

CHIRAL-ODD GENERALIZED PARTON DISTRIBUTIONS AND TRANSVERSTY IN LIGHT FRONT CONSTITUENT QUARK MODELS

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SECOND WORKSHOP ON THE QCD STRUCTURE OF THE NUCLEON
(Monte Porzio Catone, ROME, 12-16 June)



- 1 Motivation
- 2 Definition of GPDs
 - DVCS Kinematics and GPDs
- 3 Chiral-Odd GPDs
 - GPDs $\leftrightarrow$ Helicity Amplitudes
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- 4 Overlap representation
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- Very few model calculations,
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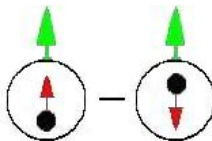
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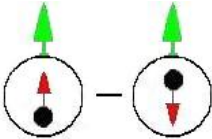
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- Test of relativistic effects, i.e. $h_1(x) \neq g_1(x) \Rightarrow$

Possibility of measure the relativistic nature of quarks inside the nucleon.

GPDs as generalization of Parton Distributions

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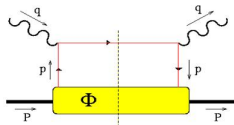
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$Q^2 \rightarrow \infty$, x_B fixed

hard process



Parton Distributions



Compton Scattering Handbag
PDDs

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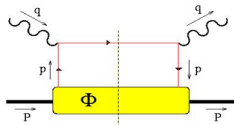
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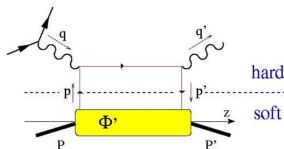
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DVCS Handbag
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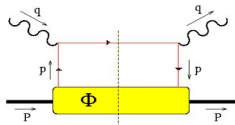
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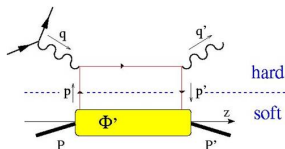
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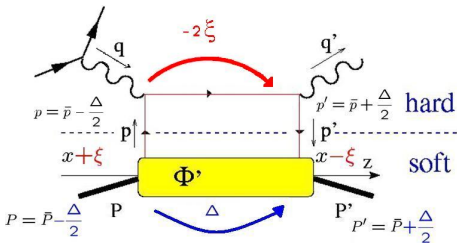
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⇒ **GPD** $\doteq$ **Nondiagonal** hadronic matrix elements of bilocal products of the light-front quark field operators. Thus, they **don't** represent probability density (like PDF), but interference between amplitudes describing different quantum fluctuation of a nucleon.

Kinematics and Correlation Functions



"Average Momentum" FRAME

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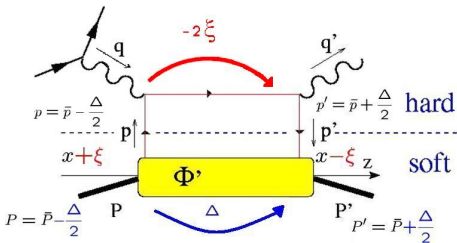
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Depending on which Γ structure we substitute $\Rightarrow$ 8 different GPDs:

Diehl, EPJ C 19 '01

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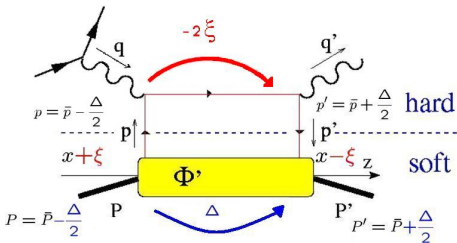
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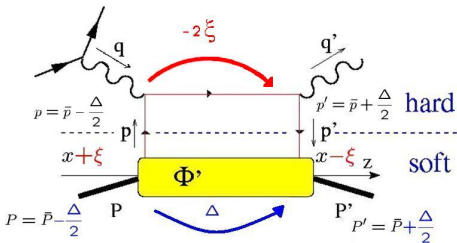
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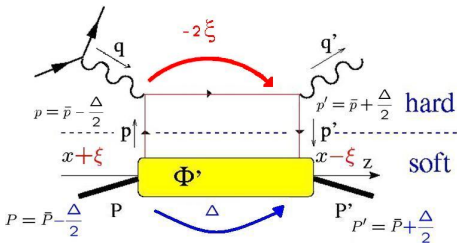
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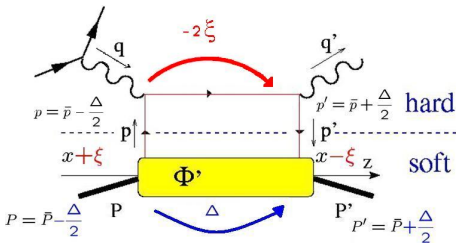
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GPDs with helicity flip (Chiral-odd)

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GPDs can be related to the following helicity amplitudes

M. Diehl, EPJ C 19, ('01)

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- $\mathbf{A}_{++,-} = (1-\xi^2)^{1/2} [\mathcal{H}_T^q + \frac{t_0-t}{4M^2} \tilde{\mathcal{H}}_T^q - \frac{\xi^2}{1-\xi^2} \mathcal{E}_T^q + \frac{\xi}{1-\xi^2} \tilde{\mathcal{E}}_T^q]$
- $\mathbf{A}_{-+,-} = -(1-\xi^2)^{1/2} \frac{(t_0-t)}{4M^2} \tilde{\mathcal{H}}_T^q$

Thus inverting:

- $\mathcal{E}_T^q = \frac{2M}{\epsilon \sqrt{t_0-t}} \left(\frac{1}{(1-\xi)} \mathbf{A}_{++,-} + \frac{1}{(1+\xi)} \mathbf{A}_{-+,-} \right) + \frac{8M^2}{(t_0-t)(1-\xi^2)\sqrt{1-\xi^2}} \mathbf{A}_{-+,-}$
- $\tilde{\mathcal{E}}_T^q = \frac{2M}{\epsilon \sqrt{t_0-t}} \left(\frac{1}{(1-\xi)} \mathbf{A}_{++,-} - \frac{1}{(1+\xi)} \mathbf{A}_{-+,-} \right) + \frac{8M^2 \xi}{(t_0-t)(1-\xi^2)\sqrt{1-\xi^2}} \mathbf{A}_{-+,-}$
- $\mathcal{H}_T^q = \frac{1}{\sqrt{1-\xi^2}} (\mathbf{A}_{++,-} + \mathbf{A}_{-+,-}) + \frac{2M\xi}{\epsilon \sqrt{t_0-t}(1-\xi^2)} (\mathbf{A}_{-+,-} - \mathbf{A}_{++,-})$
- $\tilde{\mathcal{H}}_T^q = \frac{-4M^2}{\sqrt{1-\xi^2}(t_0-t)} (\mathbf{A}_{-+,-})$

(cont'ed)



- $\mathbf{A}_{++,-} = \epsilon \frac{(t_0-t)^{1/2}}{2M} [\tilde{\mathcal{H}}_T^q + (1-\xi) \frac{\mathcal{E}_T^q + \tilde{\mathcal{E}}_T^q}{2}]$
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- $\tilde{\mathcal{H}}_T^q = \frac{-4M^2}{\sqrt{1-\xi^2}(t_0-t)} (\mathbf{A}_{-+,-})$

Change of base

Helicity Basis

$$\phi_R = P_R \phi = \frac{1}{\sqrt{2}}(1 + \gamma_5) \phi$$

$$\phi_L = P_L \phi = \frac{1}{\sqrt{2}}(1 - \gamma_5) \phi$$

Transversity Basis

$$Q_{\pm} = \frac{1}{\sqrt{2}}(1 \pm \gamma^1 \gamma_5)$$

$$Q_+ \phi \equiv \phi_{\uparrow}$$

$$Q_- \phi \equiv \phi_{\downarrow}$$

Introducing the transversity basis also for the nucleon spin states, i.e.

$$|p, \uparrow\rangle = \frac{1}{\sqrt{2}}(|p, +\rangle + |p, -\rangle)$$

$$|p, \downarrow\rangle = \frac{1}{\sqrt{2}}(|p, +\rangle - |p, -\rangle)$$

and defining the following matrix elements

$$T_{\lambda'_t \lambda_t}^q = \langle p', \lambda'_t | \int \frac{dz^-}{2\pi} e^{i\bar{x}P^+z^-} \bar{\psi}(-z/2) \gamma^+ \gamma^1 \gamma_5 \psi(z/2) | p, \lambda_t \rangle$$

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where $\lambda_t(\lambda'_t)$ labels the transverse polarization of initial (final) nucleon in the $\uparrow(\downarrow)$ direction.

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(cont'ed)

we find:

$$\begin{aligned}
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 \end{aligned}
 \quad \text{“Parity invariance”}$$

and in terms of the matrix elements “A”

$$\begin{aligned}
 \star \quad T_{\uparrow\uparrow}^q &= A_{++,-} + A_{-+,-} & T_{\uparrow\downarrow}^q &= A_{++,-} - A_{-+,-} \\
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Finally, we obtain the **chiral-odd GPDs** from **the transverse matrix elements** through the relations

$$\begin{aligned}
 \bullet \quad \mathcal{H}_T^q &= \frac{1}{(1-\xi^2)^{1/2}} T_{\uparrow\uparrow}^q - \frac{2M\xi}{\epsilon\sqrt{t_0-t}(1-\xi^2)} T_{\uparrow\downarrow}^q \\
 \bullet \quad \mathcal{E}_T^q &= \frac{2M\xi}{\epsilon\sqrt{t_0-t}} \frac{1}{1-\xi^2} T_{\uparrow\downarrow}^q + \frac{2M}{\epsilon\sqrt{t_0-t}(1-\xi^2)} \tilde{T}_{\uparrow\uparrow}^q - \frac{4M^2}{(t_0-t)(1-\xi^2)^{3/2}} (\tilde{T}_{\uparrow\downarrow}^q - T_{\uparrow\uparrow}^q) \\
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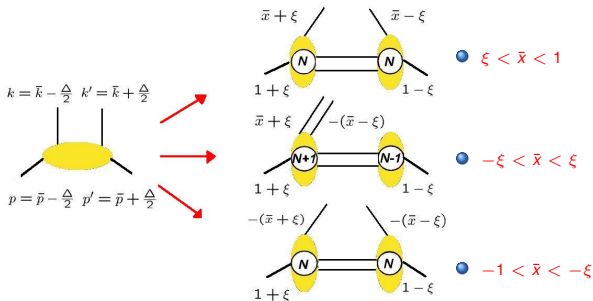
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OVERLAP REPRESENTATION

M. Diehl, Th. Feldmann, R. Jakob and P. Kroll,
Eur. Phys. J. **C 8**, '99; Nucl. Phys. **B 596**, '01.



We restrict our calculation to the region $\xi < \bar{x} < 1$ of plus-momentum fraction, where the GPDs describe the emission of a quark with plus-momentum $(\bar{x} + \xi)\bar{P}^+$ and its reabsorption with plus-momentum $(\bar{x} - \xi)\bar{P}^+$.

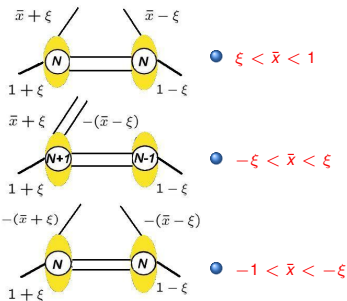
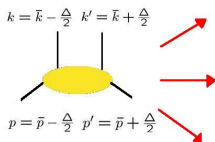
At the light-cone time $z^+ = 0$, Fourier expansion of the free quark field (Fock Decomp.):

$$\phi_{\uparrow}(z^-, \vec{z}_{\perp}) = \int \frac{dk^+ d^2 \vec{k}_{\perp}}{k^+ 16\pi^3} \Theta(k^+) \left\{ b_{\uparrow}(k^+, \vec{k}_{\perp}) u_+(k, \uparrow) \exp(-i k^+ z^- + i \vec{k}_{\perp} \cdot \vec{z}_{\perp}) + d_{\uparrow}^{\dagger}(k^+, \vec{k}_{\perp}) v_+(k, \uparrow) \exp(+i k^+ z^- - i \vec{k}_{\perp} \cdot \vec{z}_{\perp}) \right\}$$

with $b_{\uparrow}(d_{\uparrow}^{\dagger})$ annihilation (creation) operator for an on-shell (anti)quark with transverse pol. and $u_+(v_+)$ "good" projection of the (anti)quark spinor.

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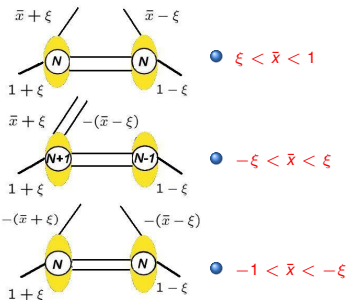
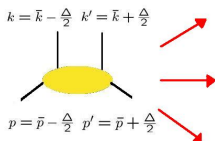
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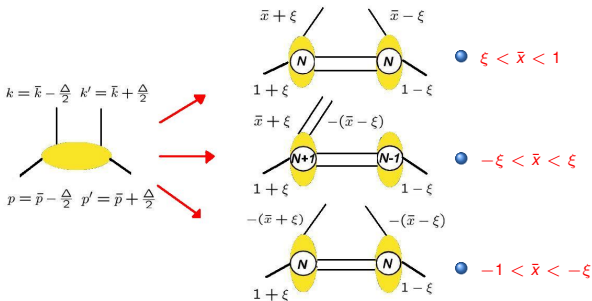
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with $b_{\uparrow}(d_{\uparrow}^{\dagger})$ annihilation (creation) operator for an on-shell (anti)quark with transverse pol. and $u_+(v_+)$ “good” projection of the (anti)quark spinor.

The representation of the nucleon state reads:

$$|p, \lambda_t\rangle = \sum_{N, \beta} \int [dx]_N [d^2 \vec{k}_\perp]_N \Psi_{\lambda_t, N, \beta}(r) |N, \beta; k_1, \dots, k_N\rangle,$$

- $\Psi_{\lambda_t, N, \beta}$, Light-Cone Wave Function (LCWF) of the N-parton Fock state;
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After some calculations we find the overlap representation for the matrix elements

$$\begin{aligned} T_{\lambda'_t \lambda_t}^q &= \sum_{N, \beta=\beta'} \left(\sqrt{1-\xi} \right)^{2-N} \left(\sqrt{1+\xi} \right)^{2-N} \sum_{j=1}^N \text{sign}(\mu_j^t) \delta_{s_j q} \\ &\times \int [d\bar{x}]_N [d^2 \vec{k}_\perp]_N \delta(\bar{x} - \bar{x}_j) \Psi_{\lambda'_t, N, \beta'}^*(\hat{r}') \Psi_{\lambda_t, N, \beta}(\tilde{r}) \end{aligned}$$

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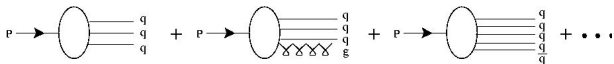
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Light-Cone Wave Function in Constituent Quark Models

$$|\Psi\rangle = \psi_{3q}|qqq\rangle + \psi_{3q,g}|qqq, g\rangle + \psi_{3q,q\bar{q}}|3q, q\bar{q}\rangle + \dots$$



CQM are quantomechanical models with a fixed number of constituents and relativity-consistent, based on **two** fundamental hypotheses:

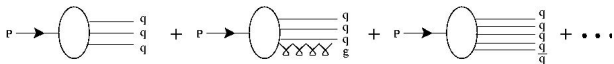
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In such models “relativity” can be incorporated in a straightforward way and the wave functions can be calculated solving the Hamiltonian eigenvalues equation in **different forms** of relativistic dynamic linked by an **unitary transformation**.

S. Boffi, B. Pasquini and M. Traini, Nucl. Phys. **B 649**,('03).

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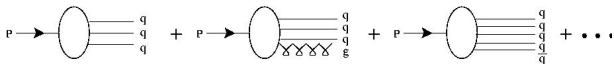
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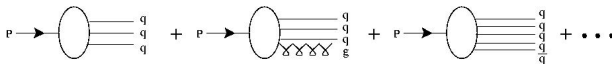
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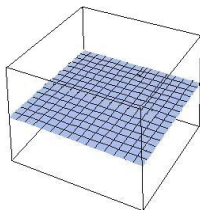
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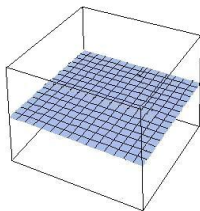
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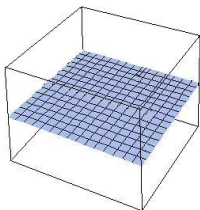
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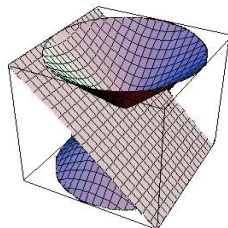


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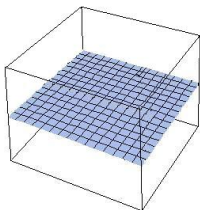
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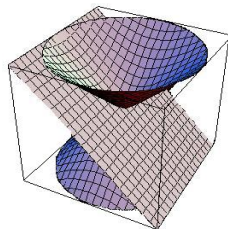


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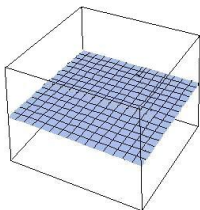
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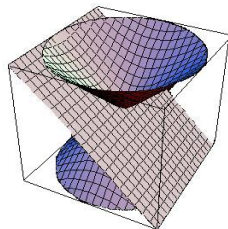


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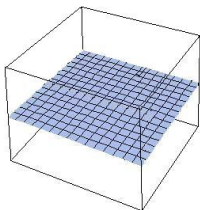
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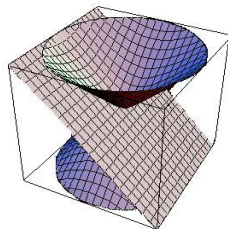


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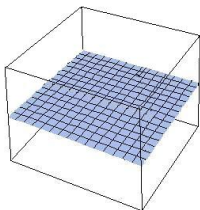
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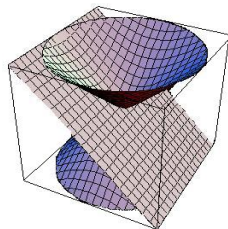


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RESULTS

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Phys. Rev **D 72**, '05

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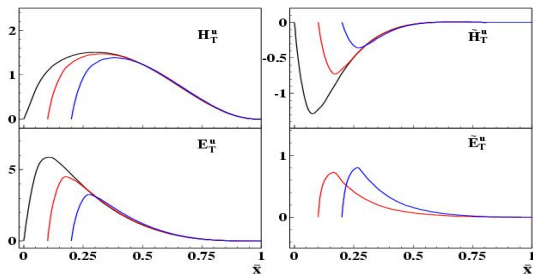
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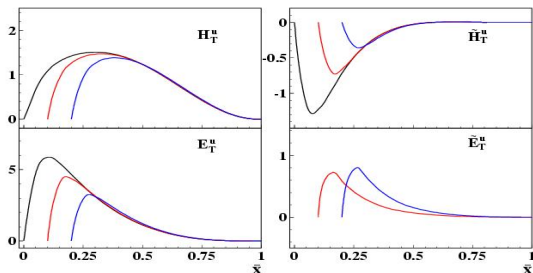
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$\Rightarrow$ “Not only $\mathcal{H}_T^q(x, 0, 0) \neq 0$, (Scopetta in MIT bag model)”.

RESULTS

B. Pasquini, M. P. and S. Boffi,
Phys. Rev D **72**, '05

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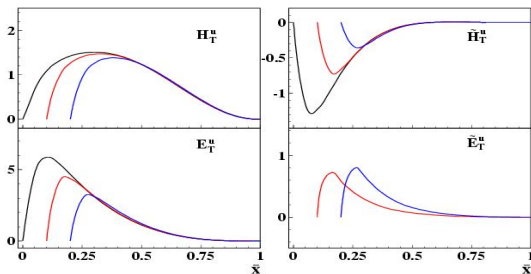
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Forward Limit:

$$\mathcal{H}_T^q(x, 0, 0) = h_1^q(x) = \sum_{\lambda_i^t \tau_i} \sum_{j=1}^3 \delta_{\tau_j \tau_q} \text{sign}(\lambda_j^t) \int [d\bar{x}]_3 [d\mathbf{k}_\perp]_3 \delta(x - x_j) \left| \psi_{\lambda^t}^{[f]}(x_i, \mathbf{k}_{\perp i}; \lambda_j^t, \tau_i) \right|^2$$

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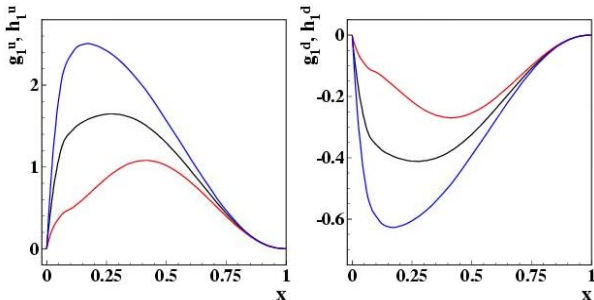
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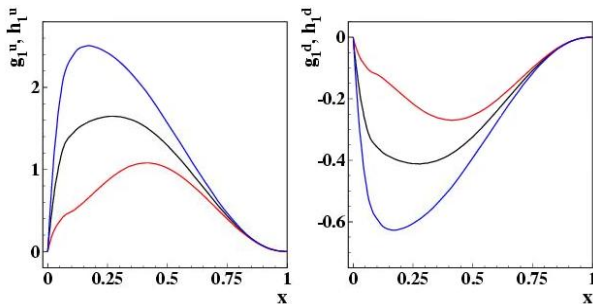


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Helicity and Transversity distributions for the u (left panel) and d (right panel) quark. The **blue** lines correspond to h_1^q , the **red** lines show g_1^q , and the black lines are the **nonrelativistic** results when Melosh rotations reduce to the identity ($h_1^q = g_1^q$).

- **Axial** (Δq) and **Tensor** (δq) “Charges”

$$\Delta q = \int_{-1}^1 dx g_1^q(x), \quad \delta q = \int_{-1}^1 dx h_1^q(x).$$

The nucleon **axial** / **tensor** charge measures the net number of **longitudinally** / **transversely** polarized valence quarks in a **longitudinally** / **transversely** polarized nucleon.

	NR	HO	HYP
Δu	4/3	1.0	0.61
Δd	-1/3	-0.25	-0.15
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$$\langle \delta^x J_q^x \rangle = \langle J_{q,+\hat{x}}^x - J_{q,-\hat{x}}^x \rangle = \frac{1}{2} \left[A_{T20}(0) + 2\tilde{A}_{T20}(0) + B_{T20}(0) \right], \quad (1)$$

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Using LCWFs derived from the hypercentral CQM we obtain

$$\langle \delta^x J_u^x \rangle = 0.39, \quad \langle \delta^x J_d^x \rangle = 0.10, \quad (\text{HYP}) \quad (2)$$

while using the Schmidt-Soffer harmonic oscillator wave function of the nucleon (PLB **407**, '97), we obtain:

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Summary

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...and Outlook

In the **CQM** we have taken into account only valence quarks and this limits the average longitudinal momentum fraction $\bar{x}$ between ξ and 1. Nevertheless the inclusion of “**sea**” contributions (to access **ERBL** region) is possible following, e.g., the lines of the papers: B. Pasquini and S. Boffi, **PRD 71** ('05) and **PRD 73** ('06).

THANKS TO THE ORGANIZERS OF THE WORKSHOP



FOR THE OPPORTUNITY GRANTED TO ME



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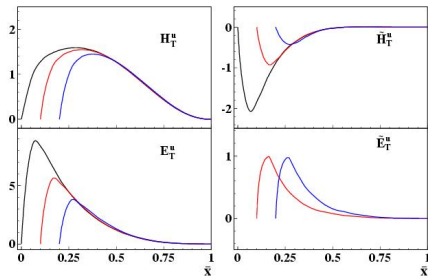
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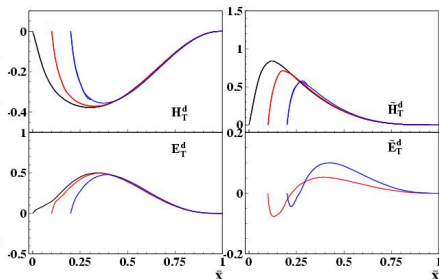
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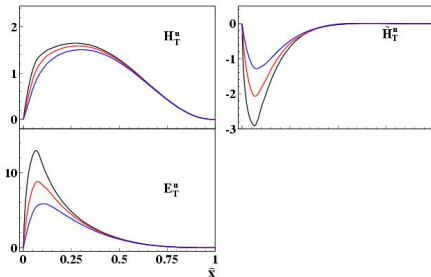
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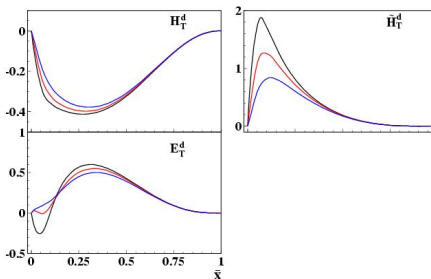
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Hypercentral Model

P. Faccioli, M. Traini and V. Vento, *Nucl. Phys. A* **656**, '99

$$H = \sum_{i=1}^3 \sqrt{\mathbf{k}_i^2 + m_i^2} - \frac{\tau}{\sqrt{\rho^2 + \lambda^2}} + k\sqrt{\rho^2 + \lambda^2},$$

with

$$\rho = \frac{\mathbf{r}_1 - \mathbf{r}_2}{\sqrt{2}} \quad \text{e} \quad \lambda = \frac{\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3}{\sqrt{6}}.$$

Features:

- Low energy spectrum, Form Factors;
- $SU(6)$ symmetric nucleon wave function;
- For $x \rightarrow 1$ (valence quark dominance), GPDs are obtained in a covariant approach and exhibit the exact forward limit reproducing the parton distribution with the correct support and automatically fulfilling the particle number and momentum sum rule.