Light-quark and charm interplay in the Dalitz-plot analysis of hadronic decays in Focus

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Why the charm community had

to face the problem of the light mesons

- certainly a complication for the interpretation of charm decay dynamics
- more difficult Dalitz plot analysis

and how it started

to be educated in the general s-wave formalism

- two-body unitarity
- Breit-Wigner approx. limits, K-matrix approach etc...

Over the last decade *charm* Dalitz-plot analysis has emerged as an excellent tool to study

- **Dynamics of hadronic decays**
  - substructures in the Dalitz plots
  - interference & branching ratios

- **FSI effects in three-body decays**
  - phase shifts between different resonant amplitudes
  - extension of the isospin analysis from the two-body system

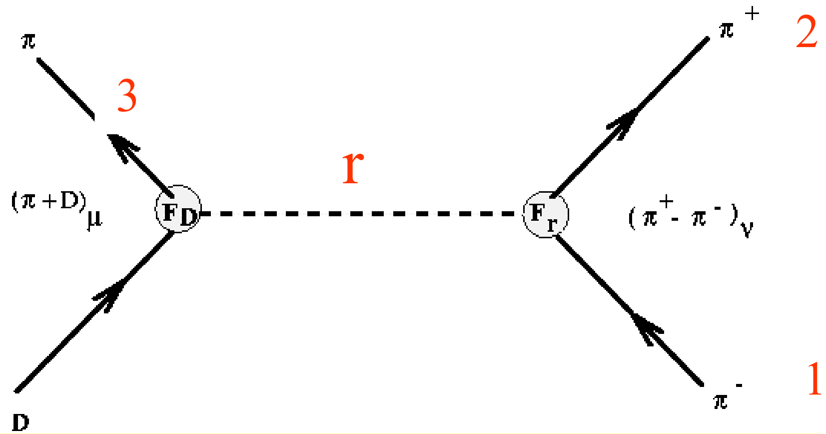
- **Role of non-spectator diagrams in charm decays**

  e.g. $D_s^{\pm} \rightarrow \pi^{\pm}\pi^{\mp}\pi^{\pm}$

The high statistics now available demands proper parametrization

How can we formulate the problem?

$$D^+ \to r\ 3$$
$$\quad \hookrightarrow 1\ 2$$



The problem is to write the propagator for the resonance r

For a well-defined wave with specific isospin and spin *(IJ)* characterized by narrow and well-isolated resonances, we know how.

- the propagator is of the simple Breit-Wigner type and the amplitude is

$$A = F_D F_r \times \left|\overrightarrow{p_1}\right|^J \left|\overrightarrow{p_3}\right|^J P_J(\cos \vartheta_{13}^r) \times \frac{1}{m_r^2 - m^2 - i m_r \Gamma_r}$$

In contrast

when the specific ***IJ***–wave is characterized by large and heavily overlapping resonances (just as the scalars!), the problem is not that simple.

Indeed, it is very easy to realize that the propagation is no longer dominated by a single resonance but is the results of complicated interplay among resonances.

In this case, it can be demonstrated on very general grounds that the propagator may be written in the context of the K-matrix approach as

$$(I - iK \cdot \rho)^{-1}$$

where K is the matrix for the scattering of particles 1 and 2.

$\downarrow$

i.e., to write down the propagator we need the scattering matrix

We may summarize by saying that:

The decay amplitude may be written, in general, as a coherent sum of BW terms for waves with well-isolated resonances plus K-matrix terms for waves with overlapping resonances.

$$A(D) = a_0 e^{i\delta_0} + \sum_{i=1}^{m} a_i e^{i\delta_i} F_i^{BW} + \sum_{i=m+1}^{n} a_i e^{i\delta_i} F_i^{K}$$

Where the general form of F_i^K for scalars is

$$F_k^K = (I - iK\cdot\rho)^{-1}_{kj} \left\{ \sum_\alpha \frac{\beta_\alpha g_j^\alpha}{m_\alpha^2 - m^2} + f_j \frac{1-s_0}{s-s_0} \right\} \times \frac{s - s_A/2m_\pi^2}{(s-s_{A0})(1-s_{A0})}$$

In the light of this

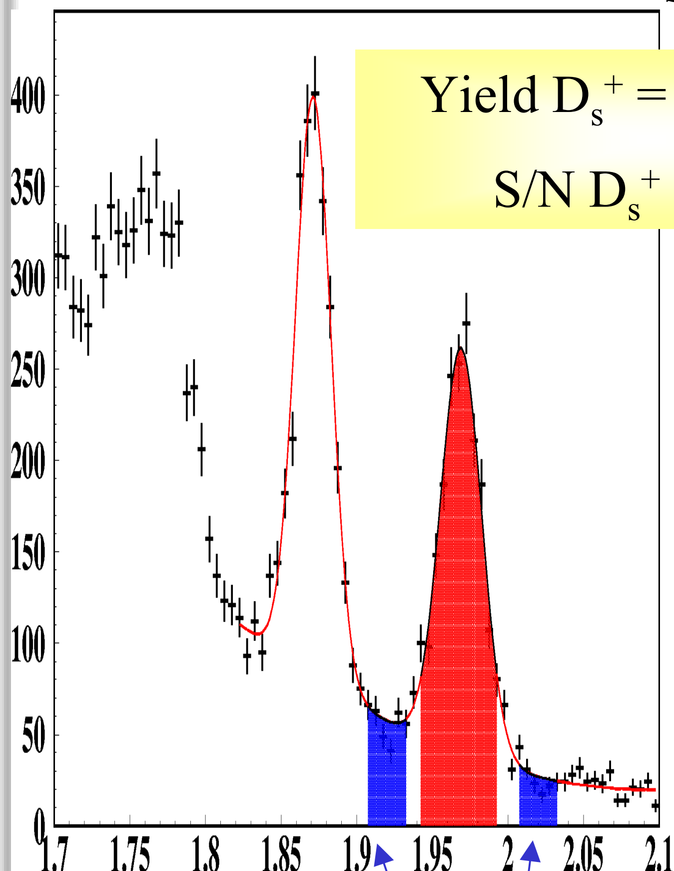
if we try to fit a generic Dalitz plot with the simple isobar model, we have to let the masses and widths of overlapping resonances in the same waves float freely, in order to get a decent fit.

But now the question arises:

Is this a meaningful description of the underlying physics or just a mirage?

We shall see this problem in specific examples of FOCUS data

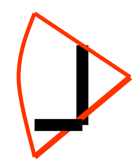
FOCUS $D_s^+ \to \pi^+\pi^+\pi^-$ analysis



Sideband

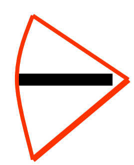
Observe:

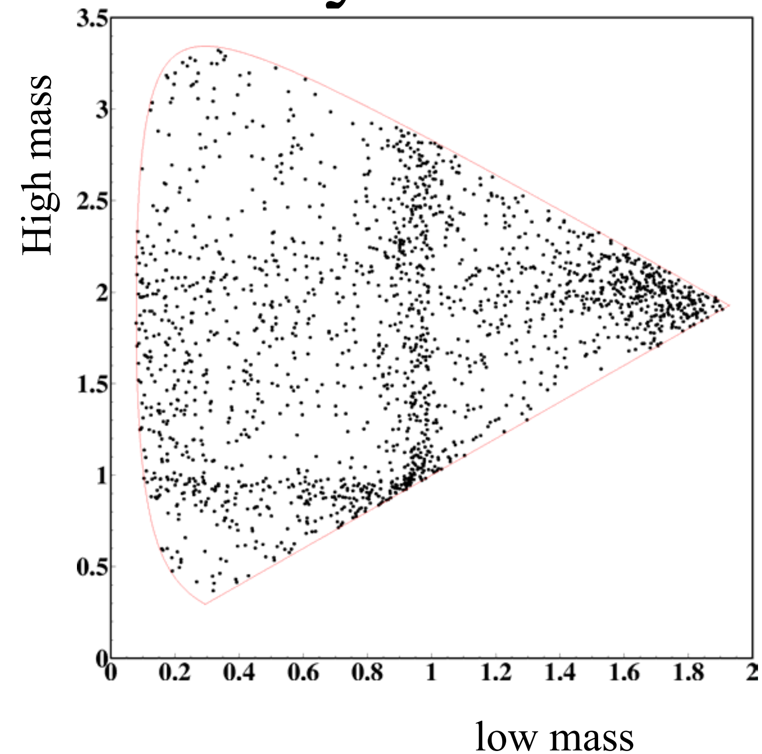
•$f_0(980)$



•$f_2(1270)$

•f_0 (1500)

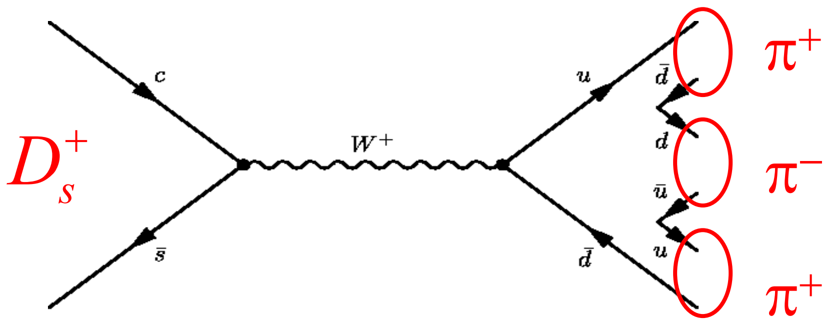


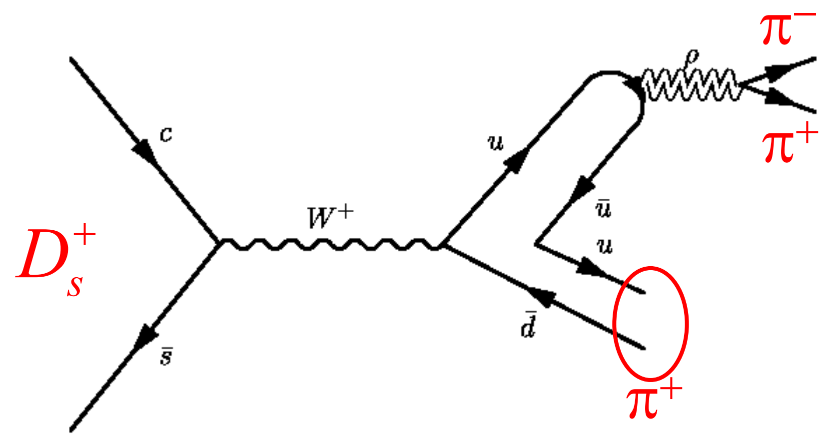


Dominated by ***weird*** *resonances with simultaneous KK and ππ couplings*

Decay processes

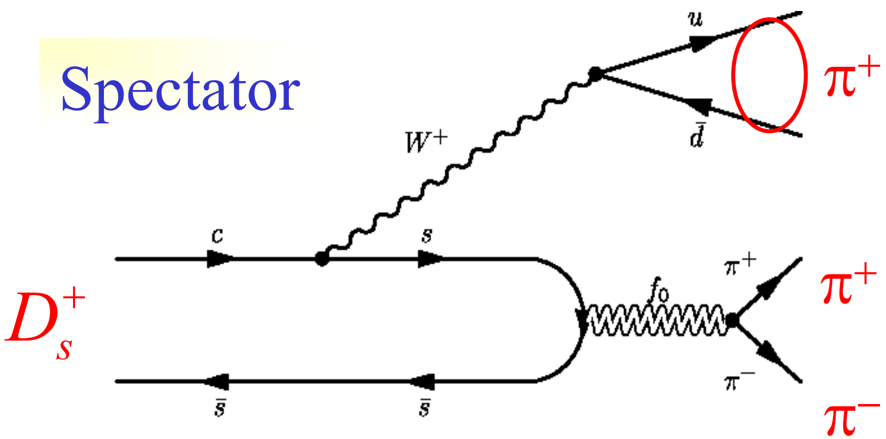
Annihilation





$$\rho = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$$

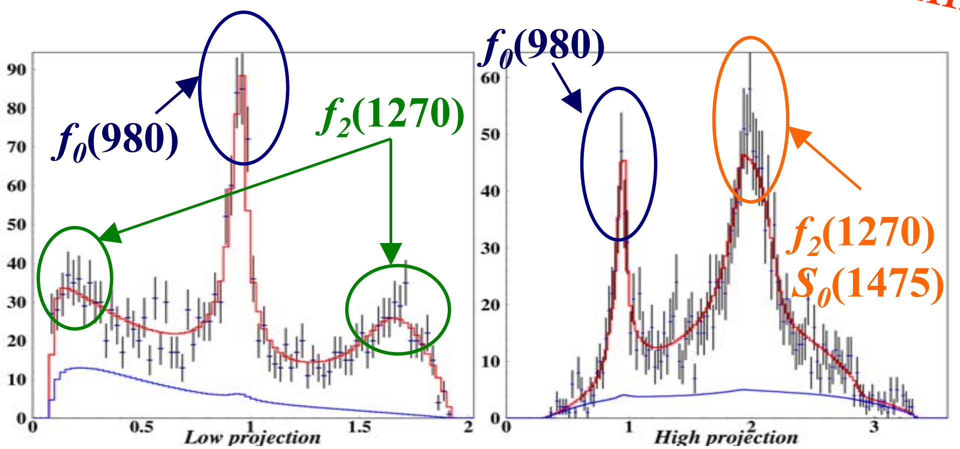
Spectator



$$f_0 = c_1\left(u\bar{u} + d\bar{d}\right) + c_2\left(s\bar{s}\right)$$

Isobar approach

Preliminary

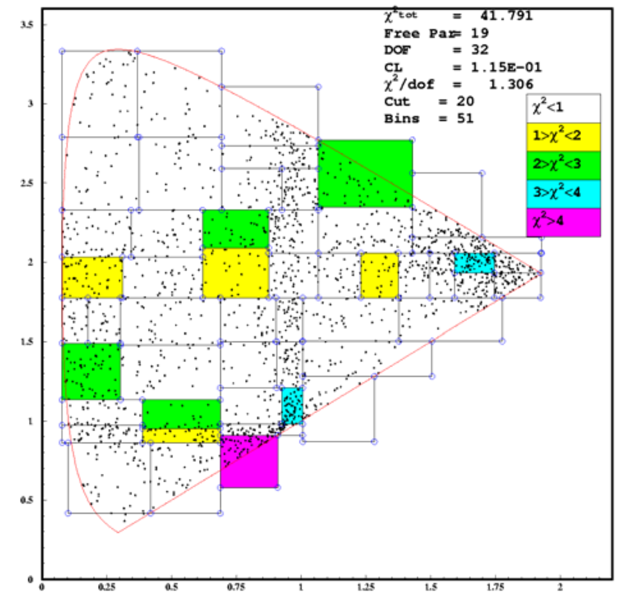


Coupled-channel BW

$$m = 975 \pm 10 \quad MeV$$
$$\Gamma_{\pi\pi} = 90 \pm 30 \quad MeV$$
$$\Gamma_{KK} = 36 \pm 15 \text{ MeV}$$

$$m = 1475 \pm \mathbf{10} \quad MeV$$
$$\Gamma = 112 \pm 24 \quad MeV$$

resonances	fit fraction (%)	phase ϕ_j	amplitude a_j
NR	25.5 ± 4.6	246.5 ± 4.7	0.520 ± 0.046
$f_2(1275)$	9.8 ± 1.3	140.2 ± 9.2	0.323 ± 0.022
$f_0(980)$	$\boxed{94.4 \pm 3.8}$	0 ($fixed$)	1 ($fixed$)
$S_0(1475)$	17.4 ± 3.1	249.7 ± 6.4	0.429 ± 0.043
$\rho^0(1450)$	4.1 ± 1.0	187.3 ± 15.3	0.208 ± 0.028



$$f_r = \frac{\int \left| \boxed{a_r} e^{i\boxed{\delta_r}} A_r \right|^2 dm_{12}^2\, dm_{13}^2}{\int \left| \sum_j a_j e^{i\delta_j} A_j \right|^2 dm_{12}^2\, dm_{13}^2} \qquad \sum_r f_r = 150\%$$

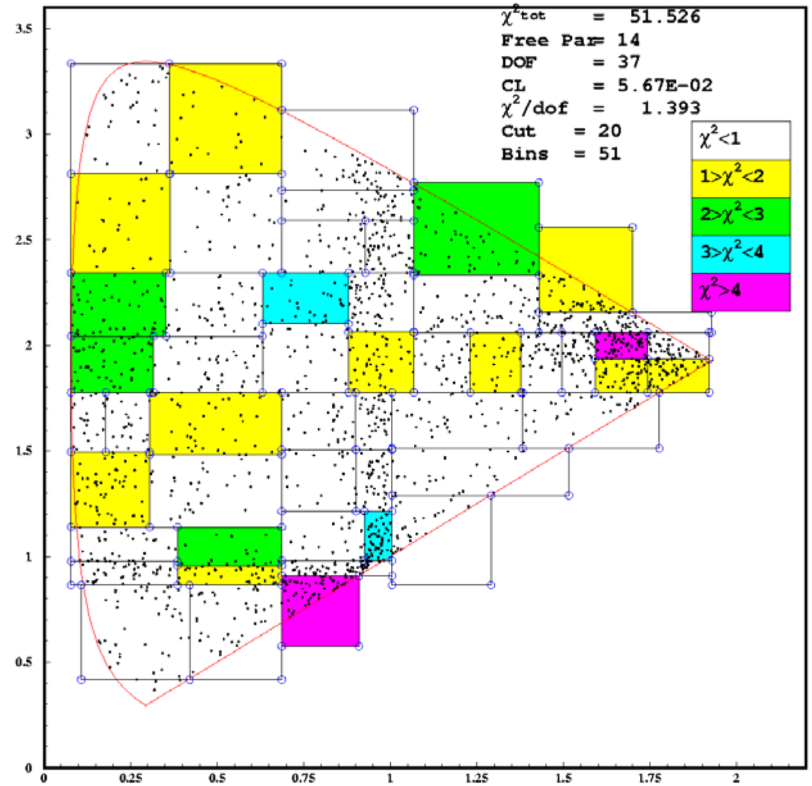
Preliminary

PDG Value for f0(1500)

m = 1507 ± 5 MeV

Γ = 109 ± 7 MeV

resonances	*fit fraction (%)*	*phase* ϕ_i
NR	36.9 ± 4.4	241.7 ± 3.8
$f_2(1275)$	9.5 ± 1.2	141.9 ± 7.4
$f_0(980)$	94.0 ± 3.6	0 *(fixed)*
$f_0(1500)$	17.8 ± 3.1	270.2 ± 5.4
$\rho(1450)$	4.6 ± 0.8	196.4 ± 14.6

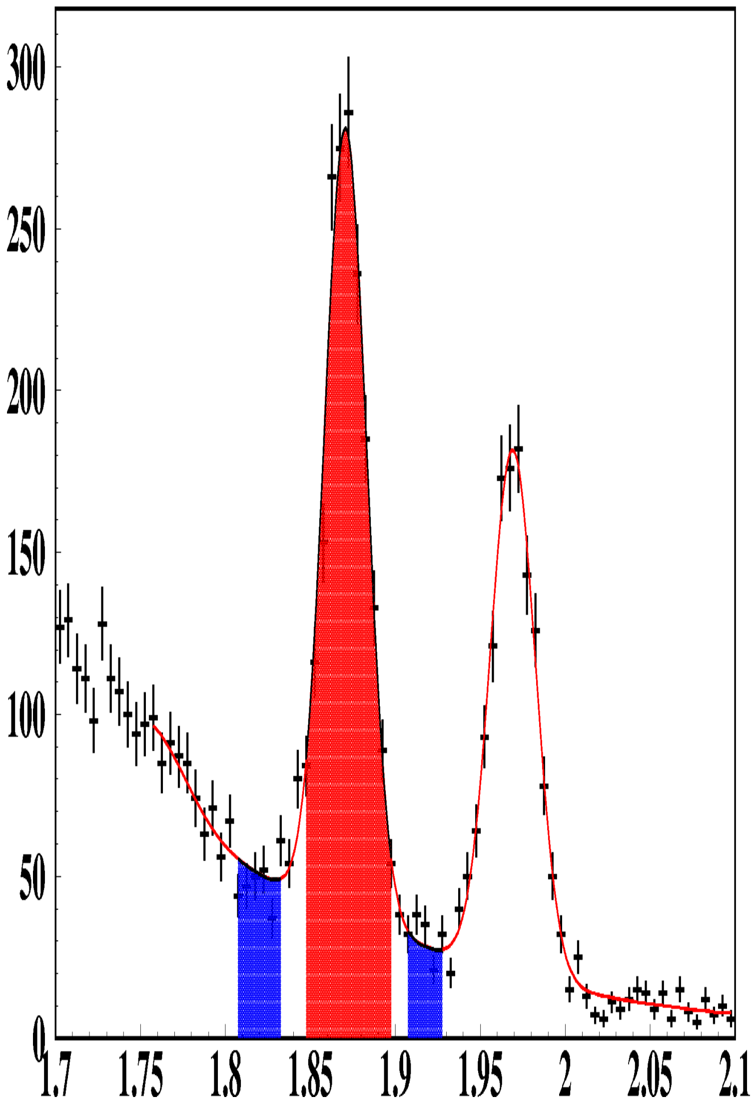


S_0(1475) (Focus)

m = 1475 ± 10 MeV

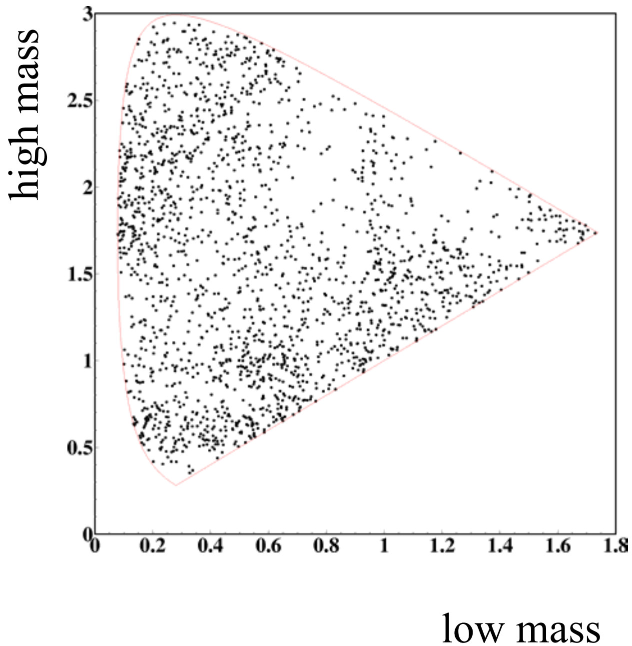
Γ = 112 ± 24 MeV

FOCUS $D^+ \rightarrow \pi^+ \pi^+ \pi^-$



Yield $D^+ = 1527 \pm 51$

S/N $D^+ = 3.64$



Isobar approach

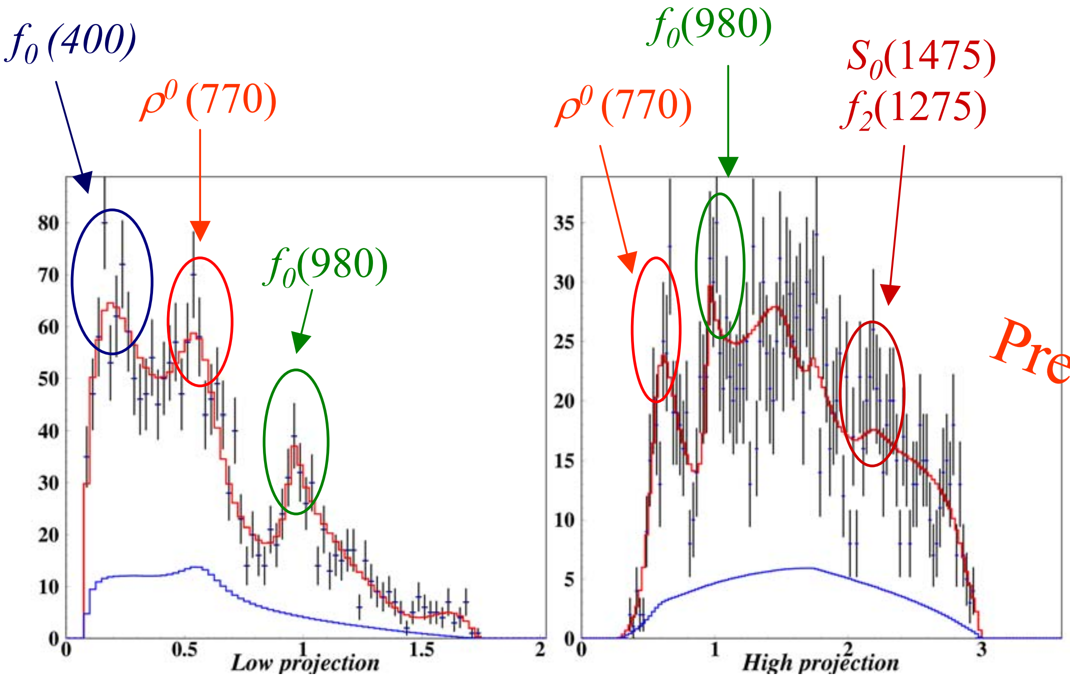
Single BW for $f_0(400)$

$m = 443 \pm 27\ MeV$
$\Gamma = 443 \pm 80\ MeV$

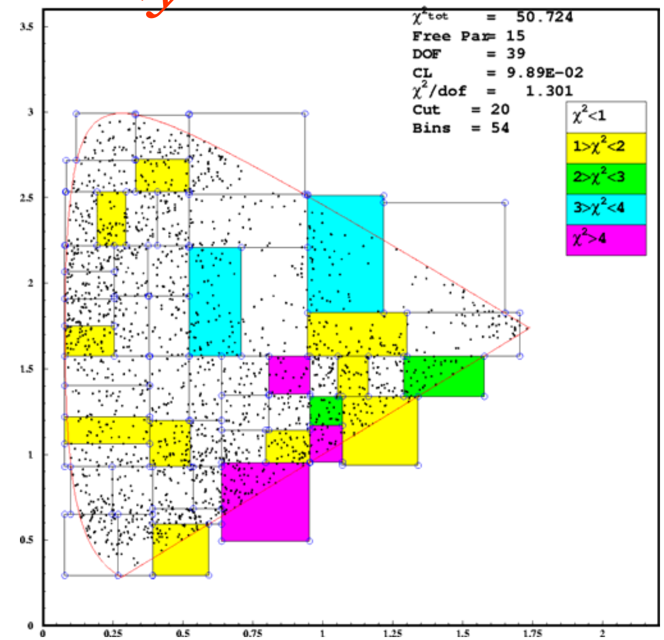
E791 Results :

$m = 478^{+24}_{-23} \pm 17\,MeV$

$\Gamma = 324^{+42}_{-40} \pm 21\,MeV$

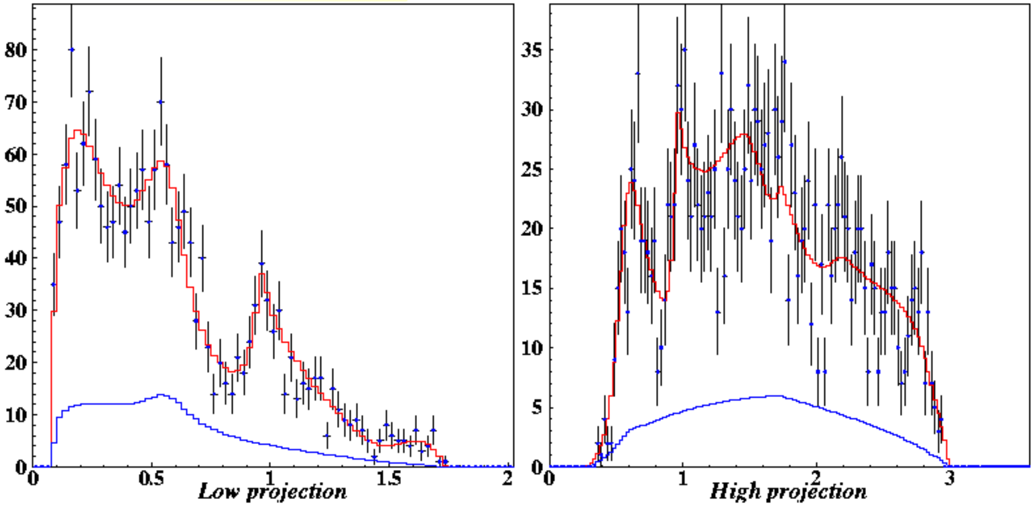


Preliminary

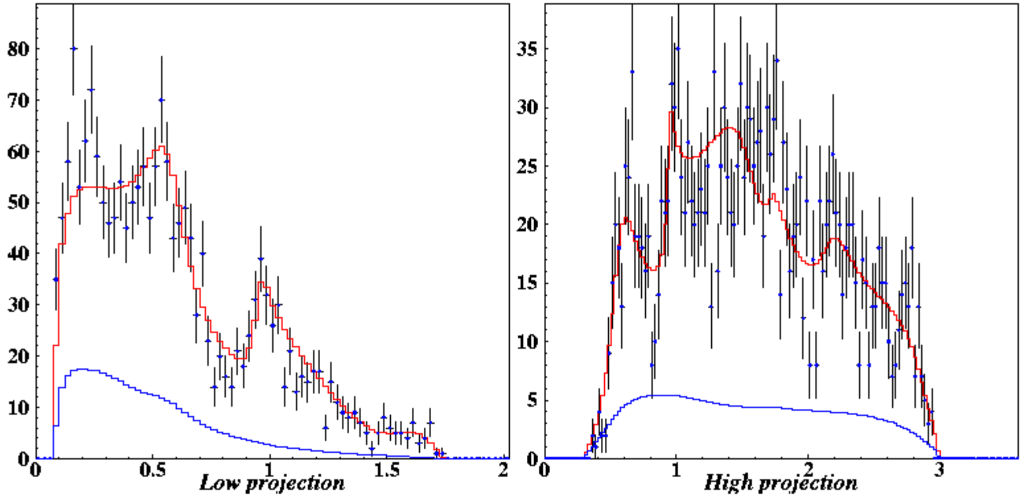


resonances	*fit fraction (%)*	*phase ϕ_j*	*amplitude a_j*
NR	9.8 ± 4.3	$0\ (fixed)$	$1\ (fixed)$
$\rho^0(770)$	32.8 ± 3.8	62.9 ± 16.8	1.830 ± 0.408
$f_2(1275)$	12.3 ± 2.1	-213.3 ± 17.7	1.120 ± 0.306
$f_0(980)$	6.7 ± 1.5	-145.9 ± 17.7	0.827 ± 0.239
$S_0(1475)$	1.8 ± 1.2	242.3 ± 25.8	0.425 ± 0.208
$f_0(400)$	18.9 ± 5.3	-96.9 ± 30.7	1.389 ± 0.468

With $f_0(400)$



Without $f_0(400)$



Preliminary

C.L. ~ 1 %

C.L. ~ 10^{-8} %

Resonances in K-matrix formalism

S.U. Chung et al.
Ann.Physik 4(1995) 404

$$K_{ij} = \sum_a \frac{g_{ai}(m) g_{aj}(m)}{(m_a^2 - m^2)}$$

where

$$g^2_{ai}(m) = m_a \Gamma_{ai}(m)$$

Unitarity ...

$$K_{ij} = \sum_a \frac{g_{ai}(m) g_{aj}(m)}{(m_a^2 - m^2)} + c_{ij}(m^2)$$

$$T = (I - iK \cdot \rho)^{-1} K$$

... from scattering to production: the P-vector approach

I.J.R. Aitchison
Nucl.Phys. A189 (1972) 514

$$P_i = \sum_{\alpha} \frac{\beta_\alpha g_{\alpha i}(m)}{(m_\alpha^2 - m^2)}$$

β (complex) carries the production information

$$P_i = P_i + d_i$$

$$F = (I - iK \cdot \rho)^{-1} P$$

known from scattering data

We need a description of the scattering ...

"K-matrix analysis of the 00++-wave in the mass region below 1900 MeV''

V.V Anisovich and A.V.Sarantsev Eur.Phys.J.A16 (2003) 229

- GAMS — $pp\rightarrow p^0p^0n$, hhn e hh'n, $|t|<0.2$ (GeV/c^2)
- GAMS — $pp\rightarrow p^0p^0n$, $0.30<|t|<1.0$ (GeV/c^2)
- BNL — $pp^- \rightarrow K\bar{K}n$
- CERN-Munich — $p^+p^- \rightarrow p^+p^-$
- Crystal Barrel — $p\bar{p} \rightarrow p^0p^0p^0$, p^0p^0h , p^0hh
- Crystal Barrel — $p\bar{p} \rightarrow p^0p^0p^0$, p^0p^0h
- Crystal Barrel — $p\bar{p} \rightarrow p^+p^-p^0$, $K^+K^-p^0$, $K_sK_sp^0$, $K^+K_sp^-$
- Crystal Barrel — $n\bar{p} \rightarrow p^0p^0p^-$, $p^-p^-p^+$, $K_sK^-p^0$, $K_sK_sp^-$
- E852 — $p^-p\rightarrow p^0p^0n$, $0<|t|<1.5$ (GeV/c^2)

$$K_{ij}^{00}(s) = \left(\sum_\alpha \frac{g_i^{(\alpha)} g_j^{(\alpha)}}{m^2_\alpha - s} + f_{ij}^{scatt} \frac{1 - s_0^{scatt}}{s - s_0^{scatt}} \right) \frac{s - s_A / 2m_\pi^2}{(s - s_{A0})(1 - s_{A0})}$$

K_{ij}^{IJ} is a 5x5 matrix (i,j=1,2,3,4,5)

A&S

1=ππ, 2=$K\overline{K}$ 3=4π 4= $\eta\eta$ 5= $\eta\eta'$

$g_i^{(\alpha)}$ is the coupling constant of the bare state α to the meson channel

f_{ij}^{scatt} and s_0 describe a smooth part of the K-matrix elements

$(s - s_A/2m_\pi^2)/(s - s_{A0})(1 - s_{A0})$ suppresses the false kinematical singularity at s = 0 near the ππ threshold

A&S K-matrix poles, couplings etc.

$Poles$	$g_{\pi\pi}$	g_{KK}	$g_{4\pi}$	$g_{\eta\eta}$	$g_{\eta\eta'}$
0.65100	0.24844	–0.52523	0	–0.38878	–0.36397
1.20720	0.91779	0.55427	0	0.38705	0.29448
1.56122	0.37024	0.23591	0.62605	0.18409	0.18923
1.21257	0.34501	0.39642	0.97644	0.19746	0.00357
1.81746	0.15770	–0.17915	–0.90100	–0.00931	0.20689
s_0^{scatt}	f_{11}^{scatt}	f_{12}^{scatt}	f_{13}^{scatt}	f_{14}^{scatt}	f_{15}^{scatt}
–3.30564	0.26681	0.16583	–0.19840	0.32808	0.31193
s_A	s_{A0}				
1.0	–0.2				

A&S T-matrix poles and couplings

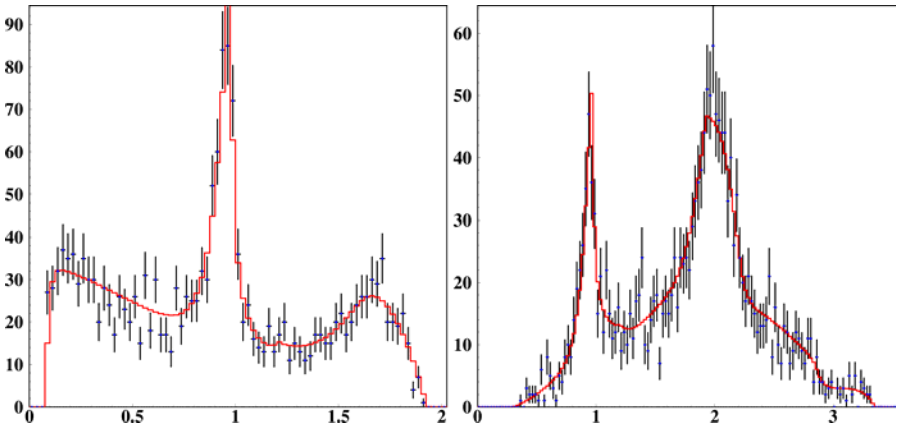
$(m, \Gamma/2)$	$g_{\pi\pi}$	g_{KK}	$g_{4\pi}$	$g_{\eta\eta}$	$g_{\eta\eta'}$
$(1.019, 0.038)$	$0.415e^{i\,13.1}$	$0.580e^{i\,96.5}$	$0.1482\,e^{i\,80.9}$	$0.484\,e^{i\,98.6}$	$0.401\,e^{i\,102.1}$
$(1.306, 0.167)$	$0.406\,e^{i\,116.8}$	$0.105\,e^{i\,100.2}$	$0.8912\,e^{-i\,61.9}$	$0.142\,e^{i\,140.0}$	$0.225\,e^{i\,133.0}$
$(1.470, 0.960)$	$0.758\,e^{i\,97.8}$	$0.844e^{i\,97.4}$	$1.681\,e^{i\,91.1}$	$0.431\,e^{i\,115.5}$	$0.175\,e^{i\,152.4}$
$(1.489, 0.058)$	$0.246\,e^{i\,151.5}$	$0.134\,e^{i\,149.6}$	$0.4867\,e^{-i\,123.3}$	$0.100\,e^{-i\,170.6}$	$0.115\,e^{-i\,133.9}$
$(1.749, 0.165)$	$0.536\,e^{i\,101.6}$	$0.072\,e^{i\,134.2}$	$0.7334\,e^{-i\,123.6}$	$0.160\,e^{i\,126.7}$	$0.313\,e^{i\,101.1}$

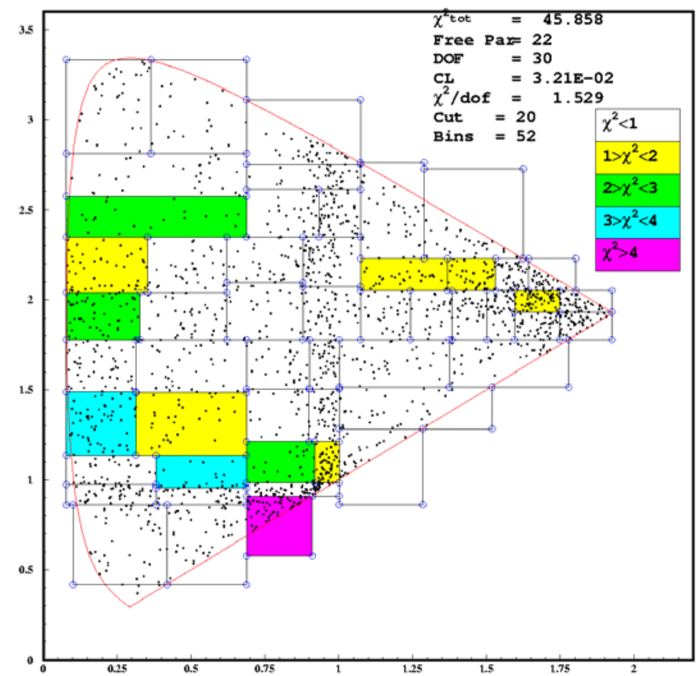
A&S fit does not need the σ

First fits to charm Dalitz plots in the K-matrix approach!

preliminary

$D_s \rightarrow \pi\pi\pi$



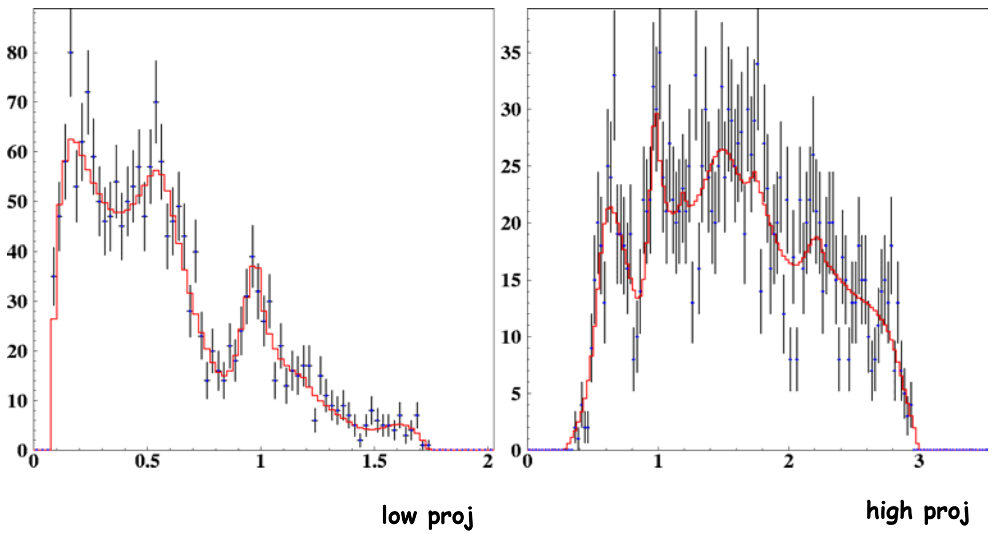


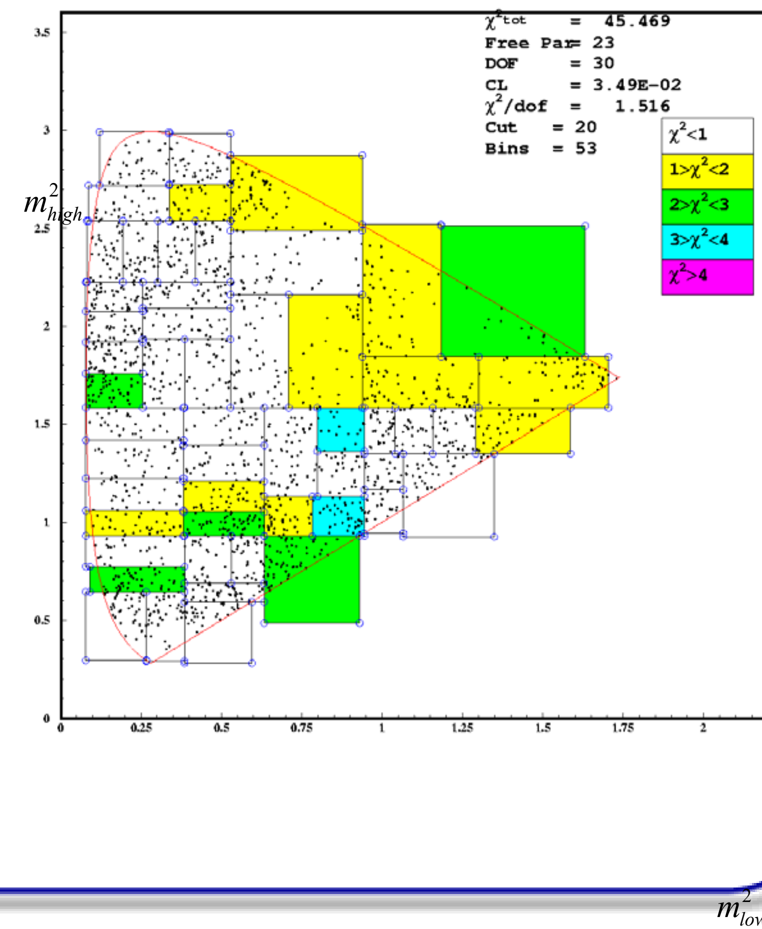
	fit fractions	phases
Γ SWave	= 0.8776 ± 0.0171	(0.0 ± 0.0)
Γ $\rho(1450)$	= 0.0542 ± 0.0114	(-161.4 ± 9.3)
Γ $f_2(1275)$	= 0.1153 ± 0.0119	(120.3 ± 5.4)

$$\sum_r f_r \sim 105\%$$

$D^+ \to \pi\pi\pi$

preliminary





	fit fractions	phases
Γ Swave	= 0.6647 ± 0.0416	(101.8 ± 22.5)
Γ ρ(770)	= 0.2116 ± 0.0436	(0.0 ± 0.0)
Γ f_2(1275)	= 0.1143 ± 0.0142	(−113.0 ± 9.0)

$$\sum_r f_r \sim 99\%$$

Conclusions

• The presence of scalar resonances in D-meson decays required a revision of the Dalitz plot parametrization

• A self-consistent description of s-wave isoscalar scattering in the energy range of interest is given in the K-matrix approach by A&S

• This is the first application of K-matrix approach to charm decays.

The results are extremely encouraging since the same parametrization of two-body resonances coming from light-quark experiments works for charm decays too

Such a result was not at all obvious!

- K-matrix analysis of $D^{+} \rightarrow \pi\pi\pi$ **does not need the σ but I doubt it is the last word on this puzzle !**

Program for the near future

- **perform an isoscalar s-wave global fit including charm data as well**

- **study the KKπ channel**

•A great deal of work still to be performed