Light-quark and charm interplay in the Dalitz-plot analysis of hadronic decays in Focus

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I.N.F.N. Milano

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LNF 7-11 April 2003
Why the charm community had
to face the problem of the light mesons
  • certainly a complication for the interpretation of charm decay dynamics
  • more difficult Dalitz plot analysis

and how it started
to be educated in the general s-wave formalism
  • two-body unitarity
  • Breit-Wigner approx. limits, K-matrix approach etc...
Over the last decade charm Dalitz-plot analysis has emerged as an excellent tool to study

- Dynamics of hadronic decays
  - substructures in the Dalitz plots
  - interference & branching ratios

- FSI effects in three-body decays
  - phase shifts between different resonant amplitudes
  - extension of the isospin analysis from the two-body system

- Role of non-spectator diagrams in charm decays
  e.g. \( D_s^\pm \rightarrow \pi^\pm \pi^0 \pi^\pm \)

The high statistics now available demands proper parametrization
How can we formulate the problem?

\[ D^+ \rightarrow r \, \, 3 \]

\[ \downarrow \quad 1 \, \, 2 \]

**The problem is to write the propagator for the resonance \( r \)**

For a well-defined wave with specific isospin and spin \((IJ)\) characterized by narrow and well-isolated resonances, we know how.

- the propagator is of the simple Breit-Wigner type and the amplitude is

\[
A = F_D F_r \times \left| p_1 \right|^J \left| p_3 \right|^J P_J (\cos \vartheta_{13}) \times \frac{1}{m_r^2 - m^2 - i m_r \Gamma_r}
\]
In contrast

when the specific $IJ$–wave is characterized by large and heavily overlapping resonances (just as the scalars!), the problem is not that simple.

Indeed, it is very easy to realize that the propagation is no longer dominated by a single resonance but is the result of complicated interplay among resonances.

In this case, it can be demonstrated on very general grounds that the propagator may be written in the context of the K-matrix approach as

$$(I - iK \cdot \rho)^{-1}$$

where $K$ is the matrix for the scattering of particles 1 and 2.

i.e., to write down the propagator we need the scattering matrix
We may summarize by saying that:

The decay amplitude may be written, in general, as a coherent sum of BW terms for waves with well-isolated resonances plus K-matrix terms for waves with overlapping resonances.

\[
A(D) = a_0 e^{i\delta_0} + \sum_{i=1}^{m} a_i e^{i\delta_i} F_{i}^{BW} + \sum_{i=m+1}^{n} a_i e^{i\delta_i} F_{i}^{K}
\]

Where the general form of \( F_{i}^{K} \) for scalars is

\[
F_{k}^{K} = (I - iK \cdot \rho)^{-1}_{kj} \left\{ \sum_{\alpha} \frac{\beta_\alpha g^\alpha_j}{m_\alpha^2 - m^2} + f_j \frac{1-s_0}{s-s_0} \right\} \times \frac{s-s_A/2m_\pi^2}{(s-s_A)(1-s_A)}
\]
In the light of this

if we try to fit a generic Dalitz plot with the simple isobar model, we have to let the masses and widths of overlapping resonances in the same waves float freely, in order to get a decent fit.

But now the question arises:

Is this a meaningful description of the underlying physics or just a mirage?

We shall see this problem in specific examples of FOCUS data
FOCUS $D_s^+ \rightarrow \pi^+\pi^+\pi^-$ analysis

Yield $D_s^+ = 1475 \pm 50$

S/N $D_s^+ = 3.41$

Observe:

- $f_0(980)$
- $f_2(1270)$
- $f_0(1500)$

Dominated by weird resonances with simultaneous $KK$ and $\pi\pi$ couplings
Decay processes

Annihilation

\[
\begin{align*}
D_s^+ & \\
W^+ & \\
c & \rightarrow u & \bar{d} & \rightarrow \pi^+ & \pi^- & \pi^+ \\
\end{align*}
\]

Spectator

\[
\begin{align*}
D_s^+ & \\
W^+ & \\
c & \rightarrow u & d & \rightarrow \pi^+ & \pi^+ \\
\end{align*}
\]

\[
\rho = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})
\]

\[
f_0 = c_1(uu + d\bar{d}) + c_2(s\bar{s})
\]
Isobar approach

<table>
<thead>
<tr>
<th>Resonances</th>
<th>Fit Fraction (%)</th>
<th>Phase $\phi_j$</th>
<th>Amplitude $a_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_R$</td>
<td>25.5 ± 4.6</td>
<td>246.5 ± 4.7</td>
<td>0.520 ± 0.046</td>
</tr>
<tr>
<td>$f_2(1275)$</td>
<td>9.8 ± 1.3</td>
<td>140.2 ± 9.2</td>
<td>0.323 ± 0.022</td>
</tr>
<tr>
<td>$f_0(980)$</td>
<td>94.4 ± 3.8</td>
<td>0 (fixed)</td>
<td>1 (fixed)</td>
</tr>
<tr>
<td>$S_0(1475)$</td>
<td>17.4 ± 3.1</td>
<td>249.7 ± 6.4</td>
<td>0.429 ± 0.043</td>
</tr>
<tr>
<td>$\rho^0(1450)$</td>
<td>4.1 ± 1.0</td>
<td>187.3 ± 15.3</td>
<td>0.208 ± 0.028</td>
</tr>
</tbody>
</table>

Coupled-channel BW

$m = 975 \pm 10 \text{ MeV}$
$\Gamma_{\pi\pi} = 90 \pm 30 \text{ MeV}$
$\Gamma_{KK} = 36 \pm 15 \text{ MeV}$

$m = 1475 \pm 10 \text{ MeV}$
$\Gamma = 112 \pm 24 \text{ MeV}$

\[ f_r = \frac{\int \left| a_r e^{i\delta_r} A_r \right|^2 d m_{12}^2 d m_{13}^2}{\int \left| \sum_j a_j e^{i\delta_j} A_j \right|^2 d m_{12}^2 d m_{13}^2} \sum_r f_r = 150\% \]
PDG Value for f0(1500)
\[ m = 1507 \pm 5 \text{ MeV} \]
\[ \Gamma = 109 \pm 7 \text{ MeV} \]

<table>
<thead>
<tr>
<th>resonances</th>
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<th>phase $\phi_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NR</td>
<td>36.9 ± 4.4</td>
<td>241.7 ± 3.8</td>
</tr>
<tr>
<td>$f_2(1275)$</td>
<td>9.5 ± 1.2</td>
<td>141.9 ± 7.4</td>
</tr>
<tr>
<td>$f_1(980)$</td>
<td>94.0 ± 3.6</td>
<td>0 (fixed)</td>
</tr>
<tr>
<td>$f_0(1500)$</td>
<td>17.8 ± 3.1</td>
<td>270.2 ± 5.4</td>
</tr>
<tr>
<td>$\rho(1450)$</td>
<td>4.6 ± 0.8</td>
<td>196.4 ± 14.6</td>
</tr>
</tbody>
</table>

$S_0(1475)$ (Focus)
\[ m = 1475 \pm 10 \text{ MeV} \]
\[ \Gamma = 112 \pm 24 \text{ MeV} \]
FOCUS $D^+ \rightarrow \pi^+\pi^+\pi^-$

Yield $D^+ = 1527 \pm 51$

S/N $D^+ = 3.64$
Isobar approach

Single BW for $f_0(400)$

$m = 443 \pm 27 \text{ MeV}$
$\Gamma = 443 \pm 80 \text{ MeV}$

E791 Results:
$m = 478^{+24}_{-23} \pm 17 \text{MeV}$
$\Gamma = 324^{+42}_{-40} \pm 21 \text{MeV}$

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<th>amplitude $a_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NR</td>
<td>9.8 ± 4.3</td>
<td>0 (fixed)</td>
<td>1 (fixed)</td>
</tr>
<tr>
<td>$\rho^0(770)$</td>
<td>32.8 ± 3.8</td>
<td>62.9 ± 16.8</td>
<td>1.830 ± 0.408</td>
</tr>
<tr>
<td>$f_2(1275)$</td>
<td>12.3 ± 2.1</td>
<td>−213.3 ± 17.7</td>
<td>1.120 ± 0.306</td>
</tr>
<tr>
<td>$f_0(980)$</td>
<td>6.7 ± 1.5</td>
<td>−145.9 ± 17.7</td>
<td>0.827 ± 0.239</td>
</tr>
<tr>
<td>$S_0(1475)$</td>
<td>1.8 ± 1.2</td>
<td>242.3 ± 25.8</td>
<td>0.425 ± 0.208</td>
</tr>
<tr>
<td>$f_0(400)$</td>
<td>18.9 ± 5.3</td>
<td>−96.9 ± 30.7</td>
<td>1.389 ± 0.468</td>
</tr>
</tbody>
</table>
Sandra Malvezzi - Dalitz plot in the charm sector

With $f_0(400)$

C.L. $\sim 1\%$

Without $f_0(400)$

C.L. $\sim 10^{-8}\%$

Preliminary
Resonances in K-matrix formalism

\[ K_{ij} = \sum_a g_{ai}(m) g_{aj}(m) \left( \frac{m_a^2 - m^2}{m_a^2 - m^2} \right) \]

where

\[ g_{ai}^2(m) = m_a \Gamma_{ai}(m) \]

Unitarity ...

\[ K_{ij} = \sum_a \frac{g_{ai}(m) g_{aj}(m)}{(m_a^2 - m^2)} + c_{ij}(m^2) \]

\[ T = (I - iK \cdot \rho)^{-1} K \]
... from scattering to production: the P-vector approach

\[ P_i = \sum_\alpha \beta_\alpha g_{\alpha i} \left( \frac{m}{m_\alpha^2 - m^2} \right) \]

\[ P_i = P_i + d_i \]

\[ F = \left( I - iK \cdot \rho \right)^{-1} P \]

\( \beta \) (complex) carries the production information

known from scattering data

I.J.R. Aitchison
Nucl.Phys. A189 (1972) 514
We need a description of the scattering...

“K-matrix analysis of the 00++-wave in the mass region below 1900 MeV”


* GAMS  \[ pp \rightarrow p^0p^0n, hn e hh' n, |t|<0.2 \text{ (GeV/c}^2\text{)} \]
* GAMS  \[ pp \rightarrow p^0p^0n, 0.30<|t|<1.0 \text{ (GeV/c}^2\text{)} \]
* BNL \[ pp \rightarrow KK n \]
* CERN-Munich \[ p^+p^- \rightarrow p^+p^- \]
* Crystal Barrel  \[ pp \rightarrow p^0p^0p^0, p^0p^0h, p^0hh \]
* Crystal Barrel  \[ pp \rightarrow p^0p^0p^0, p^0p^0h \]
* Crystal Barrel  \[ pp \rightarrow p^+p^-p^0, K^+K^-p^0, K_sK^-p^0, K^+K^-p^- \]
* Crystal Barrel  \[ np \rightarrow p^0p^0p^-, p^+p^-p^+ , K_sK^-p^0, K^-K^0p^- \]
* E852  \[ p^+p^- \rightarrow p^0p^0n, 0<|t|<1.5 \text{ (GeV/c}^2\text{)} \]
\[ K_{ij}^{00}(s) = \left( \sum_{\alpha} \frac{g_i^{(\alpha)} g_j^{(\alpha)}}{m_{\alpha}^2 - s} + f_{ij}^{scatt} \frac{1 - s_0^{scatt}}{s - s_0^{scatt}} \right) \frac{s - s_A/2m_{\pi}^2}{(s - s_{A0})(1 - s_{A0})} \]

\[ K_{ij}^{IJ} \] is a 5x5 matrix (i,j=1,2,3,4,5)  
\[ 1=\pi\pi, \ 2=K\bar{K} \ 3=4\pi \ 4=\eta\eta \ 5=\eta\eta' \]

\( g_i^{(\alpha)} \) is the coupling constant of the bare state \( \alpha \) to the meson channel

\( f_{ij}^{scatt} \) and \( s_0 \) describe a smooth part of the K-matrix elements

\[ (s - s_A/2m_{\pi}^2)/(s - s_{A0})(1 - s_{A0}) \] suppresses the false kinematical singularity at \( s = 0 \) near the \( \pi\pi \) threshold
### A&S K-matrix poles, couplings etc.

<table>
<thead>
<tr>
<th>Poles</th>
<th>$g_{\pi\pi}$</th>
<th>$g_{KK}$</th>
<th>$g_{4\pi}$</th>
<th>$g_{\eta\eta}$</th>
<th>$g_{\eta\eta'}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.65100</td>
<td>0.24844</td>
<td>-0.52523</td>
<td>0</td>
<td>-0.38878</td>
<td>-0.36397</td>
</tr>
<tr>
<td>1.20720</td>
<td>0.91779</td>
<td>0.55427</td>
<td>0</td>
<td>0.38705</td>
<td>0.29448</td>
</tr>
<tr>
<td>1.56122</td>
<td>0.37024</td>
<td>0.23591</td>
<td>0.62605</td>
<td>0.18409</td>
<td>0.18923</td>
</tr>
<tr>
<td>1.21257</td>
<td>0.34501</td>
<td>0.39642</td>
<td>0.97644</td>
<td>0.19746</td>
<td>0.00357</td>
</tr>
<tr>
<td>1.81746</td>
<td>0.15770</td>
<td>-0.17915</td>
<td>-0.90100</td>
<td>-0.00931</td>
<td>0.20689</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$s_{0}^{\text{scatt}}$</th>
<th>$f_{11}^{\text{scatt}}$</th>
<th>$f_{12}^{\text{scatt}}$</th>
<th>$f_{13}^{\text{scatt}}$</th>
<th>$f_{14}^{\text{scatt}}$</th>
<th>$f_{15}^{\text{scatt}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.30564</td>
<td>0.26681</td>
<td>0.16583</td>
<td>-0.19840</td>
<td>0.32808</td>
<td>0.31193</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$s_A$</th>
<th>$s_{A0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>-0.2</td>
</tr>
</tbody>
</table>
### A&S T-matrix poles and couplings

<table>
<thead>
<tr>
<th>$(m, \Gamma/2)$</th>
<th>$g_{\pi\pi}$</th>
<th>$g_{KK}$</th>
<th>$g_{4\pi}$</th>
<th>$g_{\eta\eta}$</th>
<th>$g_{\eta\eta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.019, 0.038)</td>
<td>$0.415 e^{i13.1}$</td>
<td>$0.580 e^{i96.5}$</td>
<td>$0.1482 e^{i80.9}$</td>
<td>$0.484 e^{i98.6}$</td>
<td>$0.401 e^{i102.1}$</td>
</tr>
<tr>
<td>(1.306, 0.167)</td>
<td>$0.406 e^{i116.8}$</td>
<td>$0.105 e^{i100.2}$</td>
<td>$0.8912 e^{-i61.9}$</td>
<td>$0.142 e^{i140.0}$</td>
<td>$0.225 e^{i133.0}$</td>
</tr>
<tr>
<td>(1.470, 0.960)</td>
<td>$0.758 e^{i97.8}$</td>
<td>$0.844 e^{i97.4}$</td>
<td>$1.681 e^{i91.1}$</td>
<td>$0.431 e^{i115.5}$</td>
<td>$0.175 e^{i152.4}$</td>
</tr>
<tr>
<td>(1.489, 0.058)</td>
<td>$0.246 e^{i151.5}$</td>
<td>$0.134 e^{i149.6}$</td>
<td>$0.4867 e^{-i123.3}$</td>
<td>$0.100 e^{-i170.6}$</td>
<td>$0.115 e^{-i133.9}$</td>
</tr>
<tr>
<td>(1.749, 0.165)</td>
<td>$0.536 e^{i101.6}$</td>
<td>$0.072 e^{i134.2}$</td>
<td>$0.7334 e^{-i123.6}$</td>
<td>$0.160 e^{i126.7}$</td>
<td>$0.313 e^{i101.1}$</td>
</tr>
</tbody>
</table>

**A&S fit does not need the σ**
First fits to charm Dalitz plots in the K-matrix approach!

\[ D_s \rightarrow \pi\pi\pi \]

**fit fractions**

- \( \Gamma_{S^{\text{wave}}} = 0.8776 \pm 0.0171 \)
- \( \Gamma_{\rho(1450)} = 0.0542 \pm 0.0114 \)
- \( \Gamma_{f_2(1275)} = 0.1153 \pm 0.0119 \)

\[ \sum_{r} f_{r} \sim 105\% \]

**phases**

- \( \phi_{S^{\text{wave}}} = 0.0 \pm 0.0 \)
- \( \phi_{\rho(1450)} = -161.4 \pm 9.3 \)
- \( \phi_{f_2(1275)} = 120.3 \pm 5.4 \)

\[ \chi^2/dof = 1.529 \]

**Summary**

- Free Parms: 22
- D.O.F: 30
- Cl: 3.215E-02
- \( \chi^2/dof = 1.529 \)
- Cut: 50
- Hins: 52

Sandra Malvezzi - Dalitz plot in the charm sector
$D^+ \rightarrow \pi\pi\pi$

fit fractions

$$\Gamma_{Swave} = 0.6647 \pm 0.0416 \quad \{101.8 \pm 22.5\}$$

$$\Gamma_{\rho(770)} = 0.2116 \pm 0.0436 \quad \{0.0 \pm 0.0\}$$

$$\Gamma_{f_2(1275)} = 0.1143 \pm 0.0142 \quad \{-113.0 \pm 9.0\}$$

$$\sum f_r \sim 99\%$$

phases

preliminary

Sandra Malvezzi - Dalitz plot in the charm sector
Conclusions

• The presence of scalar resonances in D-meson decays required a revision of the Dalitz plot parametrization

• A self-consistent description of s-wave isoscalar scattering in the energy range of interest is given in the K-matrix approach by A&S

• This is the first application of K-matrix approach to charm decays.

  The results are extremely encouraging since the same parametrization of two-body resonances coming from light-quark experiments works for charm decays too

Such a result was not at all obvious!
• K-matrix analysis of $D^+ \rightarrow \pi\pi\pi$ does not need the $\sigma$ but I doubt it is the last word on this puzzle!

Program for the near future

• perform an isoscalar s-wave global fit including charm data as well

• study the KK$\pi$ channel

• A great deal of work still to be performed