Round Table: TMD distributions and Orbital Angular Momentum

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some points

- TMD's: $f_{1T}^{\perp}$ (Sivers) and $h_1^{\perp}$ (Boer-Mulders)
  and $h_{1T}^{\perp}$, $h_{1L}^{\perp}$, $g_{1T}^{\perp}$.
- clear access:
  $f_{1T}^{\perp}$: q from SIDIS, DY [A_N];
  gluon Sivers pdf from $p^{\uparrow}p \to jet\, jet\, X$, $p^{\uparrow}p \to CX$ (to be discussed)
- What do we learn from them? What do we learn from them on OAM?
- $\boldsymbol{k}_{\perp}$ from TMD's and connection to OAM
- Longitudinal/Transverse OAM [Ji, Jaffe / Leader sum rules] in
  a longitudinally/ transversely polarized hadron

A short reminder (partonic interpretation)

In a transversely polarized hadron,

$$\boldsymbol{P}^A = \cos\phi_{S_A}\hat{\boldsymbol{X}} + \sin\phi_{S_A}\hat{\boldsymbol{Y}}$$

$$\hat{f}_{a/A,S_A}(x_a, k_{\perp a}) = \hat{f}_{a/A}(x_a, k_{\perp a}) + \frac{1}{2}\,\Delta^N \hat{f}_{a/A^\uparrow}\,(x_a, k_{\perp a})\,(\hat{\boldsymbol{p}}_A \times \hat{\boldsymbol{k}}_{\perp a}) \cdot \hat{\boldsymbol{P}}^A$$

$$\hat{f}_{a/S_A}\,(x_a, \boldsymbol{k}_{\perp a}) - \hat{f}_{a/-S_A}\,(x_a, \boldsymbol{k}_{\perp a}) = \Delta^N \hat{f}_{a/A^\uparrow}\,(x_a, k_{\perp a})\,\sin(\phi_{S_A} - \phi_a)$$

In an unpolarized hadron:

$$\hat{f}_{s_a/A}(x_a, k_{\perp a}) = \frac{1}{2}\hat{f}_{a/A}(x_a, k_{\perp a}) + \frac{1}{2}\,\Delta^N \hat{f}_{a^\uparrow/A}\,(x_a, k_{\perp a})\,(\hat{\boldsymbol{p}}_A \times \hat{\boldsymbol{k}}_{\perp a}) \cdot \hat{\boldsymbol{P}}^a$$

$$\hat{f}^a_{s/A}(x_a, \boldsymbol{k}_{\perp a}) - \hat{f}^a_{-s/A}(x_a, \boldsymbol{k}_{\perp a}) = \Delta^N \hat{f}_{a^\uparrow/A}\,(x_a, k_{\perp a})\,\sin\phi_{s_a}$$

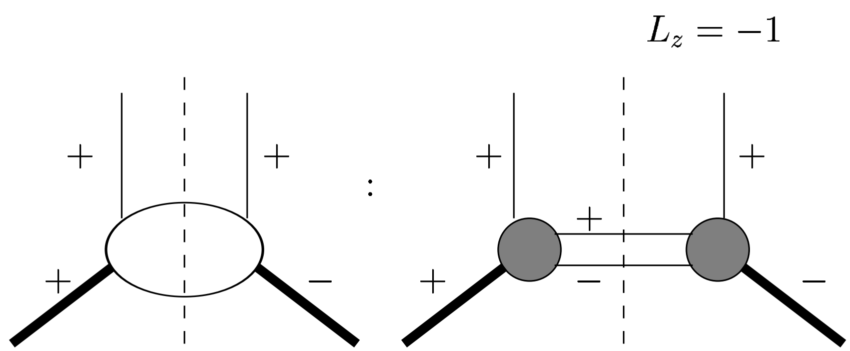
Notice: $(\hat{\boldsymbol{p}}_A \times \hat{\boldsymbol{k}}_{\perp a}) = \hat{\boldsymbol{y}}$

Being $|\uparrow/\downarrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle \pm i|-\rangle)$ we have $\quad [F^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}\ ]$

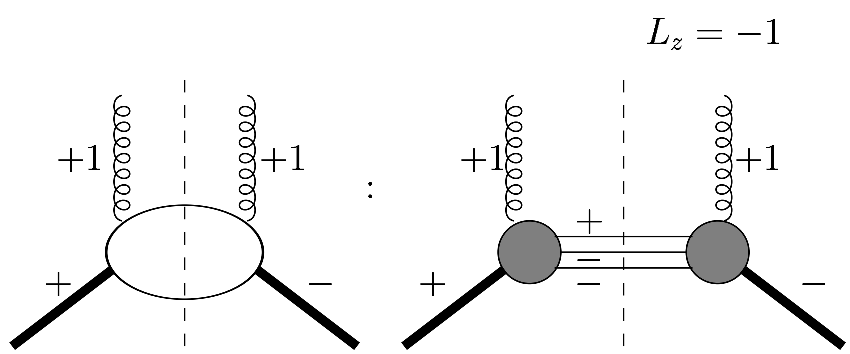
$$\Delta^N \hat{f}_{a/A^\uparrow} = \frac{-2k_{\perp a}}{M}\, f_{1T}^{\perp a} = 4\,\mathrm{Im}F^{++}_{+-} \qquad \text{(Sivers)}$$

$$\Delta^N \hat{f}_{q^\uparrow/A} = -\frac{k_{\perp q}}{M}\, h_1^{\perp q} = -2\,\mathrm{Im}F^{+-}_{++} \qquad \text{(Boer − Mulders)}$$

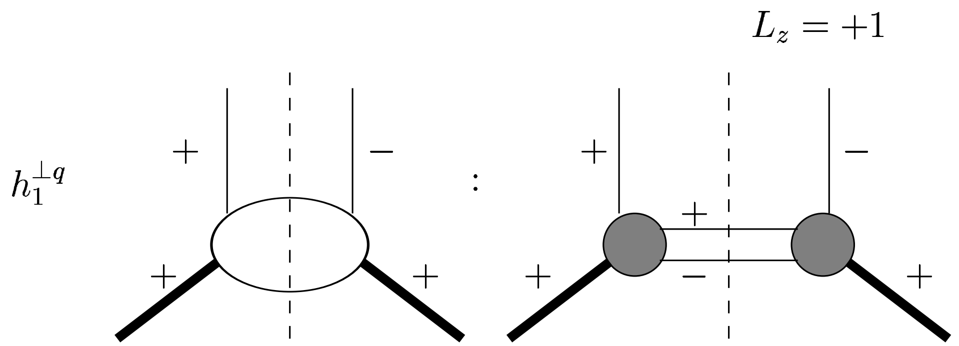
$f_{1T}^{\perp q}$:



$f_{1T}^{\perp g}$:



Rescattering: Brodsky et al. '02 and then
gauge link: Ji, Belitzky, Yuan, Collins, Mulders...



Notice: Immaginary part $\rightarrow h_1^{\perp}$; Real part $\rightarrow h_{1L}^{\perp}$.

As for the Sivers diagram: Immaginary part $\rightarrow f_{1T}^{\perp}$, Real part $\rightarrow g_{1T}^{\perp}$.

Constraint on $f_{1T}^{\perp a}$: Burkardt sum rule [04] (rephrased)

$$\sum_a \int dx\, d^2\boldsymbol{k}_\perp\, \boldsymbol{k}_\perp\, \hat{f}_{a/S_A}(x, \boldsymbol{k}_\perp) = \sum_a \boldsymbol{k}_\perp^a = 0$$

$$\boldsymbol{k}_\perp^a = (\sin\phi_{S_A} \hat{\boldsymbol{X}} - \cos\phi_{S_A} \hat{\boldsymbol{Y}})\, \pi/2 \int dx\, dk_\perp\, k_\perp^2 \Delta^N \hat{f}_{a/A^\uparrow}(x, k_\perp)$$

Notice: $\boldsymbol{k}_\perp^a \perp \boldsymbol{S}_A$.

a) checked by model calcul. (Goeke et al. '06)

b) from fit on SIDIS (Anselmino et al. [05]) one gets:

$k_\perp^u \simeq 0.14$ GeV/c $\quad k_\perp^d \simeq -0.13$ GeV/c (depend. on low x extrapolation)

c) from fit on $pp \to \pi X$ (UD, Murgia [04]) one gets:

$k_\perp^u \simeq 0.032$ GeV/c $\quad k_\perp^d \simeq -0.036$ GeV/c (Valence like behaviour (safer));

$\boldsymbol{k}_\perp^u + \boldsymbol{k}_\perp^d \approx 0$?

$\Rightarrow \boldsymbol{k}_\perp^g \approx 0$?

$\Rightarrow \boldsymbol{k}_\perp^g + \boldsymbol{k}_\perp^{sea} \approx 0$?

No analog sum rule for $h_1^\perp$:

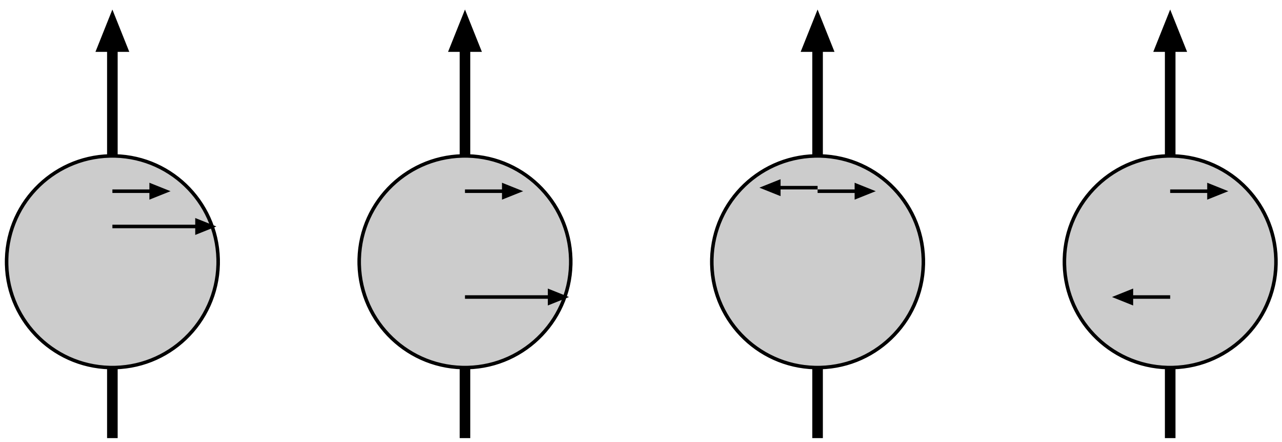
a) transverse polarization depends on $\boldsymbol{k}_\perp$ direction;

b) gluons with their linear polarization.

$\boldsymbol{k}_\perp$ from $f_{1T}^\perp$ vs. orbital angular momentum

$$\boldsymbol{S}_A = \hat{\boldsymbol{Y}} \to \hat{\boldsymbol{k}}_\perp^a = \hat{\boldsymbol{X}}$$

2 quarks same/different flavour $\to$ same/opposite sign:



$\boldsymbol{k}_\perp^1 + \boldsymbol{k}_\perp^2 \neq 0$	$\boldsymbol{k}_\perp^1 + \boldsymbol{k}_\perp^2 \neq 0$	$\boldsymbol{k}_\perp^1 + \boldsymbol{k}_\perp^2 = 0$	$\boldsymbol{k}_\perp^1 + \boldsymbol{k}_\perp^2 = 0$
$L_z^1 + L_z^2 \neq 0$	$L_z^1 + L_z^2 = 0$	$L_z^1 + L_z^2 = 0$	$L_z^1 + L_z^2 \neq 0$

- $\boldsymbol{k}_\perp^1 + \boldsymbol{k}_\perp^2 \neq 0 \to L_z^1 + L_z^2 = 0, \neq 0?$
dependence on space distribution...
fixed flavour; total contribution

- help from GPD's $f_{q/p^\uparrow}(x, \boldsymbol{b})$?

- change of sign of $f_{1T}^{\perp a}$ from SIDIS to DY:
$\to$ change of sign of $\boldsymbol{k}_\perp^a$.
contributions to L_z^a from SIDIS vs. DY. ?

- role of other TMD's:
i.e. $g_{1T}^\perp$ can give net $\boldsymbol{k}_\perp \neq 0$
$\to$ contribution to L_z + its helicity contribution:
new constraints on TMD's?

- role of gluons (need of processes involving gluons at LO): $\boldsymbol{k}_\perp$, OAM

- TMD's vs. Bakker, Leader, Trueman OAM ($L_\perp$)?

- ...