Nucleon-nucleon, $\gamma p$ and $\gamma\gamma$ scattering, using factorization: the Aspen Model and analytic amplitude analysis

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Outline

• Part I: fitting $\sigma_{\text{tot}}$, $\rho$ values and B, nuclear slopes of accelerator data--p-p and pbar-p--using a QCD-inspired model (Aspen Model)  
  \textit{M.M. Block, E.M. Gregores, F. Halzen & G. Pancheri}

• Part II: global fit using both cosmic ray and accelerator data simultaneously  
  \textit{M.M. Block, F. Halzen & T. Stanev}

• Part III: predictions of forward scattering parameters for LHC (14 TeV)---$\sigma_{\text{tot}}$, $\sigma_{\text{elastic}}$, $\rho$, and B
• Part IV-The **factorization hypothesis**: relating n-n, $\gamma p$ and $\gamma\gamma$ collisions
  
  *M. M. Block and A. B. Kaidalov*

• Part V-Experimental evidence for **factorization**, **quark counting** and **vector dominance** in n-n, $\gamma p$ and $\gamma\gamma$ collisions, using the Aspen Model

  *M. M. Block, F. Halzen, A. B. Kaidalov and G. Pancheri*

• Part VI-Experimental evidence for **factorization**, using **real analytical amplitudes**, for n-n, $\gamma p$ and $\gamma\gamma$ collisions

  *M. M. Block and K. Kang*
Part I: fitting $\sigma_{\text{tot}}$, $\rho$ values and $B$, nuclear slopes, of accelerator data--p-p and pbar-p-- using a QCD-inspired model (Aspen Model)

M.M. Block, E.M. Gregores, F. Halzen & G. Pancheri

Eduardo, Martin, Francis
3 of 4 authors working hard!

Giulia, #4 author
Eikonal Formalism

We assume that the scattering amplitude in impact parameter space \( b \) is given by

\[
a(b, s) = \frac{i}{2} \left( 1 - e^{i\chi(b, s)} \right) \\
= \frac{i}{2} \left( 1 - e^{-\chi_{\text{Im}}(b, s) + i\chi_{\text{Re}}(b, s)} \right),
\]

The total cross section \( \sigma_{\text{tot}}(s) \), \( \rho \)-value, elastic cross section \( \sigma_{\text{elastic}}(s) \) and differential elastic scattering cross section \( \frac{d\sigma}{dt} \) are given by:

\[
\sigma_{\text{tot}}(s) = 4 \int \text{Im} a(b) \, d^2\vec{b},
\]

\[
\rho(s) = \frac{\text{Re} \left[ \int a(b, s) \, d^2\vec{b} \right]}{\text{Im} \left[ \int a(b, s) \, d^2\vec{b} \right]},
\]

\[
\sigma_{\text{elastic}}(s) = 4 \int |a(b)|^2 \, d^2\vec{b},
\]

\[
\frac{d\sigma}{dt} = \frac{1}{\pi} \left| \int e^{ij\vec{q}\cdot\vec{b}} a(b, s) \, d^2\vec{b} \right|^2, \quad t = -q^2.
\]
Theoretical Conditions Required

We insist that our model satisfies:

1. Crossing symmetry, i.e., be either even or odd under the transformation $E \rightarrow -E$, where $E$ is the laboratory energy. This allows us to simultaneously describe $\bar{p}p$ and $pp$ scattering.

2. Unitarity. We will use an eikonal formalism to guarantee this.

3. Analyticity. We need this to calculate the phase of the forward scattering amplitude and hence, the $\rho$ value.

4. The Froissart bound. Asymptotically, we expect that the total cross section will rise as $(\log s)^2$. 
\[ \chi_{gg}(b, s) = \sigma_{gg}^{QCD}(s)W_{gg}(b; \mu_{gg}) \]

with

\[ \sigma_{gg}^{QCD}(s) = \int d\left(\frac{\hat{s}}{s}\right) F_{gg}(x_1 x_2 = \frac{\hat{s}}{s}) \sigma_{gg}(\hat{s}), \]

\[ F_{gg} = \int dx_1 dx_2 f_g(x_1) f_g(x_2) \delta(x_1 x_2 - \tau), \]

using the gluon structure function

\[ f_g(x) \sim \frac{(1 - x)^5}{x^J}, \]

with \( J = 1 + \epsilon \), in Regge language, the Pomeron intercept, and

\[ \sigma_{gg}(\hat{s}) = C_{gg} \Sigma \theta(\hat{s} - m_0^2), \] where \( \Sigma = \frac{9\pi \alpha_s^2}{m_0^2} \), \( \alpha_s = 0.5 \), and \( C_{gg} \) is a real constant fitted by experiment.
The high energy behavior is controlled by

$$\lim_{s \to \infty} \int_{m_0^2/s}^{1} d\tau F_{gg}(\tau) \sim \int_{m_0^2/s}^{1} d\tau \frac{-\log \tau}{\tau J}$$

$$\sim \left( \frac{s}{m_0^2} \right)^\epsilon \log \frac{s}{m_0^2}.$$

The cut-off impact parameter $b_c$ is given by

$$c W_{gg}(b_c, \mu_{gg}) s^\epsilon \log(s/m_0^2) \sim 1,$$

where $c$ is a constant. For large values of $\mu b$,

$$c'(\mu_{gg} b_c)^{5/2} e^{-\mu_{gg} b_c} s^\epsilon \sim 1$$

with $c'$ another constant, and therefore,
If \( J > 1 \), and \( s \to \infty \), we get the Froissart bound,

\[
\sigma_{tot} = 2\pi \left( \frac{\epsilon}{\mu_{gg}} \right)^2 \log^2 \frac{s}{s_0}.
\]

The usual Froissart bound coefficient of the \( \log^2 \frac{s}{s_0} \) term, \( 1/m_{\pi}^2 = 20 \text{ mb} \), is now replaced by \( (\epsilon/\mu_{gg})^2 \sim 0.002 \text{ mb} \). Note that \( \mu_{gg} \) controls the size of the area occupied by the gluons inside the nucleon.
To reproduce the cross section at lower energies one must consider the \(qq\) and \(qg\) contributions. Using toy structure functions \(f_q \sim x^{-1/2}(1 - x)^3\) and \(f_g \sim x^{-1}(1 - x)^5\), we find

\[
\chi_{qq} = \left[ \frac{m_0}{\sqrt{s}} \log \frac{s}{s_0} + \mathcal{P}\left(\frac{m_0}{\sqrt{s}}\right) \right] W_{qq}(b; \mu_{qq})
\]

and

\[
\chi_{qq} = \left[ a' \log \frac{s}{s_0} + \mathcal{P}'\left(\frac{m_0}{\sqrt{s}}\right) \right] W_{qg}(b; \sqrt{\mu_{qq}\mu_{gg}}).
\]

\(\mathcal{P}\) and \(\mathcal{P}'\) are polynomials in \(m_0/\sqrt{s}\).
Since we *only* attempt to decipher the high energy behavior, we replace the above $\chi$’s by

$$
\chi_{qq} = \sum \left[ C + C_{\text{Regge}} \frac{m_q}{\sqrt{s}} \right] W(b; \mu_{qq}), \\
\chi_{qg} = \sum \left[ C_{\log} \log \frac{s}{s_0} \right] W(b; \sqrt{\mu_{qq}\mu_{gg}}),
$$

as suggested by standard Regge arguments. Treating the real coefficients $C$, $C_{\text{Regge}}$ and $C_{\log}$ as parameters, this low energy parametrization can also accommodate elastic and diffractive contributions.
We set

\[ \chi_{qq} = \sum \left[ C + C_{\text{Regge}} \frac{m_0}{\sqrt{s}} \right] W(b; \mu_{qq}), \]
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as suggested by standard Regge arguments. Treating the real coefficients \( C, \ C_{\text{Regge}} \text{ and } C_{\log} \) as parameters, this low energy parametrization can also accommodate elastic and diffractive contributions.
We insure the correct analyticity properties of our even amplitudes by substituting

\[ s \rightarrow s \ e^{-i\pi/2} \]

throughout.

We introduce an \textit{ad hoc} crossing odd amplitude parametrizing the difference between the \( pp \) and \( \bar{p}p \) scattering amplitudes

\[ \chi_{\text{odd}}(s, b) = -C_{\text{odd}} \frac{m_0}{\sqrt{s}} e^{-i\pi/4} W(b; \mu_{\text{odd}}), \]

where the real constant \( C_{\text{odd}} \) is fixed by experiment.
Results of the $\chi^2$ Fit to $\sigma$, $\rho$ and $B$ data from pp and pbar-p
Part II: global fit using both cosmic ray and accelerator data simultaneously

*M.M. Block, F. Halzen & T. Stanev*
Accelerator data give good fit

The published cosmic ray data (the Diamond and Triangles) are the problem
EXPERIMENTAL PROCEDURE

Fly’s Eye Shower Profile

Monte Carlo Example

Fig. 1 An extensive air shower that survives all data cuts. The curve is a Gaisser-Hillas shower-development function: shower parameters $E=1.3\ \text{EeV}$ and $X_{\text{max}} = 727 \pm 33 \ \text{g cm}^{-2}$ give the best fit.

Fig. 7 $X_{\text{max}}$ distribution with exponential trailing edge.
Extraction of $\sigma_{\text{tot}}(pp)$ from Cosmic Ray Extensive Air Showers

Two significant drawbacks of the extraction method are the need for:

1. a model of proton–air interactions to complete the loop between the measured attenuation length $\Lambda_m$ and the cross section $\sigma_{p-\text{air}}^{\text{inel}}$, i.e., the value of $k$, in

$$\Lambda_m = k\lambda_{p-\text{air}} = k\frac{14.5m_p}{\sigma_{p-\text{air}}^{\text{inel}}}.$$ 

2. a simultaneous relation between $B$ and $\sigma_{pp}$ at very high energies—well above the region currently accessed by accelerators.
Published Fly’s Eye Result:

\[ \sigma_{\text{tot}}(p\text{-air}) = 540 \pm 50 \text{ mb, at } 30 \text{ TeV} \]

See R. Engel et al., Phys. Rev. D58, 014019 (1998), for \( \sigma_{\text{tot}}(p\text{-air}) \) curves, using Glauber theory.
Results of Global Fit, Cosmic Ray and Accelerator Data

\[ \Lambda = k \cdot \lambda_{p\text{-air}} = k \cdot 14.5 m_p / \sigma_{p\text{-air}} \]

- \( \sigma_{\text{inel}}(p\text{-air}) \) vs. energy
- \( \sigma_{\text{tot}}(p\text{-p}) \) vs. \( \sigma_{\text{inel}}(p\text{-air}) \)
Monte Carlo Results:
C.L. Pryke, astro-ph/0003442

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<th>Model</th>
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<td>CORSIKA-SIBYLL</td>
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<td>MOCCA–SIBYLL</td>
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<td>CORSIKA-QGSjet</td>
<td>1.30 ± 0.04</td>
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<tr>
<td>MOCCA–Internal</td>
<td>1.32 ± 0.03</td>
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</table>

Measured $k = 1.349 ± 0.045 ± 0.028$
Result of Global fit for $\sigma_{\text{tot}}(pp)$ and $\sigma_{\text{tot}}(p\bar{p}-p)$, using both Accelerator and Cosmic Ray Data

Measured $k = 1.349 \pm 0.045 \pm 0.028$
LHC Predictions

Our predictions for the LHC (14 TeV) are:

\[ \sigma_{\text{tot}} = 107.4 \pm 1.5 \text{ mb} \]

\[ \sigma_{\text{elastic}} = 30.9 \pm 0.5 \text{ mb} \]

\[ \rho = 0.112 \pm 0.002 \]

\[ B = 19.41 \pm 0.15 \text{ GeV}^{-2} \]
LHC Predictions

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\[ B = 19.41 \pm 0.15 \text{ GeV}^{-2} \]

GapSurvivalProbability \[ < |S|^2 > = 12.6 \pm 0.1 \% \]
Elastic Scattering at the LHC

p-p elastic scattering at $\sqrt{s} = 14$ TeV

d$\sigma$/d$t$, in mb/(GeV/c)$^2$ vs. $|t|$, in (GeV/c)$^2$
A Useful High Energy Parameterization of $\sigma_{\text{inel}} (p\text{-air})$:

$$\sigma_{\text{inel}} (p\text{-air}) = 177.43 + 15.44 \ln s - 0.0204 \ln^2 s,$$

$\sigma$ in mb, $s$ in GeV$^2$. 

\[
\begin{align*}
\sigma_{\text{inel}} \text{ (pair),} \\
\text{in mb}
\end{align*}
\]
A High Energy Parameterization of $\sigma_{\text{inel}}(p\text{-air})$, useful for calculating Neutrino Fluxes

$$\sigma_{\text{inel}}(p\text{-air}) = \sigma_0 [1 + a_0 \ln(E_{\text{lab}} / 1000)],$$
$$\sigma_0 = 297.20 \text{ mb} , \quad a_0 = 0.04879, \quad E_{\text{lab}} \text{ in GeV.}$$
Consequences of the Factorization Hypothesis in nucleon-nucleon, $\gamma$-p and $\gamma-\gamma$ Collisions,

M. M. Block, Northwestern University

A. B. Kaidalov, Moscow University

Alexei
If \( \frac{\sigma_{el}}{\sigma_{tot}}_{nn} = \frac{\sigma_{el}}{\sigma_{tot}}_{\gamma p} = \frac{\sigma_{el}}{\sigma_{tot}}_{\gamma \gamma} \), all \( s \)

and

\[
\chi^\text{even}(s, b) = i \left[ \sigma_{gg}(s)W(b; \mu_{gg}) + \sigma_{qg}(s)W(b; \mu_{qg}) + \sigma_{qq}(s)W(b; \mu_{qq}) \right],
\]

\[
W(b; \mu) = \frac{\mu^2}{96\pi}(\mu b)^3 K_3(\mu b),
\]

with \( \int W(b; \mu)d^2 b = 1 \),

then we substitute \( \sigma \rightarrow \kappa \sigma, \mu \rightarrow \sqrt{\frac{1}{\kappa}} \mu \) in \( \chi^\text{even} \) to get \( \chi^\gamma p \)

\[
\chi^\gamma p(s, b) = i \left[ \kappa \sigma_{gg}(s)W(b; \sqrt{\frac{1}{\kappa}} \mu_{gg}) + \kappa \sigma_{qg}(s)W(b; \sqrt{\frac{1}{\kappa}} \mu_{qg}) + \kappa \sigma_{qq}(s)W(b; \sqrt{\frac{1}{\kappa}} \mu_{qq}) \right],
\]
Next, we substitute $\sigma \to \kappa \sigma$, $\mu \to \sqrt{\frac{1}{\kappa}} \mu$ in $\chi^{\gamma p}$ to get $\chi^{\gamma \gamma}$

$$
\chi^{\gamma \gamma}(s, b) = i \left[ \kappa^2 \sigma_{gg}(s) W(b; \frac{1}{\kappa^2} \mu_{gg}) + \kappa^2 \sigma_{qq}(s) W(b; \frac{1}{\kappa} \mu_{qq}) + \kappa^2 \sigma_{qq}(s) W(b; \frac{1}{\kappa} \mu_{qq}) \right],
$$

where $\kappa$ is an energy-independent constant. The additive quark model has $\kappa = 2/3$.

Since the ratios of elastic to total cross sections are the same, the three opacities, $\chi_{b=0}$, are all the same, i.e.,

$$
O^{nn} = O^{\gamma p} = O^{\gamma \gamma} = \frac{i}{12\pi} \left[ \sigma_{gg}(s) \mu_{gg}^2 + \sigma_{qq}(s) \mu_{qq}^2 + \sigma_{qq}(s) \mu_{qq}^2 \right].
$$

This yields the three factorization theorems:
Factorization Theorems

\[
\begin{align*}
\sigma_{\gamma\gamma}(s) &= \kappa P_{\text{had}}^\gamma \sigma_{\gamma p}(s) = (\kappa P_{\text{had}}^\gamma)^2 \sigma_{nn}(s) \\
B_{\gamma\gamma}(s) &= \kappa B_{\gamma p}(s) = \kappa^2 B_{nn}(s) \\
\rho_{\gamma\gamma}(s) &= \rho_{\gamma p}(s) = \rho_{nn}(s),
\end{align*}
\]

where \( P_{\text{had}}^\gamma \) is the probability that a photon transforms into a hadron, assumed to be independent of energy and \( \kappa \) is a proportionality constant, also independent of energy, but is model-dependent

- \( \kappa = 2/3 \) for the additive quark model.
- The theorems survive exponentiation of the eikonal.
Measurement of $\kappa$, using $B_{\gamma p}(s) = \kappa B_{nn}(s)$

From $\chi^2$ fit, $\kappa = 0.661 \pm 0.008$
Experimental Verification of Factorization Theorem
\[ \rho_{\gamma p}(s) = \rho_{\gamma n}(s) \]

- ‘Elastic’ Scattering Reactions: \[ \gamma + p \rightarrow V + p \rightarrow V + p, \]
  \[ V = \rho, \omega, \phi \]
- True Elastic (Compton) Scattering: \[ \gamma + p \rightarrow \gamma + p \]
Forward Compton Scattering, using Real Analytic Amplitudes

\[ \gamma + p \rightarrow \gamma + p, \quad \text{with } \nu = E_\gamma \]

Forward high energy (real analytic) scattering amplitude, \( f_+ \)

\[
f_+ = i \frac{\nu}{4\pi} \left\{ A + \beta \left[ \ln(s/s_0) - i\pi/2 \right]^2 + cs^{\mu-1} e^{i\pi(1-\mu)/2} \right\} + C_{\text{subtraction}}
\]

\( C_{\text{subtraction}} = f_+(0) = -\alpha/m, \) Thompson scattering limit,

\( A, \beta, c, s_0 \) and \( \mu \) are real constants to be fit.

\( \mu = 0.5 \) corresponds to a descending Regge trajectory
Using optical theorem,

\[
\sigma_{\text{tot}} = A + \beta \left[ \ln^2 \frac{s}{s_0} - \frac{\pi^2}{4} \right] + c \sin\left(\frac{\pi \mu}{2}\right)s^\mu - 1
\]

\[\chi^2/\text{d.f.} = 0.98, \text{ for 40 d.f., for } \log^2(s) \text{ fit}\]

\[\rho = \frac{\beta \pi \ln \frac{s}{s_0} - c \cos\left(\frac{\pi \mu}{2}\right)s^\mu - 1 + \frac{4\pi}{\nu} f_+(0)}{\sigma_{\text{tot}}}\]
Experimental Verification of Factorization Theorem
\[ \rho_{\gamma p}(s) = \rho_{nn}(s) \]

- True Elastic (Compton) Scattering: \( \gamma + p \rightarrow \gamma + p \)
$\sigma(\gamma\gamma)$, fitted with log$^2$s

$\chi^2$/d.f. = 0.06, for 9 d.f.
$\rho_{\gamma \gamma}$, using Real Analytic Amplitudes
Compton Scattering, using Real Analytic Amplitudes

\[ \rho_{\text{analytic}} = -\frac{70.2/\nu^{1/2} + 38.07/\nu}{96.6 + 70.2/\nu^{1/2}} \]

\[ \nu = E_\gamma (\text{lab}), \text{ in GeV} \]

\[ \gamma + p \rightarrow \gamma + p, \text{ using real analytic amplitudes} \]
Vector Dominance, using $\gamma p$ Cross Sections, in
\[
\sigma_{\gamma p}(s) = \frac{2}{3} P^\gamma_{\text{had}} \sigma_{\text{nn}}(s)
\]

Using $f_\rho^2/4\pi = 2.2$, $f_\omega^2/4\pi = 23.6$ and $f_\phi^2/4\pi = 18.4$, we find \(\Sigma_V(4\pi\alpha/f_V^2) = 1/249\), where \(V = \rho, \omega, \phi\).
We find $P_{\text{had}}^\gamma = 1/240$ normalizing to the low energy cross sections $\sigma_{\gamma p}$. This value is approximately 4% greater than that derived from vector dominance, $1/249$. 

Frascati Photon 2005
Verification of Factorization using $\gamma\gamma$ Scattering:

$$\sigma_{\gamma\gamma}(s) = \frac{2}{3} P_{\text{had}}^\gamma \sigma_{\text{p}} = \left(\frac{2}{3} P_{\text{had}}^\gamma \right)^2 \sigma_{\text{nn}}(s), \quad P_{\text{had}}^\gamma = 1/240$$
Does factorization depend on using a dipole-dipole form factor?

\[
W_{\text{nn}}(b; \mu) = \frac{\mu^2}{96\pi} (\mu b)^3 K_3(\mu b)
\]

Could we also use a monopole-monopole for $\gamma\gamma$, with a monopole-dipole for $\gamma$p?
\[ W_{nn}(b; \mu) = \frac{1}{(2\pi)^2} \int \frac{\mu^8}{(q^2 + \mu^2)^4} e^{i\vec{q} \cdot \vec{b}} d^2 \vec{q}, \]
\[ W_{\gamma p}(b; \mu, \nu) = \frac{1}{(2\pi)^2} \int \frac{\mu^4 \nu^2}{(q^2 + \mu^2)^2 (q^2 + \nu^2)} e^{i\vec{q} \cdot \vec{b}} d^2 \vec{q}, \]
\[ W_{\gamma\gamma}(b; \nu) = \frac{1}{(2\pi)^2} \int \frac{\nu^4}{(q^2 + \nu^2)^2} e^{i\vec{q} \cdot \vec{b}} d^2 \vec{q}. \]

**dipole-dipole**

**dipole-monopole**

**monopole-monopole**

**Fourier transforms**

\[ W_{nn}(b; \mu) = \frac{\mu^2}{96\pi} (\mu b)^3 K_3(\mu b), \]
\[ W_{\gamma p}(b; \mu, \nu) = \frac{\mu^2 \nu^2}{4\pi (\mu^2 - \nu^2)} \left( \frac{2\mu^2}{\mu^2 - \nu^2} (K_0(\nu b) - K_0(\mu b)) - (\mu b) K_1(\mu b) \right), \]
\[ W_{\gamma\gamma}(b; \nu) = \frac{\nu^2}{4\pi}(\nu b) K_1(\nu b), \]

\[ <b^2> = \int b^2 W(b) d^2 \vec{b} = \frac{16}{\mu^2} = \frac{8}{\nu^2}, \]

set \[ <b^2>_{nn} = <b^2>_{\gamma\gamma} \]

yields energy–independent relation: \[ \nu = \mu / \sqrt{2} \]
$W(b)$, dipole-dipole, dipole-monopole, monopole-monopole
Total cross section and the ratio of elastic to total cross section

\[ \sigma_{\text{tot}} = 2 \int \left[ 1 - e^{-\sigma_0 W(b)} \right] d^2 \vec{b} \]

\[ \frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} = \frac{\int \left[ 1 - e^{-\sigma_0 W(b)} \right]^2 d^2 \vec{b}}{\sigma_{\text{tot}}} \].

\( \text{nn} \) uses dipole-dipole, \( \gamma p \) uses monopole-dipole, \( \gamma \gamma \) uses monopole-monopole

<table>
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<th>Reaction</th>
<th>( \sqrt{s} = 1500 \text{ GeV} )</th>
<th>( \sqrt{s} = 400 \text{ GeV} )</th>
<th>( \sqrt{s} = 25 \text{ GeV} )</th>
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<td>( \sigma_{\text{el}}/\sigma_{\text{tot}} )</td>
<td>( \sigma_{\text{tot}}, \text{ in mb} )</td>
<td>( \sigma_{\text{el}}/\sigma_{\text{tot}} )</td>
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<td>nn</td>
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<td>( \gamma p )</td>
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<td>( \sigma_{\gamma p}/\sigma_{\gamma \gamma} )</td>
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<td>( \sqrt{s} = 25 \text{ GeV} )</td>
<td>1.016</td>
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To test factorization, we will utilize *real analytical amplitudes*.

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Part VI: Experimental evidence for factorization, using real analytical amplitudes, for n-n, γp and γγ collisions

*M. M. Block and K. Kang*

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Kyungsik Kang, In Paris

April 4-11, 2003

Frascati Photon 2003
Real Analytic Amplitudes

\[
\begin{align*}
\text{even:} & \quad \frac{4\pi}{p} f^+(s) = i \left\{ A + \beta [\ln(s/s_0) - i\pi/2]^2 + cs^{\mu-1}e^{i\pi(1-\mu)/2} \right\} \\
\text{odd:} & \quad \frac{4\pi}{p} f^-(s) = -Ds^{\alpha-1}e^{i\pi(1-\alpha)/2} \\
\end{align*}
\]

where \( A, \beta, c, s_0, D, \mu \) and \( \alpha \) are real constants.

\[
\begin{align*}
\sigma_{\bar{p}p}(s) &= A + \beta \left[ \ln^2 s/s_0 - \frac{\pi^2}{4} \right] + c \sin(\pi\mu/2)s^{\mu-1} - D\cos(\pi\alpha/2)s^{\alpha-1} \\
\sigma_{pp}(s) &= A + \beta \left[ \ln^2 s/s_0 - \frac{\pi^2}{4} \right] + c \sin(\pi\mu/2)s^{\mu-1} + D\cos(\pi\alpha/2)s^{\alpha-1} \\
\sigma_{nn}(s) &= A + \beta \left[ \ln^2 s/s_0 - \frac{\pi^2}{4} \right] + c \sin(\pi\mu/2)s^{\mu-1} \\
\rho_{\bar{p}p}(s) &= \frac{\beta \pi \ln s/s_0 - c \cos(\pi\mu/2)s^{\mu-1} - D\sin(\pi\alpha/2)s^{\alpha-1}}{\sigma_{\bar{p}p}} \\
\rho_{pp}(s) &= \frac{\beta \pi \ln s/s_0 - c \cos(\pi\mu/2)s^{\mu-1} + D\sin(\pi\alpha/2)s^{\alpha-1}}{\sigma_{pp}} \\
\rho_{nn}(s) &= \frac{\beta \pi \ln s/s_0 - c \cos(\pi\mu/2)s^{\mu-1}}{\sigma_{nn}}.
\end{align*}
\]
Real analytic amplitudes for $\gamma p$ and $\gamma\gamma$

\[
\frac{4\pi}{p} f_{\gamma p}(s) = iN \left\{ A + \beta [\ln(s/s_0) - i\pi/2]^2 + c s^{\mu-1} e^{i\pi(1-\mu)/2} \right\}
\]

\[
\frac{4\pi}{p} f_{\gamma\gamma}(s) = iN^2 \left\{ A + \beta [\ln(s/s_0) - i\pi/2]^2 + c s^{\mu-1} e^{i\pi(1-\mu)/2} \right\}
\]

where $N$ is the proportionality constant in the factorization relation

\[
\frac{\sigma_{nn}(s)}{\sigma_{\gamma p}(s)} = \frac{\sigma_{\gamma p}(s)}{\sigma_{\gamma\gamma}(s)} = N.
\]

Using the additive quark model and vector dominance,

\[
N = \frac{2}{3} P_{\gamma}^{\text{had}}, \quad \text{where} \quad P_{\gamma}^{\text{had}} \approx \Sigma_V \frac{4\pi \alpha}{f_V^2} = 1/249
\]

\[
\sigma_{\gamma p}(s) = \frac{2}{3} P_{\gamma}^{\text{had}} \left( A + \beta \left[ \ln^2 s/s_0 - \frac{\pi^2}{4} \right] + c \sin(\pi \mu/2) s^{\mu-1} \right)
\]

\[
\sigma_{\gamma\gamma}(s) = \left( \frac{2}{3} P_{\gamma}^{\text{had}} \right)^2 \left( A + \beta \left[ \ln^2 s/s_0 - \frac{\pi^2}{4} \right] + c \sin(\pi \mu/2) s^{\mu-1} \right)
\]

with the real constants $A, \beta, s_0, c, D$ and $P_{\gamma}^{\text{had}}$ (assuming $\alpha = \mu = 1/2$) being fitted by experiment.
\( \sigma_{\gamma\gamma} \), using *either* PHOJET or PYTHIA--data are from L3 and OPAL

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Fit 1</th>
<th>Fit 2</th>
<th>Fit 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no $\sigma_{\gamma\gamma}$</td>
<td>$\sigma_{\gamma\gamma}$, PHOJET</td>
<td>$\sigma_{\gamma\gamma}$, PYTHIA</td>
</tr>
<tr>
<td>$A$(mb)</td>
<td>37.2 ± 0.81</td>
<td>37.1 ± 0.87</td>
<td>37.3 ± 0.77</td>
</tr>
<tr>
<td>$\beta$(mb)</td>
<td>0.304 ± 0.023</td>
<td>0.302 ± 0.024</td>
<td>0.307 ± 0.022</td>
</tr>
<tr>
<td>$s_0((GeV)^2)$</td>
<td>34.3 ± 14</td>
<td>32.6 ± 16</td>
<td>35.1 ± 14</td>
</tr>
<tr>
<td>$D$(mb(GeV)$^2(1-\alpha)$)</td>
<td>−35.1 ± 0.83</td>
<td>−35.1 ± 0.85</td>
<td>−35.4 ± 0.84</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$c$(mb(GeV)$^2(1-\mu)$)</td>
<td>55.0 ± 7.5</td>
<td>55.9 ± 8.1</td>
<td>54.6 ± 7.3</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$P_\gamma^{had}$</td>
<td>$1/(233.1 ± 0.6)$</td>
<td>$1/(233.1 ± 0.6)$</td>
<td>$1/(233.1 ± 0.6)$</td>
</tr>
<tr>
<td>$N_{OPAL}$</td>
<td>—</td>
<td>0.929 ± 0.037</td>
<td>0.861 ± 0.050</td>
</tr>
<tr>
<td>$N_{L3}$</td>
<td>—</td>
<td>0.929 ± 0.025</td>
<td>0.808 ± 0.020</td>
</tr>
<tr>
<td>$\chi^2$/d.f.</td>
<td>1.62</td>
<td>1.49</td>
<td>1.87</td>
</tr>
<tr>
<td>d.f.</td>
<td>68</td>
<td>78</td>
<td>78</td>
</tr>
<tr>
<td>total $\chi^2$</td>
<td>110.5</td>
<td>115.9</td>
<td>146.0</td>
</tr>
</tbody>
</table>

**GOOD**

**BAD**
$\sigma_{pp}$, $\sigma_{p\bar{p}}$ vs. $\sqrt{s}$

The graph shows the cross-section $\sigma$, in mb, for $pp$ and $p\bar{p}$ collisions as a function of the square root of the center-of-mass energy $\sqrt{s}$, in GeV. The data points are differentiated by color: blue for $pp$ and red for $p\bar{p}$, with error bars indicating statistical uncertainties. The graph includes logarithmic fits to the data, with solid black lines for $pp$ and dashed black lines for $p\bar{p}$. The fits are labeled as $\log^2(s)$ analytic amplitude fit.
$\rho_{pp}$, $\rho_{pbar-p}$ vs. $\sqrt{s}$
\[ \sigma_{\gamma-p} = \left( \frac{2}{3}P_{\text{had}} \right) \sigma_{nn} \]
\[ \sigma_{\gamma\gamma} = \left(\frac{2}{3} \cdot P_{\text{had}}\right)^2 \sigma_{nn}, \]

L3 and OPAL data, uncorrected with PYTHIA and PHOJET
curve: $\sigma_{\gamma\gamma} = (2/3*P_{\text{had}})^2 \sigma_{\text{nn}}$

data: L3 and OPAL, renormalized by factor 0.929, using PHOJET

\[ \sqrt{s}, \text{ in GeV} \]

\[ \sigma(\gamma\gamma), \text{ in nb} \]
solid line: $\rho_{nn} = \rho_{\gamma p} = \rho_{\gamma\gamma}$
dotted line: $\rho_{nn}$ from QCD-Inspired Fit (Aspen Model)
Conclusions

The available data on $nn$, $\gamma p$ and $\gamma \gamma$ reactions lend strong experimental support to the factorization hypotheses of

- the well-known cross section factorization theorem:
  \[ \frac{\sigma_{nn}}{\sigma_{\gamma p}} = \frac{\sigma_{\gamma p}}{\sigma_{\gamma \gamma}} \]

- the lessor-known nuclear slope factorization theorem:
  \[ \frac{B_{nn}}{B_{\gamma p}} = \frac{B_{\gamma p}}{B_{\gamma \gamma}} \]

- the relatively obscure requirement:
  \[ \rho_{nn} = \rho_{\gamma p} = \rho_{\gamma \gamma} \]
• verifying the additive quark model:

by measuring $\kappa = 0.661 \pm 0.008$, a result within $\approx 1\%$ of the value of $2/3$ expected for the quark model, using $B_{\gamma p}$ measurements over a wide span of energies, $3 \leq \sqrt{s} \leq 200$ GeV.

• confirming vector dominance:

using $\sigma_{nn}$, $\sigma_{\gamma p}$ and $\sigma_{\gamma\gamma}$ over an energy region of $3 \leq \sqrt{s} \leq 100$ GeV and a cross section factor of over $10^5$. 
Factorization Theorem for Differential Elastic Scattering

We note that:

\[
\frac{d\sigma_{nn}}{dt}(s, t) = \frac{1}{4\pi} \left| \int J_0(qb) \left(1 - e^{i\chi_{\text{even}}(b,s)}\right) d^2b \right|^2, \quad t = -q^2
\]

It is straightforward to prove the factorization theorem for differential elastic scattering:

\[
\frac{d\sigma_{nn}}{dt}(s, t) / \frac{d\sigma_{\gamma p}}{dt}(s, \frac{3}{2}t) = \frac{d\sigma_{\gamma p}}{dt}(s, \frac{3}{2}t) / \frac{d\sigma_{\gamma \gamma}}{dt}(s, \frac{9}{4}t)
\]
Differential Elastic Scattering

The differential cross section for the “elastic” scattering reaction is

\[
\frac{d\sigma_{\gamma p}^{\nu}}{dt}(s, t) = 4 \frac{d\sigma_{nn}^{\gamma}}{9 P_{V_i}^{\gamma}} dt(s, \frac{2}{3} t), \quad \gamma + p \rightarrow V_i + p, \quad V_i = \rho, \omega \text{ or } \phi.
\]

For the “elastic” \(\gamma\gamma\) scattering reaction

\[
\frac{d\sigma_{\gamma\gamma}^{\nu}}{dt}(s, t) = \left(\frac{4}{9 P_{V_i}^{\gamma}}\right)^2 \frac{d\sigma_{nn}^{\gamma}}{dt}(s, \frac{4}{9} t), \quad \gamma + \gamma \rightarrow V_i + V_i.
\]

Thus, a knowledge of \(\frac{d\sigma_{nn}^{nn}}{dt}(s, t)\) for elastic nucleon-nucleon scattering determines the differential “elastic” scattering cross sections for the reactions \(\gamma + p \rightarrow V_i + p\) and \(\gamma + \gamma \rightarrow V_i + V_i\).
dσ/dt, for “Elastic” Scattering

\[ \gamma + p \rightarrow \rho + p \]

\[ \gamma + p \rightarrow \omega + p \]

\[ \gamma + p \rightarrow \phi + p \]
We note that in the Regge approach the functions $W_i(b, \mu)$ depend on $s$, since the radius of interaction ($R \sim 1/\mu$) for the Pomeron exchange which determines an eikonal grows with energy as $R^2 = R_0^2 + \alpha'_P \ln(s/s_0)$. Thus, in the Regge eikonal model, factorization breaks down in general. However, the Pomeron slope $\alpha'_P$ is very small ($\approx 0.2$ GeV$^{-2}$) and the $R_0$ for different processes are approximately in the same relation as in the Additive Quark model. As a result, the factorization relations are valid with good accuracy even in this model, in a broad region of energies.
Form Factors $W(b)$, for same $<b^2>$
Total cross sections $\sigma_{\text{tot}}$ and $\sigma_{\text{el}}/\sigma_{\text{tot}}$, for dipole-dipole (n-n), dipole-monopole ($\gamma$-p) and monopole-monopole ($\gamma$-$\gamma$) form factors, for same $<b^2>$

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$\sigma_{\text{el}}/\sigma_{\text{tot}}$</th>
<th>$\sigma_{\text{tot}}$, in mb</th>
<th>$\sigma_{\text{el}}/\sigma_{\text{tot}}$</th>
<th>$\sigma_{\text{tot}}$, in mb</th>
<th>$\sigma_{\text{el}}/\sigma_{\text{tot}}$</th>
<th>$\sigma_{\text{tot}}$, in mb</th>
</tr>
</thead>
<tbody>
<tr>
<td>nn</td>
<td>0.272</td>
<td>73.00</td>
<td>0.236</td>
<td>57.4</td>
<td>0.184</td>
<td>40.06</td>
</tr>
<tr>
<td>$\gamma p$</td>
<td>0.268</td>
<td>71.89</td>
<td>0.235</td>
<td>56.54</td>
<td>0.186</td>
<td>39.56</td>
</tr>
<tr>
<td>$\gamma \gamma$</td>
<td>0.263</td>
<td>70.68</td>
<td>0.233</td>
<td>55.54</td>
<td>0.188</td>
<td>38.93</td>
</tr>
</tbody>
</table>

| $\sigma_{\text{tot}}/\sigma_{\tau p}$ | 1.015 | 1.015 | 1.013 |
| $\sigma_{\text{tot}}/\sigma_{\gamma \gamma}$ | 1.017 | 1.018 | 1.016 |

Table 1: The energy dependence of $\sigma_{\text{el}}/\sigma_{\text{tot}}$ and $\sigma_{\text{tot}}$ for the three reactions nn, $\gamma p$ and $\gamma \gamma$, and the energy dependence of the ratios $\sigma_{\text{tot}}/\sigma_{\tau p}$ and $\sigma_{\text{tot}}/\sigma_{\gamma \gamma}$. 
Toy eikonal $\chi=\sigma W$, for n-n, $\gamma$–p and $\gamma$–$\gamma$ scattering

where $\sigma_{\text{tot}} = 2 \int [1 - e^{-\sigma_0 W(b)}] \, d^2b$
Survival Probability of Large Rapidity Gaps in nucleon-nucleon, $\gamma p$ and $\gamma\gamma$ Collisions

M.M. Block, Northwestern University  
F. Halzen, Wisconsin University

Francis, at the table
\[ a(b, s) = \frac{i}{2} \left( 1 - e^{i\chi(b, s)} \right) \]
\[ = \frac{i}{2} \left( 1 - e^{-\chi_I(b, s)} + i\chi_R(b, s) \right), \]

\[ \sigma_{\text{tot}}(s) = 2 \int \left[ 1 - e^{-\chi_I(b, s)} \cos(\chi_R(b, s)) \right] \, d^2\vec{b}, \]

\[ \sigma_{\text{elastic}}(s) = \int \left| 1 - e^{-\chi_I(b, s)} + i\chi_R(b, s) \right|^2 \, d^2\vec{b}, \]

\[ \sigma_{\text{inelastic}}(s) = \sigma_{\text{tot}}(s) - \sigma_{\text{elastic}}(s), \]

So,

\[ \sigma_{\text{inelastic}}(s) = \int \left[ 1 - e^{-2\chi_I(b, s)} \right] \, d^2\vec{b}. \]

Hence, \( e^{-2\chi_I(b, s)} \) is the probability in impact parameter space of no inelastic interaction.
Survival Probability, $< |S|^2 >$

$W(b, \mu_{qq})$, the normalized probability distribution in impact parameter space $b$ for quarks, is the Fourier transform of a dipole form factor and is given by

$$W(b, \mu_{qq}) = \frac{\mu_{qq}^2}{96\pi} (\mu_{qq} b)^3 K_3(\mu_{qq} b),$$

where $\mu_{qq} = 0.901 \pm 0.005$ GeV.

Thus, $< |S|^2 >$, the survival probability of a large rapidity gap, is given by

$$< |S|^2 > = \int W(b; \mu_{qq}) e^{-2\chi_1(s,b)} d^2 \vec{b}.$$

Note that the energy dependence of the survival probability $< |S|^2 >$ is through the energy dependence of the eikonal $\chi_1$. 
To Prove:
Survival is process-independent, i.e.,
$\langle |S|^2 \rangle_{nn} = \langle |S|^2 \rangle_{\gamma p} = \langle |S|^2 \rangle_{\gamma \gamma}$

$$\chi^{\text{even}}(s, b) = i \left[ \sigma_{gg}(s)W(b; \mu_{gg}) + \sigma_{qg}(s)W(b; \mu_{qg}) + \sigma_{qq}(s)W(b; \mu_{qq}) \right]$$

$$\chi^{\gamma p}(s, b) = i \left[ \frac{2}{3} \sigma_{gg}(s)W(b; \sqrt{\frac{3}{2}} \mu_{gg}) + \frac{2}{3} \sigma_{qg}(s)W(b; \sqrt{\frac{3}{2}} \mu_{qg}) + \frac{2}{3} \sigma_{qq}(s)W(b; \sqrt{\frac{3}{2}} \mu_{qq}) \right],$$

$$\chi^{\gamma \gamma}(s, b) = i \left[ \frac{4}{9} \sigma_{gg}(s)W(b; \frac{3}{2} \mu_{gg}) + \frac{4}{9} \sigma_{qg}(s)W(b; \frac{3}{2} \mu_{qg}) + \frac{4}{9} \sigma_{qq}(s)W(b; \frac{3}{2} \mu_{qq}) \right],$$
\[ < |S^{nn}|^2 > = \frac{1}{96\pi} \int x_n^3 K_3(x_n) \times \exp - \left[ \frac{1}{48\pi} \sigma_{gg} \mu_{gg}^2 \left( \frac{\mu_{gg}}{\mu_{qq}} x_n \right)^3 K_3 \left( \frac{\mu_{gg}}{\mu_{qq}} x_n \right) \right. \]
\[ \left. + \sigma_{qg} \mu_{qg}^2 \left( \frac{\mu_{qg}}{\mu_{qq}} x_n \right)^3 K_3 \left( \frac{\mu_{qg}}{\mu_{qq}} x_n \right) \right] \ d^2 x_n, \ x_n = \mu_{qq} b \]

\[ < |S^{\gamma p}|^2 > = \frac{1}{96\pi} \int x_q^3 K_3(x_q) \times \exp - \left[ \frac{1}{48\pi} \sigma_{gg} \mu_{gg}^2 \left( \frac{\mu_{gg}}{\mu_{qq}} x_q \right)^3 K_3 \left( \frac{\mu_{gg}}{\mu_{qq}} x_q \right) \right. \]
\[ \left. + \sigma_{qg} \mu_{qg}^2 \left( \frac{\mu_{qg}}{\mu_{qq}} x_q \right)^3 K_3 \left( \frac{\mu_{qg}}{\mu_{qq}} x_q \right) \right] \ d^2 x_q, \ x_q = \sqrt{\frac{3}{2}} \mu_{qq} b \]

\[ < |S^{\gamma \gamma}|^2 > = \frac{1}{96\pi} \int x_g^3 K_3(x_g) \times \exp - \left[ \frac{1}{48\pi} \sigma_{gg} \mu_{gg}^2 \left( \frac{\mu_{gg}}{\mu_{qq}} x_g \right)^3 K_3 \left( \frac{\mu_{gg}}{\mu_{qq}} x_g \right) \right. \]
\[ \left. + \sigma_{qg} \mu_{qg}^2 \left( \frac{\mu_{qg}}{\mu_{qq}} x_g \right)^3 K_3 \left( \frac{\mu_{qg}}{\mu_{qq}} x_g \right) \right] \ d^2 x_g, \ x_g = \frac{3}{2} \mu_{qq} b \]

Survival is process-independent!

\[ \therefore < |S|^2 >_{nn} = < |S|^2 >_{\gamma p} = < |S|^2 >_{\gamma \gamma} \]
### Gap Survival Probability for pbar-p and pp Collisions

<table>
<thead>
<tr>
<th>C.M.S. Energy (GeV)</th>
<th>Survival Probability($\bar{p}p$), in %</th>
<th>Survival Probability($pp$), in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>63</td>
<td>$37.0 \pm 0.9$</td>
<td>$37.5 \pm 0.9$</td>
</tr>
<tr>
<td>546</td>
<td>$26.7 \pm 0.5$</td>
<td>$26.8 \pm 0.5$</td>
</tr>
<tr>
<td>630</td>
<td>$26.0 \pm 0.5$</td>
<td>$26.0 \pm 0.5$</td>
</tr>
<tr>
<td>1800</td>
<td>$20.8 \pm 0.3$</td>
<td>$20.8 \pm 0.3$</td>
</tr>
<tr>
<td>14000</td>
<td>$12.6 \pm 0.06$</td>
<td>$12.6 \pm 0.06$</td>
</tr>
<tr>
<td>40000</td>
<td>$9.7 \pm 0.07$</td>
<td>$9.7 \pm 0.07$</td>
</tr>
</tbody>
</table>

### Gap Survival (%) for nucleon-nucleon, $\gamma p$ and $\gamma\gamma$ Collisions

The graph shows the dependence of $\langle |S|^2 \rangle$ on the center-of-mass energy ($\sqrt{s}$) in GeV and the c.m.s. energy. The probability decreases as the energy increases.
Consequences of the Factorization Hypothesis in nucleon-nucleon, $\gamma$-p and $\gamma-\gamma$ Collisions,

M. M. Block, Northwestern University
A. B. Kaidalov, Moscow University

Survival Probability of Large Rapidity Gaps in nucleon-nucleon, $\gamma p$ and $\gamma\gamma$ Collisions

M.M. Block, Northwestern University
F. Halzen, Wisconsin University

Alexei

Francis, at the table