GPDs from lattice QCD

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LHPC/MILC

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W. Schroers (DESY Zeuthen)

supported by

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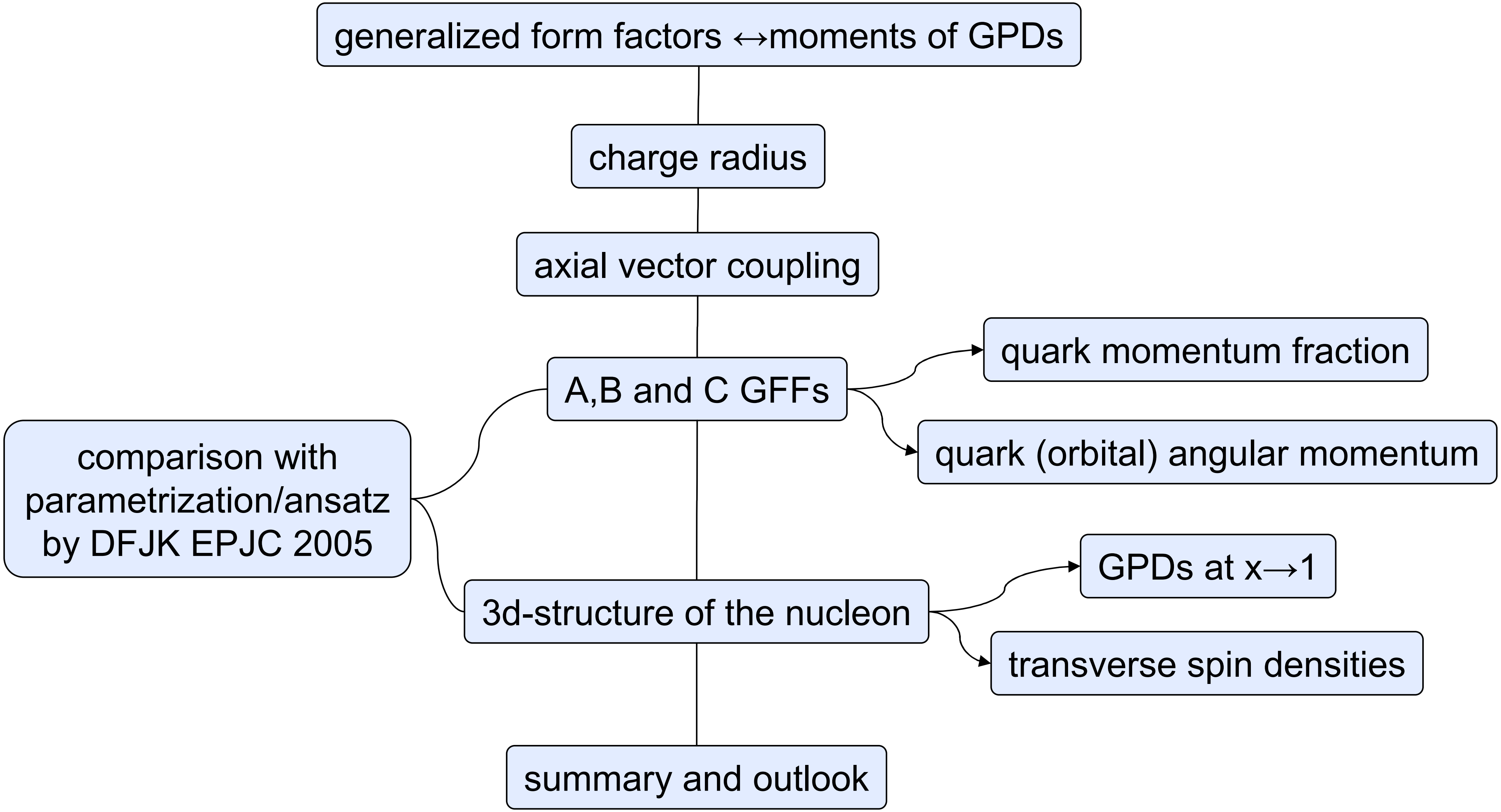
Deutsche Forschungsgemeinschaft DFG

Overview


generalized form factors ↔moments of GPDs
charge radius
axial vector coupling
A,B and C GFFs
quark momentum fraction
quark (orbital) angular momentum
comparison with parametrization/ansatz by DFJK EPJC 2005
3d-structure of the nucleon
GPDs at x→1
transverse spin densities
summary and outlook

Proton charge radius

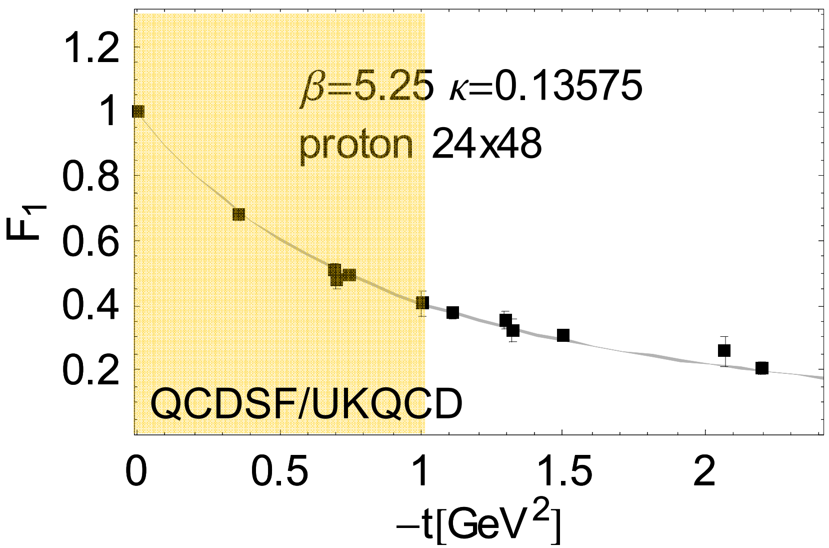
$$\langle P',\Lambda'|\bar{q}\gamma_\mu q|P,\Lambda\rangle = \bar{U}(P',\Lambda')\left\{\gamma_\mu F_1(t) + \frac{i\sigma_{\mu\nu}\Delta^\nu}{2m}F_2(t)\right\}U(P,\Lambda)$$

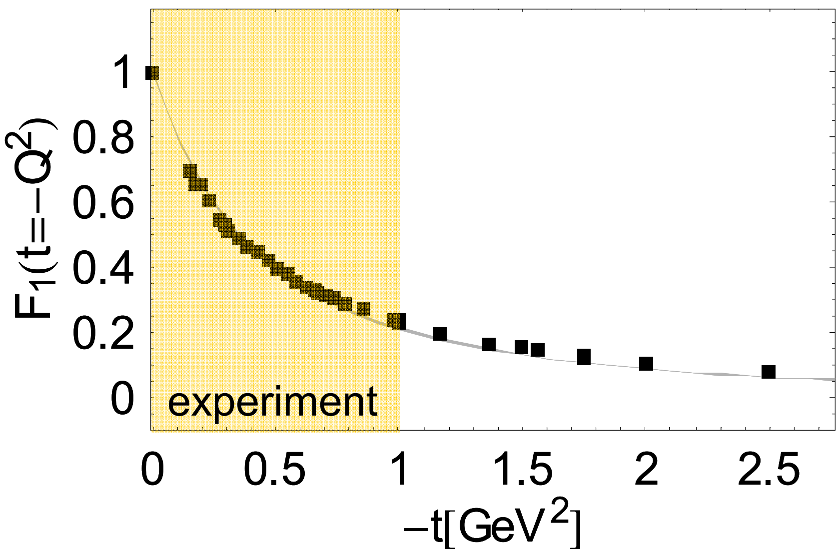
$t = -Q^2 = \Delta^2 \triangleq$ momentum transfer squared

$$\langle r^2\rangle = -6\frac{d}{dt}F_1(t)\Big|_{t=0}$$

dipole ansatz $F_1(t) = \dfrac{F_1(0)}{\left(1 - t/m_D^2\right)^2}$

$$\langle r^2\rangle = \frac{12}{m_D^2}$$





Proton charge radius

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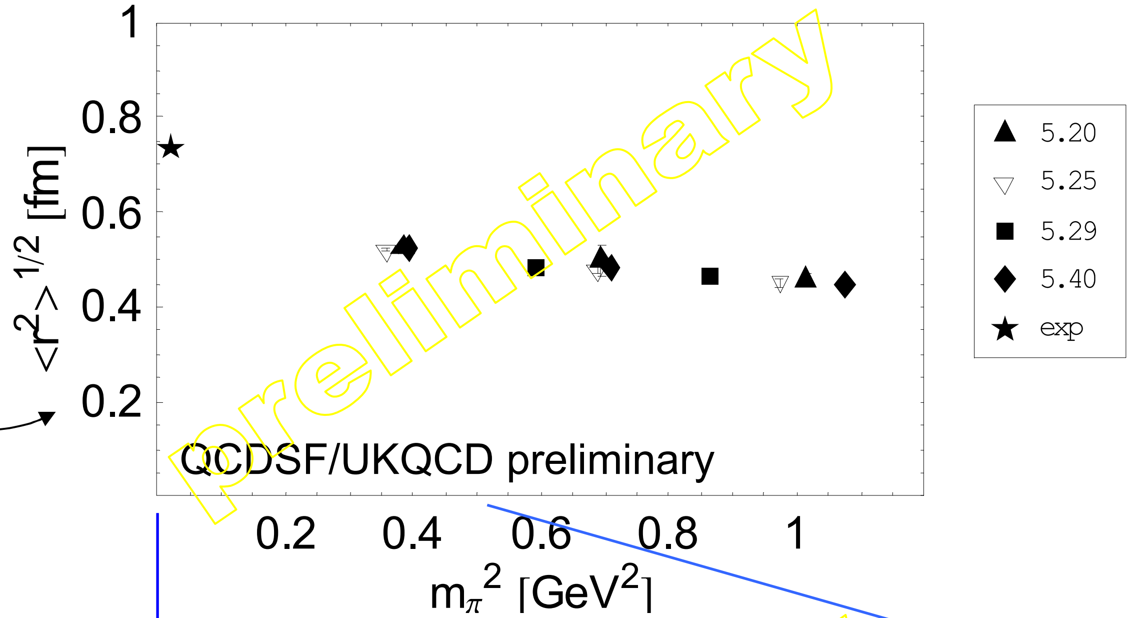
unquenched lattice simulation based on improved Wilson action

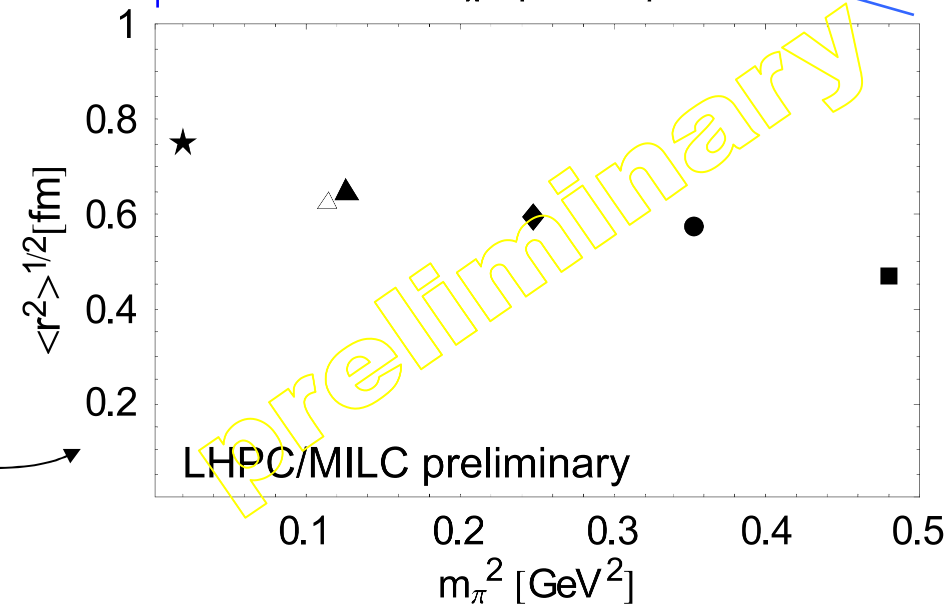
dipole fit restricted to $|t| \leq 1\,\text{GeV}^2$



LHPC/MILC

lattice simulation based on hybrid approach of staggered sea and domain wall valence quarks

dipole fit restricted to $|t| \leq 0.5\,\text{GeV}^2$



need extrapolation based on χPT

finite volume effects

discretization errors

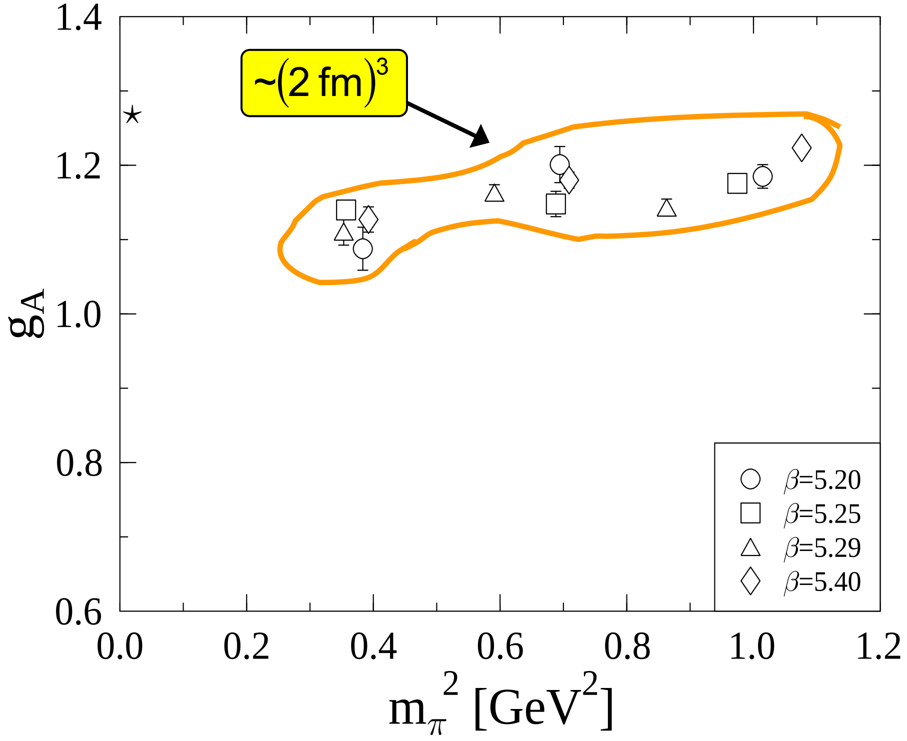
Lattice simulation of the axial vector coupling g_A (1)

iso-vector axial vector current

$$\langle N(p+\Delta)|\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d|N(p)\rangle = \bar{U}(p+\Delta)\left[\gamma_\mu\gamma_5 g_A(\Delta^2) + \Delta_\mu\gamma_5 \frac{g_p(\Delta^2)}{2m}\right]U(p)$$

QCDSF/UKQCD
hep-lat/0603028
based on improved Wilson action

and $g_A(\Delta^2 = 0) = g_A = g_A^3 = g_A^{u-d}$



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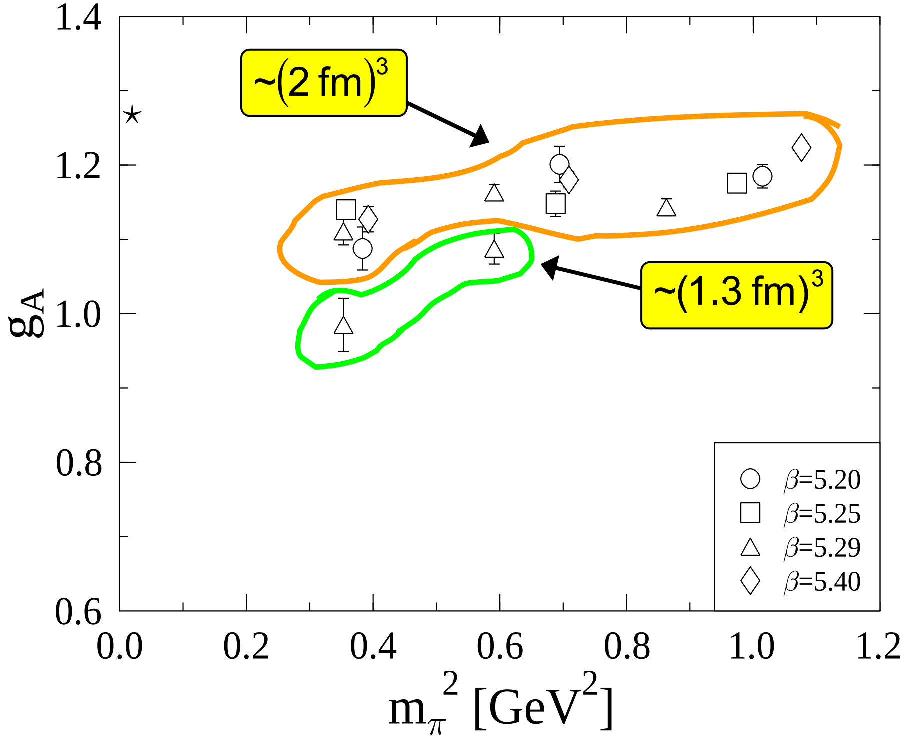

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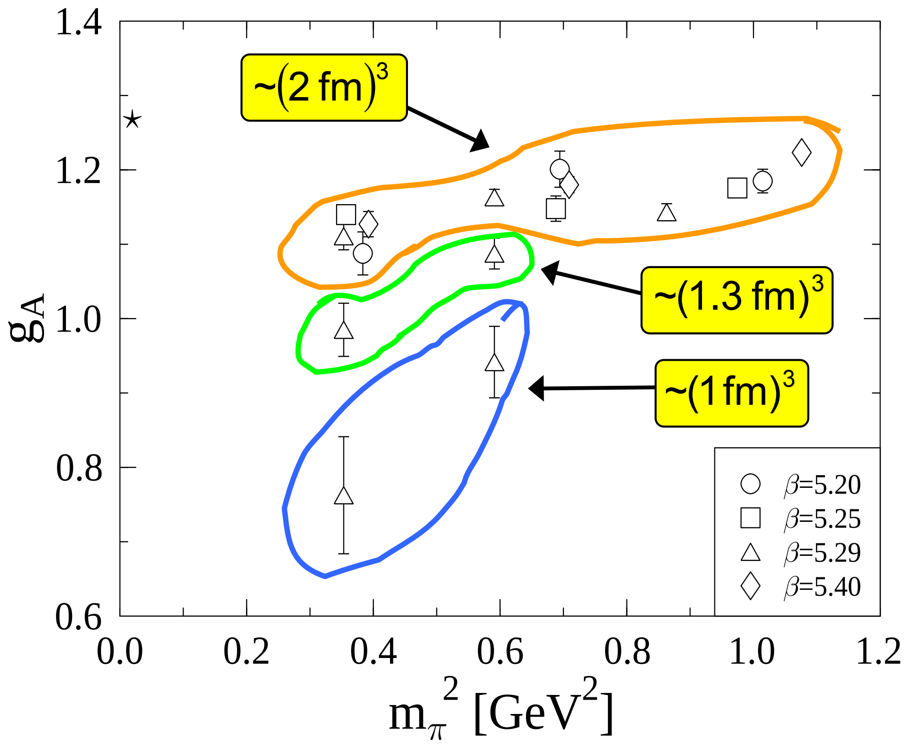

chPT (SSE with Δ-DOFs) based on Hemmert, Procura, Weise PRD 2003

Lattice simulation of the axial vector coupling g_A (1)

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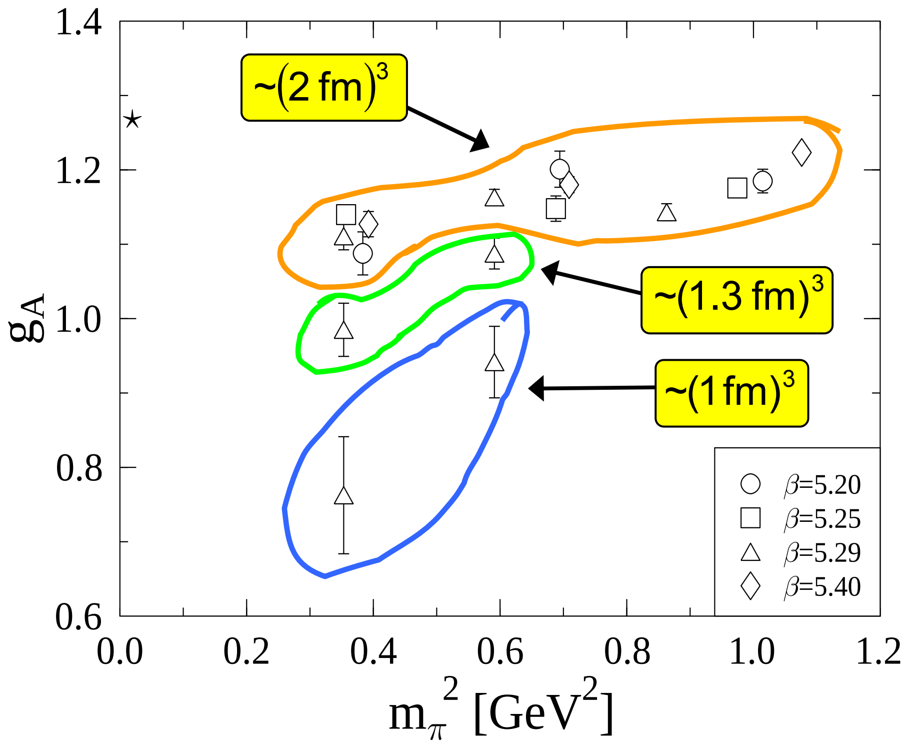

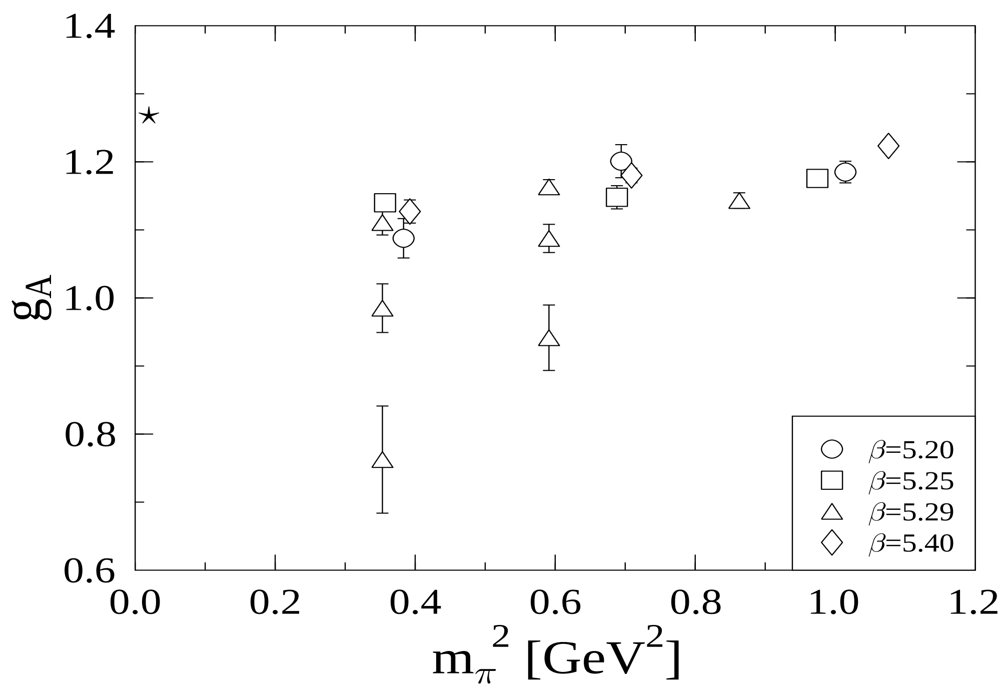

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Lattice simulation of the axial vector coupling g_A (1)

iso-vector axial vector current

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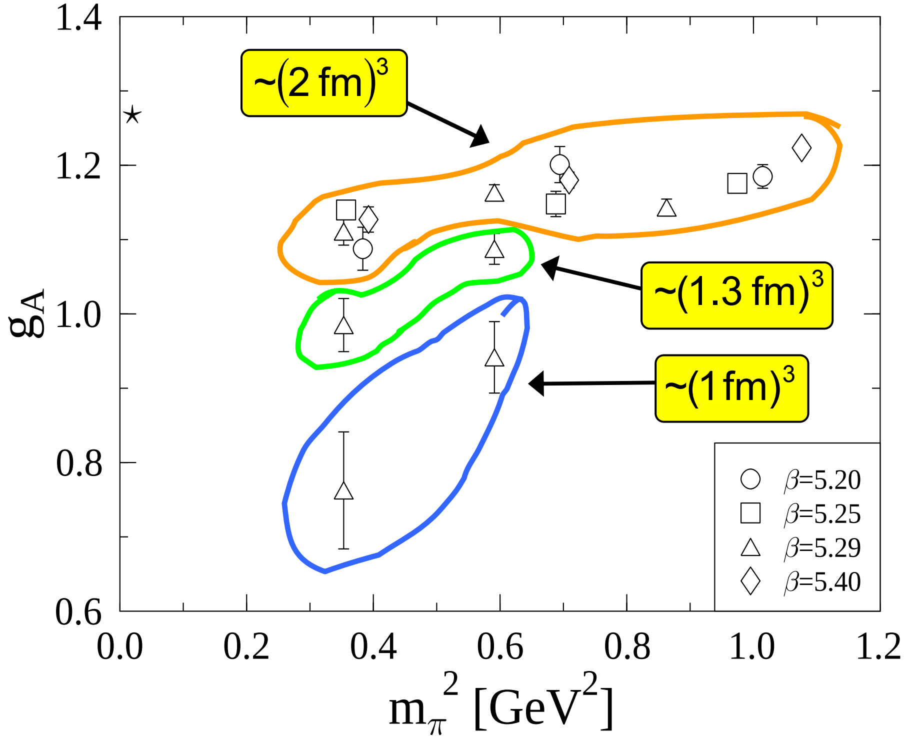
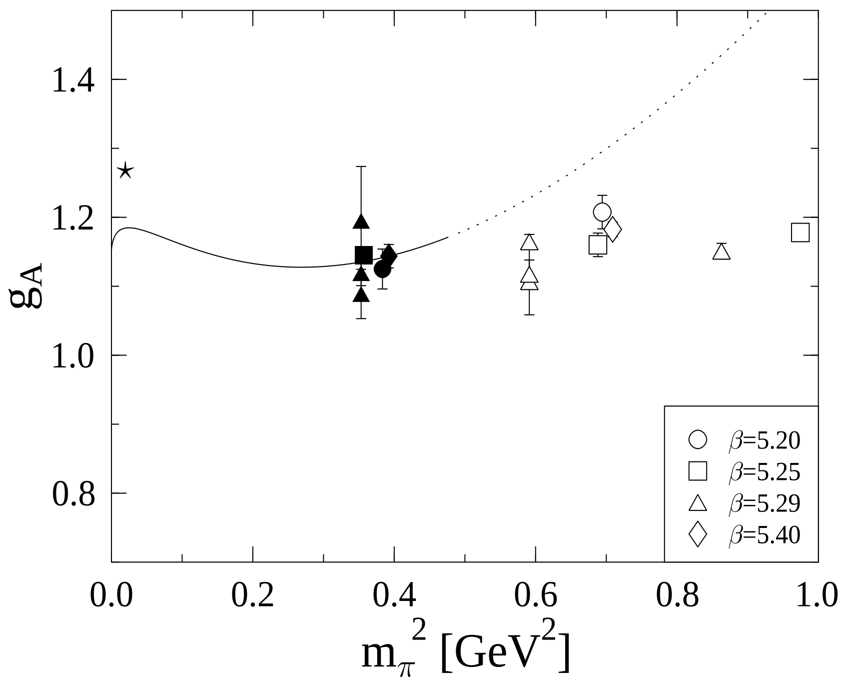
and $g_A(\Delta^2=0) = g_A = g_A^3 = g_A^{u-d}$

QCDSF/UKQCD
hep-lat/0603028
based on improved
Wilson action

fit A : $g_A(m_{\pi,phys}) = 1.18 \pm .13$

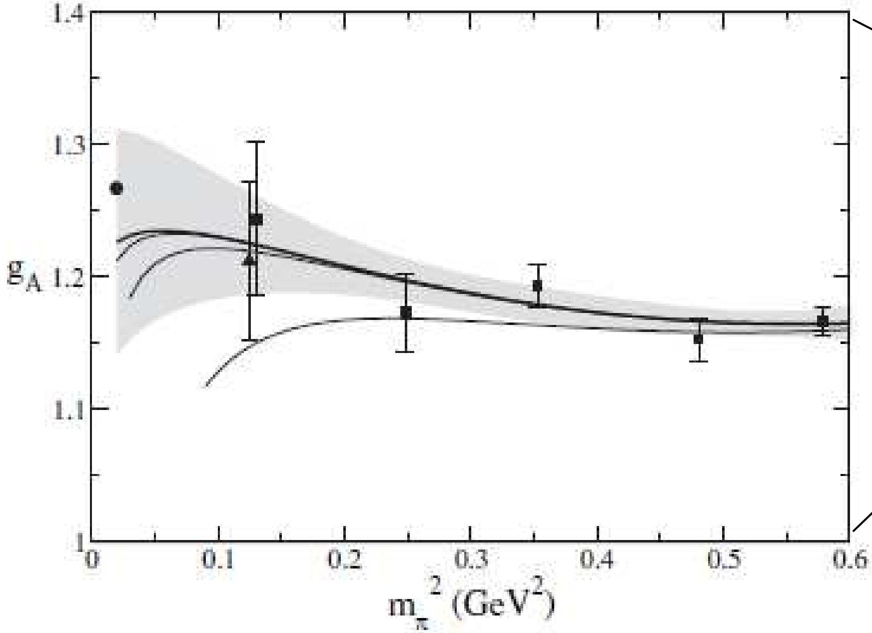




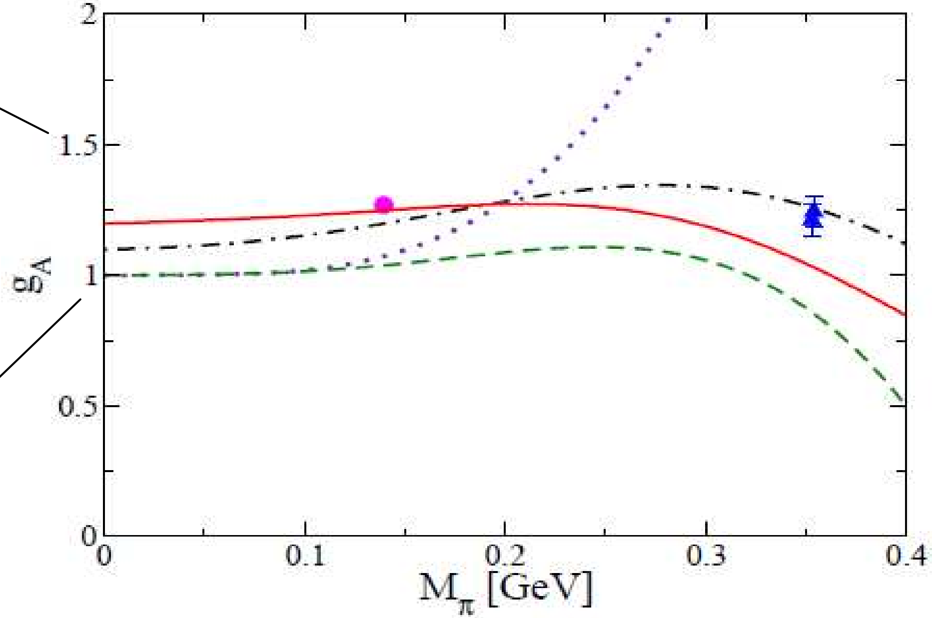
chPT (SSE with Δ-DOFs) based on Hemmert, Procura, Weise PRD 2003

Lattice simulation of the axial vector coupling g_A (2)

LHPC/MILC PRL 2006



Bernard, Meißner hep-ph/0605010



m_π as low as 350 MeV, largest volume $(3.5\ \text{fm})^3$

„beyond 1-loop“- double log contributions

fits based on Savage, Bean PRD 2004
finite volume HBχPT with Δ DOFs, no decoupling

flatness up to ~350MeV **without** explicit Δ-DOFs

possible differences in $\Delta g_A(L)$ compared to SSE

inspiring ongoing dicussion…

The GFFs A,B and C

$$\langle P',\Lambda'|\bar{q}\gamma_{\{\mu}D_{\nu\}}q|P,\Lambda\rangle = \bar{U}\left\{\gamma_{\{\mu}\bar{P}_{\nu\}}A_{20}(t) - \frac{i\Delta^{\rho}\sigma_{\rho\{\mu}\bar{P}_{\nu\}}}{2m}B_{20}(t) + \frac{\Delta_{\mu}\Delta_{\nu}}{m}C_{20}(t)\right\}U$$

LHPC/MILC (preliminary)

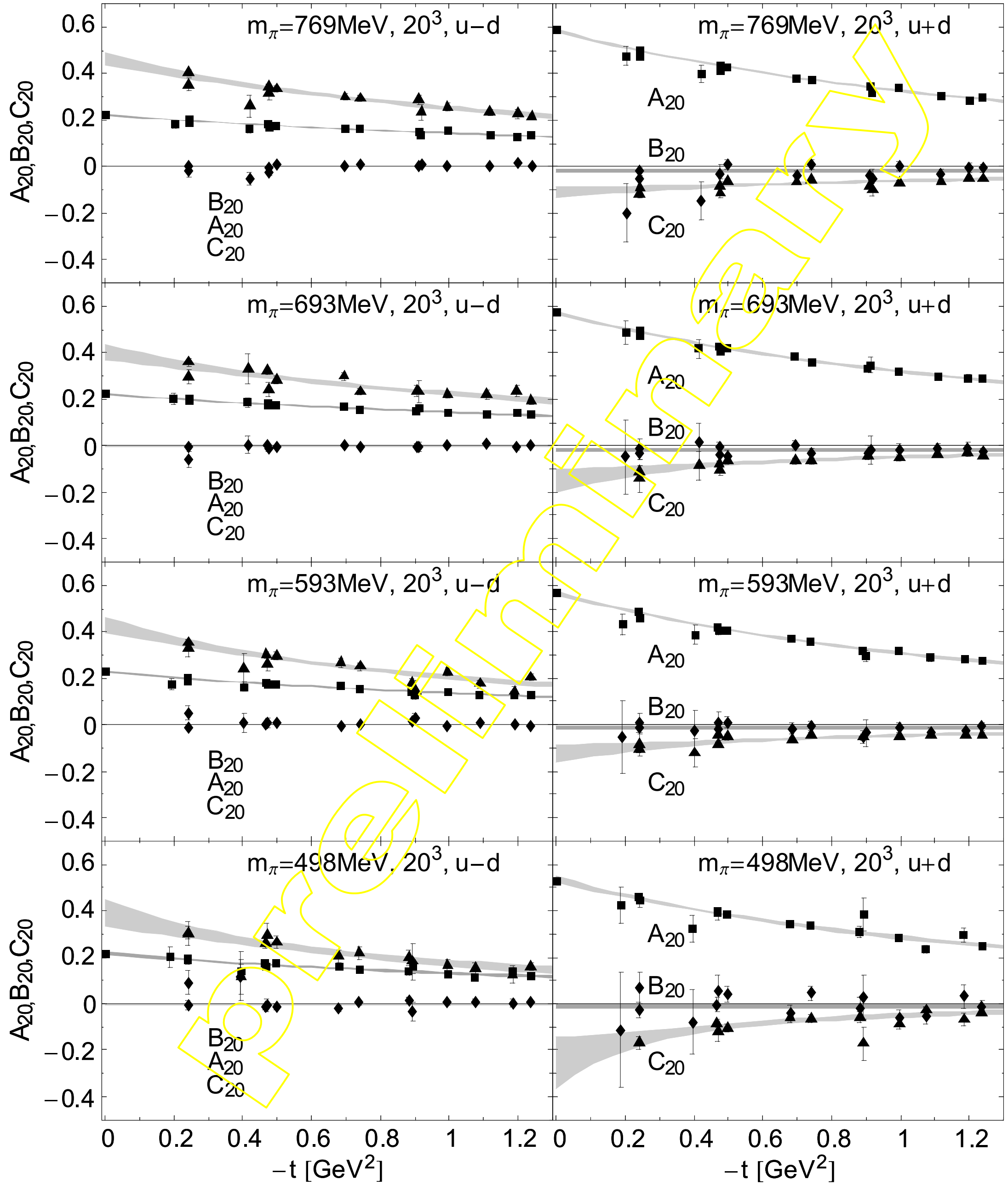
disconnected contributions
are not included↔only u-d is „exact“

in summary

$B_{20}^{u-d} > A_{20}^{u-d}$
$C_{20}^{u-d} \approx 0$

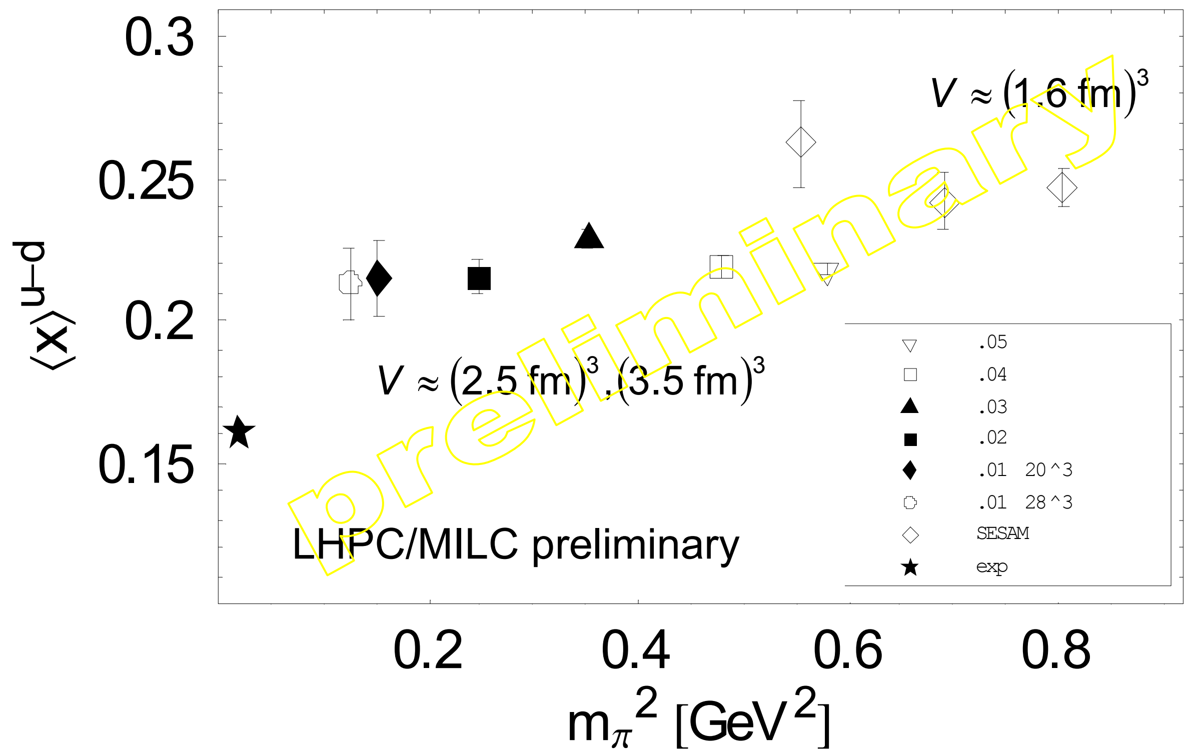
$A_{20}^{u+d} > B_{20}^{u+d} \approx 0$
$C_{20}^{u+d} < 0$

seems to be very compatible with
large N_c limit – see e.g. Goeke, Polyakov
and Vanderhaeghen PiPaNP 2001



Momentum fraction of quarks in the nucleon

$\langle P|\bar{q}(0)\gamma^{\{\mu}D^{\nu\}}q(0)|P\rangle = \bar{U}(P)\gamma^{\{\mu}\bar{P}^{\nu\}}U(P)\langle x\rangle$ ↔ $\langle x\rangle = A_{20}(0) = \int_{-1}^{1} dx\; x\, q(x) = \langle x\rangle_q + \langle x\rangle_{\bar{q}}$



at a scale of $Q^2 = 4\ \text{GeV}^2$

at $m_\pi \approx 350$ MeV, the u-d momentum fraction is ~40% larger than at $m_{\pi,phys}$

chiral extrapolation/chiral logs

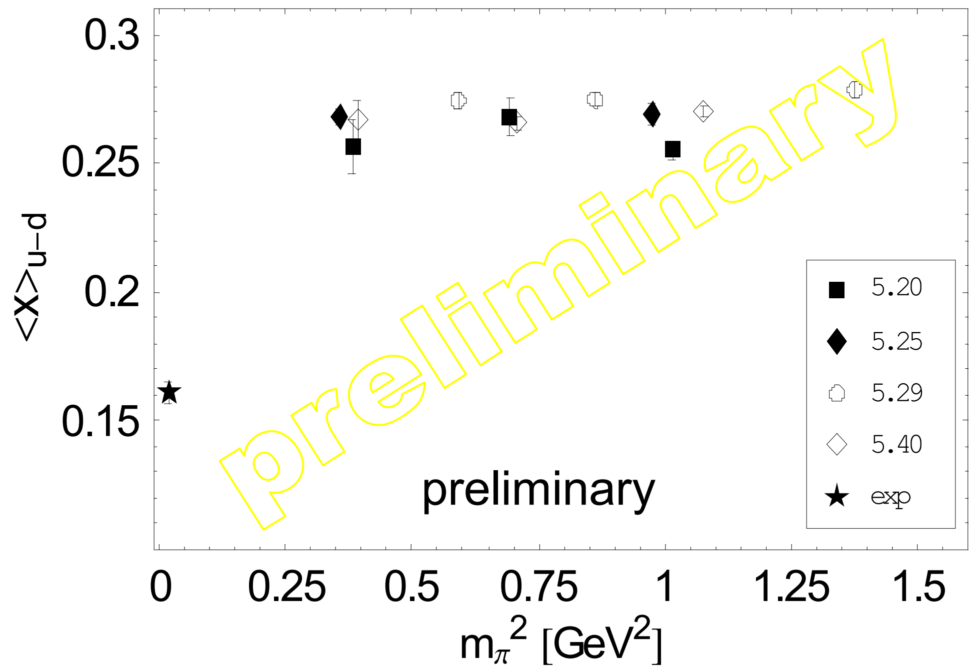
finite volume effects

discretization errors

Momentum fraction of quarks in the nucleon

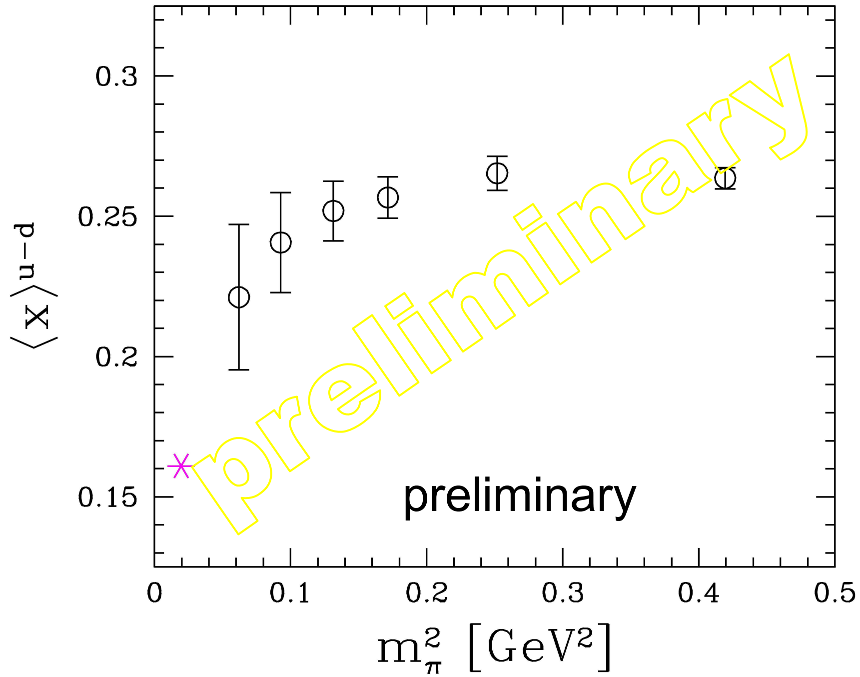
QCDSF/UKQCD

QCDSF/UKQCD unquenched improved Wilson



$V \approx (1.5 - 2.2\ \text{fm})^3$

QCDSF quenched overlap



$V \approx (2.5\ \text{fm})^3$

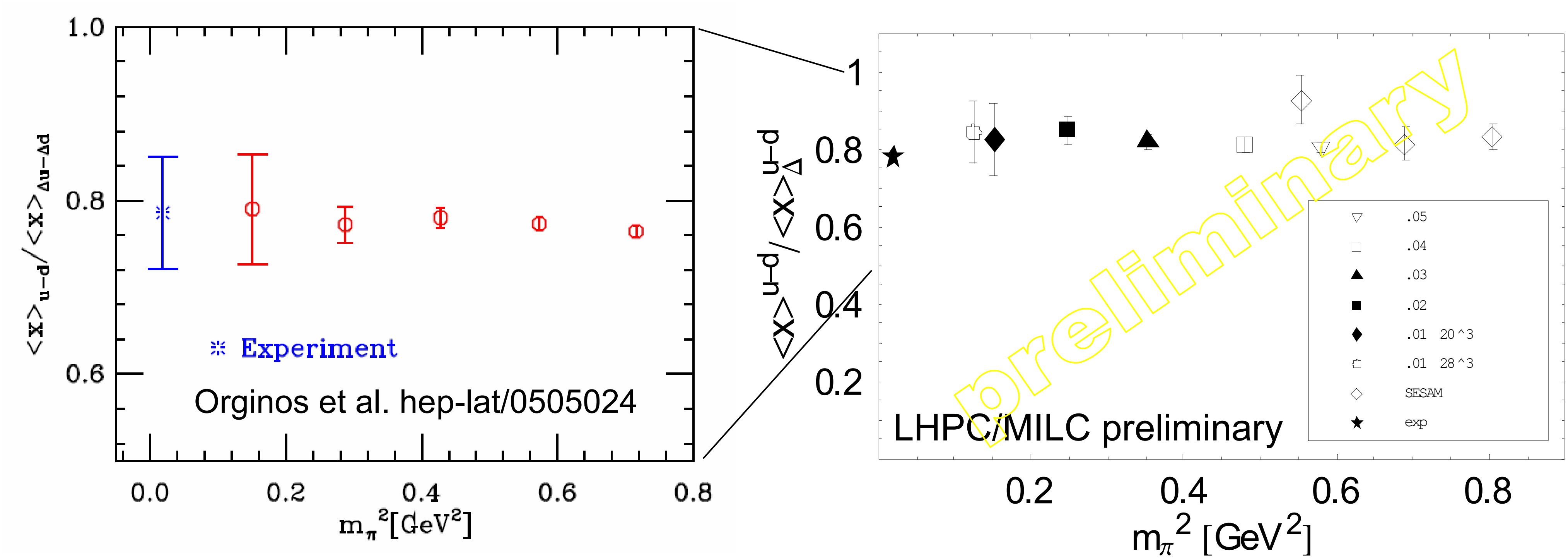
first indication of a downward bending?

unquenched calculations at m_π<300 MeV are in progress

Momentum fraction of quarks in the nucleon - ratios

quenched domain wall

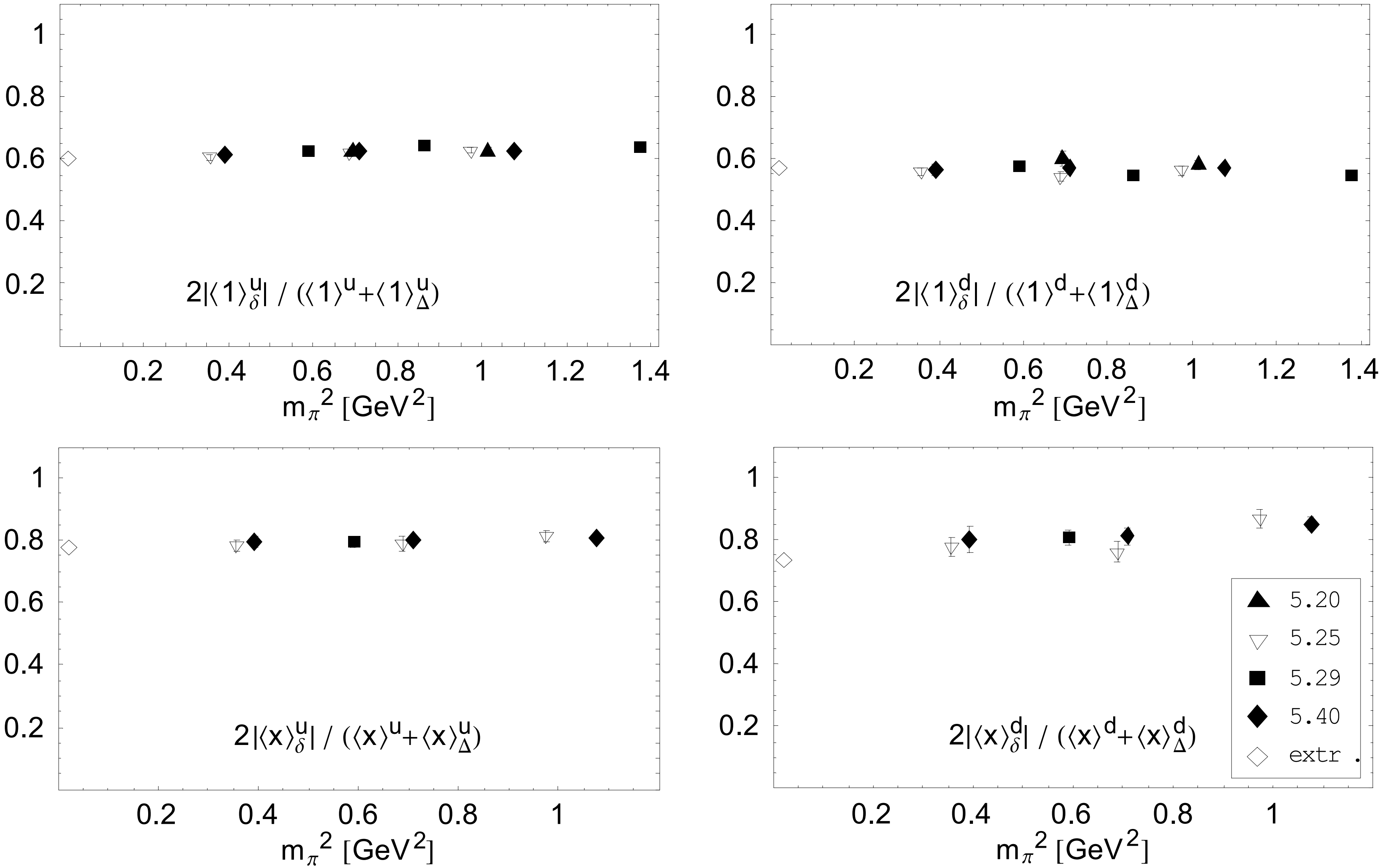
staggered sea + DW valence



linear extrapolation works…possibly due to cancellations in the pion mass dependence

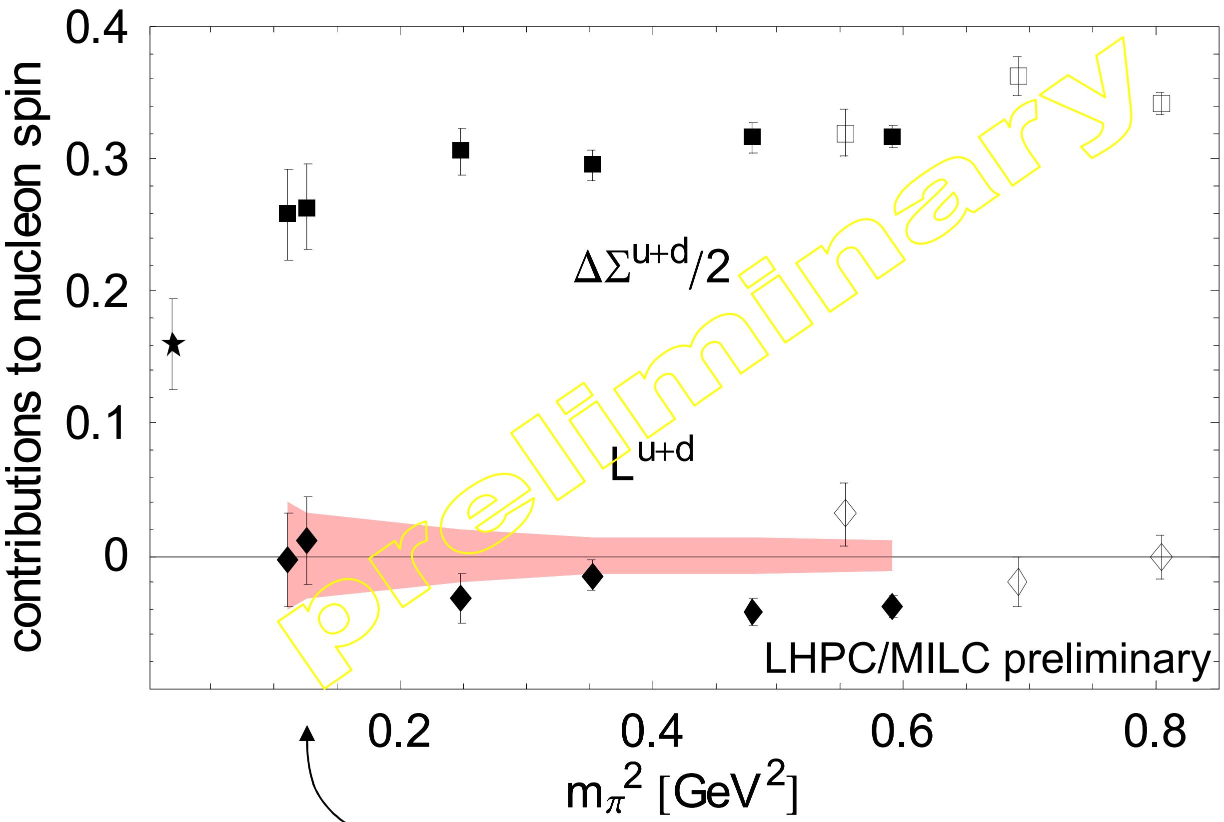
Lowest two moments of the Soffer - bound $q + \Delta q \geq 2|\delta q|$ in lattice QCD

from QCDSF/UKQCD PLB 2005, unquenched improved Wilson



if cancellation for ratios works, this is a lattice prediction for the Soffer bound

Quark spin and OAM contributions to the nucleon spin

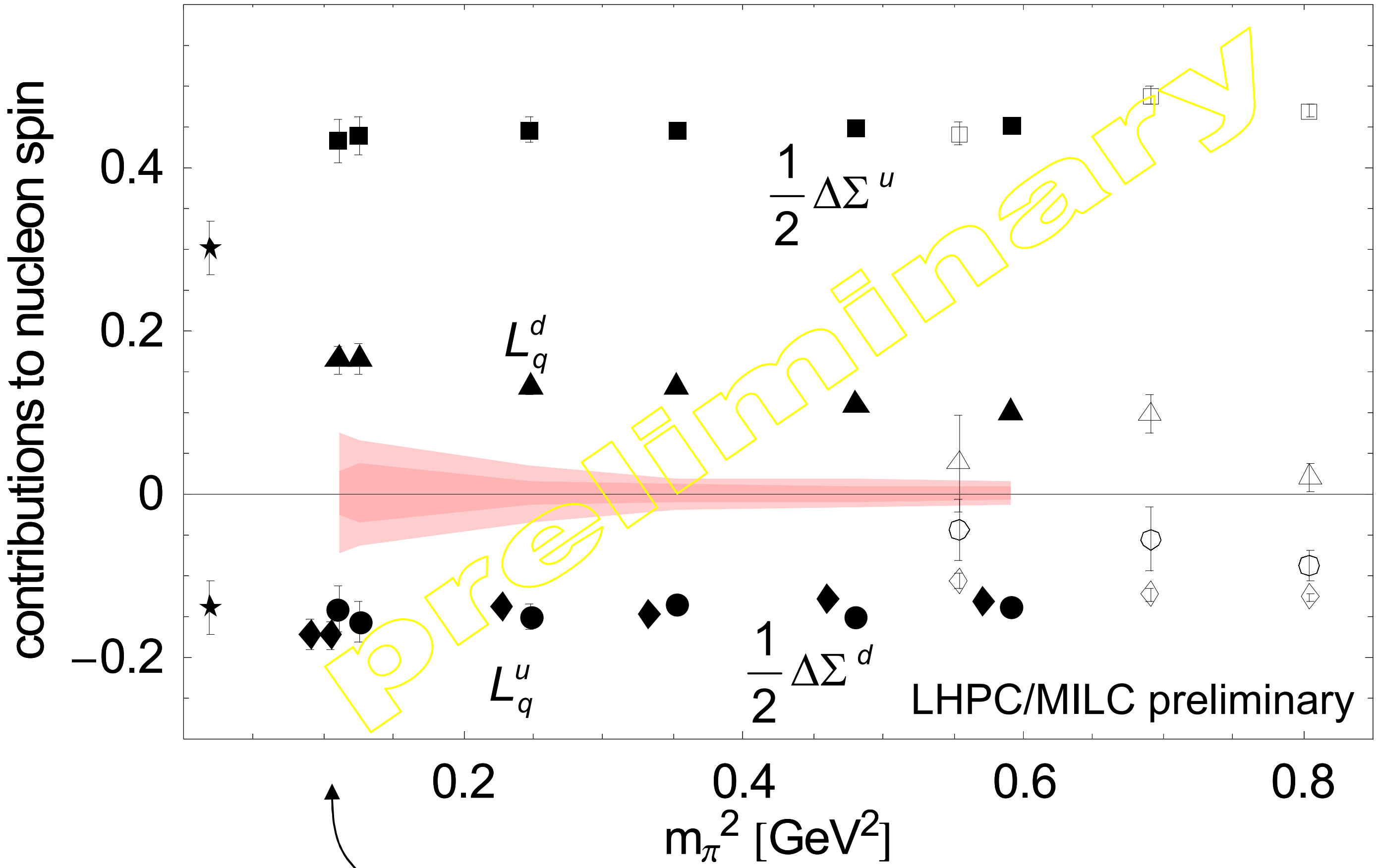


at $m_\pi \approx 350\,\mathrm{MeV}$

$$L_q^{u+d} = J_q^{u+d} - \frac{1}{2}\Delta\Sigma^{u+d} = \frac{1}{2}\left(\langle x\rangle^{u+d} + B_{20}^{u+d}(0) - \Delta\Sigma^{u+d}\right) \approx 0 \quad \longleftrightarrow \quad B_{20}^{u+d}(0) \approx 0 \text{ and } \Delta\Sigma^{u+d} \approx \langle x\rangle^{u+d}$$

interesting conjecture: $B_{20,phys}^{u+d}(0) \approx B_{20,phys}^{g}(0) \approx 0 \Rightarrow L_q^{u+d} \approx \frac{1}{2}\left(\langle x\rangle^{u+d} - \Delta\Sigma^{u+d}\right)$

Quark spin and OAM contributions to the nucleon spin



at $m_\pi \approx 350\,\mathrm{MeV}$

$L_q^d \approx -L_q^u \approx 30\,\%$ of $\frac{1}{2}$

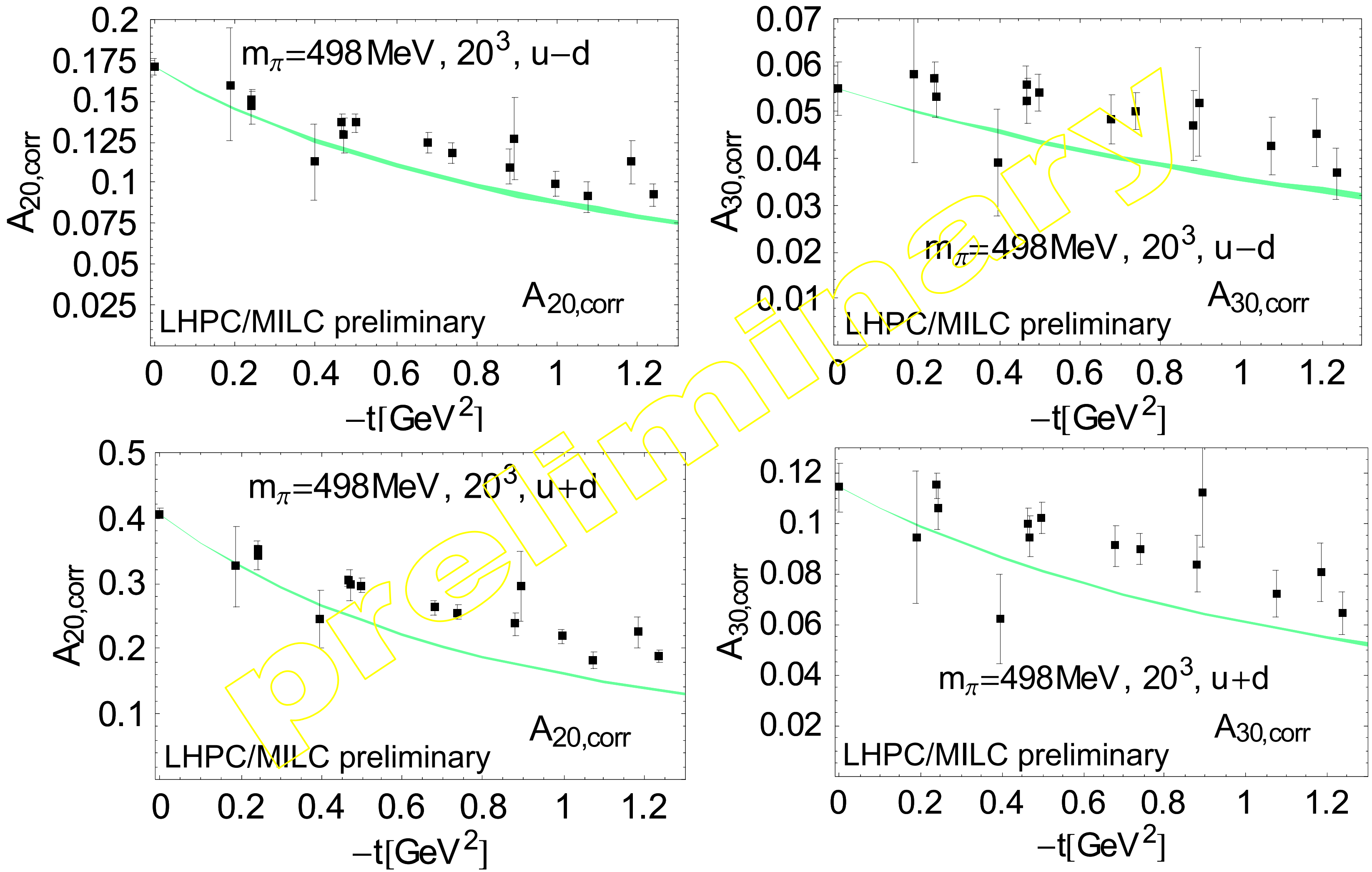
normalized lattice results compared to parametrization/ansatz by Diehl,Feldmann,Jakob,Kroll EPJC 2005

based on nucleon form - factor data, CTEQ – PDFs and the ansatz $H(x,\xi=0,t)=q_v(x)\exp\left(t\,f_v(x)\right)$

$A_{20}(t)=\int dx\,x\,H(x,0,t)$ $A_{30}(t)=\int dx\,x^2 H(x,0,t)$

u-d

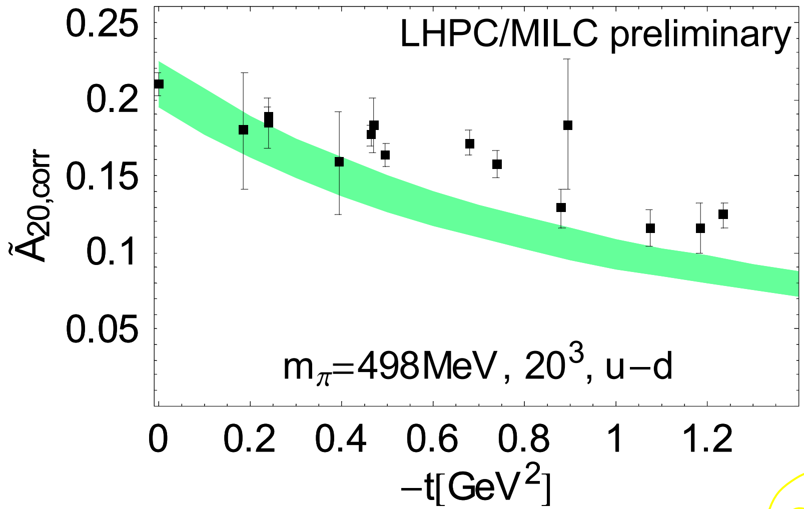
u+d

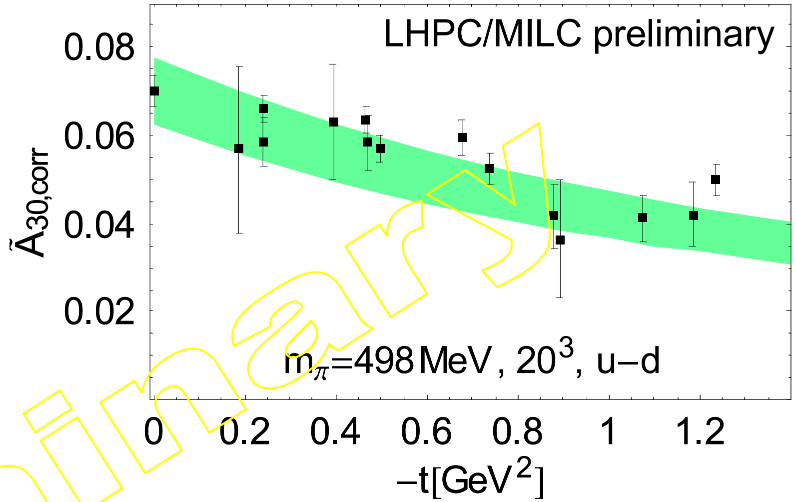


$$\tilde{A}_{20}(t) = \int dx\, x \tilde{H}(x,0,t)$$

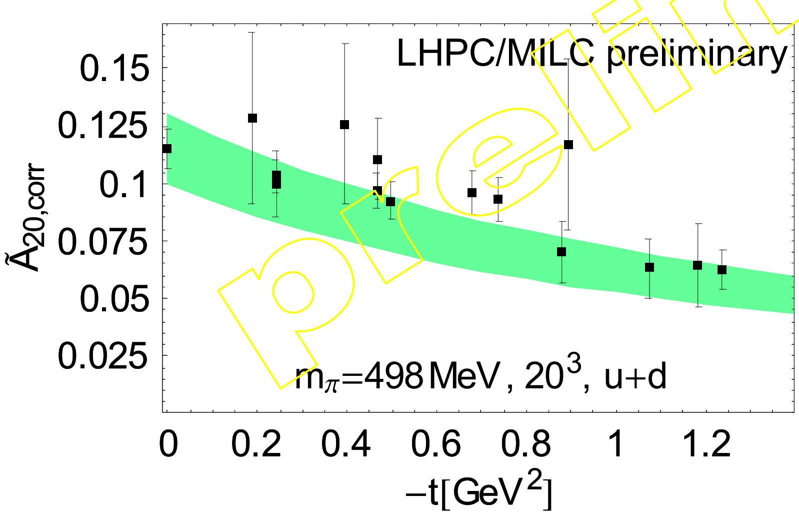
$$\tilde{A}_{30}(t) = \int dx\, x^{2} \tilde{H}(x,0,t)$$

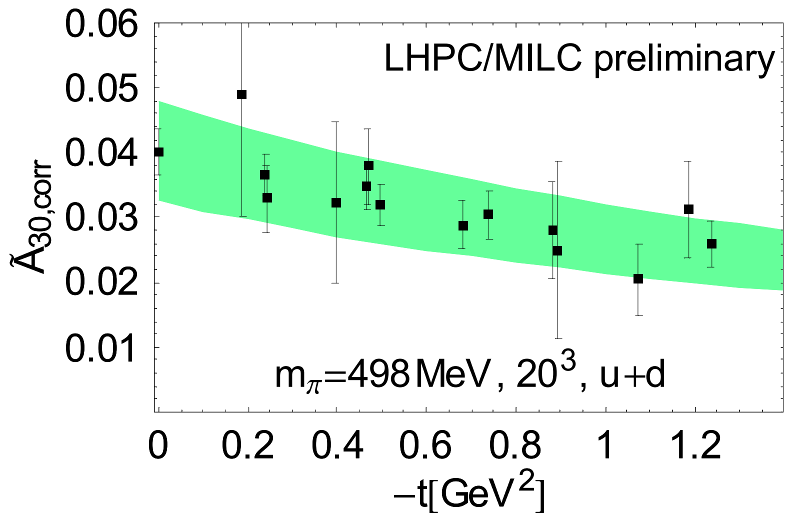
u-d





u+d





Parametrizing the GFFs: The p-pole ansatz

needed to get GPDs/GFFs in impact parameter space

p - pole ansatz $A(t) = \frac{A(0)}{\left(1 - t/m_p^2\right)^p}$ → FT → $A(b^2) = C(A(0), p)\, b^{p-1} K_{p-1}(m_p b)$

pro :
- describes almost all GFFs reasonably well over the whole range of momentum transfer
- simple relation to charge radius
- only 2(3) parameters
- simple Fourier - transform

contra :
- no sound theoretical foundation, purely phenomenological
- it is hard to determine m_p and p simultaneously from fit

(Mellin moments of) quark densities should be well behaved for $b \to 0$ and $b \to \infty$

for finite densities, we need

$H, \tilde{H}, H_T$: $p > 1$

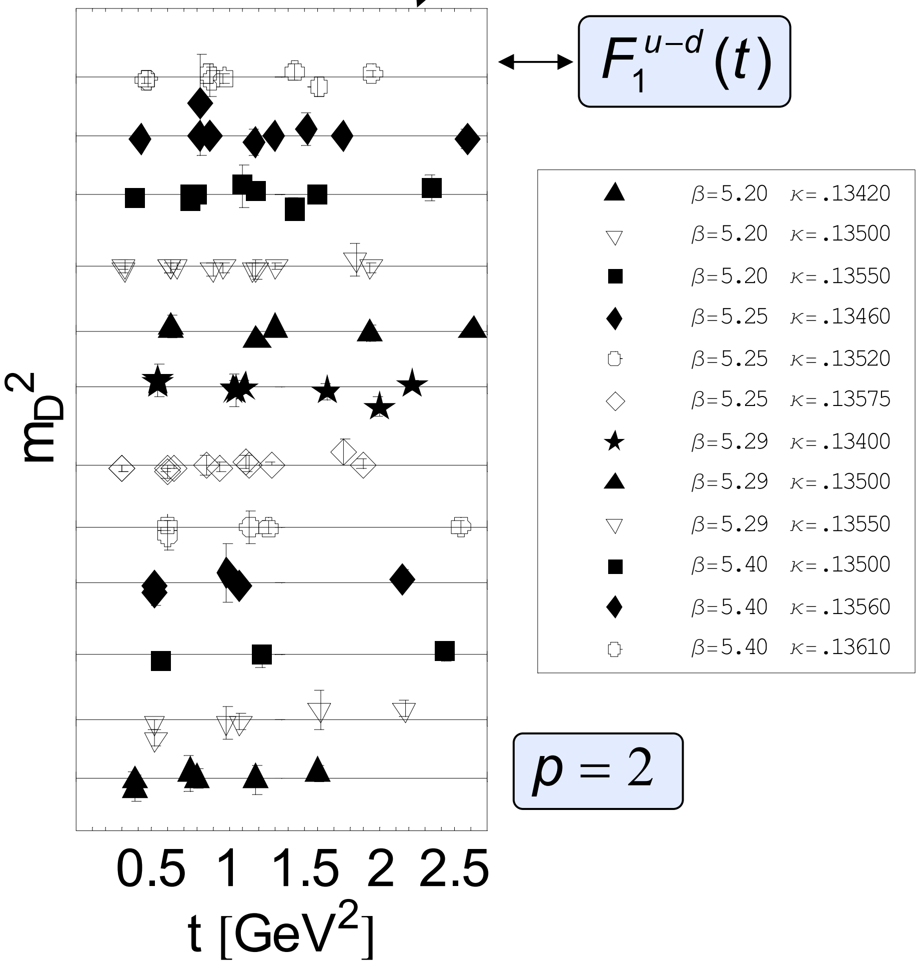
$E, \overline{E}_T$: $p > \frac{3}{2}$

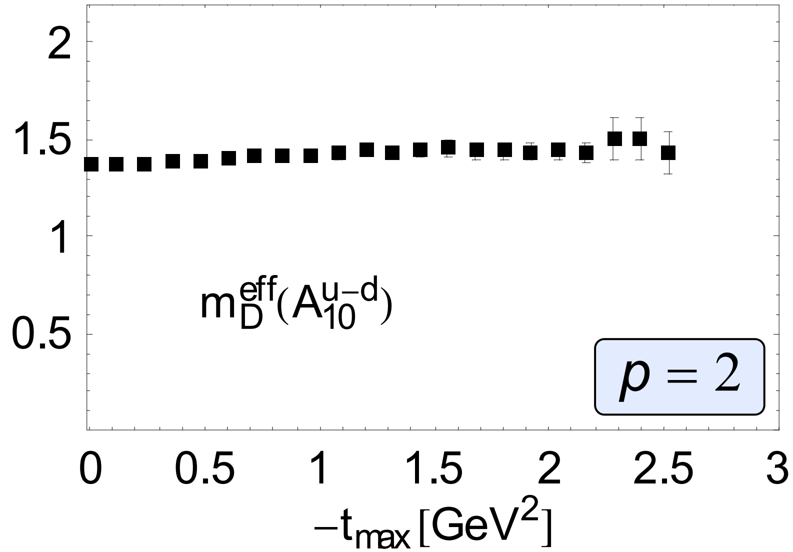
$\tilde{H}_T$: $p > 2$

Testing the dipole parametrization

QCDSF/UKQCD

$$A_{n0}(t) = \frac{A_{n0}(t)}{\left(1 - t / m_{n,p}^2\right)^p} \rightarrow m_p^2 = m_p^2\left(t, A_{n0}(t)\right)$$



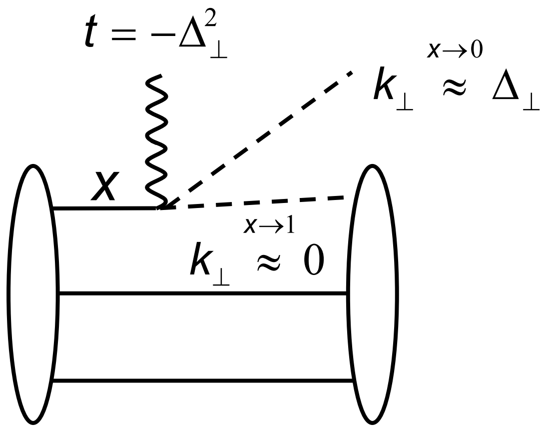


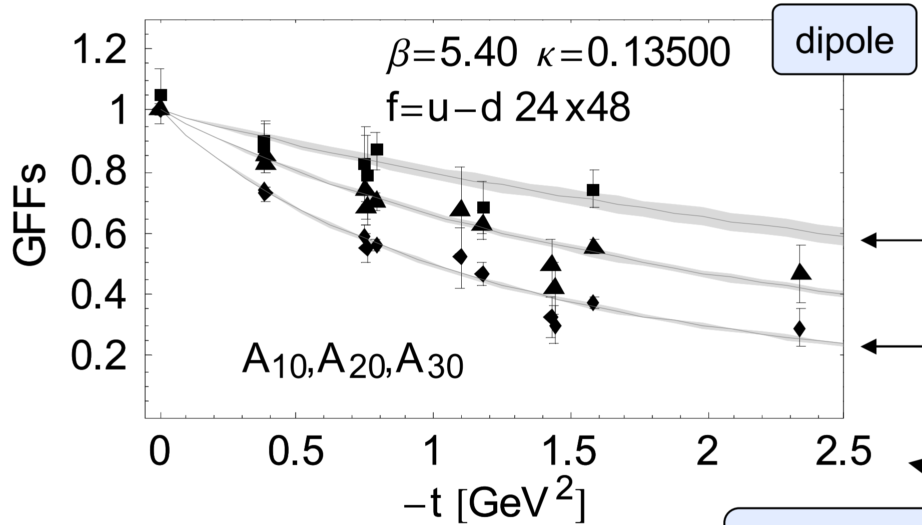
„3d“-nucleon structure (unpolarized) – momentum space

higher moments n correspond to larger momentum fraction x

$$n \to \infty \quad \Leftrightarrow \quad x \approx 1$$

$$\left\langle P^+, R_\perp = 0 \middle| \hat{\rho}(x, b_\perp) \middle| P^+, R_\perp = 0 \right\rangle = H(x, b^2) = q(x, b^2)$$

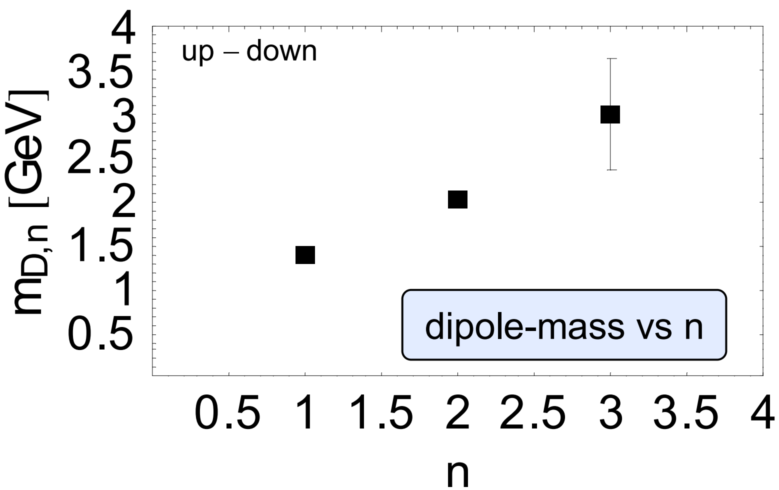




strong coupling larger <x>, n=3

weak coupling smaller <x>, n=1

first shown in LHPC/SESAM PRL 2004

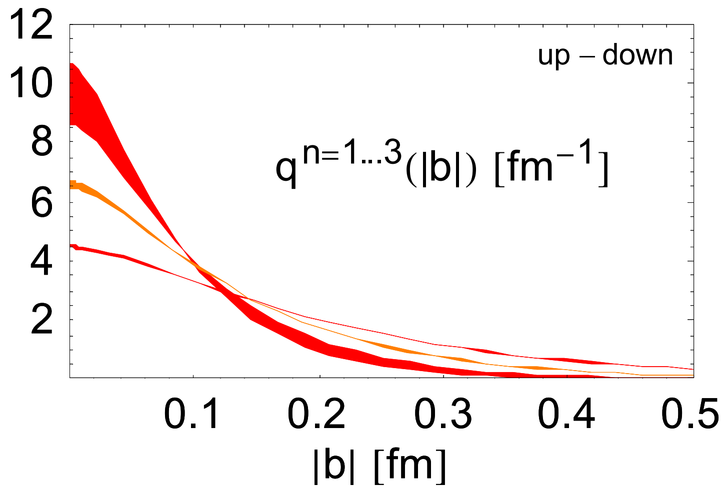


$$\int dx\, x^{n-1}$$

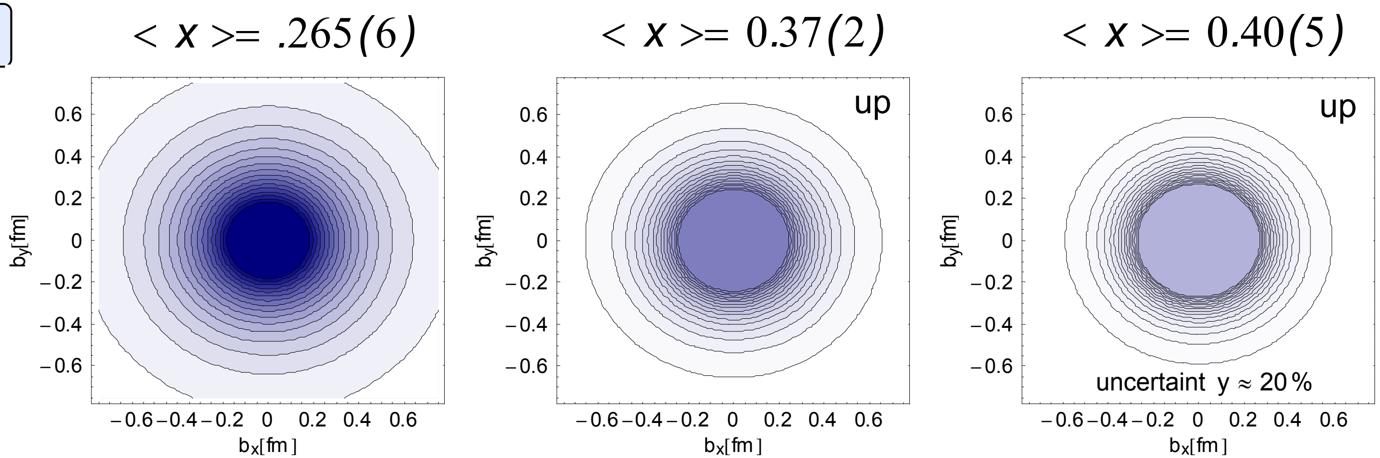
„3d"-nucleon structure (unpolarized) – coordinate space

$b_\perp \hat{=}$ distance of active quark to the CM $R_\perp = \frac{\sum_i x_i r_{\perp,i}}{\sum_i x_i}$

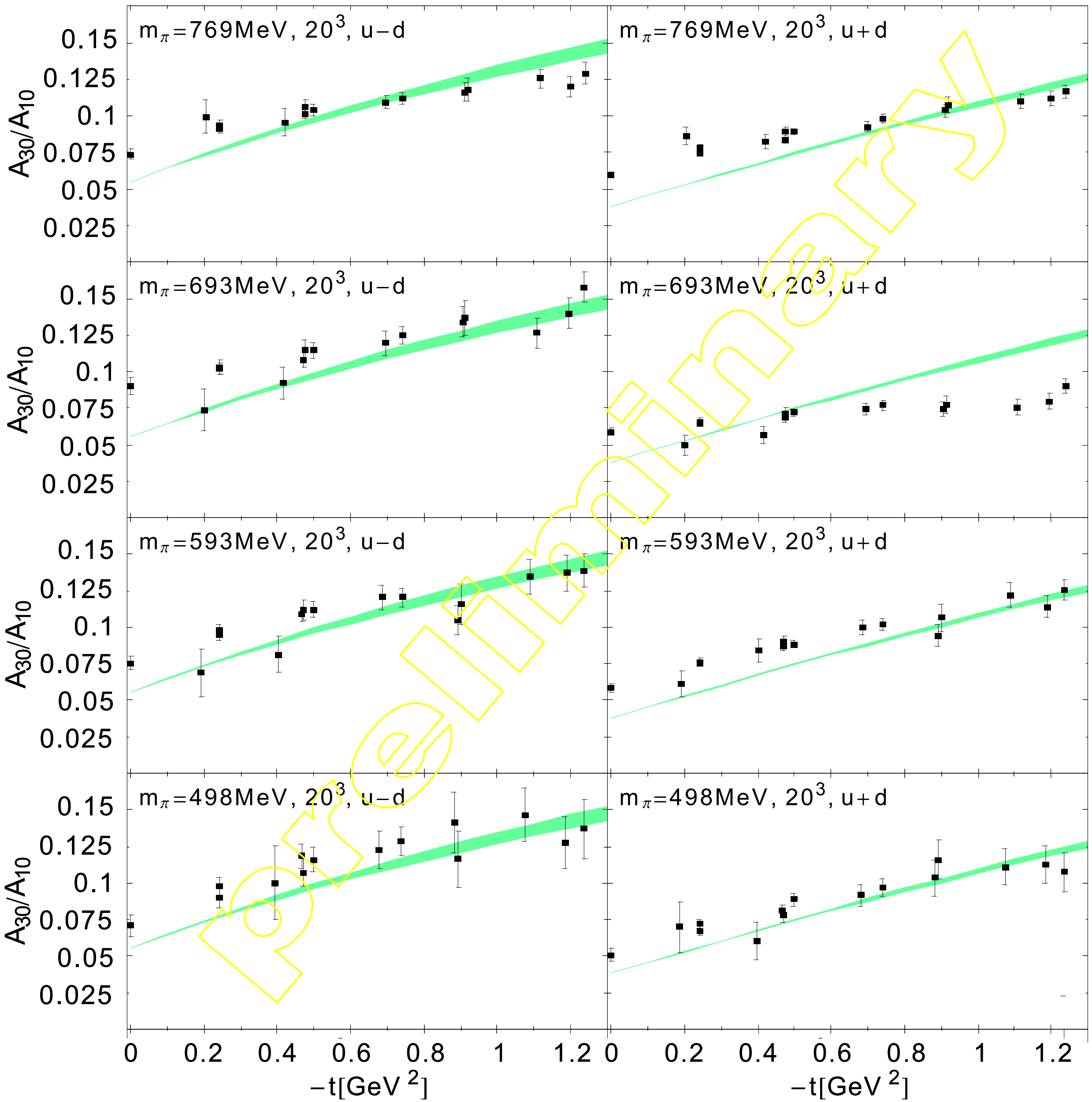
as $x \to 1$, the distribution peaks around $R_\perp$ and $\langle b_\perp^2 \rangle^{1/2} \to 0$



charge radius vs <x>



Preliminary results from LHPC/MILC for ratios of GFFs



$$\frac{A_{30}(t)}{A_{10}(t)} = \frac{A_{30}(t)}{F_{10}(t)} \quad \text{where}$$

$$A_{30}(t) = \int dx \left[x^2 H(x, \xi = 0, t) \right]$$

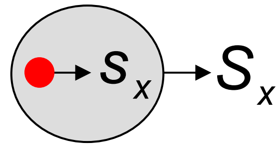
compared to parametrization/ ansatz by Diehl, Feldmann, Jakob, Kroll EPJC 2005

remarkable overall agreement at $m_\pi \approx 500$MeV

Ph. Hägler, QCDN‘06

Spin densities in the transverse plane

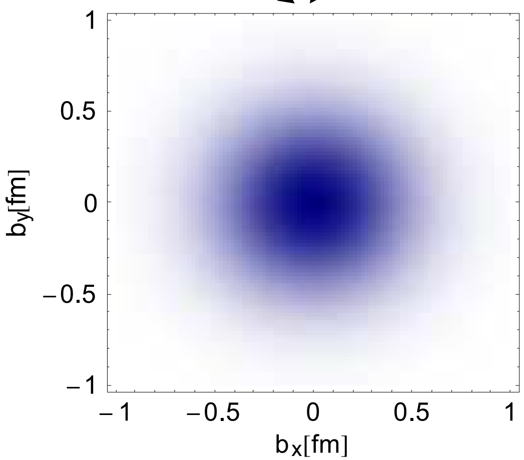
based on M. Diehl (DESY) and Ph.H., EPJC 44 (2005)



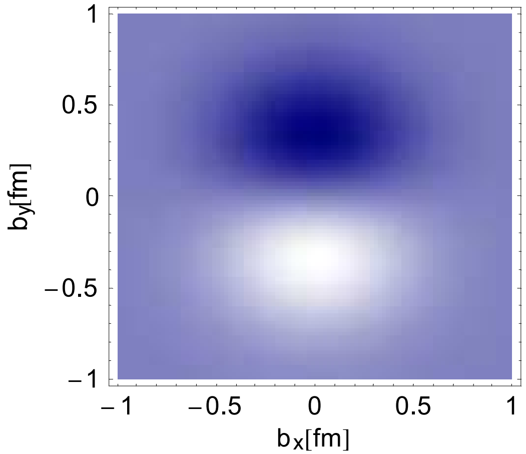
spin density for transversely polarized quarks

$$\left\langle P^+, R_\perp = 0, \Lambda, S_\perp \left| \hat{\rho}^n(b_\perp, s_\perp) \right| P^+, R_\perp = 0, \Lambda, S_\perp \right\rangle =$$

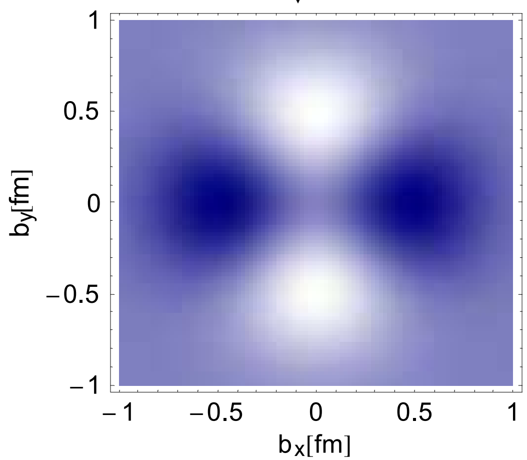
$$\frac{1}{2}\left[A_{n0} + s^i S^i \left(A_{Tn0} - \frac{1}{4M^2} \Delta_b \tilde{A}_{Tn0} \right) - \frac{S^i \epsilon^{ij} b^j}{M} B_{n0}{}' - \frac{s^i \epsilon^{ij} b^j}{M} \overline{B}_{Tn0}{}' + s^i \left(2b^i b^j - b^2 \delta^{ij} \right) S^j \frac{1}{M^2} \tilde{A}_{Tn0}{}'' \right]$$



monopole



dipole

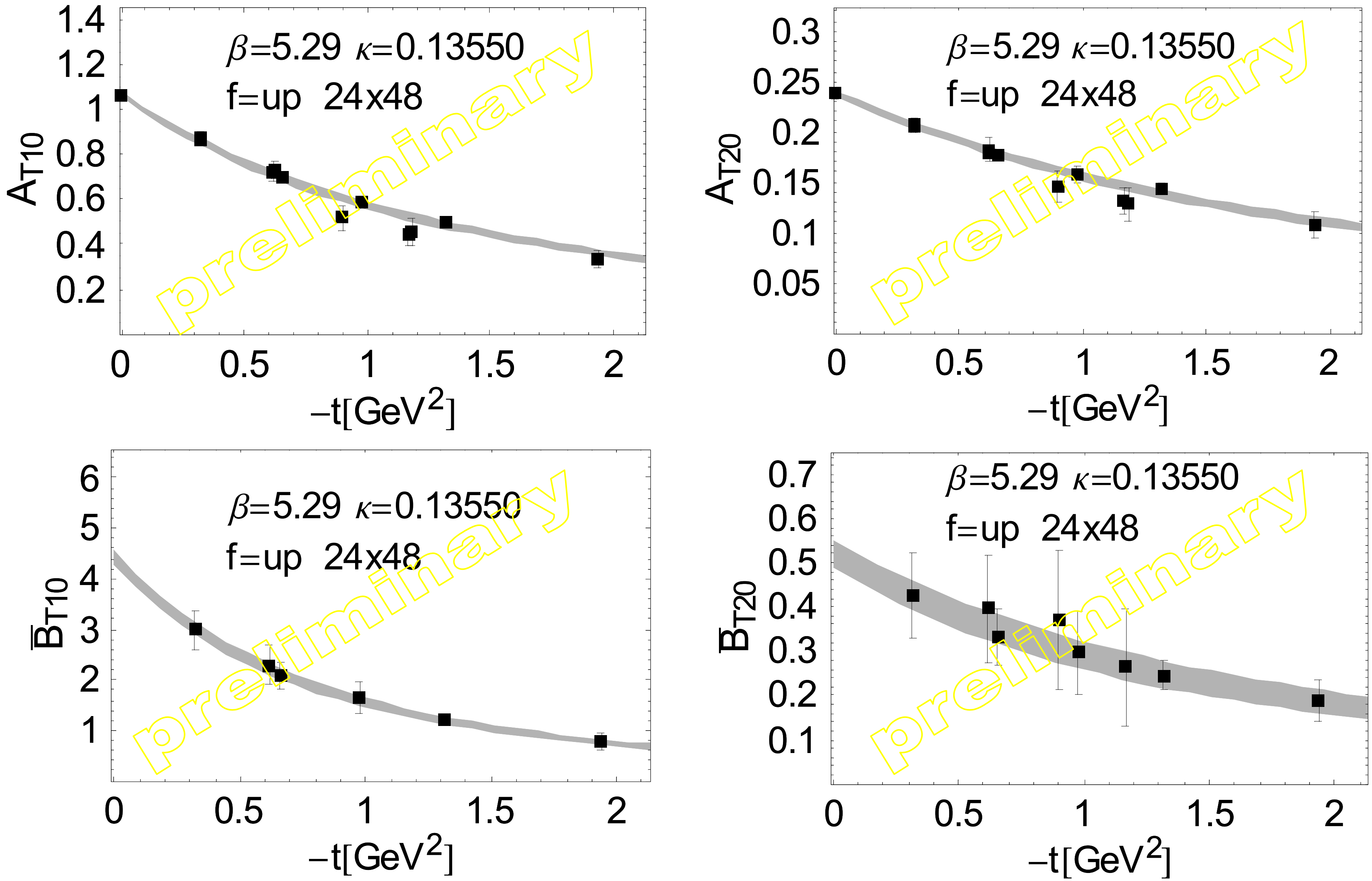


quadrupole

$$\hat{\rho}^n(b_\perp, s_\perp) = \int_{-1}^{1} dx\, x^{n-1} \int \frac{d\eta}{2\pi} e^{i\eta x}\, \overline{q}(-\eta n/2, b_\perp) \left[\gamma^+ + i s_{\perp j} \sigma^{+j} \gamma_5 \right] q(\eta n/2, b_\perp)$$

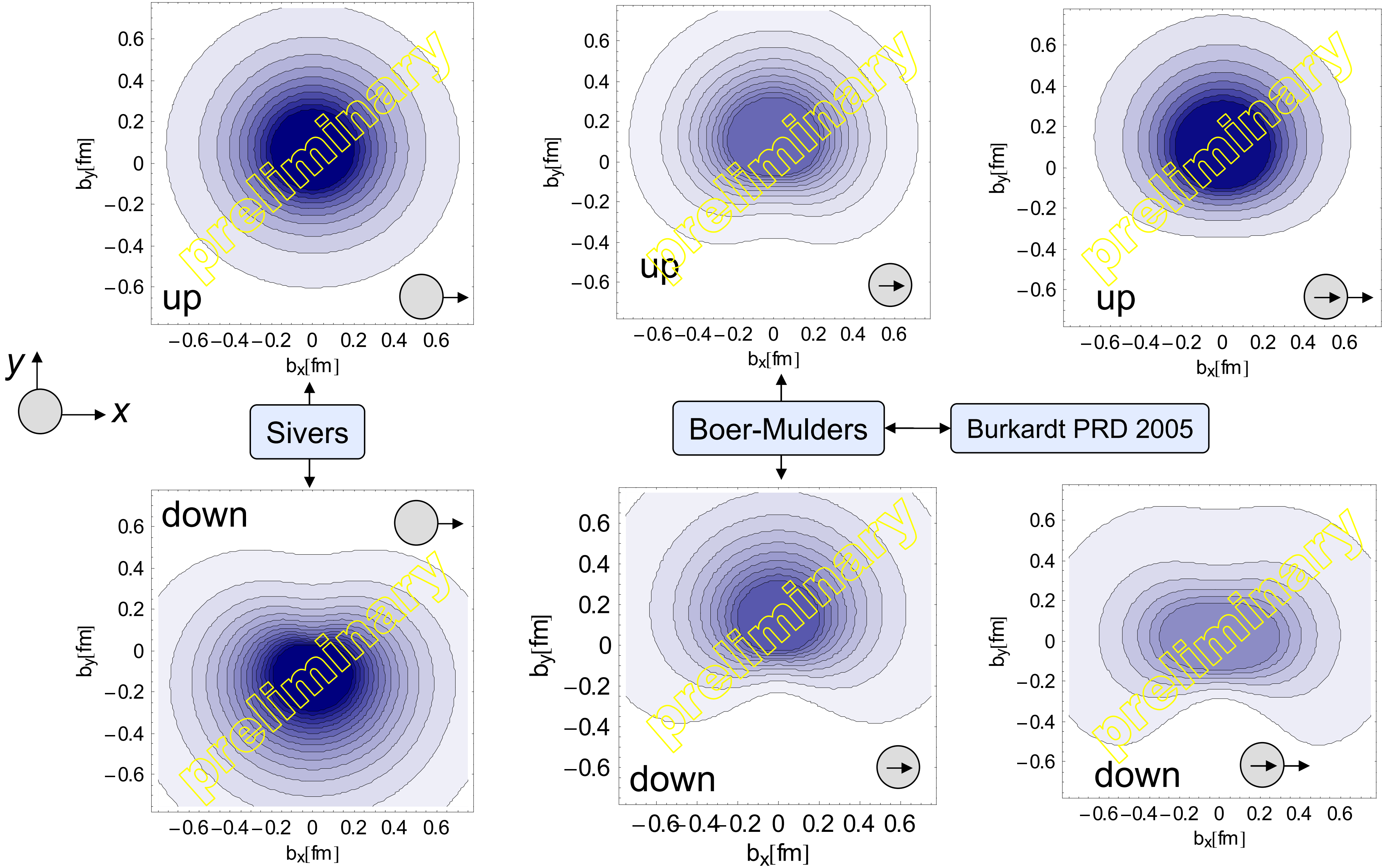
Preliminary results for the tensor GFFs

QCDSF/UKQCD



Preliminary results for the spin densities

QCDSF/UKQCD



Deformed quark densities and spin asymmetries

Sivers - function $f_{1T}^{\perp}(x,p_{\perp})$ probes correlation of transverse nucleon spin and intrinsic transverse momentum

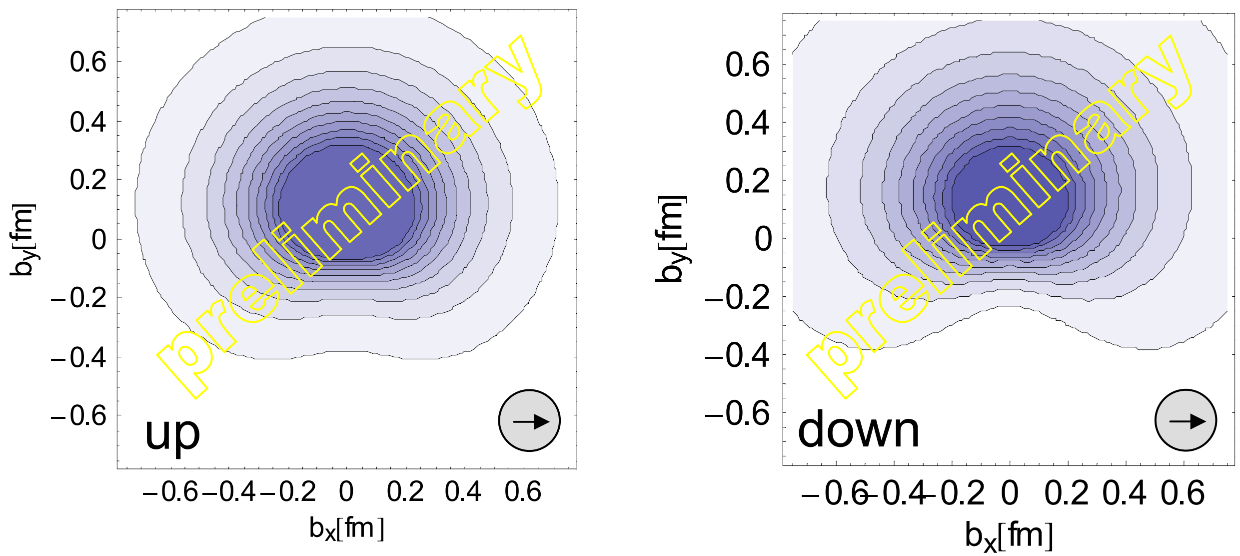
$$f_{1Tq}^{\perp}(x,p_{\perp}) \sim -\int dx E_q(x,0,0) = -B_{10,q}(0) = -\kappa_q$$

Burkardt PRD 2005

Boer - Mulders - function $h_1^{\perp}(x,p_{\perp})$ probes correlation of transverse quark spin and intrinsic transverse momentum

$$h_{1q}^{\perp}(x,p_{\perp}) \sim -\int dx \overline{E}_{Tq}(x,0,0) = -\overline{B}_{T10,q}(0)$$

$$\overline{B}_{T(u,d),lattice}(0) \approx 2 \ldots 3$$



could imply to sizeable Boer-Mulders-effect

Summary and outlook

finite volume effects in g_A can be interpreted using chPT

first indications for bending in m_π towards experimental results

overall compatibility of lattice simulations with parametrizations and model calculations

spin densities are strongly distorted for transversely polarized quarks and nucleons

possible implications for sign/size of Sivers and Boer-Mulders functions (M. Burkardt)

pointing towards sizeable (single spin) asymmetries

unquenched simulations with m_π<300MeV in progress

chPT calculations for e.g. momentum fraction in progress