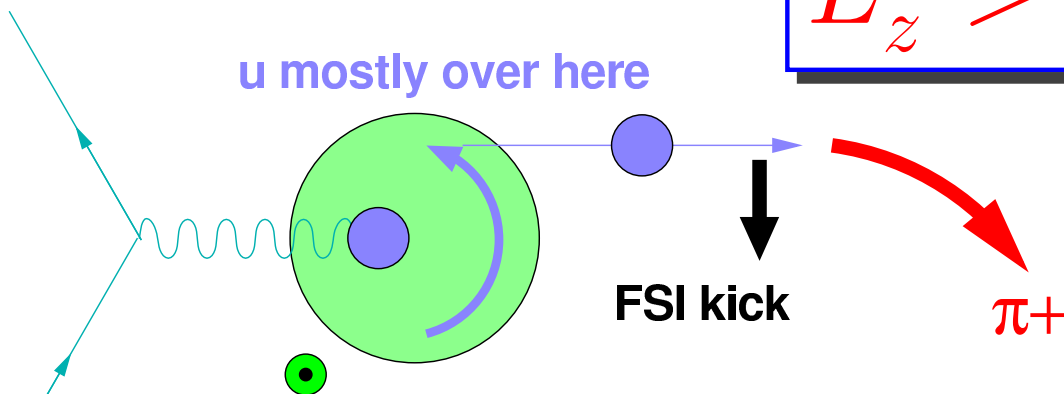
+ attractive QCD potential

(gluon exchange)

⇒ transverse asymmetries

$$L_z^u > 0$$

u mostly over here



$$\left. \begin{array}{l} \phi_S = \pi/2 \\ \phi = \pi \end{array} \right\} \sin(\phi - \phi_S) > 0$$

Longitudinal SSAs

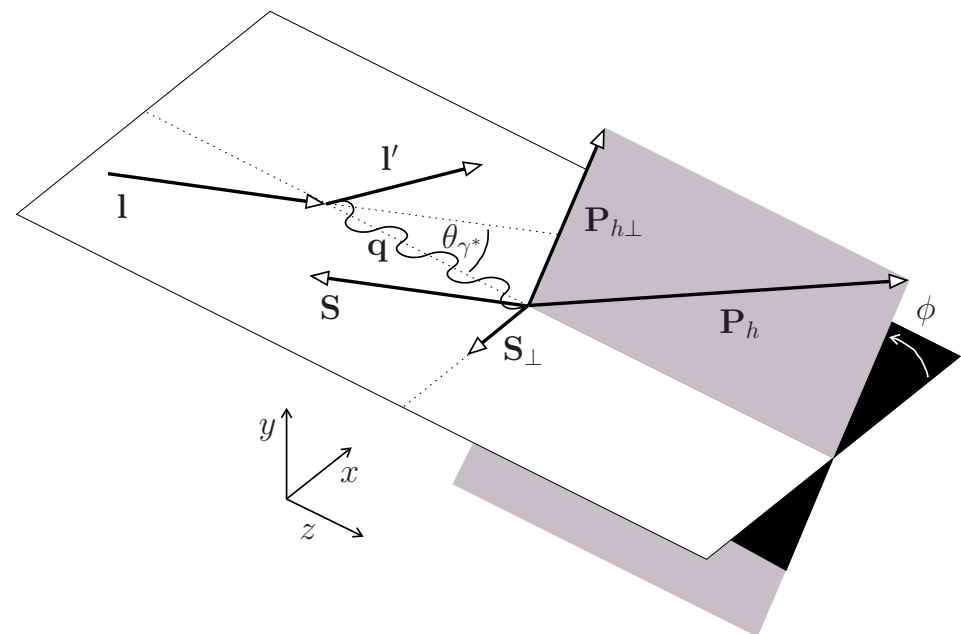
Mixing of Azimuthal Moments

Experiment: **Target Polarization w.r.t. Beam Direction (I)!**

Theory: Polarization along virtual photon direction (q)

⇒ mixing of “experimental” and “theory” asymmetries via:

[Diehl and Sapeta, Eur. Phys. J. C41 (2005)]



$$\begin{pmatrix} \langle \sin \phi \rangle_{UL}^I \\ \langle \sin(\phi - \phi_S) \rangle_{UT}^I \\ \langle \sin(\phi + \phi_S) \rangle_{UT}^I \end{pmatrix} = \begin{pmatrix} \cos \theta_{\gamma^*} & -\sin \theta_{\gamma^*} & -\sin \theta_{\gamma^*} \\ \frac{1}{2} \sin \theta_{\gamma^*} & \cos \theta_{\gamma^*} & 0 \\ \frac{1}{2} \sin \theta_{\gamma^*} & 0 & \cos \theta_{\gamma^*} \end{pmatrix} \begin{pmatrix} \langle \sin \phi \rangle_{UL}^q \\ \langle \sin(\phi - \phi_S) \rangle_{UT}^q \\ \langle \sin(\phi + \phi_S) \rangle_{UT}^q \end{pmatrix}$$

($\cos \theta_{\gamma^*} \simeq 1$, $\sin \theta_{\gamma^*}$ up to 15% at HERMES energies)

Mixing of Azimuthal Moments II

$$\begin{pmatrix} \langle \sin \phi \rangle_{UL}^I \\ \langle \sin(\phi - \phi_S) \rangle_{UT}^I \\ \langle \sin(\phi + \phi_S) \rangle_{UT}^I \end{pmatrix} = \begin{pmatrix} \cos \theta_{\gamma^*} & -\sin \theta_{\gamma^*} & -\sin \theta_{\gamma^*} \\ \frac{1}{2} \sin \theta_{\gamma^*} & \cos \theta_{\gamma^*} & 0 \\ \frac{1}{2} \sin \theta_{\gamma^*} & 0 & \cos \theta_{\gamma^*} \end{pmatrix} \begin{pmatrix} \langle \sin \phi \rangle_{UL}^q \\ \langle \sin(\phi - \phi_S) \rangle_{UT} \\ \langle \sin(\phi + \phi_S) \rangle_{UT} \end{pmatrix}$$

solve for photon-axis moments:

$$\langle \sin \phi \rangle_{UL}^q \simeq \langle \sin \phi \rangle_{UL}^I + \sin \theta_{\gamma^*} \left(\langle \sin(\phi + \phi_S) \rangle_{UT}^I + \langle \sin(\phi - \phi_S) \rangle_{UT}^I \right)$$

Mixing of Azimuthal Moments II

$$\begin{pmatrix} \langle \sin \phi \rangle_{UL}^I \\ \langle \sin(\phi - \phi_S) \rangle_{UT}^I \\ \langle \sin(\phi + \phi_S) \rangle_{UT}^I \end{pmatrix} = \begin{pmatrix} \cos \theta_{\gamma^*} & -\sin \theta_{\gamma^*} & -\sin \theta_{\gamma^*} \\ \frac{1}{2} \sin \theta_{\gamma^*} & \cos \theta_{\gamma^*} & 0 \\ \frac{1}{2} \sin \theta_{\gamma^*} & 0 & \cos \theta_{\gamma^*} \end{pmatrix} \begin{pmatrix} \langle \sin \phi \rangle_{UL}^q \\ \langle \sin(\phi - \phi_S) \rangle_{UT} \\ \langle \sin(\phi + \phi_S) \rangle_{UT} \end{pmatrix}$$

solve for photon-axis moments:

$$\begin{aligned} \langle \sin \phi \rangle_{UL}^q &\simeq \langle \sin \phi \rangle_{UL}^I + \sin \theta_{\gamma^*} \left(\langle \sin(\phi + \phi_S) \rangle_{UT}^I + \langle \sin(\phi - \phi_S) \rangle_{UT}^I \right) \\ \langle \sin(\phi \pm \phi_S) \rangle_{UT}^q &\simeq \langle \sin(\phi \pm \phi_S) \rangle_{UT}^I \\ &\quad - \underbrace{\frac{1}{2} \sin \theta_{\gamma^*} \left(\langle \sin \phi \rangle_{UL}^I \right)}_{\text{max. 0.4\% absolute}} + \underbrace{\tan \theta_{\gamma^*} \langle \sin(\phi \mp \phi_S) \rangle_{UT}^I}_{\text{max. 1\% relative}} \end{aligned}$$

correction

What About Longitudinally Polarized Targets?

$$\langle \sin \phi \rangle_{UL}^q = \langle \sin \phi \rangle_{UL}^l + \sin \theta_{\gamma^*} \left(\langle \sin(\phi + \phi_S) \rangle_{UT}^l + \langle \sin(\phi - \phi_S) \rangle_{UT}^l \right)$$

$$\langle \sin \phi \rangle_{UL}^q \propto \frac{M}{Q} \mathcal{I} \left[\frac{\hat{P}_{h\perp} k_T}{M_h} \left(\frac{M_h}{zM} g_1 G^\perp + x h_L H_1^\perp \right) + \frac{\hat{P}_{h\perp} p_T}{M} \left(\frac{M_h}{zM} h_{1L}^\perp \tilde{H} - x f_L^\perp D_1 \right) \right]$$

Bacchetta et al., Phys. Lett. B 595 (2004) 309

$\Rightarrow$ they are all subleading-twist expressions!

$$\langle \sin \phi \rangle_{UL}^l$$

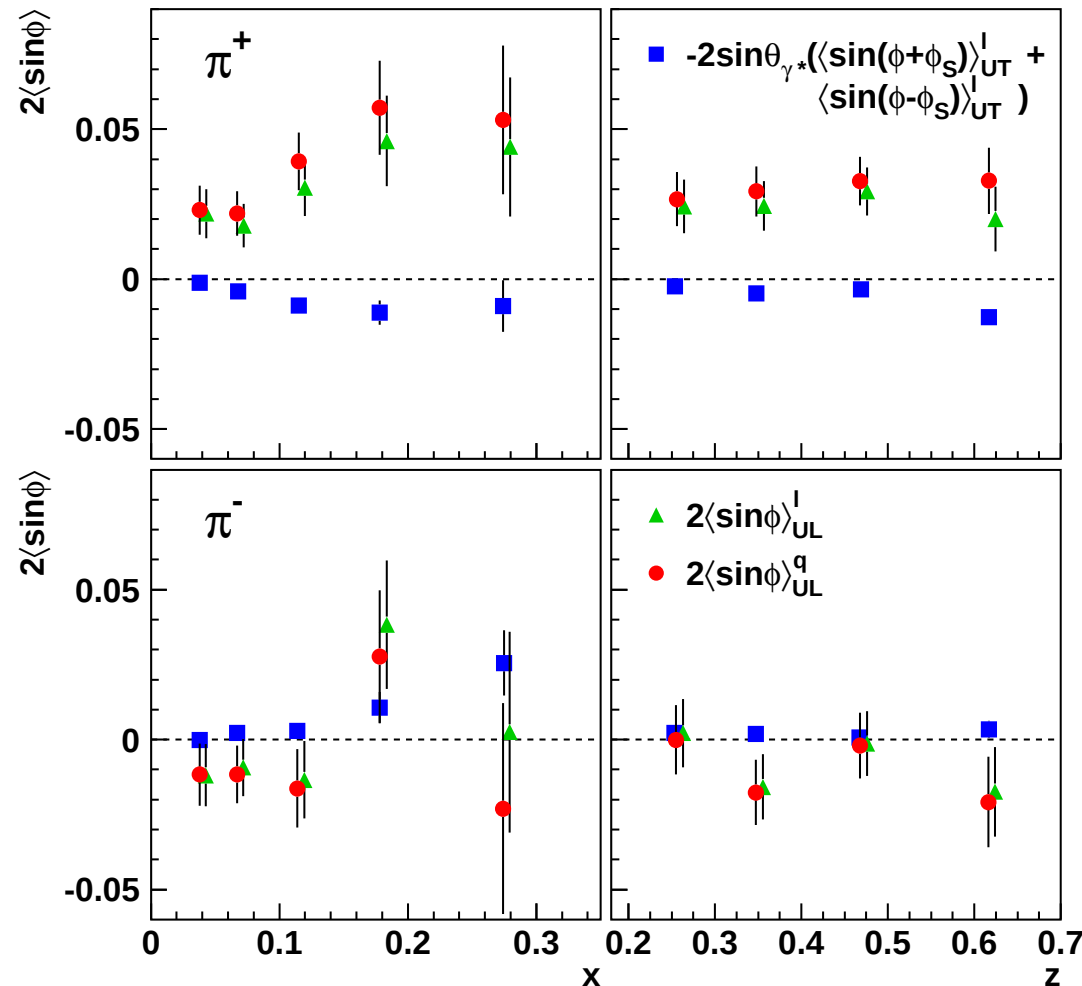
... Airapetian et al., Phys. Rev. Lett. 84 (2000) 4047

$$\langle \sin(\phi \pm \phi_S) \rangle_{UT}^l$$

... Airapetian et al., Phys. Rev. Lett. 94 (2005) 012002

What About Longitudinally Polarized Targets?

$$\langle \sin \phi \rangle_{UL}^q = \langle \sin \phi \rangle_{UL}^l + \sin \theta_{\gamma^*} \left(\langle \sin(\phi + \phi_S) \rangle_{UT}^l + \langle \sin(\phi - \phi_S) \rangle_{UT}^l \right)$$



● twist-3 dominates measured asymmetries on longitudinally polarized targets!

● significantly positive for π^+

● consistent with zero for π^-

Airapetian et al., Phys. Lett. B 622 (2005) 14

The Other Longitudinal SSA

longitudinally pol. beam & unpol. target $\Rightarrow$ subleading-twist

$$\langle \sin \phi \rangle_{LU} \propto \lambda_e \frac{M}{Q} \mathcal{I} \left[x e(x) H_1^\perp(z) - \frac{M_h}{zM} h_1^\perp(x) E(z) \right]$$

$\Rightarrow$ for long time candidate to access $e(x)$
($h_1^\perp(x)$ contribution either assumed to be zero (T-odd!) or small(?))

longitudinally pol. beam & unpol. target $\Rightarrow$ subleading-twist

$$\langle \sin \phi \rangle_{LU} \propto \lambda_e \frac{M}{Q} \mathcal{I} \left[x e(x) H_1^\perp(z) - \frac{M_h}{zM} h_1^\perp(x) E(z) \right. \\ \left. + \frac{M_h}{zM} f_1(x) G^\perp(z) - x g^\perp(x) D_1(z) \right]$$

quark-mass suppressed $\Rightarrow$
$$+ \frac{m_q}{M} h_1^\perp(x) D_1(z) - \frac{m_q}{M} f_1(x) H_1^\perp(z) \Big]$$

Bacchetta et al., Phys. Lett. B 595 (2004) 309

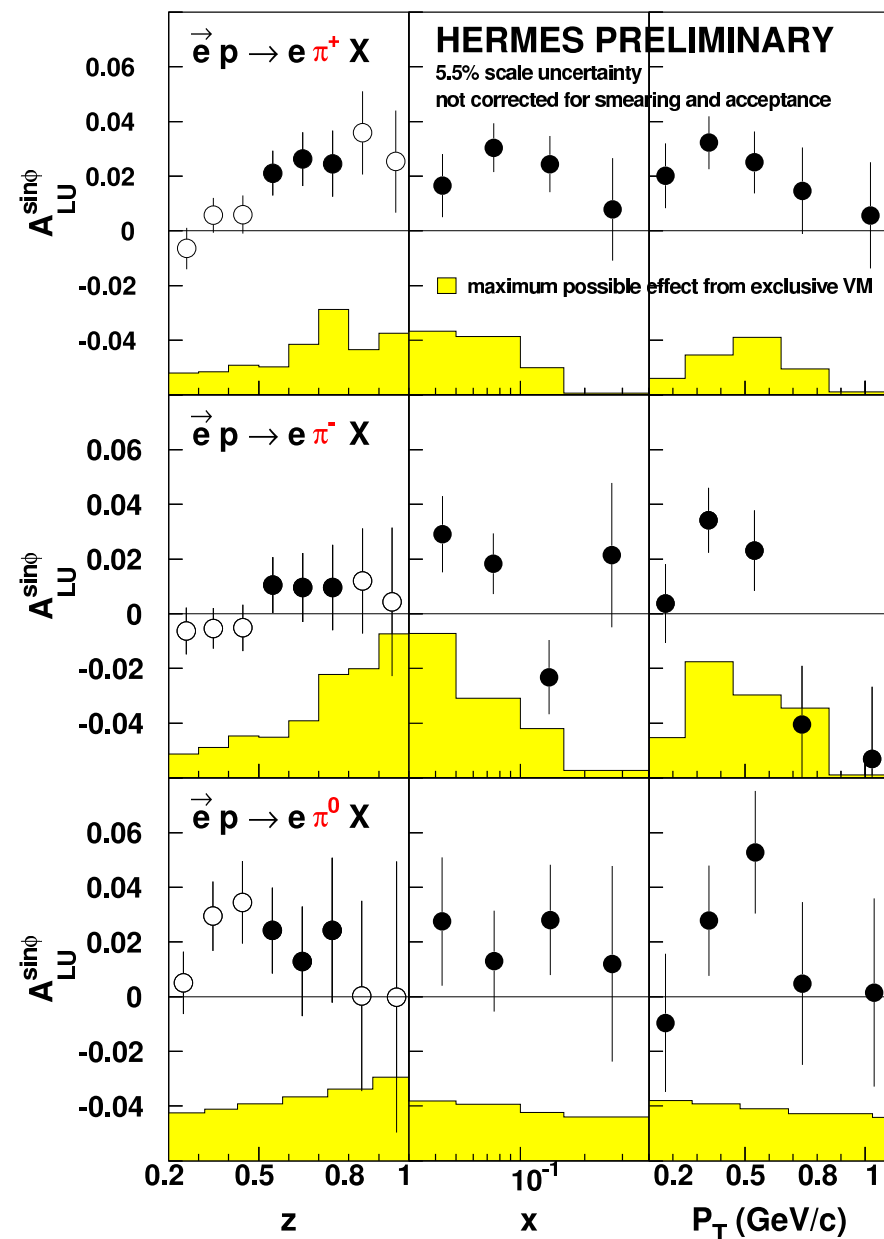
The Other Longitudinal SSA

longitudinally pol. beam & unpol. target $\Rightarrow$ subleading-twist

$$\langle \sin \phi \rangle_{LU} \propto \lambda_e \frac{M}{Q} \mathcal{I} \left[x e(x) H_1^\perp(z) - \frac{M_h}{zM} h_1^\perp(x) E(z) \right. \\ \left. + \frac{M_h}{zM} f_1(x) G^\perp(z) - x g^\perp(x) D_1(z) \right]$$

- many terms contributing – difficult to separate
- maybe some terms small?

Bacchetta et al., Phys. Lett. B 595 (2004) 309



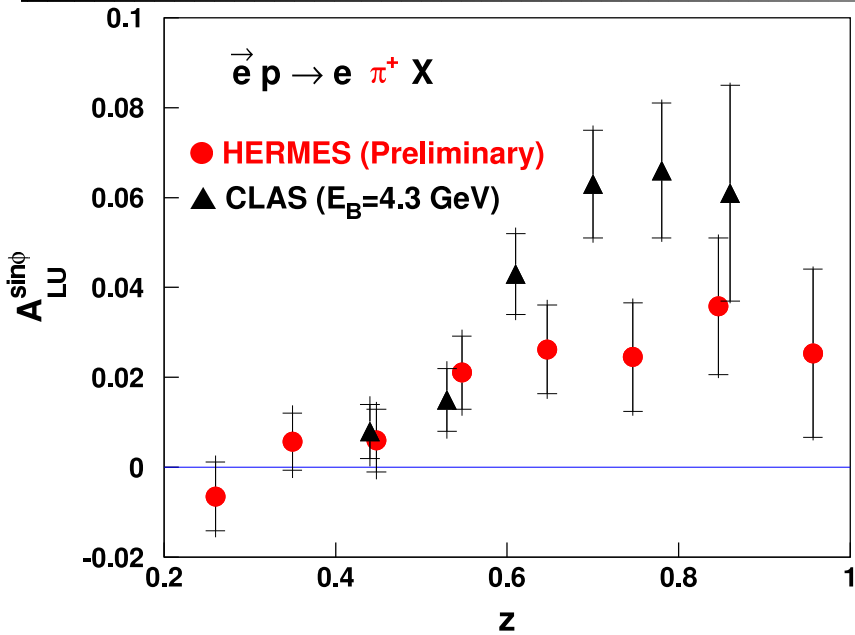
Extraction:

$$2 \langle \sin \phi \rangle_{LU} = \frac{\sum^+ \frac{\sin \phi_i}{|P_e^+|} - \sum^- \frac{\sin \phi_i}{|P_e^-|}}{\frac{1}{2}(N^+ + N^-)}$$

Vector Meson Contribution:

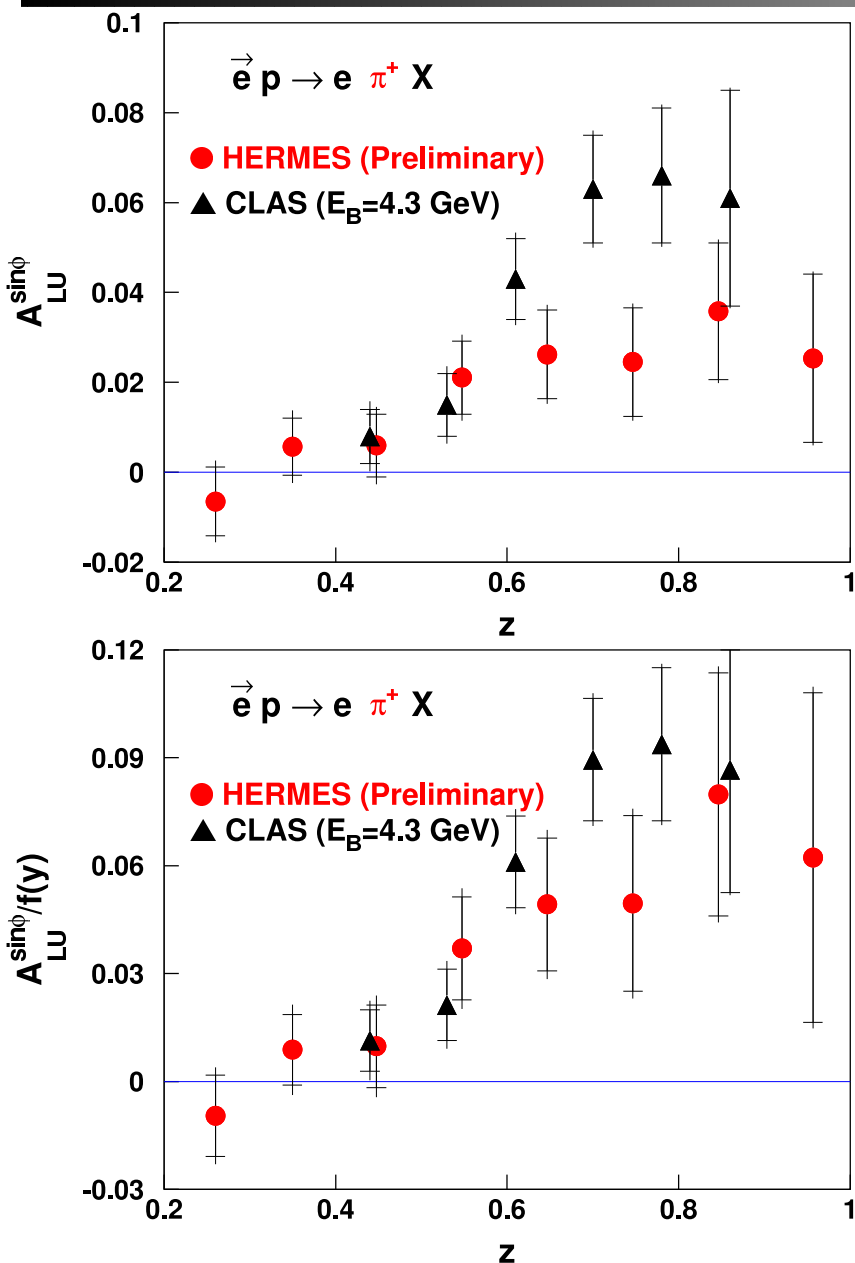
Max. possible contribution to systematic uncertainty estimated using PYTHIA MC (tuned for HERMES)

Comparisons with CLAS Results



● not so good agreement at high z

Comparisons with CLAS Results



- not so good agreement at high z
- have to correct for different y range at CLAS and HERMES:

$$\langle \sin \phi \rangle_{LU} \propto f(y) \equiv \frac{2y\sqrt{(1-y)}}{1-y+y^2/2}$$

strong suppression at HERMES for high z compared to CLAS

$\Rightarrow$ rescaling of asymmetries leads to good agreement

$$\langle \sin \phi \rangle_{LU} \propto \lambda_e \frac{M}{Q} \mathcal{I} \left[x e(x) H_1^\perp(z) - \frac{M_h}{zM} h_1^\perp(x) E(z) \right. \\ \left. - x g^\perp(x) D_1(z) + \frac{M_h}{zM} f_1(x) G^\perp(z) \right]$$

any help from other observables to separate contributions?

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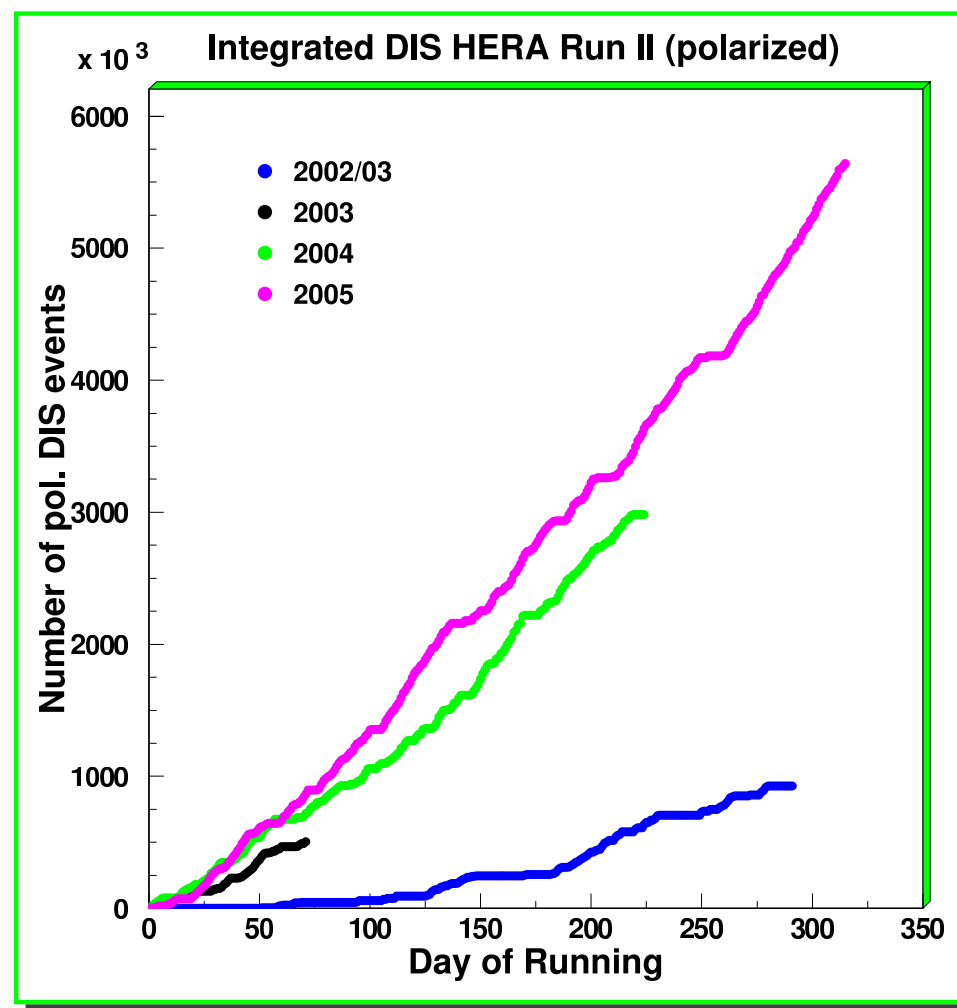
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- *jet SIDIS* $\Rightarrow$ only $g^\perp$ -term survives
- 2-hadron production:

$$\sigma_{LU} \propto \sin \phi_{R\perp} \left[x e(x) H_1^\triangleleft(z, \zeta, M_h^2) + \frac{1}{z} f_1(x) \tilde{G}^\triangleleft(z, \zeta, M_h^2) \right]$$

- First **evidence** for non-zero **Interference FF**
- **Non-vanishing Collins effect** observed for $\pi^\pm$
- Most likely scenario: $H_1^{\perp,disf} \approx -H_1^{\perp,fav}$
- First **evidence of T-odd Sivers distribution** in DIS
- Significant **positive Sivers asymmetries** for positive pions and kaons $\stackrel{?}{\Rightarrow} L_z^u > 0$
- $\sin \phi$ amplitudes on long. polar. target dominated by twist-3
- Observation of significant non-zero beam-spin asymmetries

- More data taking in 2005
⇒ **doubled statistics**



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- **Flavour decomposition** of Sivers function

$$\begin{aligned}
 A_{UT}^{\sin(\phi-\phi_S),h}(x) &= \mathcal{C} \cdot \frac{\sum_q e_q^2 f_{1T}^{\perp(1),q}(x) \int dz D_1^{q,h}(z) \mathcal{A}(x,z)}{\sum_{q'} e_{q'}^2 f_1^{q'}(x) \int dz D_1^{q',h}(z) \mathcal{A}(x,z)} \\
 &= \mathcal{C} \cdot \sum_q \frac{e_q^2 f_1^q(x) \mathcal{D}_1^{q,h}(x)}{\sum_{q'} e_{q'}^2 f_1^{q'}(x) \mathcal{D}_1^{q',h}(x)} \cdot \frac{f_{1T}^{\perp(1),q}(x)}{f_1^q(x)} \\
 &= \mathcal{C} \cdot \sum_q \mathcal{P}_q^h(x) \cdot \frac{f_{1T}^{\perp(1),q}(x)}{f_1^q(x)}
 \end{aligned}$$

- **purities** are completely **unpolarized** objects → present Monte Carlo-tunes can be used
- **probabilistic interpretation** of purities possible
- “easy”: Sivers ← fragmentation function (D_1) known

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- Collins: these purities still **depend on parametrization** of Collins FF function

Backup Slides

A Closer Look at Collins Asymmetries I

rewrite asymmetries in terms of favored and disfavored fragmentation:

- neglect strange quarks
- assume Gaussian k_T dependence of Collins FF $\rightarrow$ can resolve convolution
- employ isospin symmetry among fragmentation functions, i.e.

$$D_f \equiv D(u \rightarrow \pi^+) \simeq D(d \rightarrow \pi^-) \simeq D(\bar{d} \rightarrow \pi^+) \simeq D(\bar{u} \rightarrow \pi^-)$$

$$D_d \equiv D(d \rightarrow \pi^+) \simeq D(u \rightarrow \pi^-) \simeq D(\bar{u} \rightarrow \pi^+) \simeq D(\bar{d} \rightarrow \pi^-)$$

$$\frac{1}{2}(D_f + D_d) \simeq D(u \rightarrow \pi^0) \simeq D(d \rightarrow \pi^0) \simeq D(\bar{d} \rightarrow \pi^0) \simeq D(\bar{u} \rightarrow \pi^0)$$

$$\hookrightarrow \tilde{A}_C^{\pi^+/\pi^-}(x, z) \propto \frac{(4\delta u + \delta \bar{d})H_{f/d} + (4\delta \bar{u} + \delta d)H_{d/f}}{(4u + \bar{d})D_{f/d} + (4\bar{u} + d)D_{d/f}}$$

$$\tilde{A}_C^{\pi^0}(x, z) \propto \frac{[4(\delta u + \delta \bar{u}) + \delta d + \delta \bar{d}](H_f + H_d)}{[4(u + \bar{u}) + d + \bar{d}](D_f + D_d)}$$

A Closer Look at Collins Asymmetries II

express asymmetries in terms of flavor ratios:

$$\begin{aligned}\tilde{A}_C^{\pi^+} &= \mathcal{K}(x, z) \frac{4 + \delta r \mathcal{H}}{4 + r \mathcal{D}} \\ \tilde{A}_C^{\pi^-} &= \mathcal{K}(x, z) \frac{4 \mathcal{H} + \delta r}{4 \mathcal{D} + r} \\ \tilde{A}_C^{\pi^0} &= \mathcal{K}(x, z) \frac{(4 + \delta r)(1 + \mathcal{H})}{(4 + r)(1 + \mathcal{D})}\end{aligned}$$

Polarized Objects

$$\begin{aligned}\mathcal{H} &= \frac{H_d}{H_f} \\ \delta r &= \frac{\delta d + 4 \delta \bar{u}}{\delta u + \frac{1}{4} \delta \bar{d}}\end{aligned}$$

Unpolarized Objects

$$\begin{aligned}\mathcal{D} &= \frac{D_d}{D_f} \\ r &= \frac{d + 4 \bar{u}}{u + \frac{1}{4} \bar{d}}\end{aligned}$$

Mixed

$$\mathcal{K} = \frac{(\delta u + \frac{1}{4} \delta \bar{d}) z H_f}{(u + \frac{1}{4} \bar{d}) D_f}$$

e.g., CTEQ6, R1990 and Kretzer et al.

$\Rightarrow$ 3 constraints and 3 unknowns!

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Polarization: The three asymmetries are not independent ($C(x, z) \equiv \frac{r(x) + 4\mathcal{D}(z)}{r(x)\mathcal{D}(z) + 4}$):

$\mathcal{H}$

δr

$$\tilde{A}_C^{\pi^+}(x, z) + C(x, z)\tilde{A}_C^{\pi^-}(x, z) - (1 + C(x, z))\tilde{A}_C^{\pi^0}(x, z) = 0$$

e.g., CTEQ6,R1990 and Kretzer et al.

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2

A Closer Look at Collins Asymmetries III

eliminate $\mathcal{K}$ and relate $\mathcal{H}$ to δr

$\Rightarrow$ scan solution space for $\mathcal{H}$ and δr by sampling set of $(\tilde{A}_C^{\pi^+}, \tilde{A}_C^{\pi^-}, \tilde{A}_C^{\pi^0})$

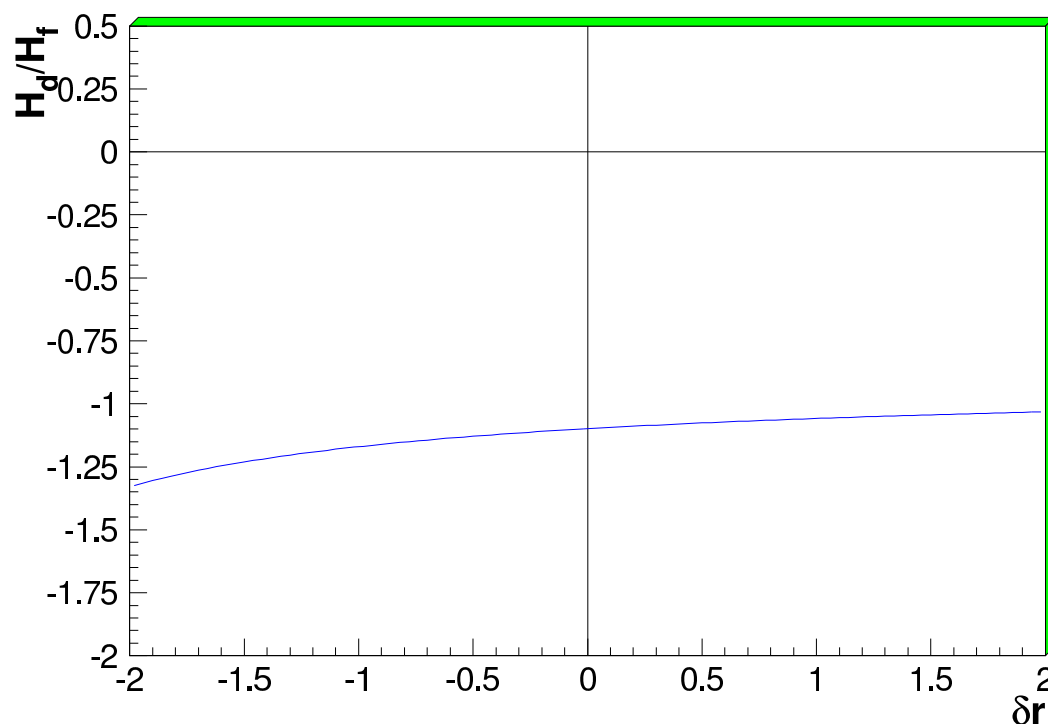
(around measured values according to statistical uncertainty)

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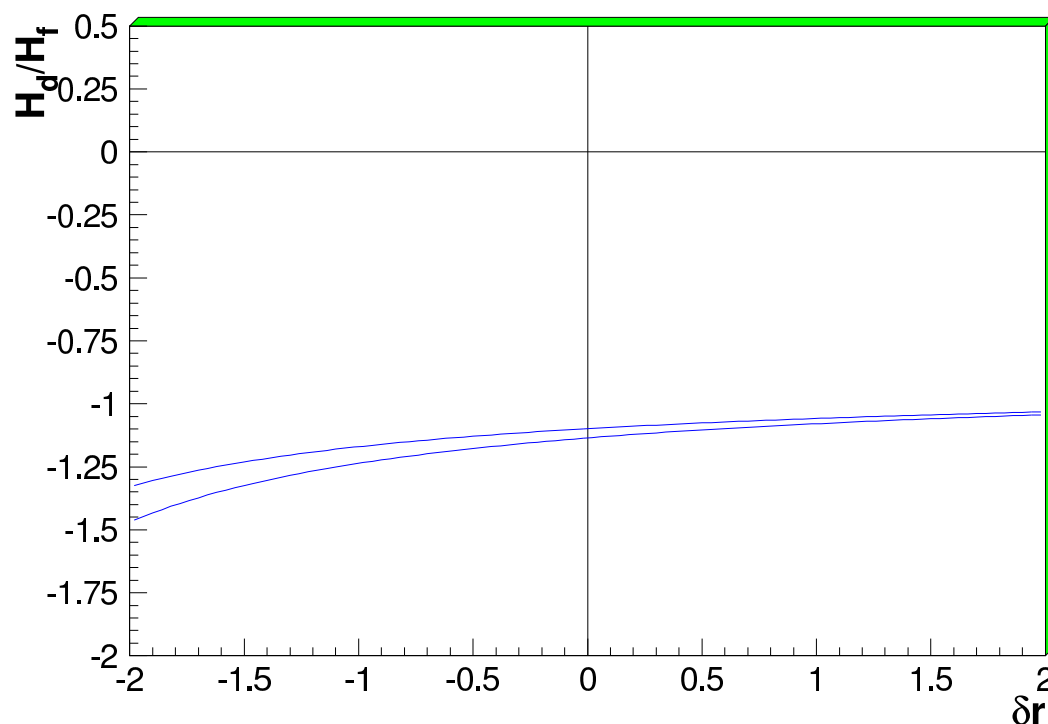


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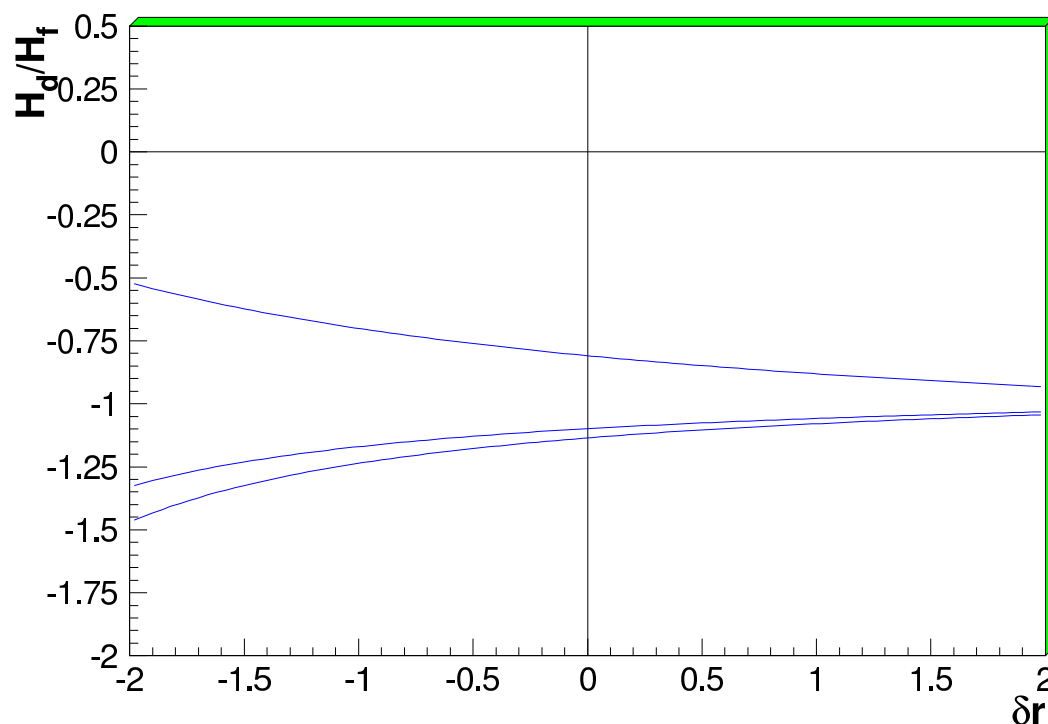


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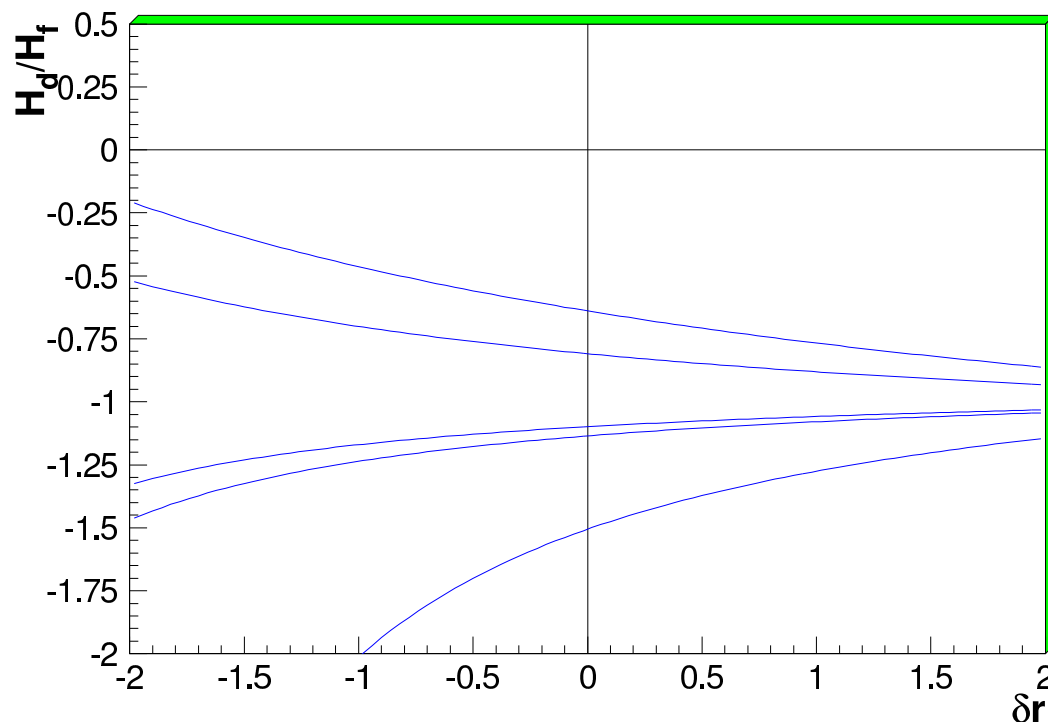


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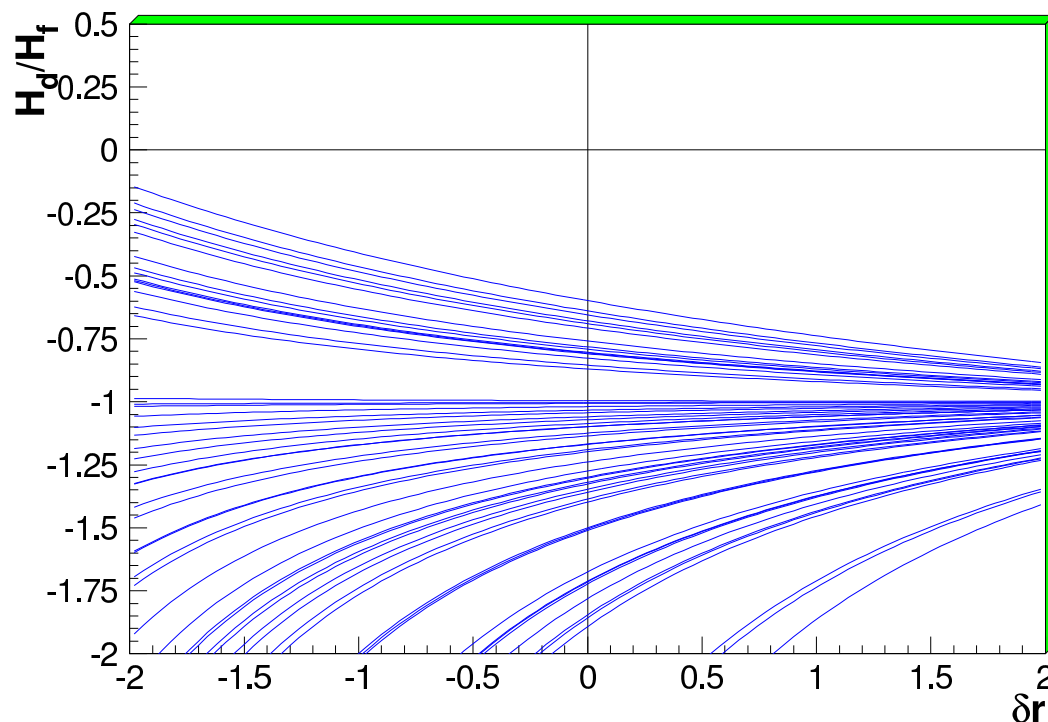


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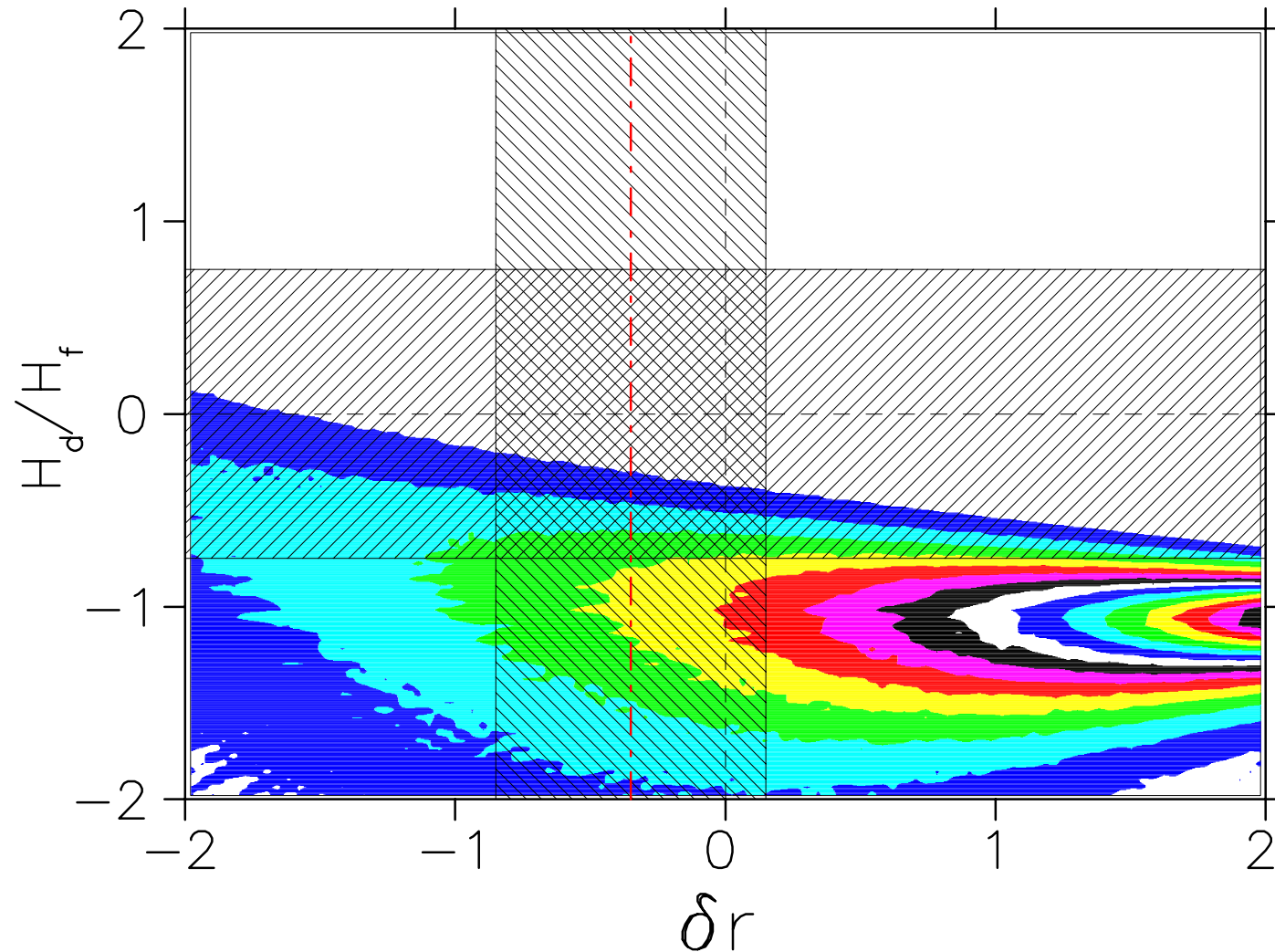
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Limits on Transversity and Collins FF

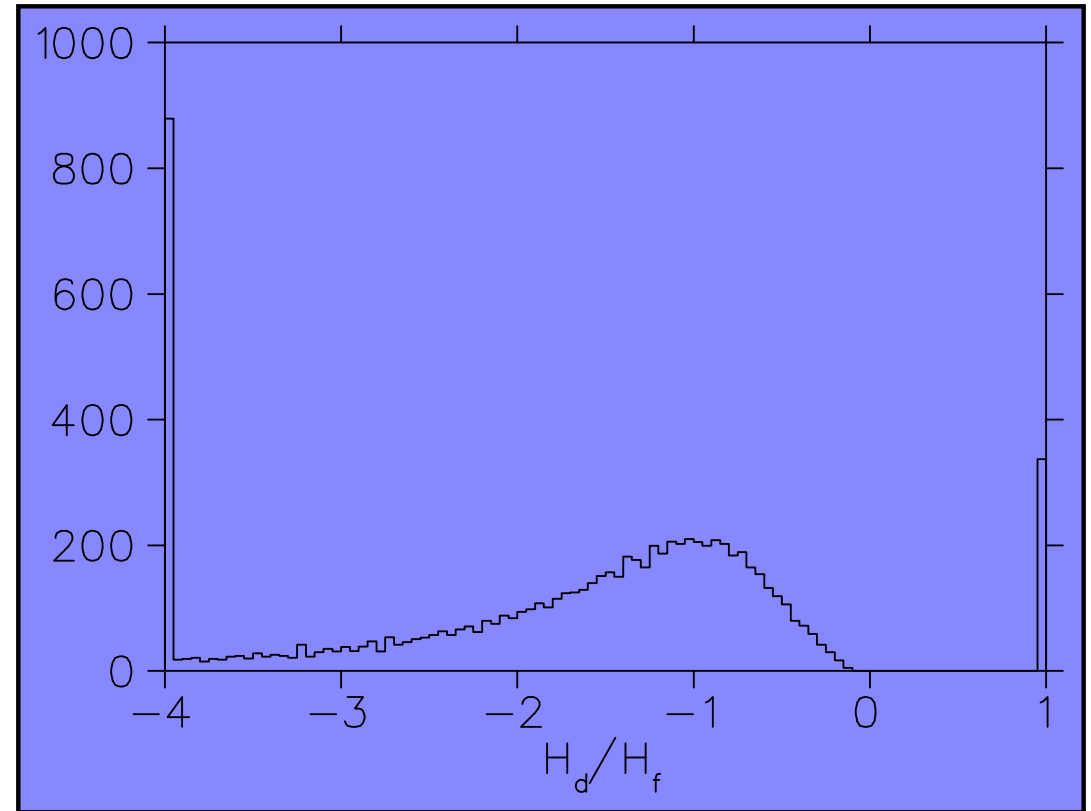
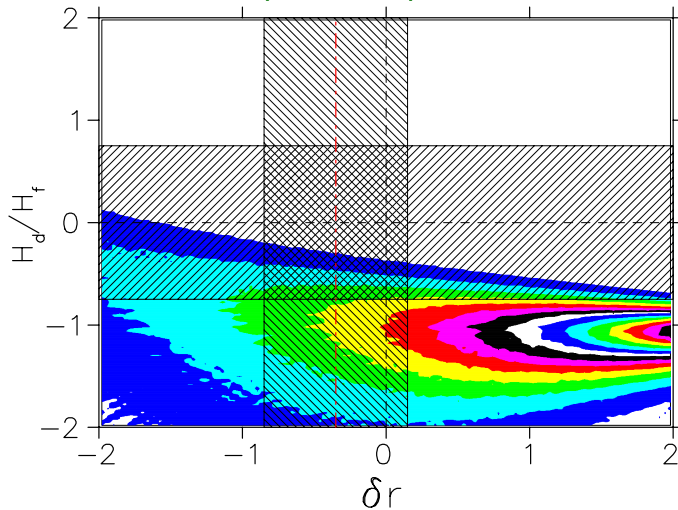
probability distribution for H_d/H_f vs. δr :



Limits on Transversity and Collins FF

$\delta r \approx \delta d / \delta u$ from χ QSM

look at slice of distribution in δr :



strong hint for H_d/H_f negative