Transverse momentum dependent correlators and transversity for experts

(Andreas Metz, Ruhr-Universität Bochum)

- Transversity measurements
- Transversity in Drell-Yan: collinear formalism
- Transversity in semi-inclusive DIS: p_T-dependent formalism
  Factorization of p_T-dependent correlators
- Universality of p_T-dependent correlators
- p_T-dependent correlators beyond leading twist
- Summary

Transversity measurements

1. Collinear formalism
   - $p^{\uparrow}\, p^{\uparrow} \longrightarrow l^{+}\, l^{-}\, X$ (Ralston, Soper, 1979)

$$\frac{d\sigma^{\uparrow\uparrow} - d\sigma^{\uparrow\downarrow}}{d\sigma^{\uparrow\uparrow} + d\sigma^{\uparrow\downarrow}} \propto \frac{h_1^q(x_1, Q^2)\, h_1^{\bar{q}}(x_2, Q^2) + \big(x_1 \leftrightarrow x_2\big)}{f_1^q(x_1, Q^2)\, f_1^{\bar{q}}(x_2, Q^2) + \big(x_1 \leftrightarrow x_2\big)}$$

   - $p^{\uparrow}\, p^{\uparrow} \longrightarrow \mathrm{jet}\, X \qquad p^{\uparrow}\, p^{\uparrow} \longrightarrow \gamma\, X$
   - $l\, p^{\uparrow} \longrightarrow l\, \Lambda^{\uparrow}\, X \qquad l\, p^{\uparrow} \longrightarrow l\, \rho\, X \qquad l\, p^{\uparrow} \longrightarrow l\, (\pi\pi)\, X$

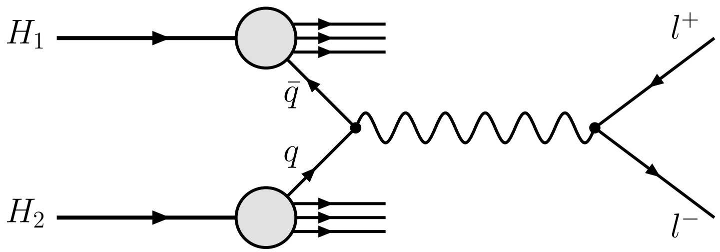
2. p_T-dependent formalism
   - $l\, p^{\uparrow} \longrightarrow l\, \pi\, X$ (Collins, 1992)

$$\frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \frac{h_1^q(x, \vec{p}_T^{\,2})\, H_1^{\perp q}(z, \vec{k}_T^{2})}{f_1^q(x, \vec{p}_T^{\,2})\, D_1^q(z, \vec{k}_T^{2})}$$

   - $p^{\uparrow}\, p \longrightarrow l^{+}\, l^{-}\, X$
   - $p^{\uparrow}\, p \longrightarrow \pi\, X$

Transversity in Drell-Yan

1. Tree level



$$A_{TT}^{H_1 H_2} = a_{TT} \frac{\sum_q e_q^2 \left(h_1^{q/H_1}(x_1, Q^2) h_1^{\bar{q}/H_2}(x_2, Q^2)\right) + \left(x_1 \leftrightarrow x_2\right)}{\sum_q e_q^2 \left(f_1^{q/H_1}(x_1, Q^2) f_1^{\bar{q}/H_2}(x_2, Q^2)\right) + \left(x_1 \leftrightarrow x_2\right)}$$

- Kinematics

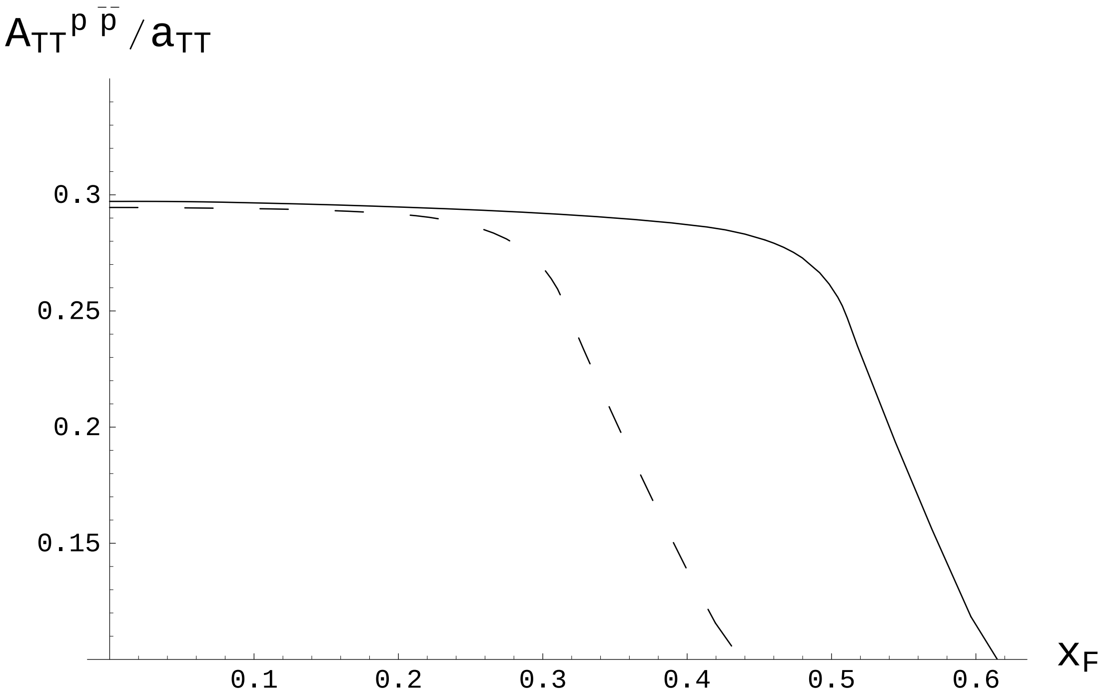
$$Q^2 \approx (x_1 P_1 + x_2 P_2)^2 \approx x_1 x_2 (P_1 + P_2)^2 = x_1 x_2 s$$

$$y = \frac{1}{2} \ln \frac{x_1}{x_2} \qquad x_1 = \sqrt{\frac{Q^2}{s}}\, e^{y} \qquad x_2 = \sqrt{\frac{Q^2}{s}}\, e^{-y} \qquad x_F = x_1 - x_2$$

- Results
  A_{TT}^{pp} small at RHIC ($\sqrt{s} = 200\,\mathrm{GeV}$)
  A_{TT}^{pp} moderate at JPARC ($\sqrt{s} = 10\,\mathrm{GeV}$)
  $A_{TT}^{p\bar{p}}$ large at GSI ($6\,\mathrm{GeV} < \sqrt{s} < 15\,\mathrm{GeV}$)

(Anselmino, Barone, Drago, Nikolaev, 2004)

$$s = 30\,\mathrm{GeV}^2 \qquad 45\,\mathrm{GeV}^2$$
$$Q^2 = 16\,\mathrm{GeV}^2$$
$$h_1^q(x, Q_0^2) = g_1^q(x, Q_0^2) \text{ at low scale } Q_0^2$$

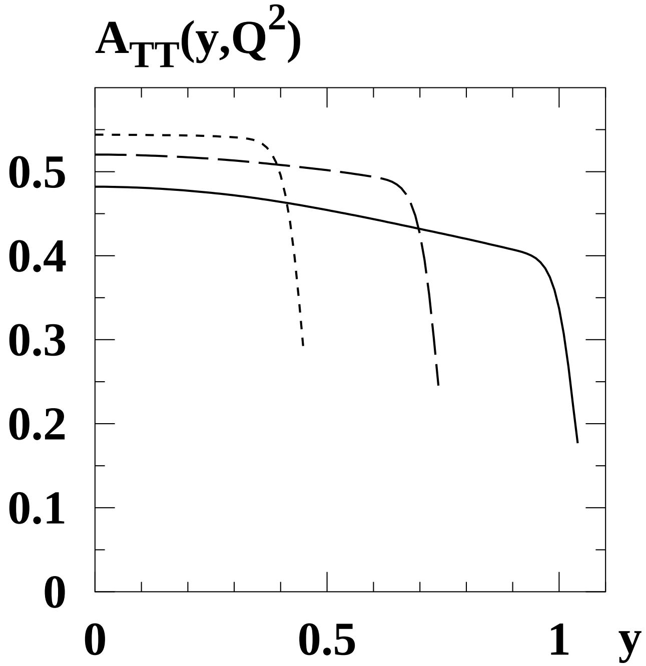


(Efremov, Goeke, Schweitzer, 2004)

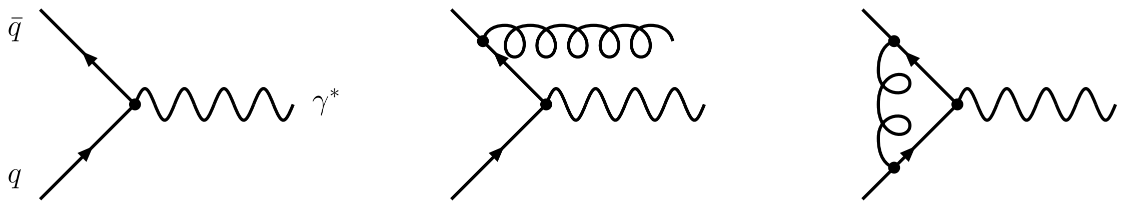
$$s = 45\,\text{GeV}^2$$

$$Q^2 = 5\,\text{GeV}^2 \qquad 9\,\text{GeV}^2 \qquad 16\,\text{GeV}^2$$

$h_1^q(x, Q_0^2)$ from chiral quark soliton model

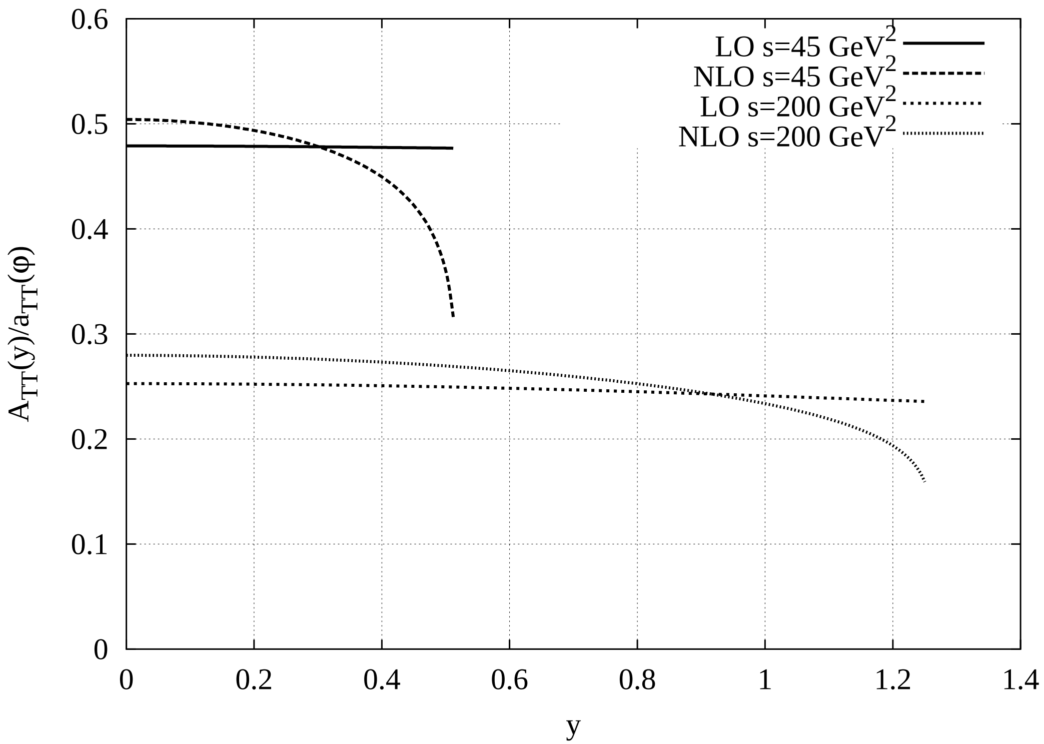


2. One-loop level



(Barone, Cafarella, Corianò, Guzzi, Ratcliffe, 2006)

$$Q^2 = 16\,\mathrm{GeV}^2$$



3. Summation of large (threshold-) logarithms

$\longrightarrow$ summation of leading logarithmic contributions to all orders in α_s

- Origin of logarithms

$$z = \frac{Q^2}{x_1 x_2 s} = \frac{Q^2}{\hat{s}} \quad \begin{cases} = 1 & \text{at tree level} \\ < 1 & \text{at } \mathcal{O}(\alpha_s) \text{ and beyond} \end{cases}$$

For k-loop calculation one finds

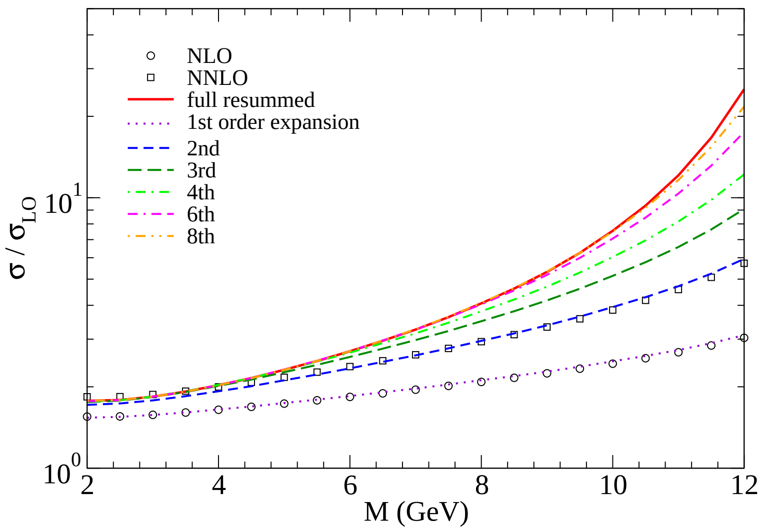
$$\alpha_s^k \left(\frac{\ln^{2k-1}(1-z)}{1-z} \right)_+$$

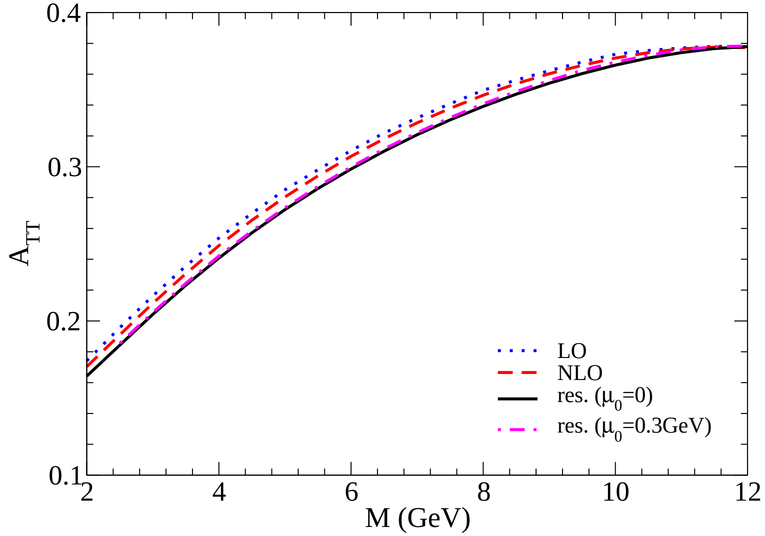
$\longrightarrow$ effect large if $z \longrightarrow 1$ (threshold for emission of real gluons)

- Unpolarized cross section and A_{TT}

(Shimizu, Sterman, Vogelsang, Yokaya, 2005)

$\sqrt{s} = 14.5\,\mathrm{GeV}$

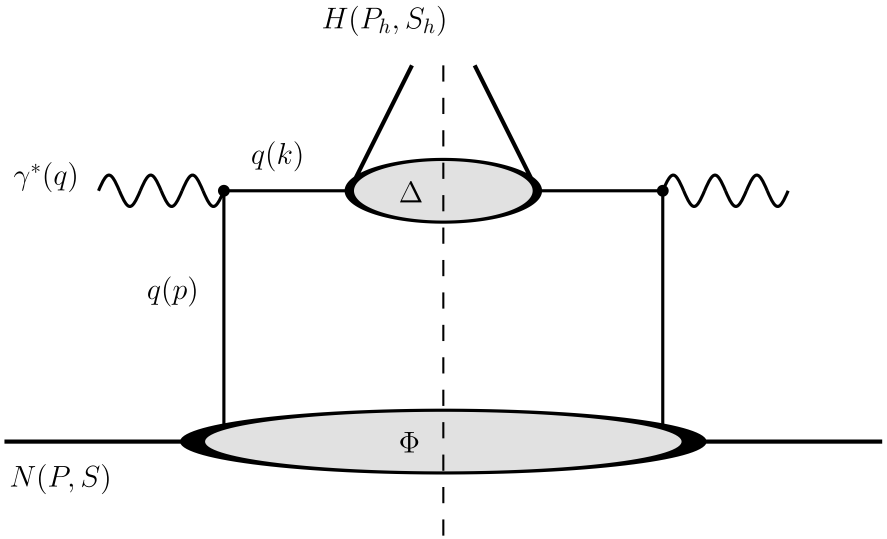




Factorization of p_T-dependent correlators

Processes: $l\,N \to l\,H\,X \qquad H_1\,H_2 \to l^+\,l^-\,X \qquad e^+\,e^- \to H_1\,H_2\,X$

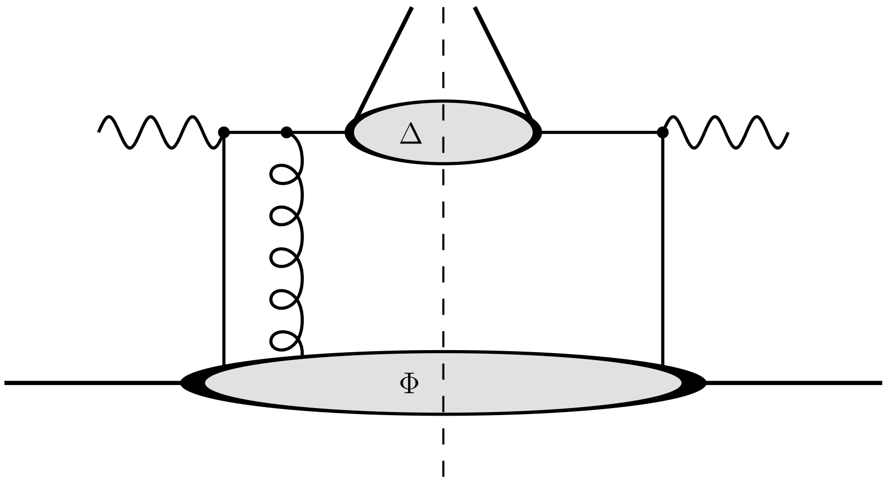
1. Tree level
   (Ralston, Soper, 1979)



$$\frac{d\sigma_{unp}}{d^3\vec{l}'\,d^3\vec{P}_h} \propto \int d^2\vec{p}_T\,d^2\vec{k}_T\,f_1(x,\vec{p}_T^{\,2})\,D_1(z,\vec{k}_T^{\,2})\,\delta^{(2)}(\vec{p}_T+\vec{q}_T-\vec{k}_T)+\ldots$$

$$f_1(x,\vec{p}_T^{\,2}) = \int \frac{d\xi^-\,d^2\vec{\xi}_T}{2(2\pi)^3}\,e^{i(xP^+\xi^- - \vec{p}_T\cdot\vec{\xi}_T)}\,\langle P,S|\,\bar{\psi}(0)\,\gamma^+\,\psi(\xi^-,\vec{\xi}_T)\,|P,S\rangle$$

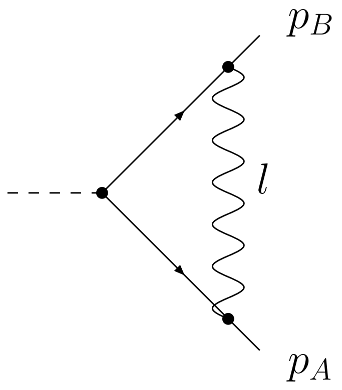
2. Tree level (gauge invariant)
   (Belitsky, Ji, Yuan, 2002; Boer, Mulders, Pijlman, 2003)



$$\frac{d\sigma_{unp}}{d^3\vec{l'}\, d^3\vec{P}_h} \propto \int d^2\vec{p}_T\, d^2\vec{k}_T\, f_1(x,\vec{p}_T^{\,2})\, D_1(z,\vec{k}_T^{\,2})\, \delta^{(2)}(\vec{p}_T+\vec{q}_T-\vec{k}_T) + \ldots$$

$$f_1(x,\vec{p}_T^{\,2}) = \int \frac{d\xi^- d^2\vec{\xi}_T}{2(2\pi)^3}\, e^{i(xP^+\xi^- - \vec{p}_T\cdot\vec{\xi}_T)}\, \langle\, |\, \bar{\psi}(0)\, \gamma^+\, \mathcal{L}(0,\vec{0}_T;\xi^-,\vec{\xi}_T)\, \psi(\xi^-,\vec{\xi}_T)\, |\, \rangle$$

3. Asymptotics of Sudakov form factor



$$Q^2 = (p_A + p_B)^2 \approx 2p_A^+ p_B^-$$

$$p_A^+ = p_B^- \approx \frac{Q}{\sqrt{2}}$$

$$\Gamma = ie^2 \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2 - m_\gamma^2 + i\varepsilon} \frac{(2p_A - l)\cdot(2p_B + l)}{[(p_A - l)^2 - m^2 + i\varepsilon]\,[(p_B + l)^2 - m^2 + i\varepsilon]}$$

- Leading regions

  Soft: $l = (m, m, m)$

  p_A-collinear: $l = (Q, \frac{m^2}{Q}, m)$

  p_B-collinear: $l = (\frac{m^2}{Q}, Q, m)$

  Hard: $l = (Q, Q, Q)$

- Soft approximation

$$S = \Gamma\big|_{\text{soft appr.}} = ie^2 \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2 - m_\gamma^2 + i\varepsilon} \frac{1}{(-l^- + i\varepsilon)\,(l^+ + i\varepsilon)}$$

- Complications

  (1) Soft and collinear factors show (light-cone) divergence
  $\longrightarrow$ e.g., non-light-like Wilson lines, drop out in final result

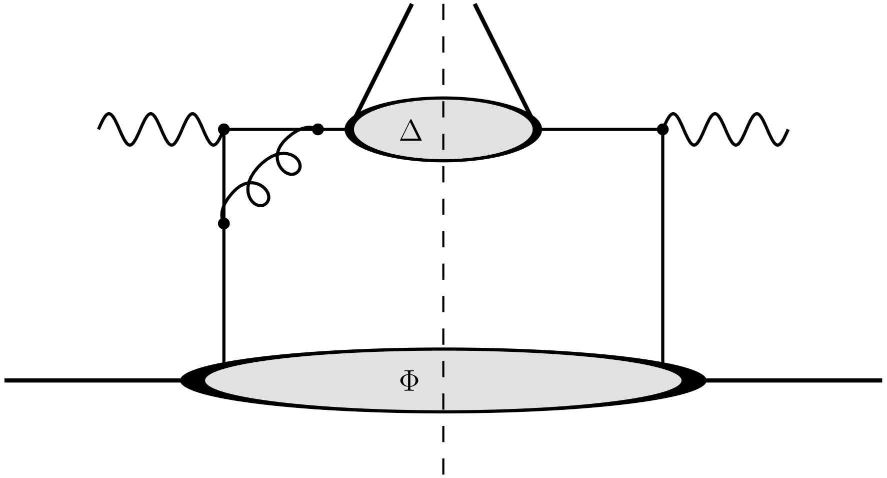
  (2) In general, leading regions overlap
  $\longrightarrow$ subtraction formalism (Collins, Hautmann, 1999)

$$\begin{aligned} S &= \Gamma\big|_{\text{soft appr.}} \\ A &= (\Gamma - S)\,\big|_{\text{coll-A appr.}} \\ B &= (\Gamma - S)\,\big|_{\text{coll-B appr.}} \\ H &= (\Gamma - S - A - B)\,\big|_{\text{hard appr.}} \end{aligned}$$

- Result

$$\Gamma = S + A + B + H + \mathcal{O}(\frac{1}{Q})$$

4. Beyond tree level
(Collins, Soper, 1981; Collins, Soper, Sterman, 1985;
Ji, Ma, Yuan, 2004; Collins, Metz, 2004)



$$\frac{d\sigma_{unp}}{d^3\vec{l}'\, d^3\vec{P}_h} \propto \int d^2\vec{p}_T\, d^2\vec{k}_T\, d^2\vec{l}_T\, f_1(x,\vec{p}_T^{\,2})\, D_1(z,\vec{k}_T^{\,2}) \times S(\vec{l}_T)\, H\, \delta^{(2)}(\vec{p}_T + \vec{q}_T + \vec{l}_T - \vec{k}_T) + \ldots$$

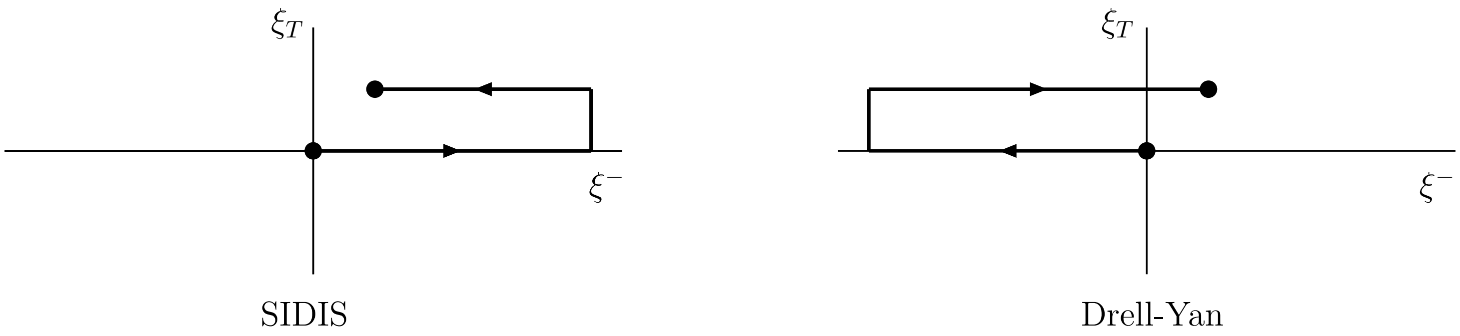
- compatible with tree level result
- checked explicitly to $\mathcal{O}(\alpha_s)$ (Ji, Ma, Yuan, 2004)
- independent check through matching with collinear factorization at high P_{hT} (Ji, Qiu, Vogelsang, Yuan, 2006)
- arguments for consistency to all orders

Universality of p_T-dependent correlators

1. Parton densities

$$\int d\xi^- d^2\vec{\xi}_T\, e^{i(xP^+\xi^- - \vec{p}_T\cdot\vec{\xi}_T)}\, \langle\, |\, \bar{\psi}(0)\, \Gamma\, \mathcal{L}(0,\vec{0}_T;\xi^-,\vec{\xi}_T)\, \psi(\xi^-,\vec{\xi}_T)\, |\, \rangle$$

- Different links for semi-inclusive DIS and Drell-Yan



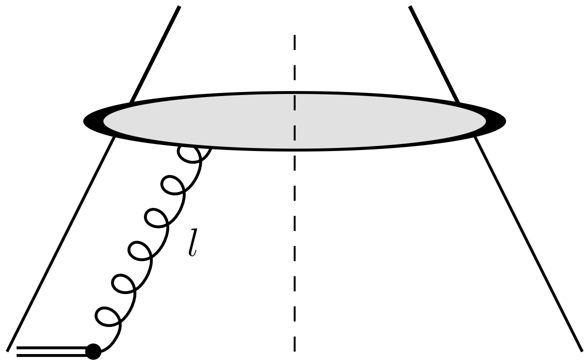
- $\longrightarrow$ Universality?
  Time-reversal: $f_{1T}^{\perp}\big|_{DY} = -f_{1T}^{\perp}\big|_{DIS}$ $\quad h_1^{\perp}\big|_{DY} = -h_1^{\perp}\big|_{DIS}$ (Collins, 2002)

2. Fragmentation functions
   - Fragmentation functions in DIS and e^+e^- *a priori* have different links, time-reversal cannot be applied because of structure

$$\sum_X |H,X,\mathrm{out}\rangle\, \langle H,X,\mathrm{out}|$$

- One-loop calculation for transverse SSA in fragmentation provides universality (Metz, 2002)

- Discussion for fragmentation functions



Comparison of two processes:

$$e^+e^- : \qquad \frac{1}{-l^- - i\varepsilon} \qquad\qquad SIDIS : \qquad \frac{1}{-l^- + i\varepsilon}$$

$\longrightarrow$ difference $\propto \delta(l^-)$
$\longrightarrow \int dl^+ \ldots$ vanishes
Kinematical argument valid for all fragmentation functions, generalizes to higher orders

3. Summary of full problem

- Semi-inclusive DIS

$$\sigma\Big|_{DIS} \propto \hat{\sigma}_{part} \otimes \mathrm{pdf} \otimes \mathrm{ff} \otimes \mathrm{soft}$$

- Drell-Yan

$$\sigma\Big|_{DY} \propto \hat{\sigma}_{part} \otimes \mathrm{pdf} \otimes \mathrm{pdf} \otimes \mathrm{soft}$$

- $e^+ e^- \longrightarrow H_1\, H_2\, X$

$$\sigma\Big|_{e^+e^-} \propto \hat{\sigma}_{part} \otimes \mathrm{ff} \otimes \mathrm{ff} \otimes \mathrm{soft}$$

$$\mathrm{pdf}\Big|_{DIS} \stackrel{?}{=} \mathrm{pdf}\Big|_{DY}$$

$$\mathrm{ff}\Big|_{DIS} \stackrel{?}{=} \mathrm{ff}\Big|_{e^+e^-}$$

$$\mathrm{soft}\Big|_{DIS} \stackrel{?}{=} \mathrm{soft}\Big|_{DY} \stackrel{?}{=} \mathrm{soft}\Big|_{e^+e^-}$$

4. Results
(Collins, Metz, 2004)

- Parton densities:

  Time-reversal: $f_{1T}^{\perp}\big|_{DY} = -f_{1T}^{\perp}\big|_{DIS}$ $h_1^{\perp}\big|_{DY} = -h_1^{\perp}\big|_{DIS}$

  6 T-even pdfs are universal

- Fragmentation functions:

  Time-reversal gives no constraint

  Analytical structure: $\mathrm{ff}\big|_{DIS} = \mathrm{ff}\big|_{e^+e^-}$ (for all 8 fragmentation functions)

- Soft factor:

  Time-reversal and analytical structure: $\mathrm{soft}\big|_{DIS} = \mathrm{soft}\big|_{DY} = \mathrm{soft}\big|_{e^+e^-}$

p_T-dependent correlators beyond leading twist

Relevant, e.g., for

Target spin asymmetry A_{UL}: data from HERMES
Beam spin asymmetry A_{LU}: data from CLAS

1. Classification

   (Mulders, Tangerman, 1995)
   (Goeke, Metz, Pobylitsa, Polyakov, 2003; Afanasev, Carlson, 2003;
   Metz, Schlegel, 2004; Bacchetta, Mulders, Pijlman, 2004;
   Goeke, Metz, Schlegel, 2005)

   Parameterization of

$$\Phi^{[\Gamma]}(x,\vec{p}_T,S) = \int \frac{d\xi^- d^2\vec{\xi}_T}{2(2\pi)^3}\, e^{i(xP^+\xi^- - \vec{p}_T\cdot\vec{\xi}_T)}\,\langle P,S\,|\,\bar{\psi}(0)\,\Gamma\,\mathcal{L}\,\psi(\xi^-,\vec{\xi}_T)\,|\,P,S\rangle$$

- Twist-2 (8 functions, 2 T-odd)

$$\Phi^{[\gamma^+]} = f_1 - \frac{\varepsilon_T^{ij} p_{Ti} S_{Tj}}{M}\, f_{1T}^{\perp}$$

$$\Phi^{[\gamma^+\gamma_5]} = \lambda\, g_{1L} + \frac{\vec{p}_T \cdot \vec{S}_T}{M}\, g_{1T}$$

$$\Phi^{[i\sigma^{i+}\gamma_5]} = S_T^i\, h_{1T} + \frac{p_T^i}{M}\left(\lambda\, h_{1L}^{\perp} + \frac{\vec{p}_T \cdot \vec{S}_T}{M}\, h_{1T}^{\perp}\right) - \frac{\varepsilon_T^{ij} p_{Tj}}{M}\, h_1^{\perp}$$

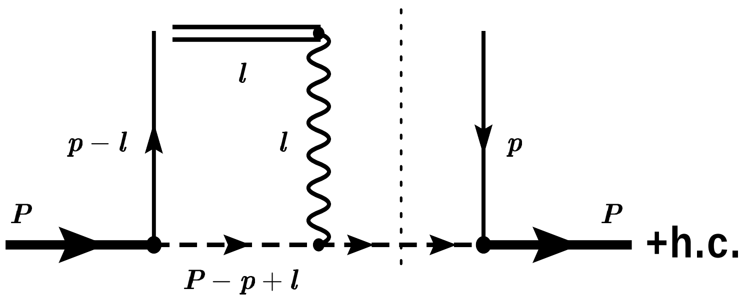
- Twist-3 (16 functions, 8 T-odd)

$$\Phi^{[1]} = \frac{M}{P^+}\left[e - \frac{\varepsilon_T^{ij} p_{Ti} S_{Tj}}{M}\, e_T^{\perp}\right]$$

$$\Phi^{[\gamma^i]} = \frac{M}{P^+}\left[\frac{p_T^i}{M}\left(f^{\perp} - \frac{\varepsilon_T^{jk} p_{Tj} S_{Tk}}{M}\, f_T^{\perp\prime}\right) + \frac{\varepsilon_T^{ij} p_{Tj}}{M}\left(\lambda\, f_L^{\perp} + \frac{\vec{p}_T \cdot \vec{S}_T}{M}\, f_T^{\perp}\right)\right]$$

$$\Phi^{[\gamma^i\gamma_5]} = \frac{M}{P^+}\left[S_T^i\, g_T^{\prime} + \frac{p_T^i}{M}\left(\lambda\, g_L^{\perp} + \frac{\vec{p}_T \cdot \vec{S}_T}{M}\, g_T^{\perp}\right) - \frac{\varepsilon_T^{ij} p_{Tj}}{M}\, g^{\perp}\right]$$

2. Calculation of $g^\perp$ in spectator model
(Gamberg, Hwang, Metz, Schlegel, 2006)



Wilson line specified by

$$v = (v^+, v^-, \vec{0}_T) \qquad \frac{v^+}{v^-} \ll 1$$

$$g^\perp \propto \sum_{\pm} \int d^4 l \, \frac{\{a_T l_T \, ; \, a^+ l^+ \, ; \, a^- l^-\}}{[(l\cdot v) \pm i\varepsilon][l^2 \mp i\varepsilon][-2p^+ l^- \, \ldots \mp i\varepsilon][2(P-p)^+ l^- \, \ldots \mp i\varepsilon]}$$

- Light-like limit

$$v^+ = 0 \, , \quad v^- = 1 \qquad\qquad l\cdot v + i\varepsilon \longrightarrow l^+ + i\varepsilon$$

$$\text{for} \quad l^+ = 0 \quad \rightarrow \quad \int^\infty dl^- \, \frac{l^-}{(l^-)^2} \quad \rightarrow \quad \text{(logarithmic) divergence}$$

- Explicit calculation

$$g^\perp \propto \ln \frac{v^+}{v^-(P^+)^2} \quad \rightarrow \quad \text{divergence}$$

- Interpretation

Light-cone divergence known from twist-2 calculations
$\longrightarrow$ non-light-like lines
dependence on these lines, in fixed order calculation of observables,
(1) either fully drops out
(2) or is suppressed

$$A_{UT}^{\rm jet} \propto f_{1T}^{\perp}\Big(\frac{v^+}{v^-} = 0\Big) \qquad\qquad f_{1T}^{\perp}\Big(\frac{v^+}{v^-}\Big) = f_{1T}^{\perp}(0) + \mathcal{O}\Big(\frac{v^+}{v^-}\Big)$$

Gauge invariant tree level formalism for twist-3:

$$A_{LU}^{\rm jet} \propto g^{\perp}\Big(\frac{v^+}{v^-} = 0\Big)$$

$g^{\perp}(0)$ undefined, $g^{\perp}(v)$ arbitrary for arbitrary vector v
$\longrightarrow$ no established factorization formalism for twist-3 p_T-dependent observables like A_{UL}, A_{LU}, etc.

Summary

1. Transversity in Drell-Yan
   - A_{TT}-measurement promising, in particular for $p\bar{p}$-collisions
   - Higher order corrections to cross sections can be significant
   - Higher order corrections hardly influence A_{TT}

2. Factorization of p_T-dependent correlators
   - All-order factorization formula exists
   - Survives all available checks

3. Universality of p_T-dependent correlators
   - T-odd parton densities in DIS and Drell-Yan have reversed sign
   - Everything else is universal
   - Situation more complicated in processes like $p\,p \to H_1\,H_2\,X$

4. p_T-dependent correlators beyond leading twist
   - Full classification of p_T-dependent correlators available
   - Status of factorization unclear