Hadron Tomography

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Outline

- GPDs: probabilistic interpretation as Fourier transforms of impact parameter dependent PDFs
  - $E(x, 0, -\mathbf{\Delta}_\perp^2)$
    $\hookrightarrow \perp$ deformation of unpol. PDFs in $\perp$ pol. target
    - physics: orbital motion of the quarks

    $\hookrightarrow$ Sivers effect
  - $2\tilde{H}_T + E_T \longrightarrow \perp$ deformation of $\perp$ pol. PDFs in unpol. target
    - correlation between quark angular momentum and quark transversity

    $\hookrightarrow$ Boer-Mulders function $h_1^\perp(x, \mathbf{k}_\perp)$
  - $N_C \longrightarrow \infty$
- Summary

Impact parameter dependent PDFs

- define $\perp$ localized state [D.Soper,PRD**15**, 1141 (1977)]

$$\left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\right\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp \left|p^+, \mathbf{p}_\perp, \lambda\right\rangle$$

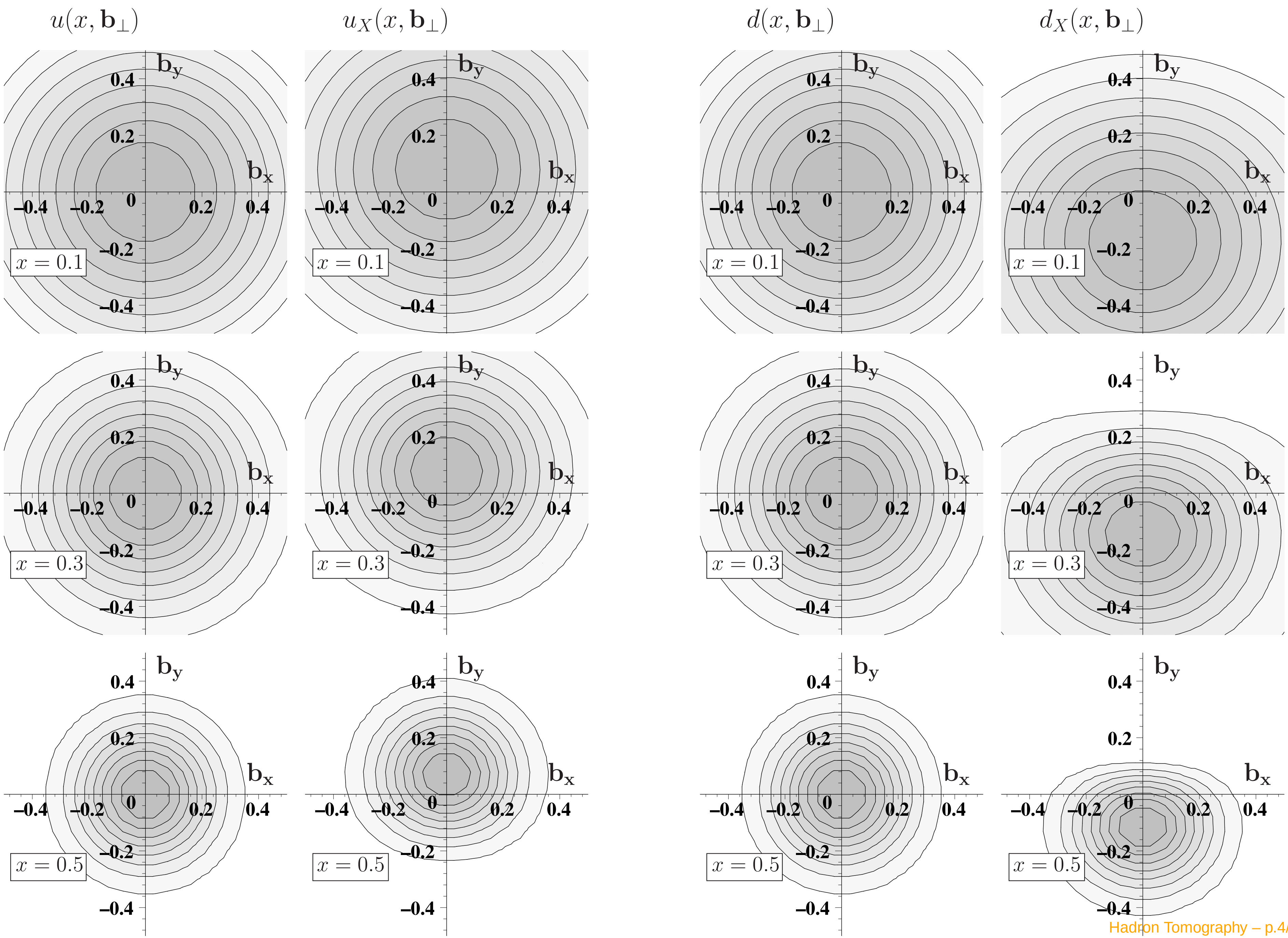
  Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has
  $\mathbf{R}_\perp \equiv \frac{1}{P^+} \int dx^- d^2\mathbf{x}_\perp \, \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{r}_{i,\perp} = \mathbf{0}_\perp$
  (cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x, \mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi} \left\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right| \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) \left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

$\hookrightarrow$

$$\begin{aligned} q(x, \mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} H(x, 0, -\mathbf{\Delta}_\perp^2), \\ \Delta q(x, \mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2), \end{aligned}$$


$u(x, \mathbf{b}_\perp)$
$u_X(x, \mathbf{b}_\perp)$
$d(x, \mathbf{b}_\perp)$
$d_X(x, \mathbf{b}_\perp)$
$x = 0.1$
$x = 0.3$
$x = 0.5$
$\mathbf{b_x}$
$\mathbf{b_y}$

Transversely Deformed Distributions and $E(x,0,-\mathbf{\Delta}_\perp^2)$

M.B., Int.J.Mod.Phys.A**18**, 173 (2003)

- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi=0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P+\Delta,\uparrow|\bar{q}(0)\,\gamma^+ q(x^-)|\,P,\uparrow\rangle = H(x,0,-\mathbf{\Delta}_\perp^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P+\Delta,\uparrow|\bar{q}(0)\,\gamma^+ q(x^-)|\,P,\downarrow\rangle = -\frac{\Delta_x-i\Delta_y}{2M} E(x,0,-\mathbf{\Delta}_\perp^2).$$

- Consider nucleon polarized in x direction (in IMF)
  $|X\rangle \equiv |p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\uparrow\rangle + |p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\downarrow\rangle$.

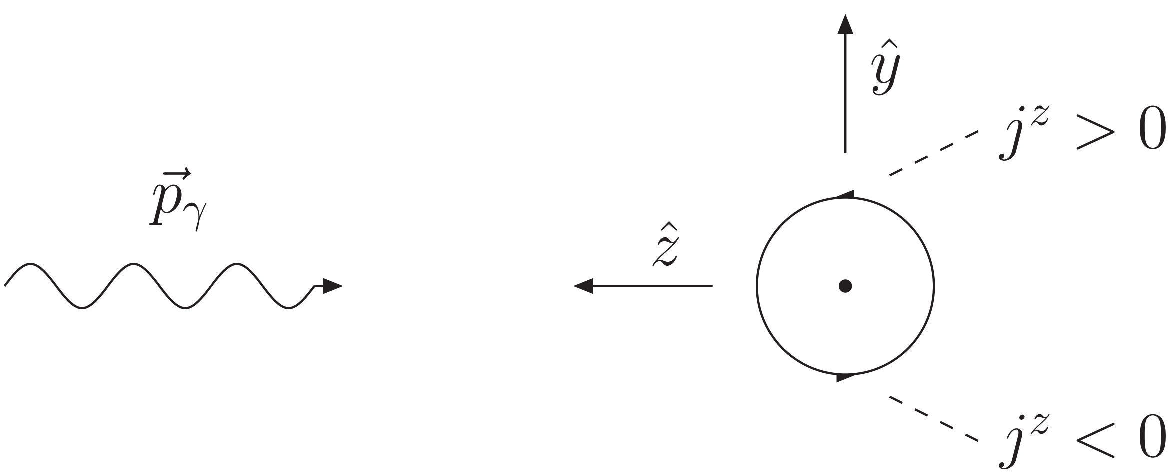
$\hookrightarrow$ unpolarized quark distribution for this state:

$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x,0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

- Physics: $j^+=j^0+j^3$, and left-right asymmetry from j^3 !
  [X.Ji, PRL **91**, 062001 (2003)]

Intuitive connection with $\vec{L}_q$

- Electromagnetic interaction couples to vector current. Due to kinematics of the DIS-reaction (and the choice of coordinates — $\hat{z}$-axis in direction of the momentum transfer) the virtual photons “see” (in the Bj-limit) only the $j^+ = j^0 + j^z$ component of the quark current
- If up-quarks have positive orbital angular momentum in the $\hat{x}$-direction, then j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side



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$\hookrightarrow$ j^+ is deformed not because there are more quarks on one side than on the other but because the DIS-photons (coupling only to j^+) “see” the quarks on the $+\hat{y}$ side better than on the $-\hat{y}$ side.

- $\perp$ deformation described by $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$

$\hookrightarrow$ not surprising to find that $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$ enters the Ji relation

$$\langle J_q^i \rangle = S^i \int dx \left[H_q(x, 0, 0) + E_q(x, 0, 0) \right] x .$$

Transversely Deformed Distributions and $E(x, 0, -\mathbf{\Delta}_\perp^2)$

- $q(x, \mathbf{b}_\perp)$ in $\perp$ polarized nucleon is deformed compared to longitudinally polarized nucleons !
- mean $\perp$ deformation of flavor q ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q_X(x, \mathbf{b}_\perp) b_y = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q^p}{2M}$$

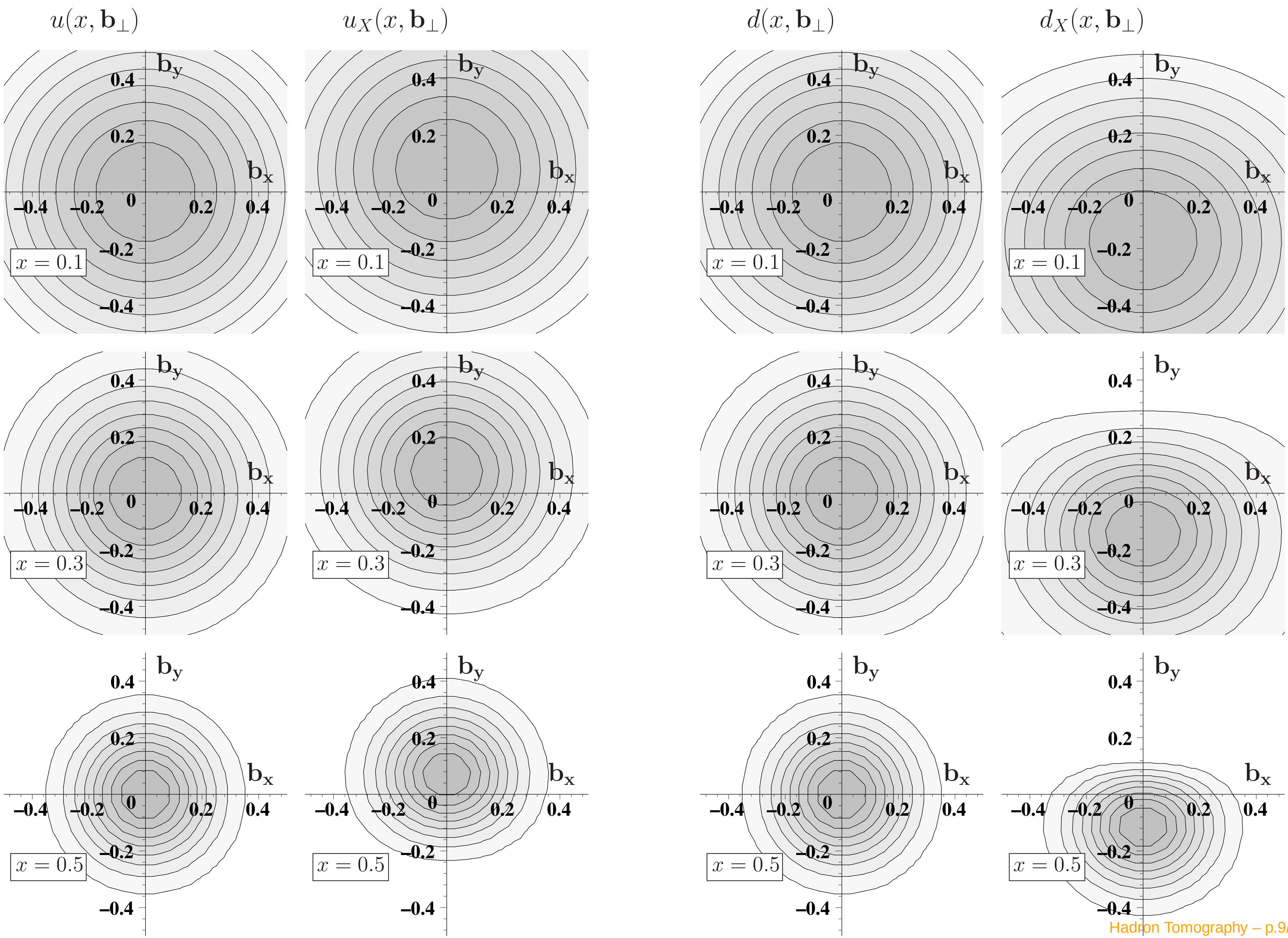
with $\kappa_{u/d}^p \equiv F_2^{u/d}(0) = \mathcal{O}(1-2) \quad \Rightarrow \quad d_y^q = \mathcal{O}(0.2fm)$

- simple model: for simplicity, make ansatz where $E_q \propto H_q$

$$\begin{aligned} E_u(x, 0, -\mathbf{\Delta}_\perp^2) &= \frac{\kappa_u^p}{2} H_u(x, 0, -\mathbf{\Delta}_\perp^2) \\ E_d(x, 0, -\mathbf{\Delta}_\perp^2) &= \kappa_d^p H_d(x, 0, -\mathbf{\Delta}_\perp^2) \end{aligned}$$

with $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \qquad \kappa_d^p = 2\kappa_n + \kappa_p = -2.033.$

- Model too simple but illustrates that anticipated deformation is very significant since κ_u and κ_d known to be large!



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$\mathbf{b_y}$
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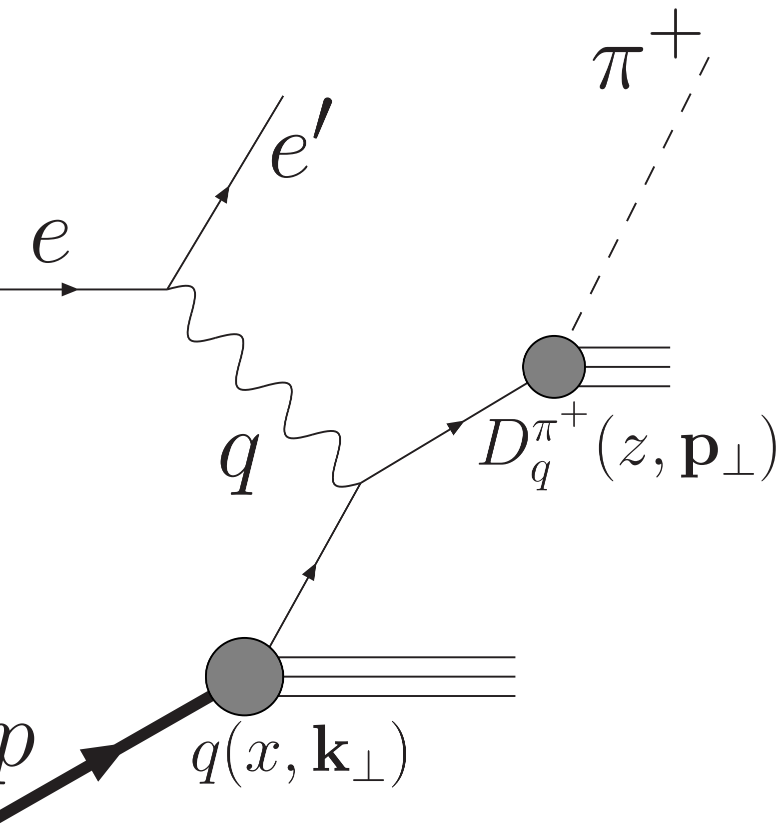
SSA ($\gamma + p \uparrow \longrightarrow \pi^+ + X$)



- use factorization (high energies) to express momentum distribution of outgoing π^+ as convolution of
  - momentum distribution of quarks in nucleon

  $\hookrightarrow$ unintegrated parton density $f_{q/p}(x, \mathbf{k}_\perp)$
  - momentum distribution of π^+ in jet created by leading quark q

  $\hookrightarrow$ fragmentation function $D_q^{\pi^+}(z, \mathbf{p}_\perp)$

- average $\perp$ momentum of pions obtained as sum of
  - average $\mathbf{k}_\perp$ of quarks in nucleon (Sivers effect)
  - average $\mathbf{p}_\perp$ of pions in quark-jet (Collins effect)

GPD $\longleftrightarrow$ SSA (Sivers)

- Sivers: distribution of **unpol.** quarks in $\perp$ pol. proton

$$f_{q/p^\uparrow}(x, \mathbf{k}_\perp) = f_1^q(x, \mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}} \times \mathbf{k}_\perp) \cdot S}{M}$$

- without FSI, $\langle \mathbf{k}_\perp \rangle = 0$, i.e. $f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2) = 0$
- with FSI, $\langle \mathbf{k}_\perp \rangle \neq 0$ (Brodsky, Hwang, Schmidt)
- FSI formally included by appropriate choice of Wilson line gauge links in gauge invariant def. of $f_{q/p}(x, \mathbf{k}_\perp)$

$\hookrightarrow$ Qiu, Sterman; Collins; Ji; Boer et al.;..

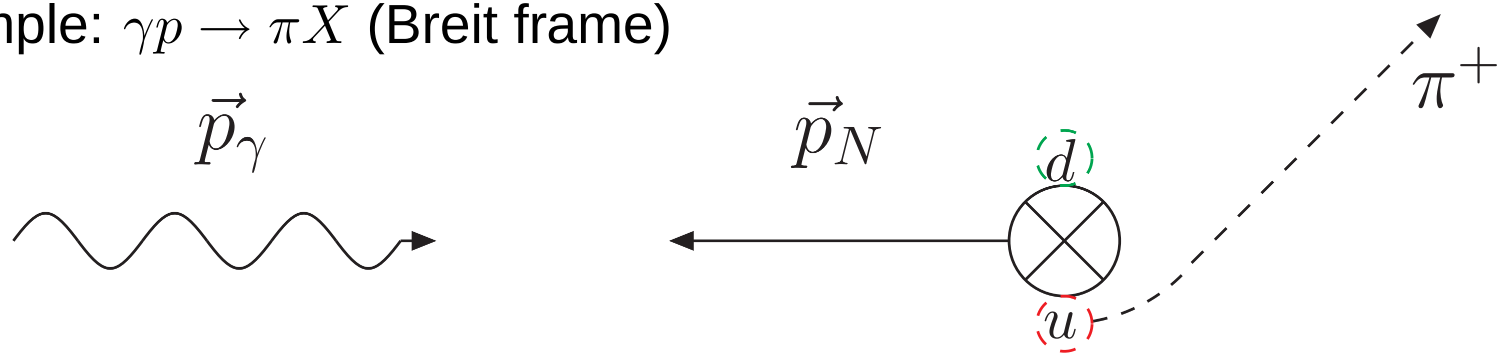
$$\langle \mathbf{k}_\perp \rangle \sim \left\langle P, S \left| \bar{q}(0)\gamma^+ \int_0^\infty d\eta^- G^{+\perp}(\eta) q(0) \right| P, S \right\rangle$$

  - $\int_0^\infty d\eta^- G^{+\perp}(\eta)$ is the $\perp$ impulse that the active quark acquires as it moves through color field of “spectators”

- What should we expect for Sivers effect in QCD ?

GPD $\longleftrightarrow$ SSA (Sivers)

- example: $\gamma p \rightarrow \pi X$ (Breit frame)



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

$\hookrightarrow$ correlation between sign of κ_q and sign of SSA: $f_{1T}^{\perp q} \sim -\kappa_q$

- $f_{1T}^{\perp q} \sim -\kappa_q$ consistent with HERMES results

Chirally Odd GPDs

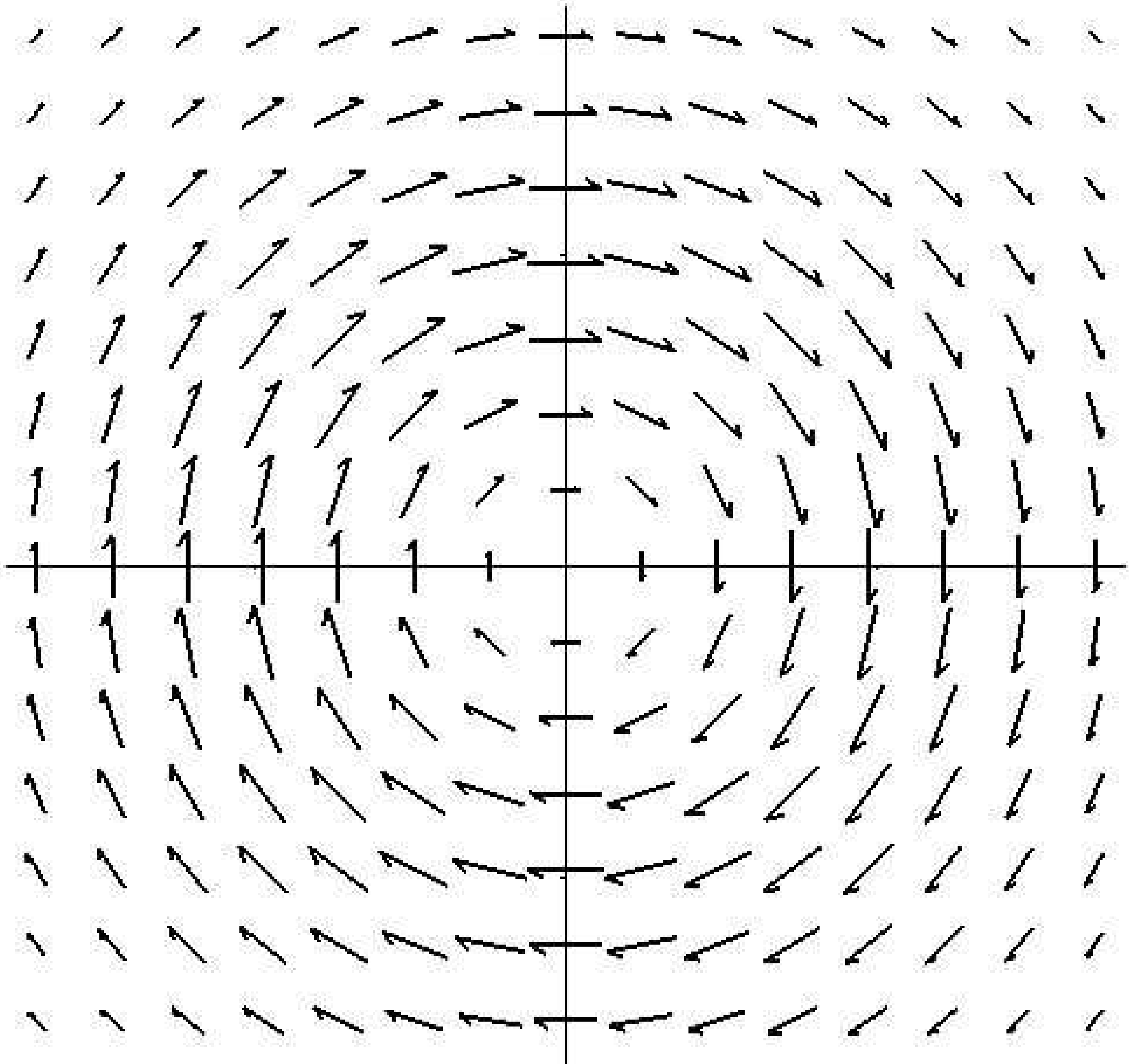
$$\int \frac{dx^-}{2\pi} e^{ixp^+x^-} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \sigma^{+j}\gamma_5 q\left(\frac{x^-}{2}\right) \right| p \right\rangle = \quad H_T \bar{u}\sigma^{+j}\gamma_5 u + \tilde{H}_T \bar{u}\frac{\varepsilon^{+j\alpha\beta}\Delta_\alpha P_\beta}{M^2} u$$
$$+ E_T \bar{u}\frac{\varepsilon^{+j\alpha\beta}\Delta_\alpha \gamma_\beta}{2M} u + \tilde{E}_T \bar{u}\frac{\varepsilon^{+j\alpha\beta}P_\alpha \gamma_\beta}{M} u$$

- See also M.Diehl+P.Hägler, hep-ph/0504175.
- Fourier trafo of $\bar{E}_T^q \equiv 2\tilde{H}_T^q + E_T^q$ for $\xi = 0$ describes distribution of transversity for unpolarized target in $\perp$ plane

$$q^i(x, \mathbf{b}_\perp) = \frac{\varepsilon^{ij}}{2M}\frac{\partial}{\partial b_j}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp} \bar{E}_T^q(x, 0, -\mathbf{\Delta}_\perp^2)$$

- origin: correlation between quark spin (i.e. transversity) and angular momentum

Transversity Distribution in Unpolarized Target



Chirally Odd GPDs

- $J^i = \frac{1}{2}\varepsilon^{ijk}\int d^3x \left[T^{0j}x^k - T^{0k}x^j\right]$
- J_q^x diagonal in transversity (does not mix), projected with $\frac{1}{2}(1 \pm \gamma^x\gamma_5)$

$\hookrightarrow$ one can derive analog to Ji's sum rule (unpol. target), e.g.

$$\left\langle J^y_{q,+\hat{y}} \right\rangle = \frac{1}{4}\int dx \left[H_T^q(x,0,0) + \bar{E}_T^q(x,0,0)\right] x$$

$\hookrightarrow$ $\bar{E}_T^q$ provides quantitative information about the correlation between quark transversity and quark angular momentum)

Boer-Mulders function

- attractive FSI expected to convert position space asymmetry into momentum space asymmetry

$\hookrightarrow$ e.g. quarks at negative b_x with spin in $+\hat{y}$ get deflected (due to FSI) into $+\hat{x}$ direction

$\hookrightarrow$ (qualitative) connection between Boer-Mulders function $h_1^\perp(x, \mathbf{k}_\perp)$ and the chirally odd GPD $\bar{E}_T$ that is similar to (qualitative) connection between Sivers function $f_{1T}^\perp(x, \mathbf{k}_\perp)$ and the GPD E.

- Boer-Mulders: distribution of $\perp$ **pol.** quarks in **unpol.** proton

$$f_{q^\uparrow/p}(x, \mathbf{k}_\perp) = \frac{1}{2}\left[f_1^q(x, \mathbf{k}_\perp^2) - h_1^{\perp q}(x, \mathbf{k}_\perp^2) \frac{(\hat{\mathbf{P}} \times \mathbf{k}_\perp) \cdot S_q}{M} \right]$$

Transversity Distribution in Unpolarized Target

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$\hookrightarrow$ semi-quantitative predictions for $h_1^\perp(x, \mathbf{k}_\perp)$

- sign of $h_1^\perp$ opposite to sign of $\bar{E}_T$
- “$\frac{h_1^\perp}{\bar{E}_T} \approx \frac{f_{1T}^\perp}{E}$”

- use measurement of $h_1^\perp$ to learn about spin-orbit correlation
- lattice ($\longrightarrow$ P.Hägler’s talk):
  - $\bar{E}_T^q > 0 \quad \Rightarrow \quad$ expect $h_1^{\perp u}$ same sign as $f_{1T}^{\perp,q}$
  - $\bar{E}_T^q > E \quad \Rightarrow \quad \left|h_1^{\perp q}\right| > \left|f_{1T}^{\perp u}\right|$

GPDs and SSAs @ large N_C

- consider ficticious world with $N_C \to \infty$, such that $g^2 N_C$ fixed

$\hookrightarrow$ baryons become infinitely heavy $M_B = \mathcal{O}(N_C)$, with finite size

$\hookrightarrow$ interaction between pair of quarks goes to zero

$\hookrightarrow$ mean field approx. becomes exact

- identify nucleon with spin/isospin $\frac{1}{2}$ state (consider only N_C odd)

$\hookrightarrow$ correlation of many spin and isospin observables as consequence of spin-flavor symmetry of large N_C baryons, (Pobylitsa et al.)

  - $H_u(x,\xi,-\mathbf{\Delta}_\perp^2) = H_d(x,\xi,-\mathbf{\Delta}_\perp^2) = \mathcal{O}(N_C^2)$
  - $E_u(x,\xi,-\mathbf{\Delta}_\perp^2) = -E_d(x,\xi,-\mathbf{\Delta}_\perp^2) = \mathcal{O}(N_C^3)$

  $\hookrightarrow$ $\perp$ deformation of PDFs in $\perp$ polarized nucleon equal and opposite for u and d quarks

  $\hookrightarrow$ equal and opposite Sivers effects for u and d, i.e. $f_{1T}^{\perp,u} = -f_{1T}^{\perp,d}$

- $\bar{E}_T^u = \bar{E}_T^d \quad \Rightarrow \quad$ equal BM-functions for u and d, i.e. $h_1^{\perp u} = h_1^{\perp d}$

GPDs and SSAs @ large N_C

- $N_C \to \infty$: Hartree approximation exact

$\hookrightarrow$ FSI depends only on position/path of active quark, but not on its flavor, spin, or x

- SSAs described by universal (spin/flavor, x indep.) function $\mathbf{I}(\mathbf{b}_\perp) = \mathbf{b}_\perp I(\mathbf{b}_\perp^2)$ satisfying

$$\bar{\mathbf{k}}_{\perp,q} \equiv \int d^2\mathbf{k}_\perp f(x,\mathbf{k}_\perp)\mathbf{k}_\perp = \int d^2\mathbf{b}_\perp \mathbf{I}(\mathbf{b}_\perp) q(x,\mathbf{b}_\perp)$$

$\hookrightarrow$ “chromodynamic lensing”

- $\mathbf{I}(\mathbf{b}_\perp)$ is the $\perp$ impulse that a quark ejected from position $\mathbf{b}_\perp$ aquires, due to FSI with spectators.

GPDs and SSAs @ large N_C

- Boer-Mulders function

$$\int d^2\mathbf{k}_\perp h_1^\perp(x,\mathbf{k}_\perp)\mathbf{k}_\perp^2 = \int d^2\mathbf{b}_\perp \mathbf{I}(\mathbf{b}_\perp)\nabla_\perp \bar{\mathcal{E}}_T(\mathbf{b}_\perp)$$

where $\bar{\mathcal{E}}_T(\mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} \bar{E}_T(x,0,\mathbf{\Delta}_\perp)e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp}$

- possible constraint on $\bar{E}_T(x,0,\mathbf{\Delta}_\perp)$ and thus on spin-orbit correlations?
- expand $I(\mathbf{b}_\perp^2) = I_0 + I_2\mathbf{b}_\perp^2 + ...$ with I_n universal (same for u, d, Sivers, Boer-Mulders)

$\hookrightarrow$

$$-\int d^2\mathbf{k}_\perp f_{1T}^\perp(x,\mathbf{k}_\perp)\mathbf{k}_\perp^2 = I_0 E(x,0,0) + I_1 \left\langle \mathbf{b}_\perp^2(x)\right\rangle_E + ...$$

$$-\int d^2\mathbf{k}_\perp h_1^\perp(x,\mathbf{k}_\perp)\mathbf{k}_\perp^2 = I_0 \bar{E}_T(x,0,0) + I_1 \left\langle \mathbf{b}_\perp^2(x)\right\rangle_{\bar{E}_T} + ...$$

Summary

- GPDs $\stackrel{FT}{\longrightarrow}$ PDFs in impact parameter space
- $E(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs for $\perp$ polarized target

$\hookrightarrow$ origin: orbital motion of the quarks [decomposotion of quark spin (Ji relation) w.r.t. quark transversity]

$\hookrightarrow$ simple mechanism (attractive FSI) to predict sign of f_{1T}^{q}

- distribution of $\perp$ polarized quarks in unpol. target described by chirally odd GPD $\bar{E}_T^q = 2\bar{H}_T^q + \tilde{E}_T^q$

$\hookrightarrow$ origin: correlation between orbital motion and spin of the quarks

$\hookrightarrow$ simple mechanism (attractive FSI) to predict sign of $h_1^\perp$

$\hookrightarrow$ or, use lensing mechanism to qualitatively “extract” $\bar{E}_T^q$ from measurement of $h_1^\perp$

$\perp$ Single Spin Asymmetry (Sivers)

- Naive definition of unintegrated parton density

$$f(x,\mathbf{k}_\perp) \propto \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ip\cdot\xi} \left\langle P,S \left| \bar{q}(0)\gamma^+ q(\xi) \right| P,S \right\rangle \Big|_{\xi^+=0} .$$

- Time-reversal invariance $\Rightarrow f(x,\mathbf{k}_\perp) = f(x,-\mathbf{k}_\perp)$

$\hookrightarrow$ Asymmetry $\int d^2\mathbf{k}_\perp f(x,\mathbf{k}_\perp)\,\mathbf{k}_\perp = 0$

- Same conclusion for gauge invariant definition with straight Wilson line $U_{[0,\xi]} = P\exp\left(ig\int_0^1 ds\xi_\mu A^\mu(s\xi)\right)$

$\perp$ Single Spin Asymmetry (Sivers)

- Naively (time-reversal invariance) $f(x, \mathbf{k}_\perp) = f(x, -\mathbf{k}_\perp)$
- However, including the final state interaction (FSI) results in nonzero asymmetry of the ejected quark! (Brodsky, Hwang, Schmidt)
- Gauge invariant definition requires quark to be connected by gauge link. Choice of path not arbitrary but must be chosen along path of outgoing quark to incorporate FSI

$$f(x, \mathbf{k}_\perp) \propto \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ip\cdot\xi} \left\langle P, S \left| \bar{q}(0) U_{[0,\infty]} \gamma^+ U_{[\infty,\xi]} q(\xi) \right| P, S \right\rangle \Big|_{\xi^+=0}$$

with $U_{[0,\infty]} = P \exp\left(ig \int_0^\infty d\eta^- A^+(\eta) \right)$

Sivers Mechanism in $A^+ = 0$ gauge

- Gauge link along light-cone trivial in light-cone gauge

$$U_{[0,\infty]} = P\exp\left(ig\int_0^\infty d\eta^- A^+(\eta)\right) = 1$$

$\hookrightarrow$ Puzzle: Sivers asymmetry seems to vanish in LC gauge (time-reversal invariance)!

- X.Ji: fully gauge invariant definition for $P(x,\mathbf{k}_\perp)$ requires additional gauge link at $x^- = \infty$

$$\begin{aligned} f(x,\mathbf{k}_\perp) &= \int \frac{dy^- d^2\mathbf{y}_\perp}{16\pi^3} e^{-ixp^+y^- + i\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \\ &\times \left\langle p,s\left|\bar{q}(y)\gamma^+ U_{[y^-,\mathbf{y}_\perp;\infty^-,\mathbf{y}_\perp]} U_{[\infty^-,\mathbf{y}_\perp,\infty^-,\mathbf{0}_\perp]} U_{[\infty^-,\mathbf{0}_\perp;0^-,\mathbf{0}_\perp]} q(0)\right|p,s\right\rangle \end{aligned}$$

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