The Fall & Rise of Total X-sections

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- Total & Elastic X-sections in QCD

↪ "soft" physics

i.e. non-perturbative QCD

- To enforce s-chanel unitarity:

eikonal formalism → $\boxed{\Omega(s,b)}$

b impact parameter

- From QCD, we need some inputs:

(1) Analytic $\alpha_s(s)$ for all s

fulfilling the notion of

confinement

+

integrability

② Some understanding within QCD of the parameters of the leading Regge trajectories :

$\boxed{P} \longrightarrow$ the Pomeron

$\boxed{R} \longrightarrow$ quasi-degenerate (ρ, ω, f_0, A_2)

Why is P half-a-unit ahead of R ?

Why is the P-slope about a half of the R-slope ?

③ Bloch-Nordsieck summation of soft-gluons which includes some mechanism for taming too fast a rise of the cross-sections

Notation : General Formulae:

- partial wave S-matrix

$$f_\ell(k) = \frac{\eta e^{2i\delta_R} - 1}{2ik}$$

- inelasticity factor $\eta = e^{-2\delta_I}$ $(0 \le \eta \le 1)$

- $f(k,\vartheta) = \sum_{\ell=0}^{\infty} (2\ell+1) f_\ell(k) P_\ell(\cos\vartheta)$

- Dictionary to go from partial waves to the impact parameter (for small angles ϑ)

$$\boxed{(\ell + 1/2) \longrightarrow (bk) \; ; \; P_\ell(\cos\vartheta) \longrightarrow J_0(b\sqrt{-t})}$$

$t = -q^2$ momentum transfer $\boxed{q \simeq k\vartheta}$

- the differential cross-section

$$\left(\frac{d\sigma}{dt}\right) = \pi \, |A(s,t)|^2$$

$$A(s,t) = \left(\frac{-i}{2k}\right) f(k,\vartheta) = \int_0^\infty b\,db\, J_0(b\sqrt{-t})\, \tilde{A}(s,b)$$

$$\tilde{A}(s,b) = 1 - \eta e^{2i\delta_R}$$

$$= (1 - \eta \cos 2\delta_R) - i\eta \sin(2\delta_R)$$

- Total X-section

$$\sigma_T(s) = (4\pi)\,\mathrm{Re}\,A(s,0)$$
$$= (4\pi)\int_0^\infty b\,db\,[1-\eta\cos 2\delta_R] = (4\pi)\int_0^\infty b\,db\,F_T(s,b)$$

- Elastic X-section

$$\sigma_{el}(s) = (2\pi)\int_0^\infty b\,db\left[(1-\eta\cos 2\delta_R)^2+(\eta\sin 2\delta_R)^2\right] = (2\pi)\int_0^\infty b\,db\,F_{el}(s,b)$$

- Define : __average__ # of collisions $\boxed{n(s,b)}$

$$e^{-n/2} = \eta\cos 2\delta_R$$

$$\begin{cases} \sigma_T(s) = (4\pi)\displaystyle\int_0^\infty b\,db\left[1-e^{-n/2}\right] \\ \sigma_{el}(s) = (2\pi)\displaystyle\int_0^\infty b\,db\left[(1-e^{-n/2})^2 + e^{-n}\tan^2\delta_R\right] \end{cases}$$

- __Forward__ __differential__ __X-section__:

$$\begin{cases} \left(\dfrac{d\sigma}{dt}\right)_{t=0} = \left(\dfrac{\sigma_T^2}{16\pi}\right)\left[1+\rho(s,0)\right] \\ \rho(s,0) = \left[\dfrac{\mathrm{Im}\,A(s,0)}{\mathrm{Re}\,A(s,0)}\right]^2 \end{cases}$$

- The __slope__ __parameter__

$$B(s) = \left[\frac{d}{dt}\ln\left(\frac{d\sigma}{dt}\right)\right]_{t=0}$$

- Expanding $A(s,t)$ for small t

$$A(s,t) \simeq A(s,0) + (t/4)\, A_2(s,0) + \cdots$$

- $$B(s) = B_0(s)\left[\frac{1+\tan\phi\,\tan\phi_2}{1+\tan^2\phi}\right]$$

$$A(s,0) = |A(s,0)|\,e^{i\phi(s)} \; ; \; A_2(s,0) = |A_2(s,0)|\,e^{i\phi_2(s)}$$

- $B_0(s)$ is the purely absorptive slope (i.e., $\delta_R = 0$)

$$\boxed{B_0(s) = \frac{\mathrm{Re}\,A_2(s,0)}{2\,\mathrm{Re}\,A(s,0)} = \frac{1}{2}\langle b^2\rangle}$$

- The ρ-parameter

$$\rho(s,0) = \tan^2\phi(s)$$

e.g. if at some energy $\rho \simeq 0.16$
$\rightarrow \tan\phi \simeq 0.4$

If some reasonable model could be made for the real part, we could say something about the phase $\phi_2(s)$ from the slope $B(s)$: of interest for slopes of Regge trajectories

Some General Remarks:

- For simplicity i) set $\phi_R = 0$

$$\therefore \quad \boxed{F_{el} = F_T^2}$$

& (ii) for large s, dimensional analysis tells us that if there is but <u>one scale</u>

<u>scaling</u> $\quad F_T(s,b) = f\left(b/b_{max}^{(s)}\right)$

$$\begin{cases} \sigma_T(s) = \left[4\pi\, b^2_{Max}(s)\right] \int_0^\infty (x\,dx)\, f(x) \\ \sigma_{el}(s) = \left[2\pi\, b^2_{max}(s)\right] \int_0^\infty (x\,dx)\, f^2(x) \end{cases}$$

$\therefore$ the <u>ratio</u> $\quad R_{el}(s) = \dfrac{\sigma_{el}(s)}{\sigma_{TOT}(s)} = \dfrac{\int_0^\infty x\,dx\, f^2(x)}{\int_0^\infty x\,dx\, f(x)}$

is <u>independent of s</u>

- Also, the <u>slope parameter</u> $B(s)$ would <u>scale with $b^2_{Max}(s)$</u>.

- While the above would not be satisfactory to describe the data accurately, it is instructive to evaluate it analytically in some simple models as limiting cases of more realistic schemes.

1. Gaussian:

$$f(x) = f_0^G e^{-x^2} \qquad (f_0^G \lesssim 1)$$

$$\boxed{R^{el}_{Gauss} = \tfrac{1}{4} f_0^G \lesssim (\tfrac{1}{4})}$$

2. Exponential (or Boltzmann):

$$f(x) = f_0^B e^{-x} \qquad (f_0^B \lesssim 1)$$

$$\boxed{R^{el}_{Boltzmann} = (\tfrac{1}{8}) f_0^B \lesssim (\tfrac{1}{8})}$$

3. Fermi Distribution:

$$f(x) = f_0^F \left(\frac{2}{e^{x}+1} \right) \qquad (f_0^F \lesssim 1)$$

$$\boxed{R^{el}_{Fermi} = f_0^F \left[1 - \frac{12 \ln 2}{\pi^2} \right] \lesssim 0.16}$$

4. Limiting Froissart Distribution:

$$n(b,s) = \begin{cases} \infty & b < b_{max}(s) \\ 0 & b > b_{max}(s) \end{cases}$$

$$\boxed{R^{el}_{Froissart} \longrightarrow \tfrac{1}{2}}$$

not tenable upto $\sqrt{s} \lesssim 2$ TeV

Moral: Fermi & Froissart not adequate

the dominant component (soft part ?) at medium energies → Boltzmann, crossing over to a Gaussian

(hard jet part)

$$\begin{cases}\underline{\text{Instability}} \text{ of the AF } \underline{\text{vacuum}} \\ \underline{\text{Confinement}} \\ \underline{\text{Analytic } \alpha_s} \\ \underline{\text{Applications}}\end{cases}$$

- In QCD, the 1-loop AF gives

$$\alpha_{1\text{-loop}}(s) = \frac{1}{b \ln(-s/\Lambda^2)}$$

has a pole at $s = -\Lambda^2$ (space-like value). Higher loops suffer from a similar disease.

- Of course, analyticity – derived from unitarity – forbids any singularity in the space-like region.

- Also, the absorptive part of the AF α_s has the "wrong" sign for <u>stability</u>. The physics behind this mysterious statement is very simple.

Consider the (static) Coulomb potential for a charged particle

in <u>vacuum</u> $\longrightarrow \dfrac{\alpha_0}{r}$

$\downarrow$

in a <u>medium</u> $\longrightarrow \dfrac{\alpha_0}{\epsilon r}$ $\quad\epsilon \to$ dielectric constant

the s-dependent (i.e., dynamical)

$$\boxed{\epsilon(s) = \frac{1}{\alpha(s)}}$$

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$\epsilon \to$ dielectric constant

the s-dependent (i.e., dynamical)

$$\boxed{\epsilon(s) = \frac{1}{\alpha(s)}}$$

$\therefore \quad \boxed{\mathrm{Im}\, \epsilon_{AF}(s) = -\pi b\, \vartheta(s)}$

sure sign of instability! In what sense?

Recall that in electrodynamics

$$\epsilon(s) = \epsilon_0 + \frac{i\,\sigma(s)}{\sqrt{s}}$$

$\sigma(s)$ is the <u>conductivity</u>. For <u>Ohm's law</u> to work, there must be <u>dissipation</u>, i.e., $\boxed{\sigma > 0}$

$\therefore \quad \boxed{\mathrm{Im}\, \epsilon(s) \geqslant 0}$

- So, how apparently disastrous result for the AF $\mathrm{Im}\, \epsilon_{AF}(s) < 0$; the system would behave as an <u>amplifier</u> and thus unstable. The point is that someone has to keep pumping in energy and hence it can only be a transient phenomena lasting for a finite time.
- Of course, this is a pleasing result since it signals that the true ground state of QCD can not be composed of quarks & glue.

- Several analytic models for α_s have been constructed.

→ <u>Shirkov et al</u>: compute $\text{Im}\,\alpha$ from AF, use dispersion relation (unsubtracted) to get the real part:

- $$\alpha_I(s) = \left(\frac{1}{b}\right)\left[\frac{1}{\ln(-s/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 + s}\right]$$

the second term cancels the unwanted pole. This procedure has been generalized upto 3 loops.

- So, why not stop here? The lacuna here is that this α_s is "<u>too tame</u>": it has not enough "<u>oomph</u>" to produce all what QCD is advertised to possess. That is, to obtain confinement, infinite # of rising Regge trajectories etc. etc., we would still have to add – ad hoc – a confining potential.

- A different analytic model is due to Nesterenko, who eliminates the pole multiplicatively

$$\alpha_{II}(s) = \left(\frac{1}{b}\right)\frac{(1 + \Lambda^2/s)}{\ln(-s/\Lambda^2)}$$

When appropriately fourier transformed, this leads to Richardson type ~~model~~

- The problem with this model is that it is too singular, so that integrals – which routinely occur in soft-gluon summations – of the type

$$\alpha_{av}(s) = \frac{1}{s}\int_0^s ds'\,\alpha_s(s')$$

are not convergent

- Taking our cue from the above, we have developed a general class of models, for which α_I & α_{II} are limiting cases.

- A simpler function to disperse is the color dielectric function $\epsilon(s) = 1/\alpha_s(s)$

$$\epsilon(s) = \left(\frac{s}{\pi}\right)\int_0^\infty \frac{ds'\,\mathrm{Im}\,\epsilon(s')}{s'\,(s'-s-i\delta)}$$

with

$$\mathrm{Im}\,\epsilon(s) = -\frac{\pi b}{\left[1+(\Lambda^2/s)^p\right]} \qquad (0 < p \leq 1)$$

- In this form, confinement is automatic provided $\epsilon(0) = 0$

- If $p = 1 \longrightarrow \alpha_{II}(s)$

Applications:

1. Intrinsic transverse momentum:

Some time ago [NPS], analytic α_s of this type were used for the transverse momentum distribution of lepton pairs, W & Z. Good agreement with data were obtained without any need of an additional intrinsic transverse momentum term.

2. Inclusive τ Decays:

- The ratio of hadronic to leptonic τ decay widths

$$R_\tau = \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)}$$

allows one to compute, with some accuracy, the value of $\alpha_s(s)$ at τ-mass: $s = M_\tau^2 \simeq (1.78\,\text{GeV})^2$

- For massless quarks

$$R_\tau = 2 \int_0^{M_\tau^2} \left(\frac{ds}{M_\tau^2}\right) \left[1 - \frac{s}{M_\tau^2}\right]^2 \left(1 + 2\frac{s}{M_\tau^2}\right) \tilde{R}(s)$$

- $\tilde{R}(s)$ is proportional to the hadronic correlator

$$\Pi(s) = \sum_q |V_{uq}|^2 \left[\Pi_{uq,V}(s) + \Pi_{uq,A}(s)\right]$$

- For massless quarks, the vector & axial-vector correlators are coincident $\therefore$ $\tilde{R}(s)$, apart from charge factors, is equal to the hadronic $R(s)$.

- $$R_\tau = \frac{1}{\pi}\int_0^1 dz\,(1-z)^3(1+z)\frac{d}{dz}\,\text{Im}\,\Pi(zM_\tau^2)$$

We can write the above as a contour integral in the complex z-plane as

$$R_\tau = \left(\frac{1}{2\pi i}\right)\oint_{|z|=1}\left(\frac{dz}{z}\right)(1-z)^3(1+z)\,D(zM_\tau^2)$$

where $D(s)$ is the Adler function

$$D(s) = -s\frac{d}{ds}\,\text{Im}\,\Pi(s) = 3\left[1+\frac{\alpha_s(s)}{\pi}\right]$$

- R_τ has been computed using two methods:

A : <u>Time-like Modulus Integration</u>

using $\quad D(s) = 3\left(1+\frac{|\alpha_s(s)|}{\pi}\right)$

$$R_\tau = 3 + 6\int_0^{M_\tau^2}\left(\frac{ds}{M_\tau^2}\right)\left[1-\frac{s}{M_\tau^2}\right]\left[1+2\frac{s}{M_\tau^2}\right]\frac{|\alpha_s(s)|}{\pi}$$

B : <u>Complex Contour Integration</u>

$$R_\tau = 3 + \frac{3}{2\pi i}\oint_{|z|=1}\left(\frac{dz}{z}\right)(1-z)^3(1+z)\,\frac{\alpha_s(zM_\tau^2)}{\pi}$$

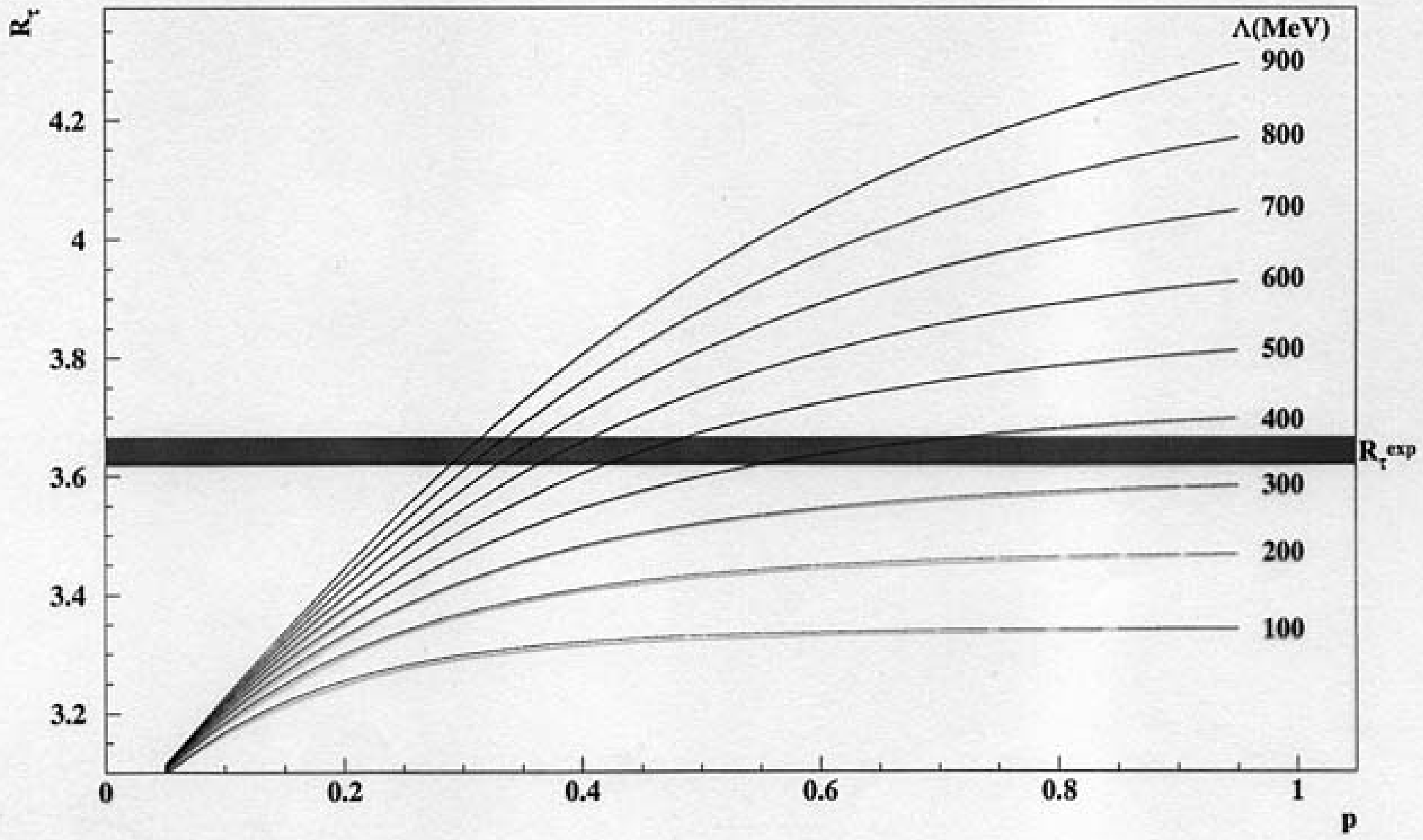
- For our α_s – with different p & Λ – the two figures show the results for R_τ obtained using both methods (A & B) and compared with experimental value (gray band). Previous analytic continuations of α_s required a large $\Lambda \sim 850$ MeV to get agreement with data. On the contrary, we get good agreement a reasonable value of

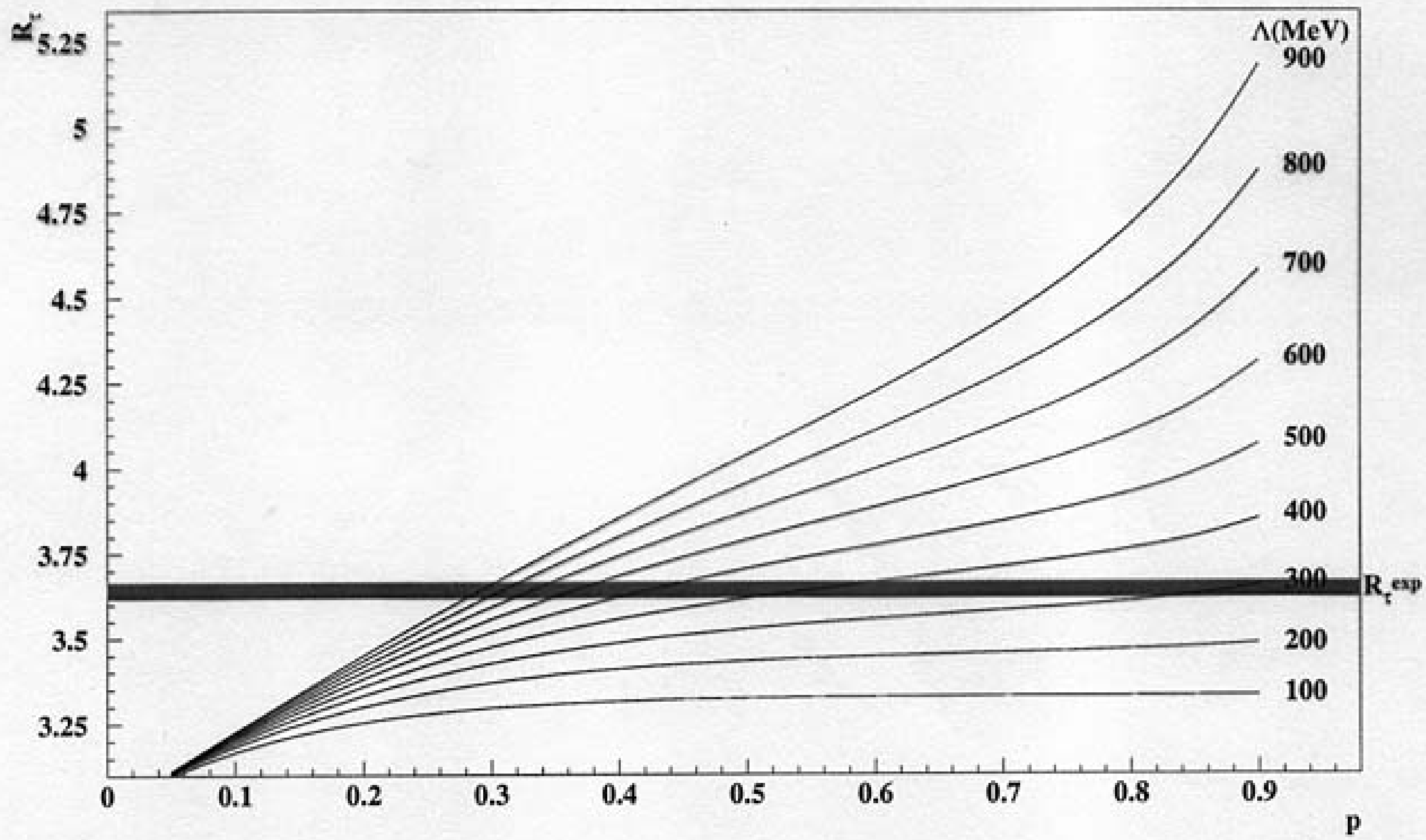
$$\Lambda \sim 300 \text{ MeV}$$

& $p \sim (0.5 \div 0.8)$ used in other analyses

Inclusive τ-lepton

(circle and time-like modulus integration)





R_τ^{exp}: D.E.Groom *et al., Eur.Phys.J.* **C15** (2000) 1.
p: A.Nakamura, G.Pancheri, Y.N.Srivastava *Z.Phys.* **C21** (1984) 243.

Ratios of slopes & intercepts of $(q\bar{q})$ & (gg) Regge Trajectories:

- A Pomeron structure ($\mathbb{P}$) in QCD is believed to be generated by 2 gluons, just as the quasi-degenerate ρ, ω, f_0, A_2 ($\mathbb{R}$) trajectories are generated by a (quasi-massless) $q\bar{q}$ pair. A partial understanding of the relationship between P & R trajectories may be obtained through the following simple <u>hadronic string</u> picture

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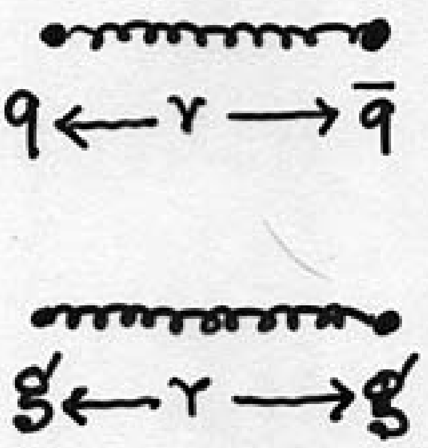

$i=R$ → massless $q\bar{q}$ pair (r)

$i=P$ → a gg pair (r)

$$E_{CM} = \underset{\text{term}}{\text{Angular Mom}} + \underset{\text{term}}{\text{Coulomb}} + \underset{\text{term}}{\text{Tension}}$$

$$\therefore \quad M_i(J) = \underset{r}{\text{Min}}\left[\frac{2(J - C_i\alpha_0)}{r} + C_i\tau_0 r\right]$$

$$\bar{r}_i = \sqrt{\frac{2(J - C_i\alpha_0)}{C_i\tau_0}}$$

$\underline{i=R} \rightarrow C_F = 4/3$

$\underline{i=P} \rightarrow C_P = 3$

Regge $\xrightarrow{\text{trajectory}}$

$$\boxed{\alpha_i(s) = C_i\alpha_i(0) + \left(\frac{1}{8C_i\tau_0}\right)s}$$

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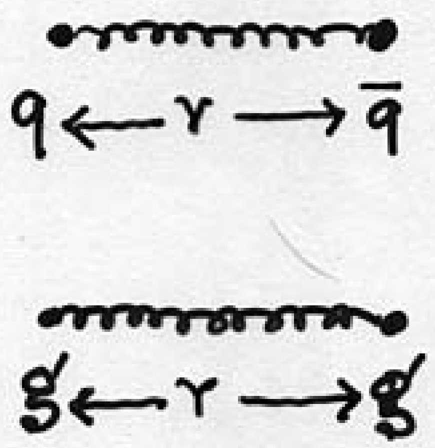

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$\left.\begin{array}{l} i = R \rightarrow C_F = 4/3 \\ i = P \rightarrow C_P = 3 \end{array}\right\}$

Regge trajectory $\longrightarrow$

$$\boxed{\alpha_i(s) = C_i \alpha_i(0) + \left(\frac{1}{8 C_i \tau_0}\right) s}$$

- $\alpha_R(0) = \frac{4}{9}\,\alpha_P(0)$
- $\alpha_P' = \frac{4}{9}\,\alpha_R'$

Both are reasonable results

This gives – to the zeroeth order – why the R-intercept is near a half if the Pomeron intercept is near 1 and why the P-intercept is about $\alpha_P' \sim 0.4\ \text{GeV}^{-2}$ if $\alpha_R' \simeq 0.9\ \text{GeV}^{-2}$.

A posteriori, it also strengthens the case for a predominantly 2-gluon makeup of the Pomeron.

• BN Summation of Soft Gluons:

In QCD, as the energy increases, there is an increasing probability of hard scattering involving hard gluons

∴ The rise can then be ascribed to the increasing # of hard gluons emitted by quarks in the colliding particles. As this # increases so does the probability of collisions.

• Quantitatively → use current parton densities to compute the energy dependence of the jet cross-section

$$\sigma_{jet} = \int_{p_{t\,min}} (d^3 \underset{\sim}{p}_t) \frac{d\sigma_{jet}(s, p_t)}{(d^3 \underset{\sim}{p}_t)}$$

Rutherford divergent

∴ artificial cutoff

For $p_{t\,min} \simeq (1 \div 2)$ GeV → X-section increases too steeply.

→ Use Eikonal formalism