Universality of single spin asymmetries in hard processes

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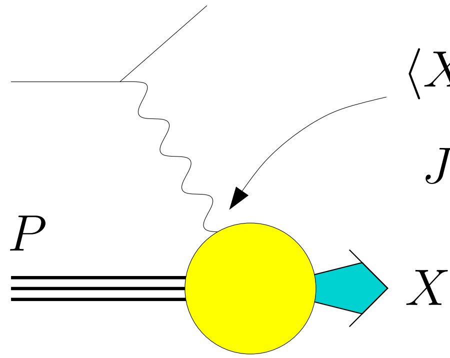
- Introduction to PDFs.
- Introduction to gauge-links and their consequences.
- $p^{\uparrow}p \to \pi\pi X$

deep inelastic scattering



$\langle X | J^\mu(0) | P \rangle$

$J^\mu(0) = \overline{\psi}(0)\gamma^\mu\psi(0)$

$$d\sigma^{\text{Hadron}} \propto \sum_q \int dx\, f(x)\, d\hat{\sigma}^{\text{parton}}(x)$$

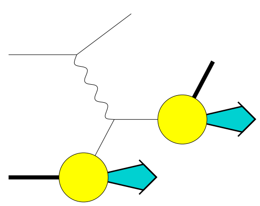
$$f(x) \propto \mathcal{FT}\, \langle P | \overline{\psi}(0)\, \mathcal{U}(0,\xi)\, \psi(\xi)\, | P \rangle$$

the $f(x)$ are universal

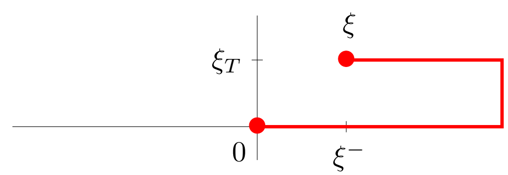
gauge-link

$$\mathcal{U}^{C}(0,\xi) = \mathcal{P}\exp ig\int_{C} \mathrm{d}z \cdot A(z)$$

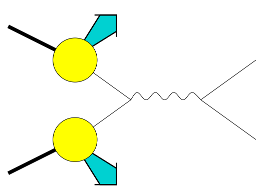
- SIDIS $(\ell + h_1 \longrightarrow \ell' + h_2 + X)$



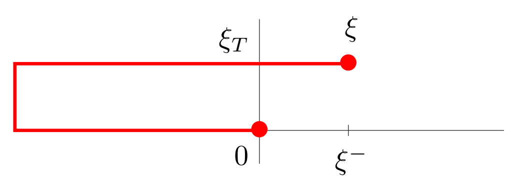
$\mathcal{U}^{[+]}$:



- Drell-Yan $(h_1 + \overline{h}_2 \longrightarrow \ell + \overline{\ell} + X)$



$\mathcal{U}^{[-]}$:



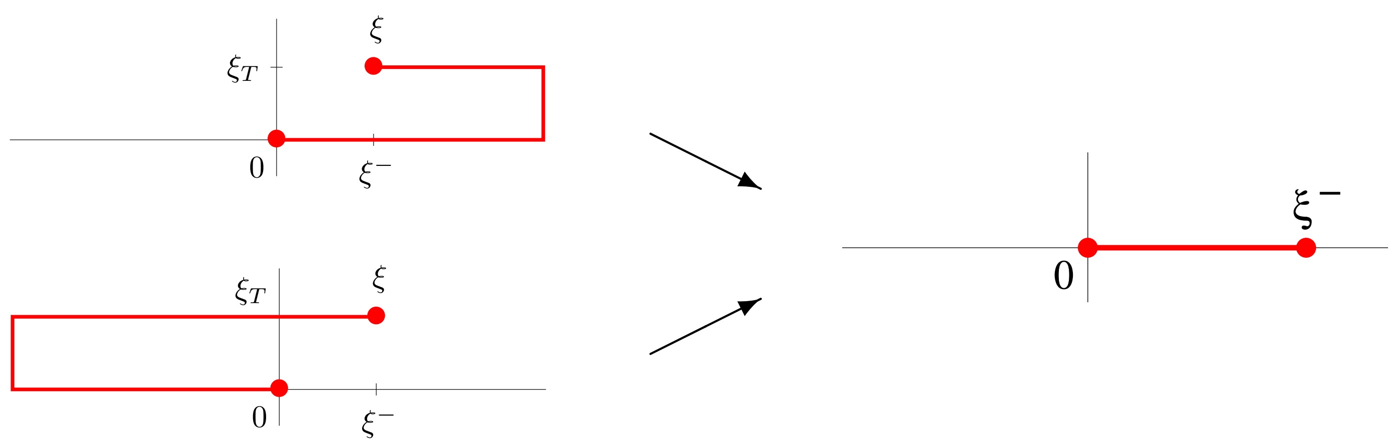
consequences of gauge-links for T-even distribution functions

$\Rightarrow$ T-even functions, e.g. $f_1(x)$:

$$f_1^{[\pm]}(x) \propto \int \mathrm{d}k^- \mathrm{d}^2 k_T \;\; \mathcal{FT} \langle\, \overline{\psi}(0)\, \mathcal{U}^{[\pm]}(0,\xi)\, \gamma^+ \psi(\xi)\, \rangle \propto f_1(x)$$

$\mathcal{U}^{[+]}$:

$\mathcal{U}^{[-]}$:

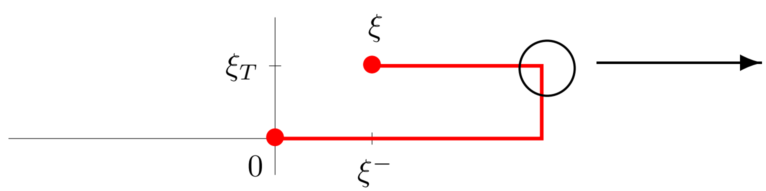


consequences of gauge-links for T-odd distribution functions

$\Rightarrow$ T-odd functions, e.g. Sivers $f_{1T}^{\perp(1)}(x)$:

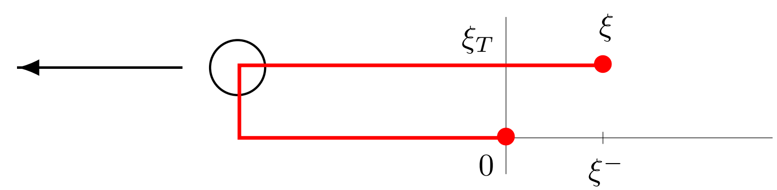
$$(f_{1T}^{\perp(1)})^{[\pm]}(x) \propto \int dk^- d^2k_T \;\; k_T \;\; \mathcal{FT}\langle\, \overline{\psi}(0)\,\mathcal{U}^{[\pm]}(0,\xi)\,\gamma^+\psi(\xi)\,\rangle$$

$$\propto \pm\, f_{1T}^{\perp(1)}(x)$$



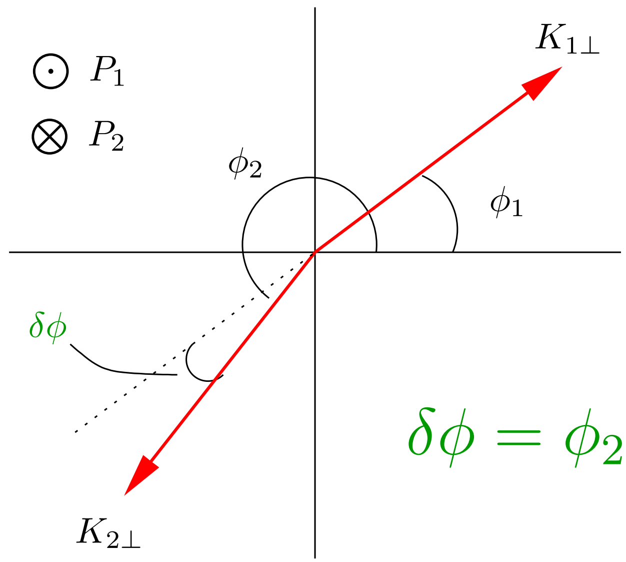
$+\,\langle\, A_T(+\infty) - A_T(-\infty)\,\rangle$ (in light-cone gauge)

$-\,\langle\, A_T(+\infty) - A_T(-\infty)\,\rangle$



azimuthal asymmetry in $p^{\uparrow}p \to \pi\pi X$

Transverse plane:

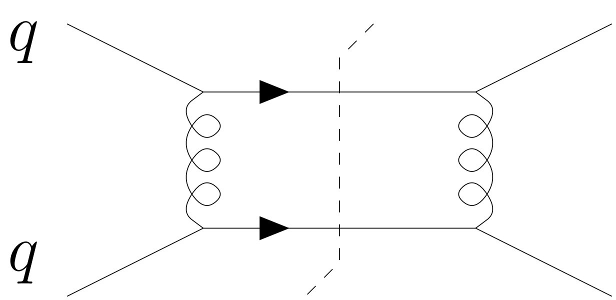


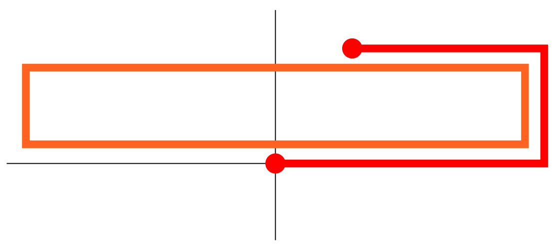
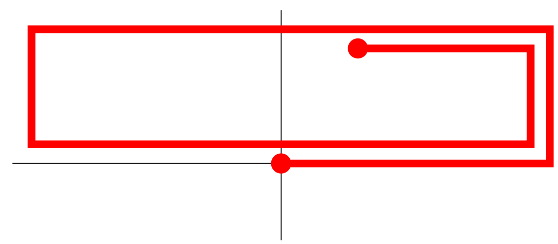
$$\delta\phi = \phi_2 - \phi_1 - \pi$$

$$\langle \sin(\delta\phi)\, \mathrm{d}\sigma \rangle = \int \frac{\mathrm{d}\phi_2}{2\pi} \sin(\delta\phi)\, \mathrm{d}\sigma$$

example of gauge-link in $p^{\uparrow}p\rightarrow\pi\pi X$

e.g. qq-scattering:

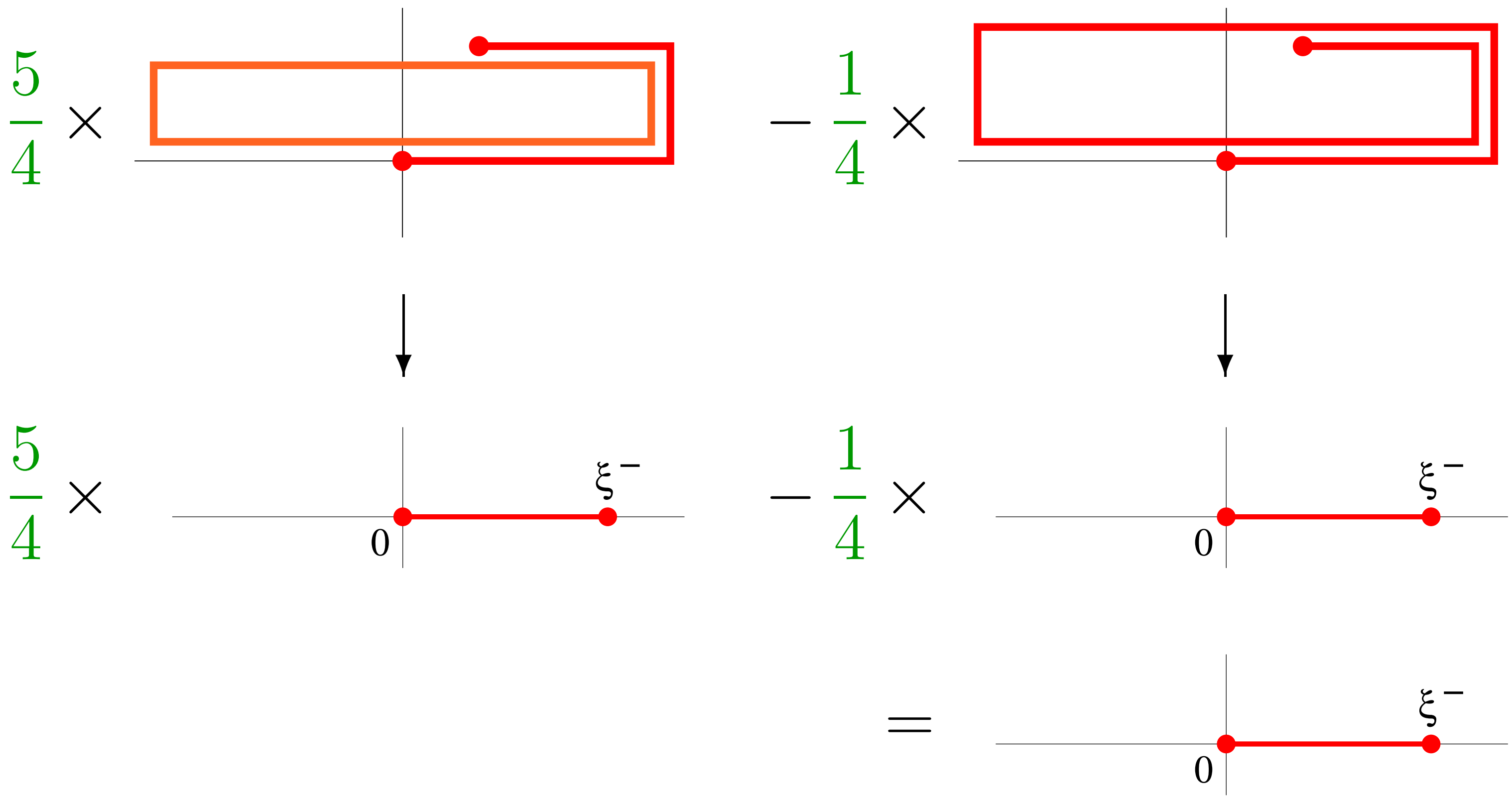


$\frac{5}{4}\times$  $-\frac{1}{4}\times$ 

$$=\frac{5}{4}\times\ \mathrm{Tr}\left(\mathcal{U}^{[\square]}\right)\mathcal{U}^{[+]}/3\ -\frac{1}{4}\times\ \mathcal{U}^{[\square]}\mathcal{U}^{[+]}$$

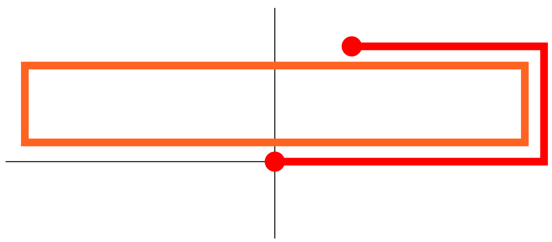
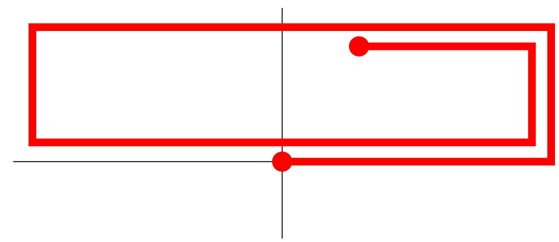
consequence for T-even functions

k_T-integration:



consequence for T-odd functions

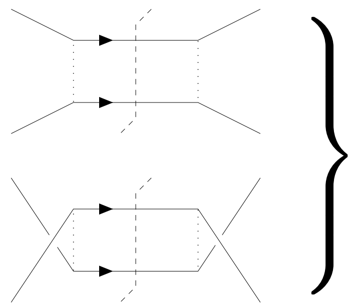
k_T-weighing:

$\frac{5}{4} \times$  $- \frac{1}{4} \times$ 

$$\frac{5}{4} \times \langle A_T(\infty) - A_T(-\infty) \rangle - \frac{1}{4} \times 3 \langle A_T(\infty) - A_T(-\infty) \rangle$$

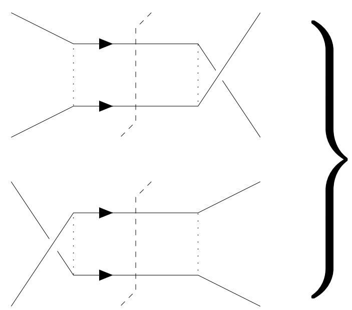
$$= \frac{1}{2} \times \langle A_T(\infty) - A_T(-\infty) \rangle$$

Sivers effects in identical-quark scattering



$$\mathcal{U} = \frac{5}{4}\,\mathrm{Tr}(\mathcal{U}^{[\Box]})\mathcal{U}^{[+]}/3 - \frac{1}{4}\,\mathcal{U}^{[\Box]}\mathcal{U}^{[-]}$$

$$\Longrightarrow \qquad \text{Sivers effect} \;=\; \frac{1}{2}\,f_{1T}^{\perp(1)}$$

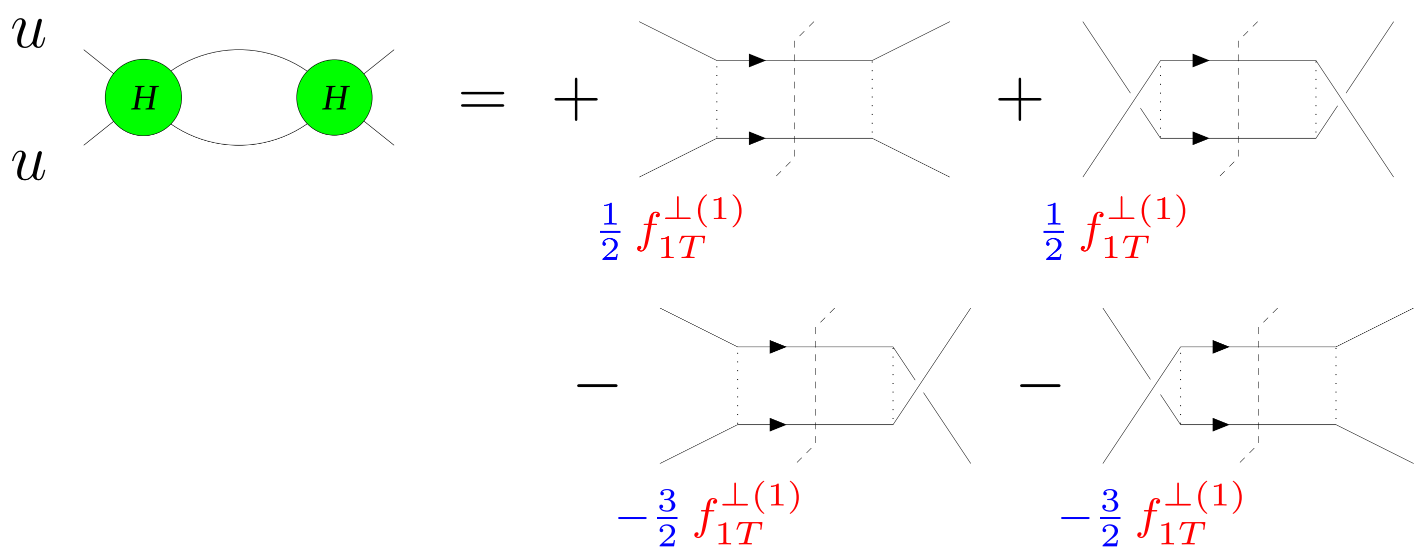


$$\mathcal{U} = \frac{9}{4}\,\mathrm{Tr}(\mathcal{U}^{[\Box]})\mathcal{U}^{[+]}/3 - \frac{5}{4}\,\mathcal{U}^{[\Box]}\mathcal{U}^{[-]}$$

$$\Longrightarrow \qquad \text{Sivers effect} \;=\; -\frac{3}{2}\,f_{1T}^{\perp(1)}$$

identical quark scattering

using the 'universal' Sivers function $f_{1T}^{\perp(1)}$

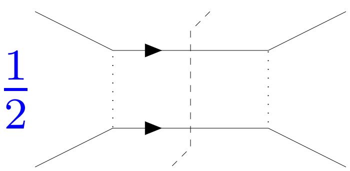
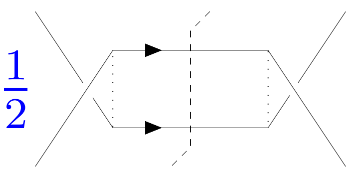


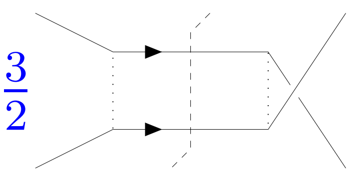
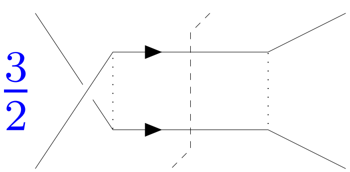
identical quark scattering

Hadronic scattering cross section

$$d\sigma^{\text{HADRON}} \propto f_{1T}^{\perp(1)}(x_1)\, f_1(x_2)\, dS^{uu\to uu}\, D_1(z_1)\, D_1(z_2)$$

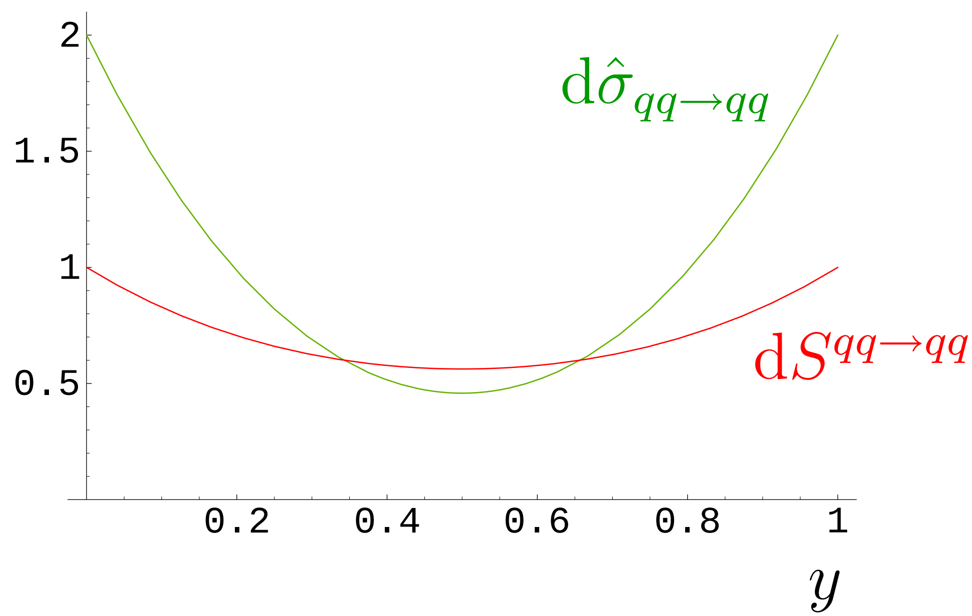
with gluonic pole cross section

$dS^{uu\to uu} \quad = \quad +\frac{1}{2}$  $+\frac{1}{2}$ 

$+\frac{3}{2}$  $+\frac{3}{2}$ 

$$\not\propto d\hat{\sigma}(uu \to uu)$$

comparison of partonic and gluonic pole cross section



$$y = -\hat{t}/\hat{s} = \cos^2(\tfrac{1}{2}\theta)$$

θ : polar angle of outgoing pion in c.o.m.

weighed scattering cross section

$$\langle \sin(\delta\phi)\, \mathrm{d}\sigma \rangle$$

$$\propto M_1 \sum_{q_i} f_{1T}^{\perp(1)}(x_1) f_1(x_2) \frac{\mathrm{d}S^{\mathrm{SIVERS}}}{\mathrm{d}\hat{t}} D_1(z_1) D_1(z_2)$$

$$+ M_2 \sum_{q_i} h_1(x_1) h_1^{\perp(1)}(x_2) \frac{\mathrm{d}S^{\mathrm{BOER\text{-}MULDERS}}}{\mathrm{d}\hat{t}} D_1(z_1) D_1(z_2)$$

+ Collins effect contributions

+ Gluon scattering contributions

summary

- Gauge-links can be calculated by resumming all initial and/or final state interactions of collinear gluons.
- Consequence of the gauge-links is that T-odd functions appear with different calculable strengths in the different scattering channels.
- Azimuthal asymmetries in $p^{\uparrow}p \rightarrow \pi\pi X$ can be written as a convolution of universal PDFs, FFs and process-dependent hard parts, the gluonic pole cross sections. The gluonic pole cross sections are, in general, different from the partonic cross sections.