Nucleon-nucleon, γp and $\gamma\gamma$ scattering, using factorization: the Aspen Model and analytic amplitude analysis

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Outline

- Part I: fitting σ_{tot}, ρ values and B, nuclear slopes of **accelerator data**--p-p and pbar-p-- **using a QCD-inspired model (Aspen Model)**
  M.M. Block, E.M. Gregores, F. Halzen & G. Pancheri

- Part II: global fit using both **cosmic ray and accelerator data simultaneously**
  M.M. Block, F. Halzen & T. Stanev

- Part III: **predictions** of forward scattering parameters for LHC (14 TeV)---σ_{tot}, $\sigma_{elastic}$, ρ, and B

- Part IV-The **factorization hypothesis:** relating n-n, γp and $\gamma\gamma$ collisions

  M. M. Block and A. B. Kaidalov

- Part V-Experimental evidence for **factorization**, **quark counting** and **vector dominance** in n-n, γp and $\gamma\gamma$ collisions, using the Aspen Model

  M. M. Block, F. Halzen, A .B. Kaidalov and G. Pancheri

- Part VI-Experimental evidence for factorization, using **real analytical amplitudes**, for n-n, γp and $\gamma\gamma$ collisions

  M. M. Block and K. Kang

Part I: fitting σ_{tot}, ρ values and B, nuclear slopes, of accelerator data--p-p and pbar-p-- using a QCD-inspired model (Aspen Model)

M.M. Block, E.M. Gregores, F. Halzen & G. Pancheri

Eduardo, Martin, Francis

3 of 4 authors working hard!

Giulia, #4 author

Eikonal Formalism

We assume that the scattering amplitude in impact parameter space b is given by

$$\boxed{\begin{aligned} a(b,s) &= \frac{i}{2}\left(1 - e^{i\chi(b,s)}\right) \\ &= \frac{i}{2}\left(1 - e^{-\chi_{\mathrm{Im}}(b,s)+i\chi_{\mathrm{Re}}(b,s)}\right), \end{aligned}}$$

The total cross section $\sigma_{\mathrm{tot}}(s)$, ρ-value, elastic cross section $\sigma_{\mathrm{elastic}}(s)$ and differential elastic scattering cross section $\frac{d\sigma}{dt}$ are given by:

$$\begin{aligned} \sigma_{\mathrm{tot}}(s) &= 4\int \mathrm{Im}\, a(b)\, d^2\vec{b}, \\ \rho(s) &= \frac{\mathrm{Re}\left[\int a(b,s)\, d^2\vec{b}\right]}{\mathrm{Im}\left[\int a(b,s)\, d^2\vec{b}\right]}, \\ \sigma_{\mathrm{elastic}}(s) &= 4\int |a(b)|^2\, d^2\vec{b}, \\ \frac{d\sigma}{dt} &= \frac{1}{\pi}\left|\int e^{i\vec{q}\cdot\vec{b}} a(b,s)\, d^2\vec{b}\right|^2, \qquad t = -q^2. \end{aligned}$$

Theoretical Conditions Required

We insist that our model satisfies:

1. Crossing symmetry, *i.e.*, be either even or odd under the transformation $E \to -E$, where E is the laboratory energy. This allows us to simultaneously describe $\bar{p}p$ and pp scattering.

2. Unitarity. We will use an eikonal formalism to guarantee this.

3. Analyticity. We need this to calculate the phase of the forward scattering amplitude and hence, the ρ value.

4. The Froissart bound. Asymptotically, we expect that the total cross section will rise as $(\log s)^2$.

COMPUTE χ_{gg} EXACTLY

$$\chi_{gg}(b,s) = \sigma_{gg}^{QCD}(s) W_{gg}(b;\mu_{gg})$$

with

$$\sigma_{gg}^{QCD}(s) = \int d\left(\frac{\hat{s}}{s}\right) F_{gg}\left(x_1 x_2 = \frac{\hat{s}}{s}\right) \sigma_{gg}(\hat{s}),$$

$$F_{gg} = \int dx_1 dx_2 \, f_g(x_1) \, f_g(x_2) \, \delta(x_1 x_2 - \tau),$$

using the gluon structure function

$$f_g(x) \sim \frac{(1-x)^5}{x^J},$$

with $J = 1 + \epsilon$, in Regge language, the Pomeron intercept, and $\sigma_{gg}(\hat{s}) = C_{gg} \Sigma \, \theta(\hat{s} - m_0^2)$,where $\Sigma = \frac{9\pi\alpha_s^2}{m_0^2}$, $\alpha_s = 0.5$, and C_{gg} is a real constant fitted by experiment.

The high energy behavior is controlled by

$$\lim_{s\to\infty} \int_{m_0^2/s}^{1} d\tau F_{gg}(\tau) \sim \int_{m_0^2/s}^{1} d\tau \frac{-\log\tau}{\tau^J}$$

$$\sim \left(\frac{s}{m_0^2}\right)^{\epsilon} \log\frac{s}{m_0^2}.$$

The cut-off impact parameter b_c is given by

$$cW_{gg}(b_c, \mu_{gg}) s^{\epsilon} \log(s/m_0^2) \sim 1,$$

where c is a constant. For large values of μb,

$$c'(\mu_{gg} b_c)^{5/2} e^{-\mu_{gg} b_c} s^{\epsilon} \sim 1$$

with c' another constant, and therefore,

$$b_c = \frac{\epsilon}{\mu_{gg}} \log \frac{s}{s_0} + O(\log \log \frac{s}{s_0'}).$$

If $J > 1$, and $s \to \infty$, we get the Froissart bound,

$$\sigma_{tot} = 2\pi (\frac{\epsilon}{\mu_{gg}})^2 \log^2 \frac{s}{s_0}.$$

The usual Froissart bound coefficient of the $\log^2 \frac{s}{s_0}$ term, $1/m_\pi^2 = 20$ mb, is now replaced by $(\epsilon/\mu_{gg})^2 \sim 0.002$ mb. Note that μ_{gg} controls the size of the area occupied by the gluons inside the nucleon.

To reproduce the cross section at lower energies one must consider the qq and qg contributions. Using toy structure functions $f_q \sim x^{-1/2}(1-x)^3$ and $f_g \sim x^{-1}(1-x)^5$, we find

$$\chi_{qq} = \left[\frac{m_0}{\sqrt{s}} \log \frac{s}{s_0} + \mathcal{P}(\frac{m_0}{\sqrt{s}})\right] W_{qq}(b; \mu_{qq})$$

and

$$\chi_{qg} = \left[a' \log \frac{s}{s_0} + \mathcal{P}'(\frac{m_0}{\sqrt{s}})\right] W_{qg}(b; \sqrt{\mu_{qq}\mu_{gg}}).$$

$\mathcal{P}$ and $\mathcal{P}'$ are polynomials in $m_0/\sqrt{s}$.

Since we *only* attempt to decipher the high energy behavior, we replace the above χ's by

$$\boxed{\begin{aligned} \chi_{qq} &= \Sigma \left[C + C_{\text{Regge}} \frac{m_0}{\sqrt{s}} \right] W(b; \mu_{qq}), \\ \chi_{qg} &= \Sigma \left[C_{\log} \log \frac{s}{s_0} \right] W(b; \sqrt{\mu_{qq}\mu_{gg}}), \end{aligned}}$$

as suggested by standard Regge arguments. Treating the real coefficients C, C_{Regge} and $C_{\log}$ as parameters, this low energy parametrization can also accommodate elastic and diffractive contributions.

We set

$$\boxed{\begin{aligned} \chi_{qq} &= \Sigma \left[C + C_{\text{Regge}} \frac{m_0}{\sqrt{s}} \right] W(b; \mu_{qq}), \\ \chi_{qg} &= \Sigma \left[C_{\log} \log \frac{s}{s_0} \right] W(b; \sqrt{\mu_{qq}\mu_{gg}}), \end{aligned}}$$

as suggested by standard Regge arguments. Treating the real coefficients C, C_{Regge} and $C_{\log}$ as parameters, this low energy parametrization can also accommodate elastic and diffractive contributions.

We insure the correct analyticity properties of our even amplitudes by substituting

$$s \longrightarrow \; s \; e^{-i\pi/2}$$

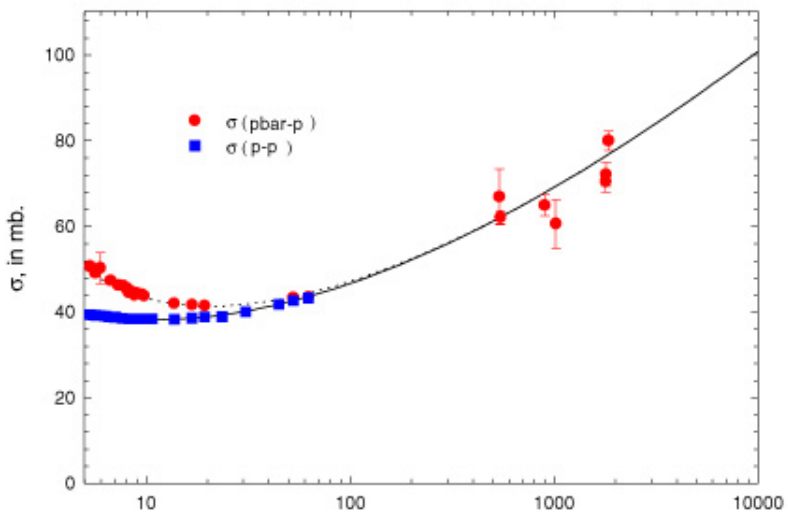
throughout.

We introduce an *ad hoc* crossing odd amplitude parametrizing the difference between the pp and $\bar{p}p$ scattering amplitudes

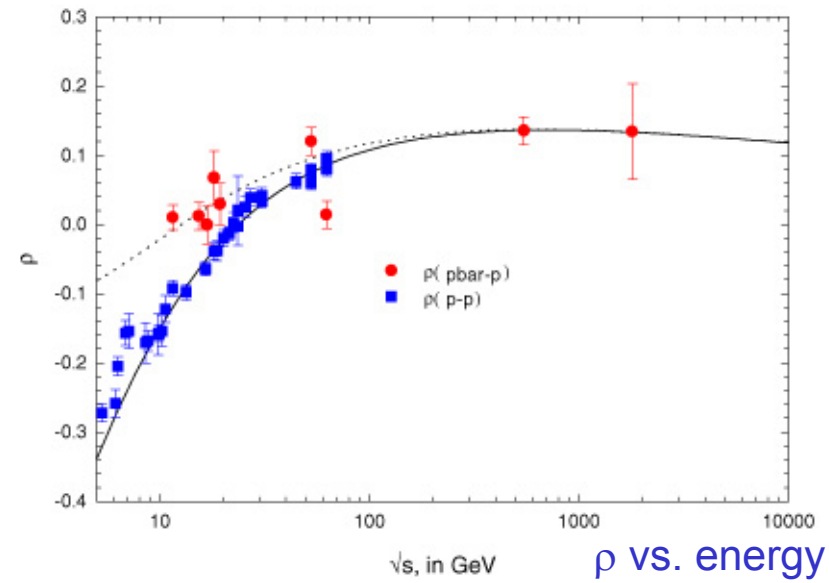
$$\chi_{odd}(s,b) = -C_{\text{odd}} \frac{m_0}{\sqrt{s}} e^{-i\pi/4} W(b;\mu_{odd}),$$

where the real constant C_{odd} is fixed by experiment.

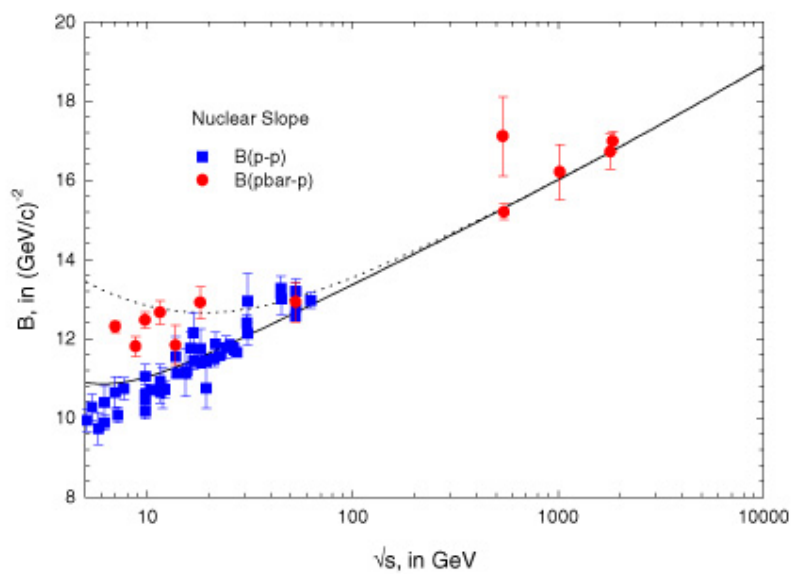
Results of the χ^2 Fit to σ, ρ and B data from pp and pbar-p



σ vs. energy



ρ vs. energy



B vs. energy

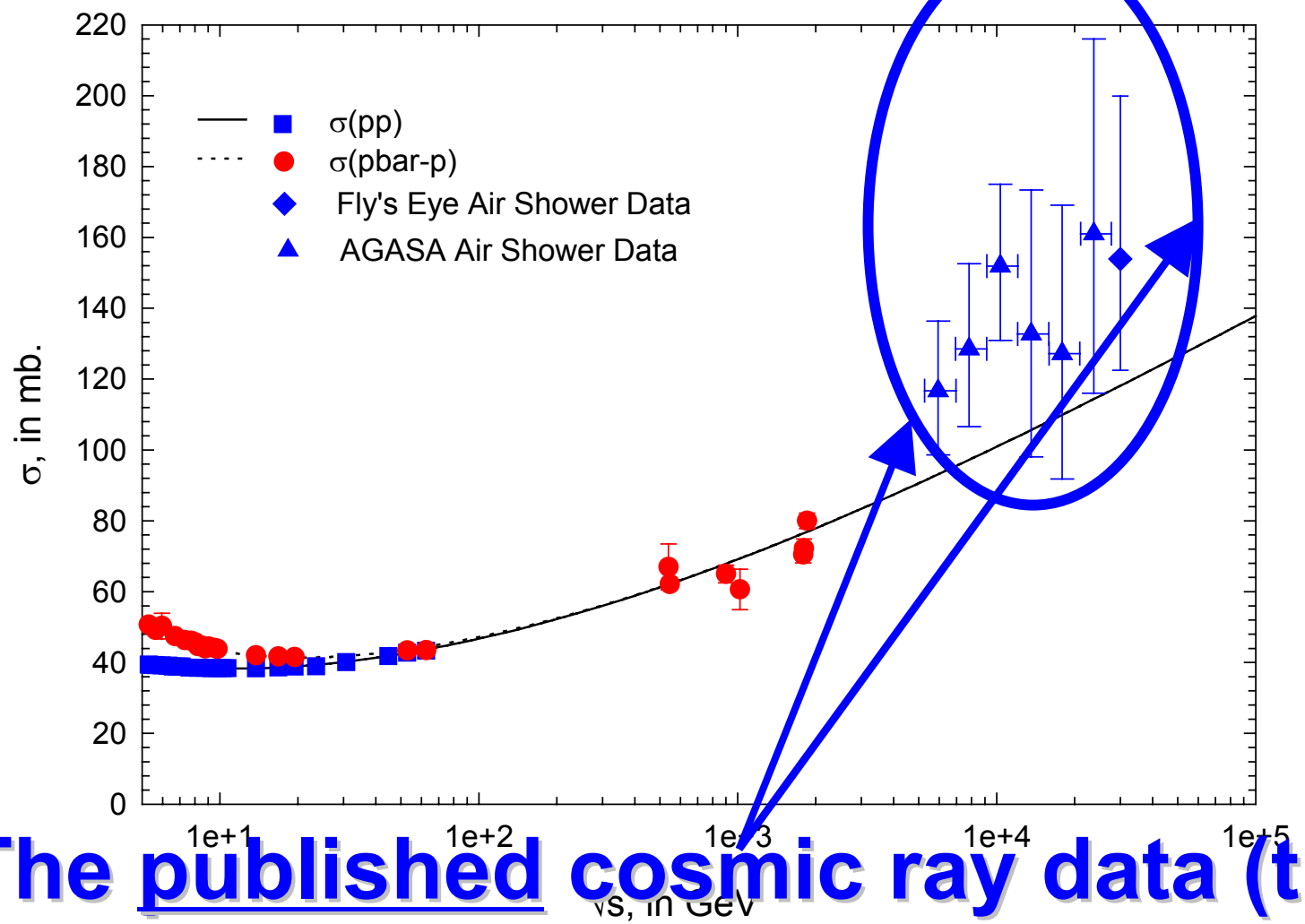
Part II: global fit using both cosmic ray and accelerator data simultaneously

M.M. Block, F. Halzen & T. Stanev

Todor thinking?

Francis wearing money tie (for ICE CUBE?)

Accelerator data give good fit



The published cosmic ray data (the Diamond and Triangles) are the problem

EXPERIMENTAL PROCEDURE

Fly's Eye Shower Profile

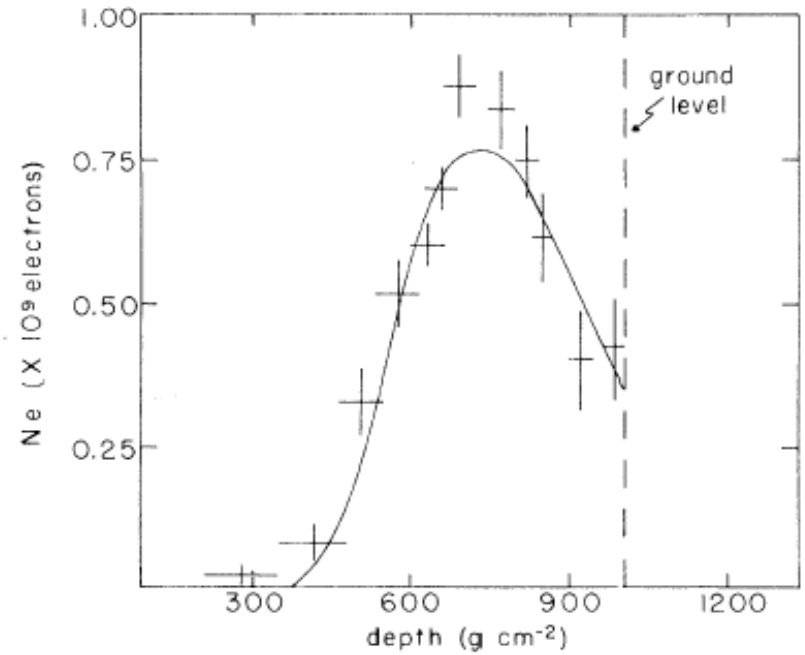


Fig. 1 An extensive air shower that survives all data cuts. The curve is a Gaisser-Hillas shower-development function: shower parameters E=1.3 EeV and X_{max} =727 ± 33 g cm^{-2} give the best fit.

Monte Carlo Example

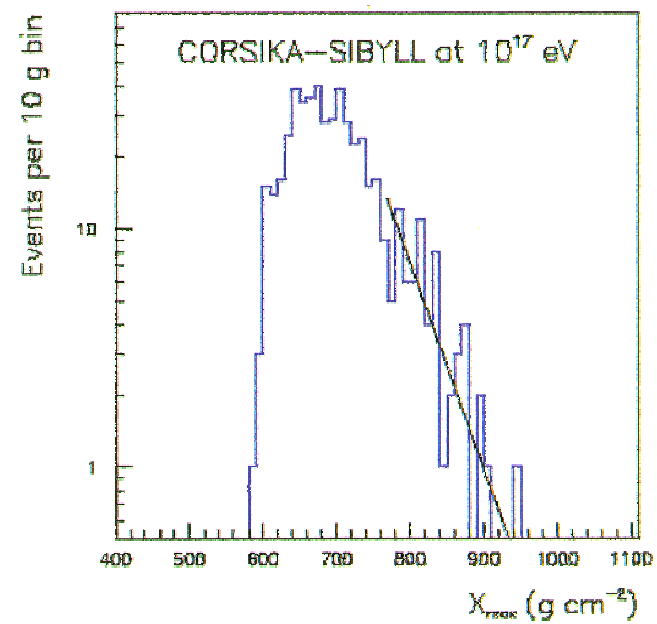


Fig. 7 X_{max} distribution with exponential trailing edge

Extraction of σ_{tot}(pp) from Cosmic Ray Extensive Air Showers

Two significant drawbacks of the extraction method are the need for:

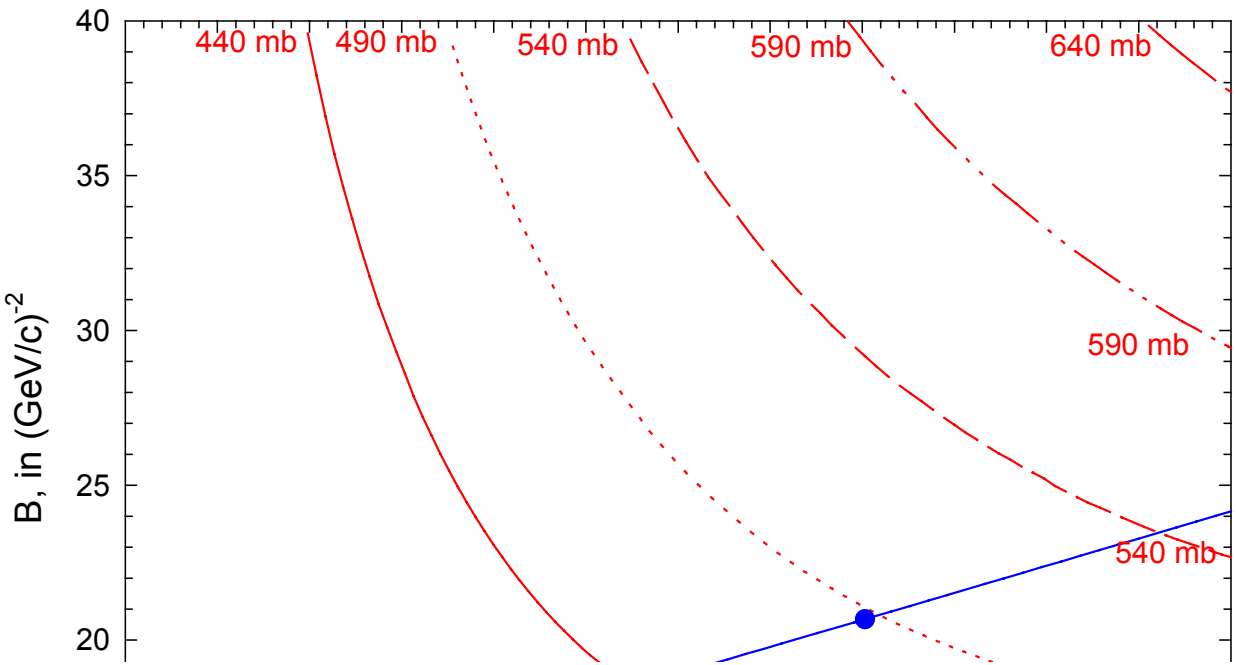
1. a model of proton–air interactions to complete the loop between the measured attenuation length Λ_m and the cross section $\sigma^{\text{inel}}_{p-\text{air}}$, *i.e.*, the value of k, in

$$\Lambda_m = k\lambda_{p-\text{air}} = k\frac{14.5 m_p}{\sigma^{\text{inel}}_{p-\text{air}}}.$$

2. a simultaneous relation between B and σ_{pp} at very high energies—well above the region currently accessed by accelerators.

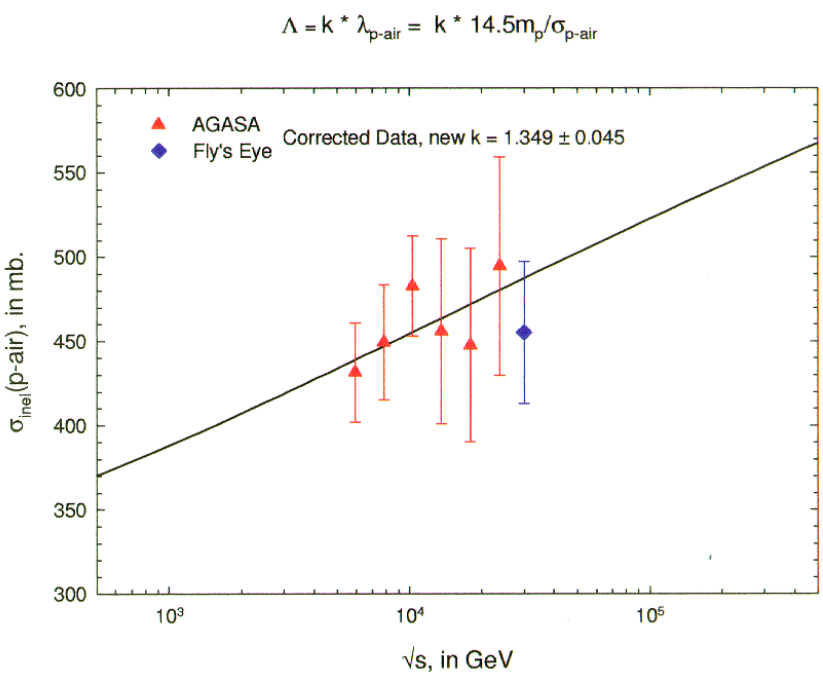
Published Fly's Eye Result:

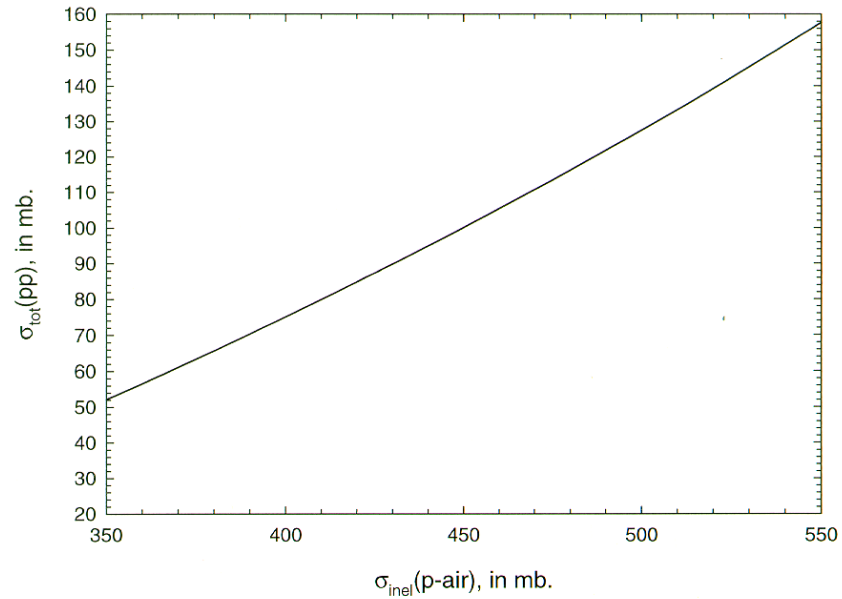
σ_{tot}(p-air) = 540 ± 50 mb, at 30 TeV



See R. Engel et al, Phys. Rev. D58, 014019 (1998), for σ_{tot}(p-air) curves, using Glauber theory

Results of Global Fit, Cosmic Ray and Accelerator Data





σ_{inel}(p-air) vs. energy

σ_{tot}(p-p) vs. σ_{inel}(p-air)

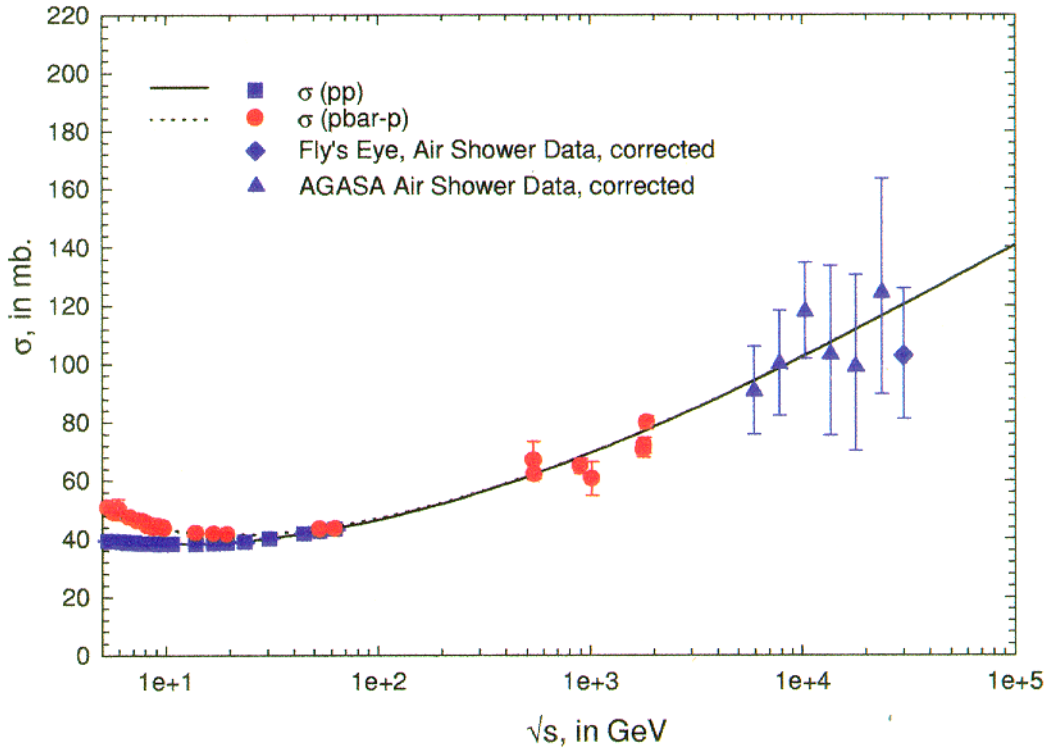
Monte Carlo Results:

C.L. Pryke, astro-ph/0003442

Model	k
CORSIKA-SIBYLL	1.15 ± 0.05
MOCCA–SIBYLL	1.16 ± 0.03
CORSIKA-QGSjet	1.30 ± 0.04
MOCCA–Internal	1.32 ± 0.03

Measured $k = 1.349 \pm 0.045 \pm 0.028$

Result of Global fit for σ_{tot}(pp) and σ_{tot}(pbar-p), using both Accelerator and Cosmic Ray Data



σ vs. energy

Measured $k = 1.349 \pm 0.045 \pm 0.028$

LHC Predictions

Our predictions for the LHC (14 TeV) are:

$$\sigma_{\text{tot}} = 107.4 \pm 1.5 \text{ mb}$$

$$\sigma_{\text{elastic}} = 30.9 \pm 0.5 \text{ mb}$$

$$\rho = 0.112 \pm 0.002$$

$$B = 19.41 \pm 0.15 \text{ GeV}^{-2}$$

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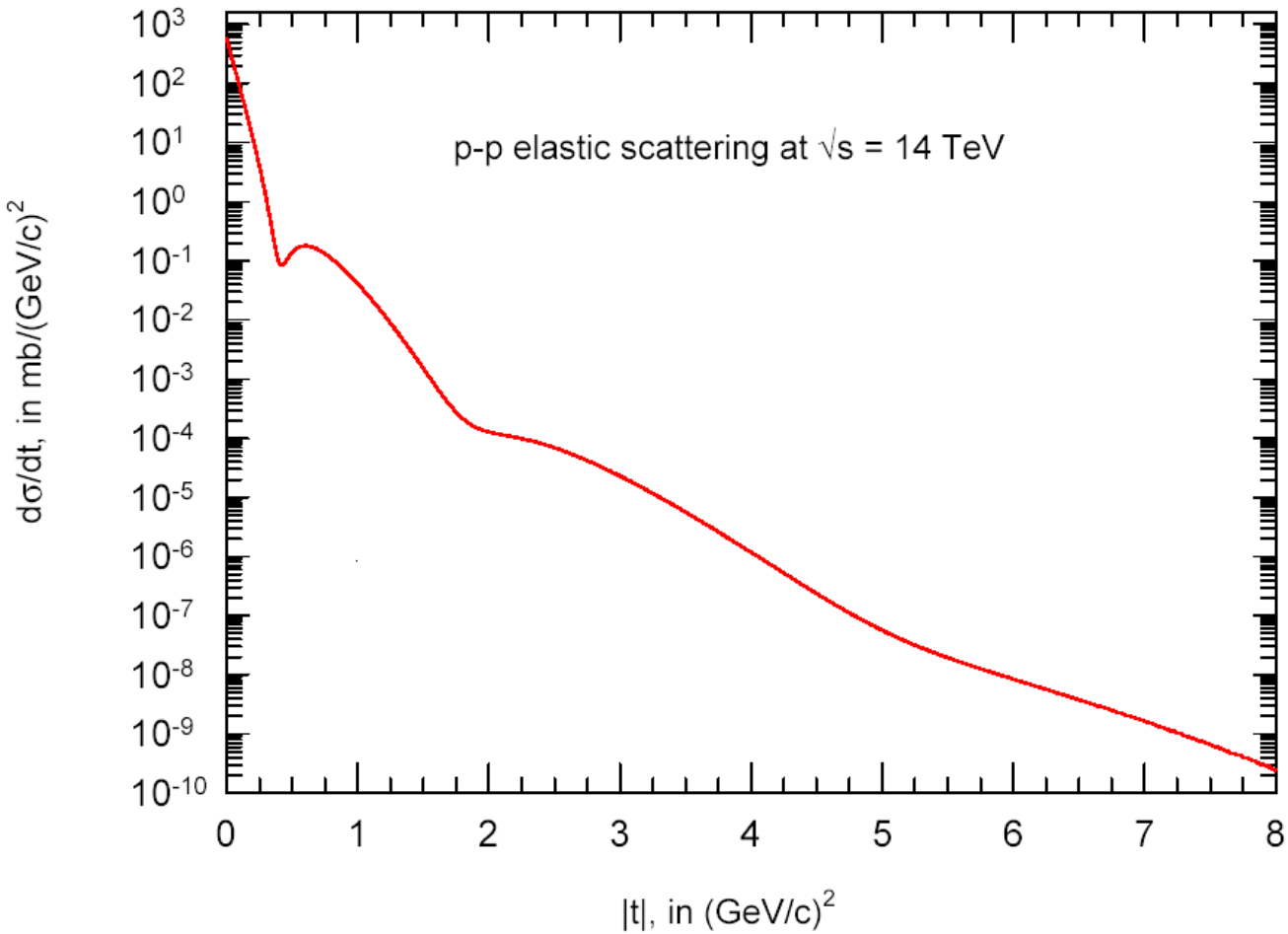
$$\rho = 0.112 \pm 0.002$$

$$B = 19.41 \pm 0.15 \text{ GeV}^{-2}$$

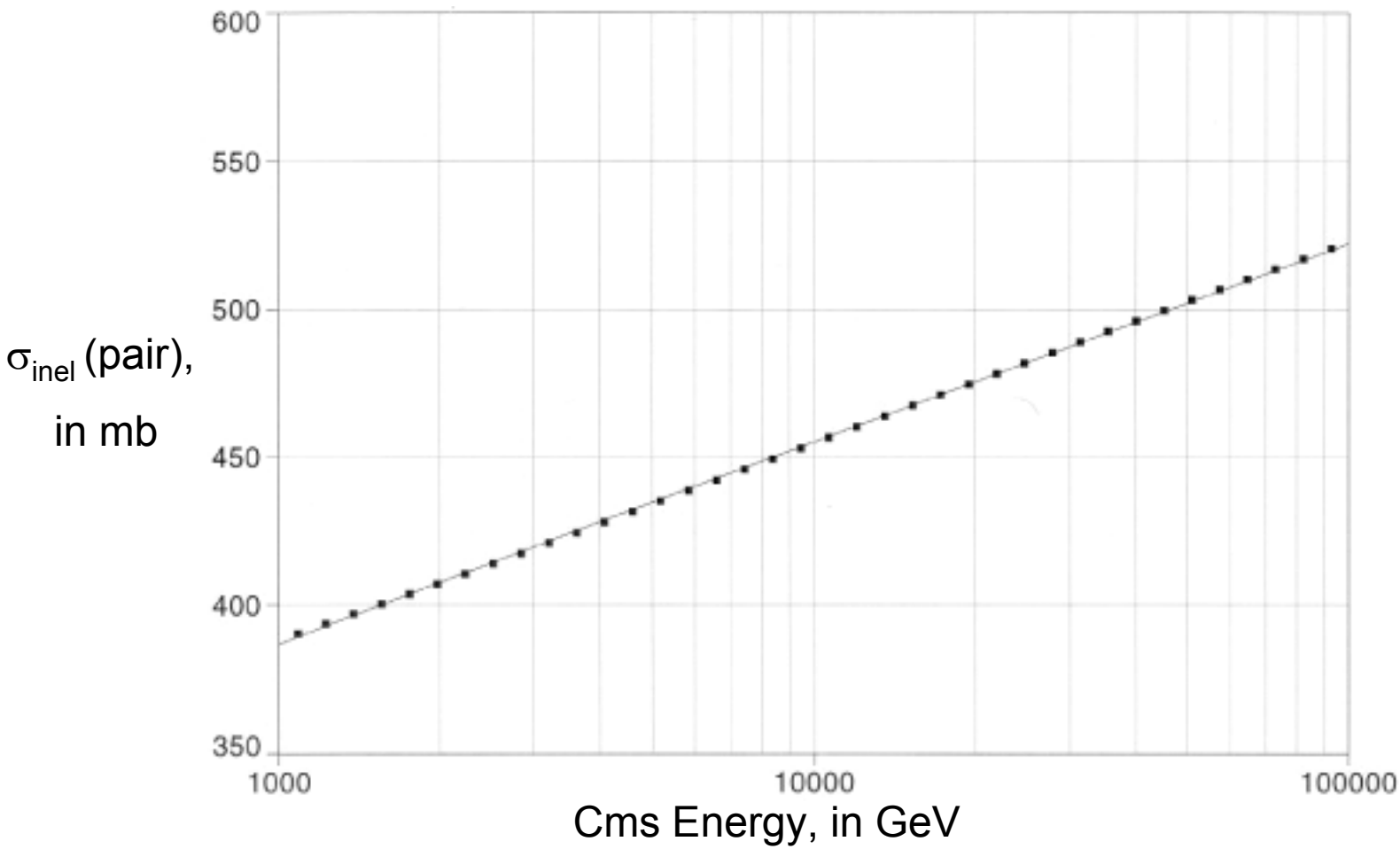
GapSurvivalProbability $< |S|^2 >= 12.6 \pm 0.1$ %

Elastic Scattering at the LHC



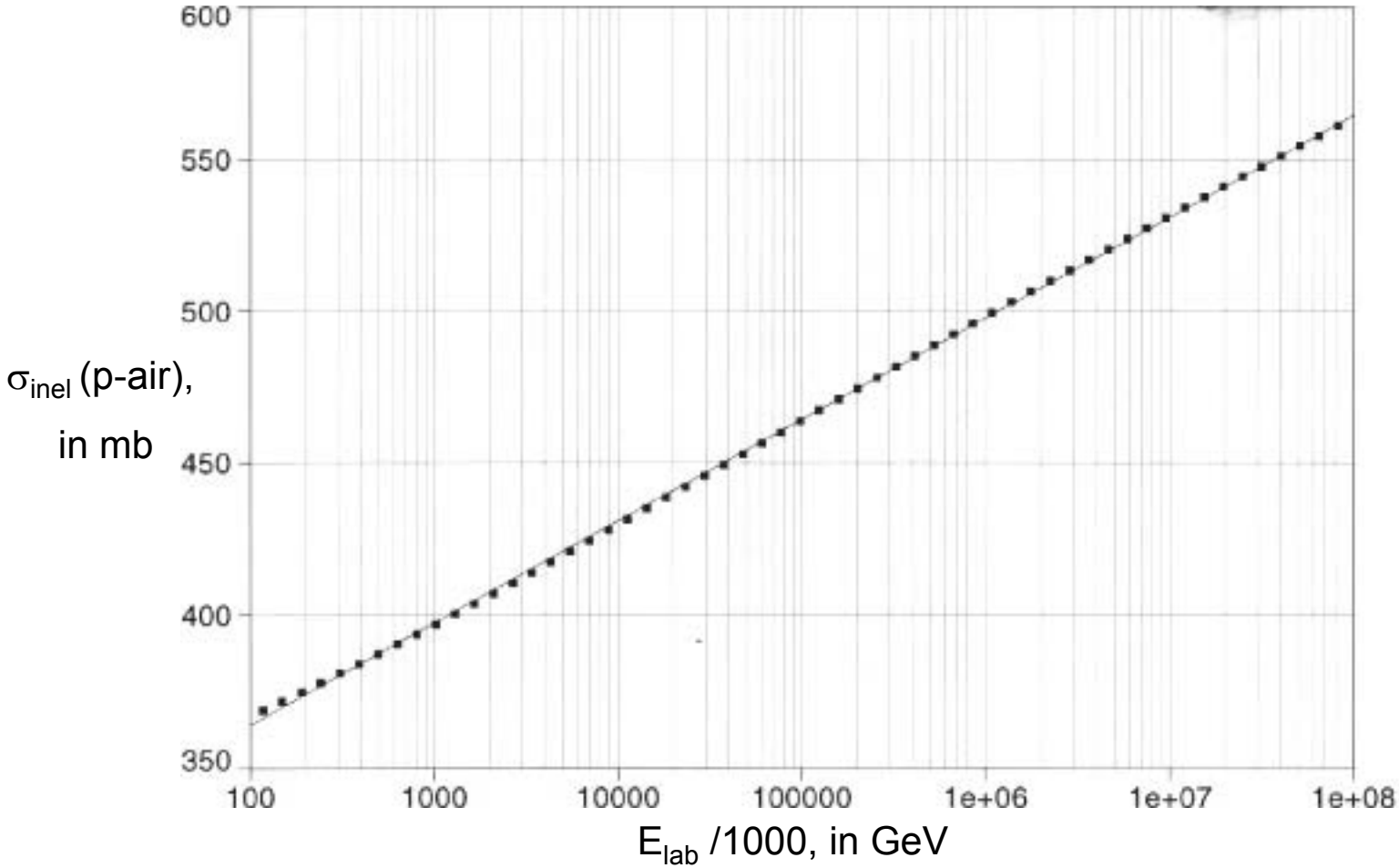
A Useful High Energy Parameterization of σ_{inel} (p-air) :

σ_{inel} (p-air) = 177.43 + 15.44 ln s - 0.0204 $\ln^2 s$, σ in mb, s in Gev^2



A High Energy Parameterization of σ_{inel} (p-air), useful for calculating Neutrino Fluxes

$$\sigma_{inel}\text{(p-air)} = \sigma_0[1 + a_0 \ln(E_{lab}/1000)],$$
$$\sigma_0 = 297.20 \text{ mb}, \; a_0 = 0.04879, \; E_{lab} \text{ in GeV.}$$

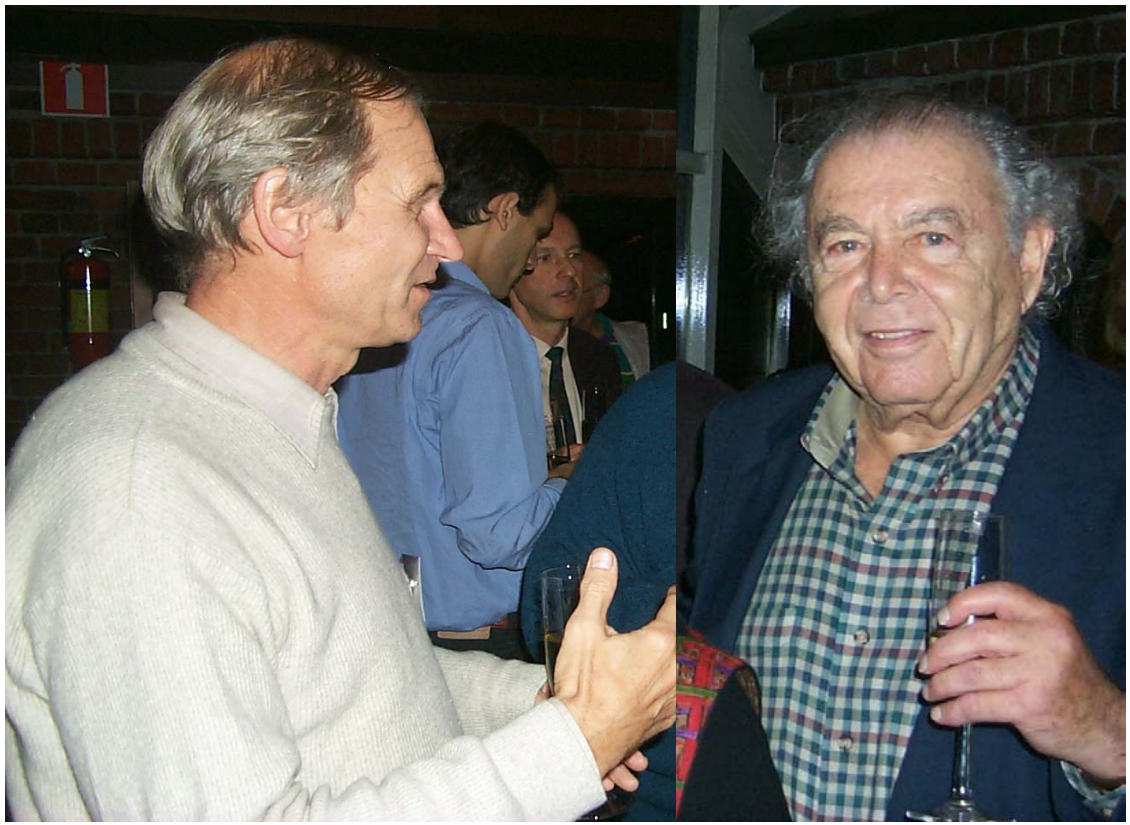


Consequences of the Factorization Hypothesis in nucleon-nucleon, γ-p and γ–γ Collisions,

M. M. Block, Northwestern University

A. B. Kaidalov, Moscow University

Alexei



If $\left(\frac{\sigma_{\rm el}}{\sigma_{\rm tot}}\right)_{\rm nn} = \left(\frac{\sigma_{\rm el}}{\sigma_{\rm tot}}\right)_{\gamma {\rm p}} = \left(\frac{\sigma_{\rm el}}{\sigma_{\rm tot}}\right)_{\gamma\gamma}$, all s

and

$$\chi^{\rm even}(s,b) = i\left[\sigma_{\rm gg}(s)W(b\,;\mu_{\rm gg}) + \sigma_{\rm qg}(s)W(b\,;\mu_{\rm qg}) + \sigma_{\rm qq}(s)W(b\,;\mu_{\rm qq})\right],$$

$$W(b\,;\mu) = \frac{\mu^2}{96\pi}(\mu b)^3 K_3(\mu b),$$

$$\text{with } \int W(b\,;\mu)d^2\vec{b} = 1,$$

then we substitute $\sigma \to \kappa\sigma,\ \mu \to \sqrt{\frac{1}{\kappa}}\mu$ in $\chi^{\rm even}$ to get $\chi^{\gamma p}$

$$\chi^{\gamma p}(s,b) = i\left[\kappa\sigma_{\rm gg}(s)W(b\,;\sqrt{\tfrac{1}{\kappa}}\mu_{\rm gg}) + \kappa\sigma_{\rm qg}(s)W(b\,;\sqrt{\tfrac{1}{\kappa}}\mu_{\rm qg}) + \kappa\sigma_{\rm qq}(s)W(b\,;\sqrt{\tfrac{1}{\kappa}}\mu_{\rm qq})\right],$$

Next, we substitute $\sigma \to \kappa\sigma,\ \mu \to \sqrt{\frac{1}{\kappa}}\mu$ in $\chi^{\gamma p}$ to get $\chi^{\gamma\gamma}$

$$\chi^{\gamma\gamma}(s,b) = i\left[\kappa^2\sigma_{gg}(s)W(b\,;\tfrac{1}{\kappa}\mu_{gg}) + \kappa^2\sigma_{qg}(s)W(b\,;\tfrac{1}{\kappa}\mu_{qg}) + \kappa^2\sigma_{qq}(s)W(b\,;\tfrac{1}{\kappa}\mu_{qq})\right],$$

where κ is an energy-independent constant. The additive quark model has $\kappa = 2/3$.

Since the ratios of elastic to total cross sections are the same, the three opacities, $\chi_{b=0}$, are all the same, *i.e.*,

$$O^{nn} = O^{\gamma p} = O^{\gamma\gamma} = \frac{i}{12\pi}\left[\sigma_{gg}(s)\mu_{gg}^2 + \sigma_{qg}(s)\mu_{qg}^2 + \sigma_{qq}(s)\mu_{qq}^2\right].$$

This yields the three factorization theorems:

Factorization Theorems

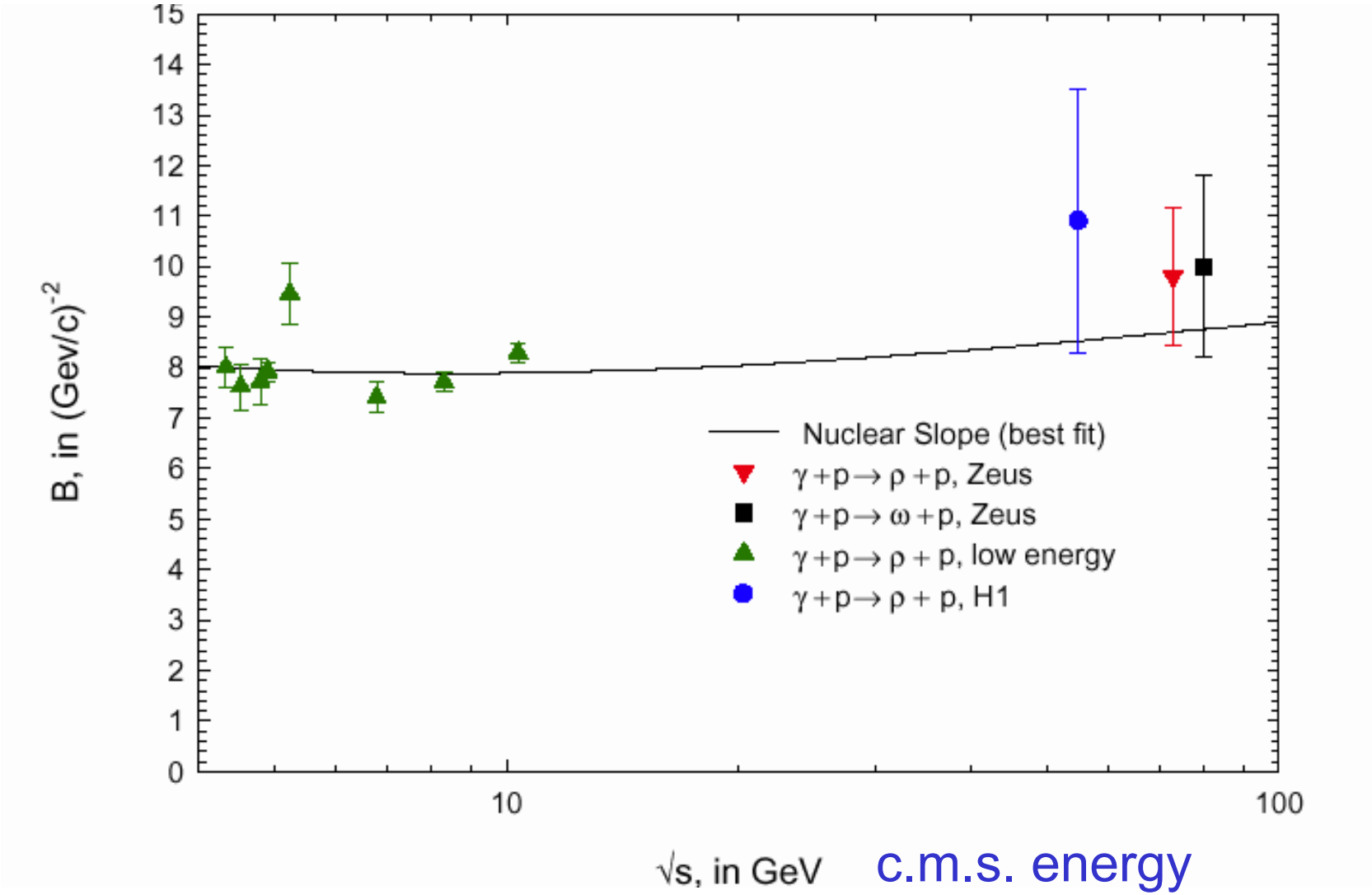
$$\begin{aligned}
\sigma_{\gamma\gamma}(s) &= \kappa P^{\gamma}_{\mathrm{had}} \sigma_{\gamma \mathrm{p}}(s) &&= \left(\kappa P^{\gamma}_{\mathrm{had}}\right)^2 \sigma_{\mathrm{nn}}(s) \\
B_{\gamma\gamma}(s) &= \kappa B_{\gamma \mathrm{p}}(s) &&= \kappa^2 B_{\mathrm{nn}}(s) \\
\rho_{\gamma\gamma}(s) &= \rho_{\gamma \mathrm{p}}(s) &&= \rho_{\mathrm{nn}}(s),
\end{aligned}$$

where $P^{\gamma}_{\mathrm{had}}$ is the probability that a photon transforms into a hadron, assumed to be independent of energy and κ is a proportionality constant, also independent of energy, but is model-dependent

- $\kappa = 2/3$ for the additive quark model.
- The theorems survive *exponentiation* of the eikonals.

Measurement of κ, using $B_{\gamma p}(s) = \kappa B_{nn}(s)$

B



c.m.s. energy

From χ^2 fit, $\kappa = 0.661 \pm 0.008$

Experimental Verification of Factorization Theorem $\rho_{\gamma \mathrm{p}}(s) = \rho_{\mathrm{nn}}(s)$

- 'Elastic' Scattering Reactions: $\gamma + p \to V + p \to V + p$, $V = \rho, \omega, \phi$
- True Elastic (Compton) Scattering: $\gamma + p \to \gamma + p$

Forward Compton Scattering, using Real Analytic Amplitudes

$\gamma + p \rightarrow \gamma + p$, with $\nu = E_\gamma$

Forward high energy (real analytic) scattering amplitude, f_+

$$f_+ = i\frac{\nu}{4\pi}\left\{A + \beta[\ln(s/s_0) - i\pi/2]^2 + cs^{\mu-1}e^{i\pi(1-\mu)/2}\right\} + C_{\text{subtraction}}$$

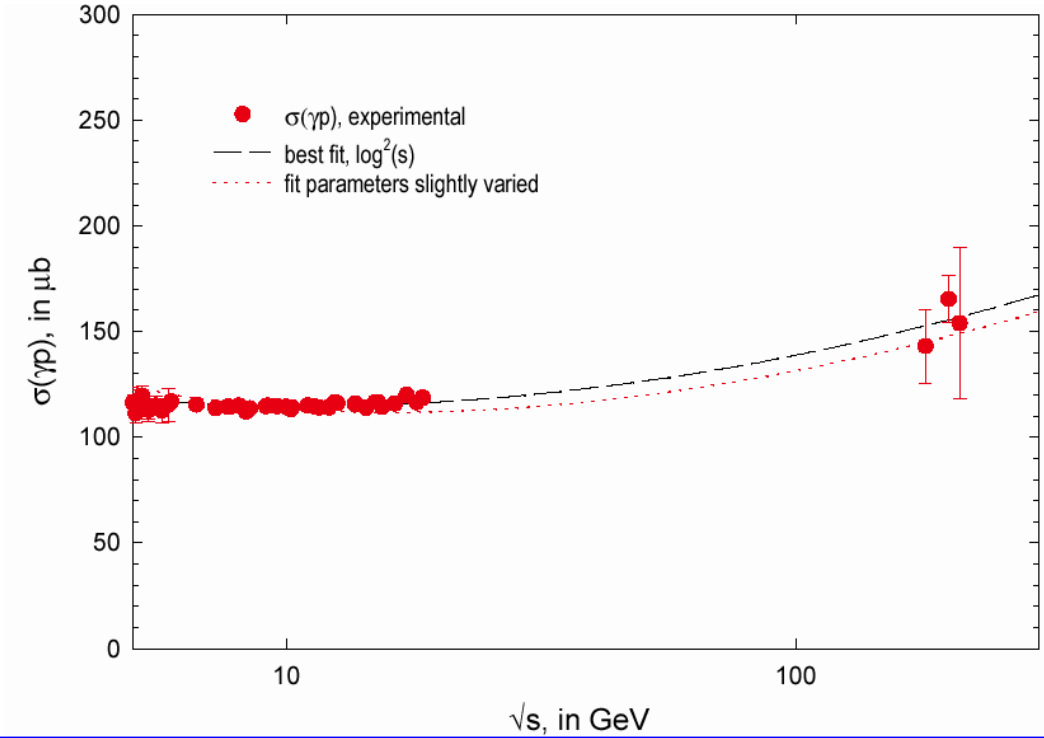
$C_{\text{subtraction}} = f_+(0) = -\alpha/m$, Thompson scattering limit,

A, β, c, s_0 and μ are real constants to be fit.

μ=0.5 corresponds to a descending Regge trajectory

Using optical theorem,

$$\sigma_{\rm tot} = A + \beta\left[\ln^2 s/s_0 - \frac{\pi^2}{4}\right] + c\,\sin(\pi\mu/2)s^{\mu-1}$$

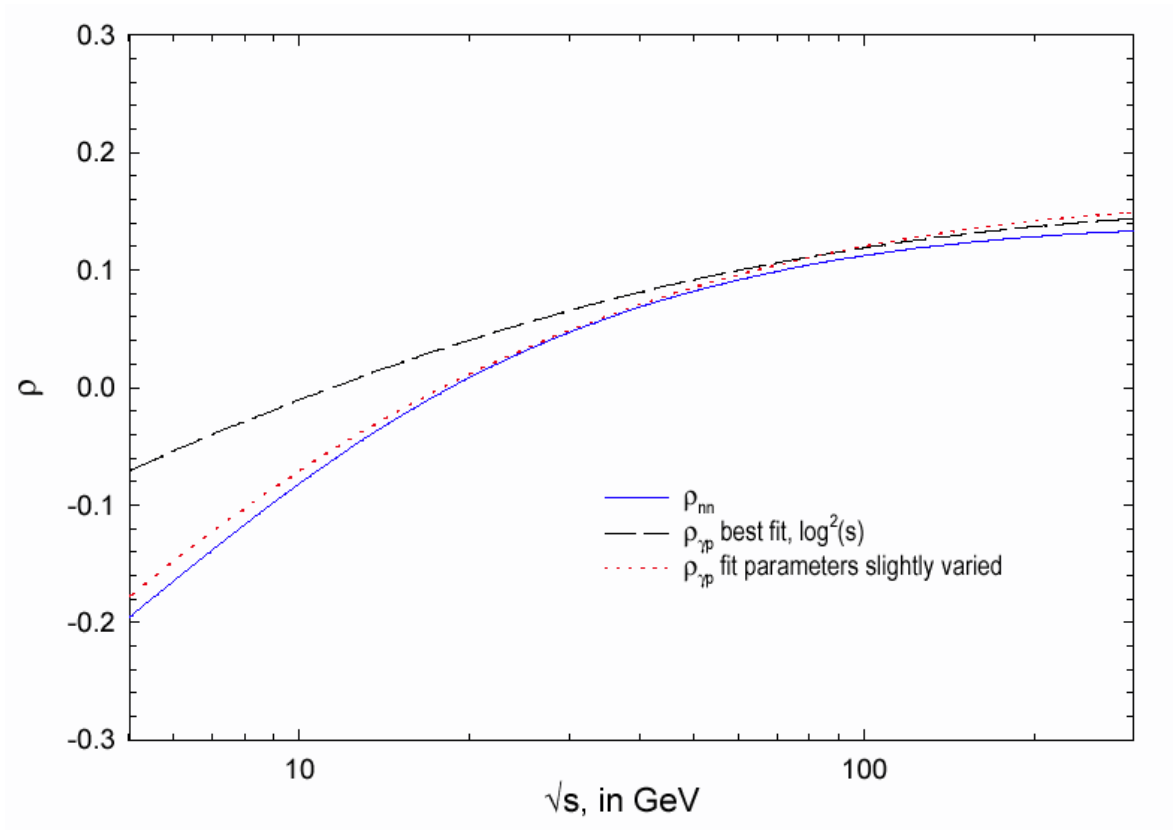


χ^2/d.f.=0.98,
for 40 d.f.,
for $\log^2(s)$ fit

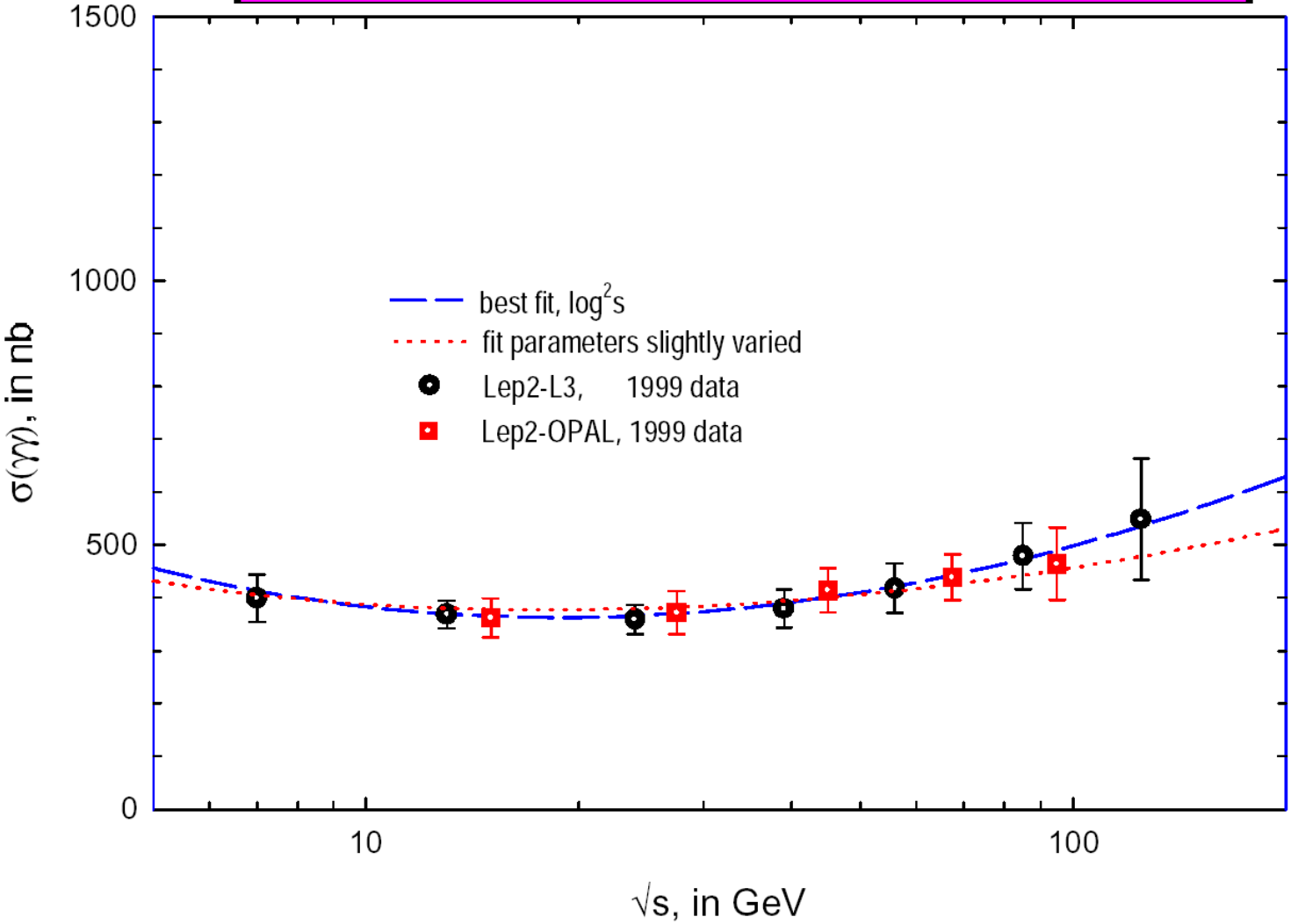
$$\rho = \frac{\beta\,\pi\ln s/s_0 - c\,\cos(\pi\mu/2)s^{\mu-1} + \frac{4\pi}{\nu}f_+(0)}{\sigma_{\rm tot}}$$

Experimental Verification of Factorization Theorem $\rho_{\gamma p}(s) = \rho_{nn}(s)$

- True Elastic (Compton) Scattering: $\gamma + p \rightarrow \gamma + p$



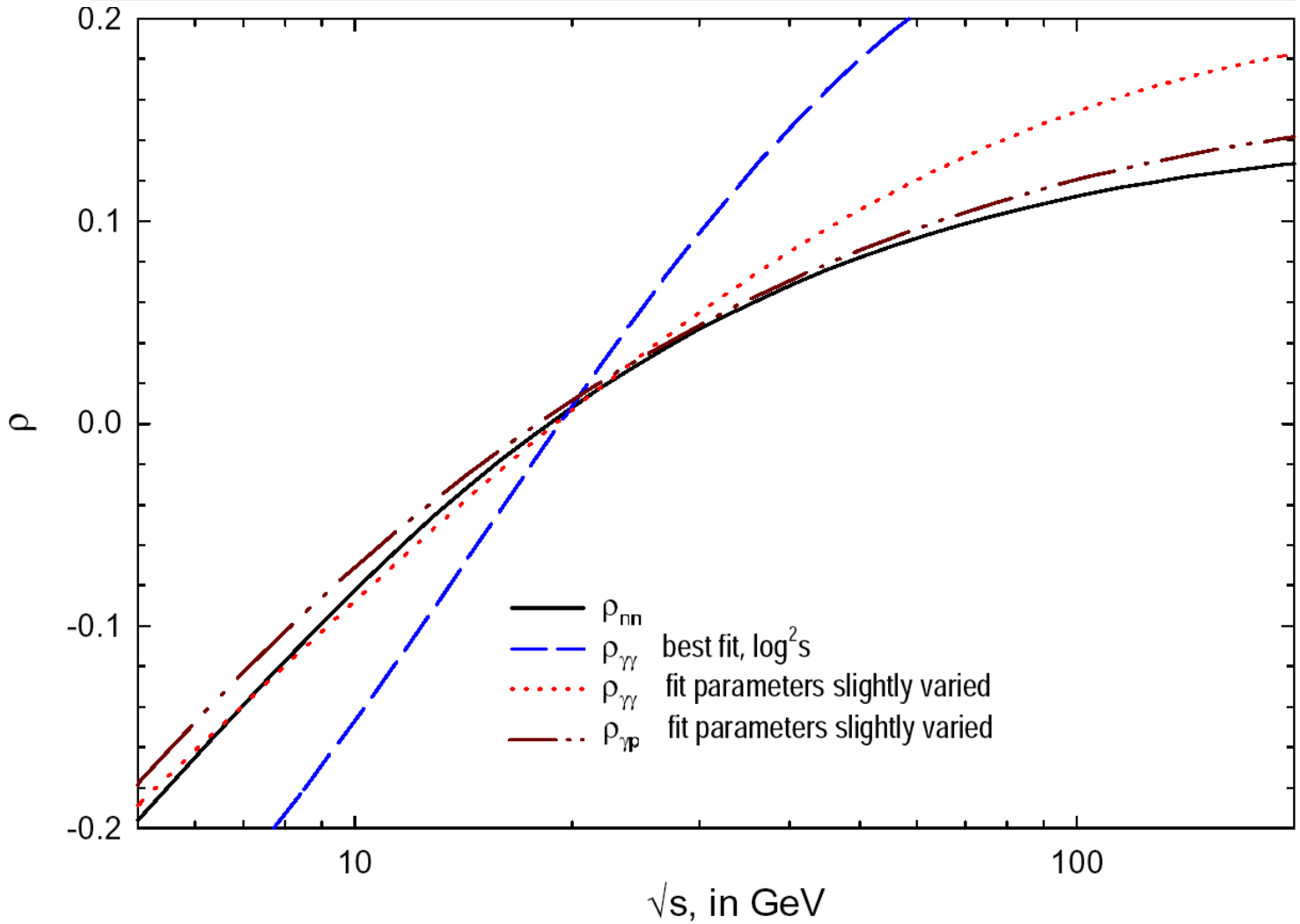
$\sigma(\gamma\gamma)$, fitted with $\log^2 s$



χ^2/d.f.=0.06,

for 9 d.f

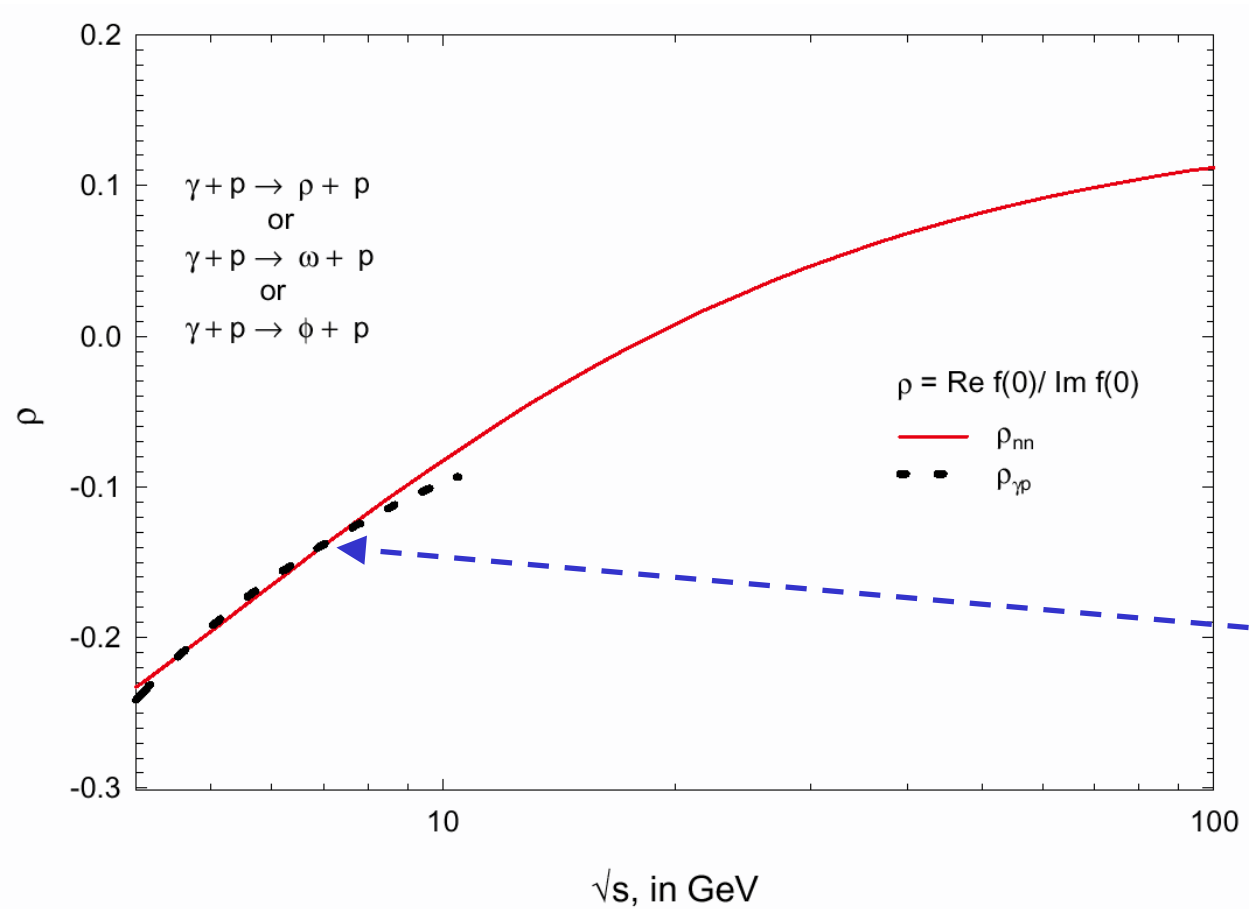
$\rho_{\gamma\gamma}$, using Real Analytic Amplitudes



Compton Scattering, using Real Analytic Amplitudes

$$\rho_{\text{analytic}} = -\frac{70.2/\nu^{1/2} + 38.07/\nu}{96.6 + 70.2/\nu^{1/2}}$$

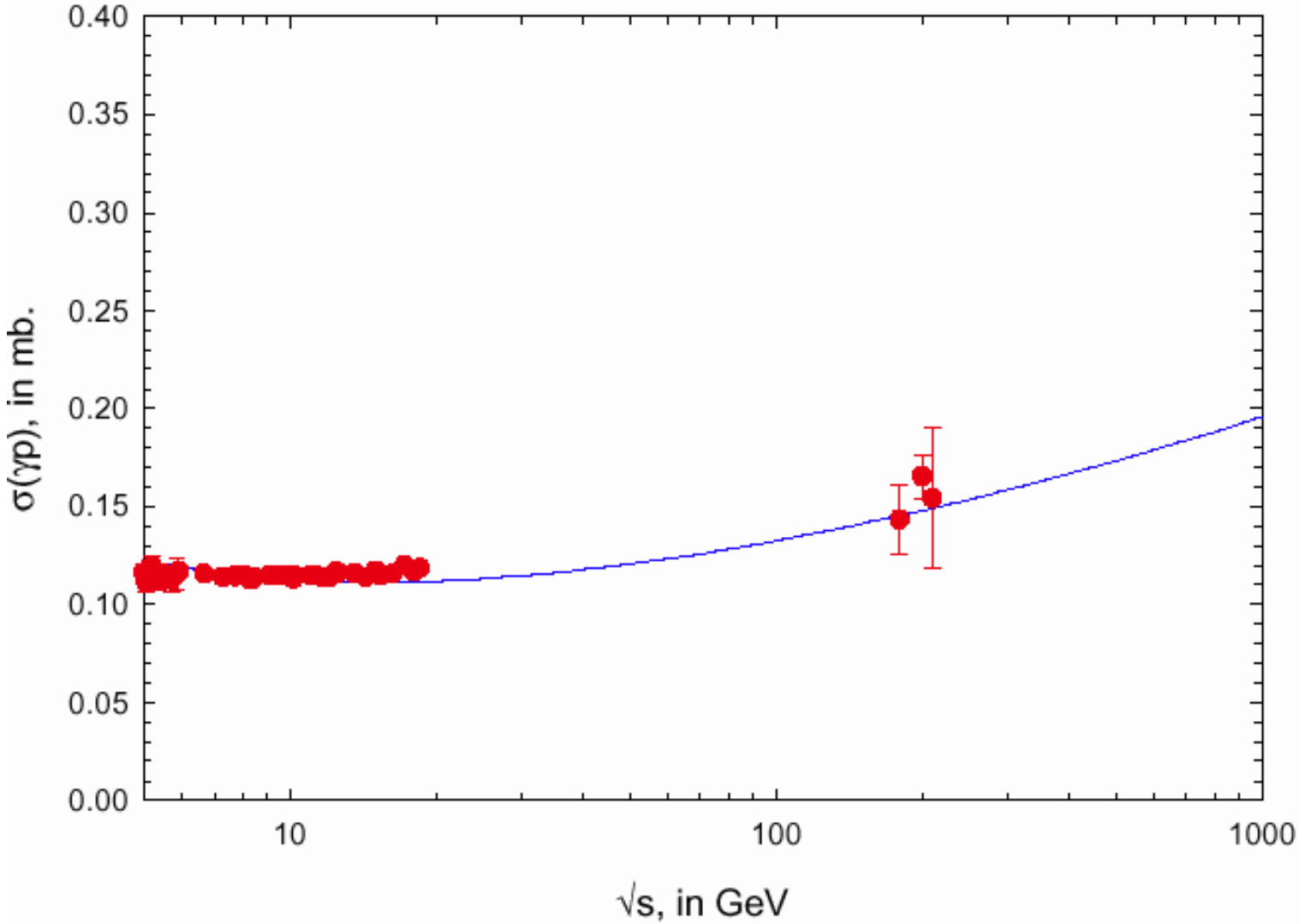
$\nu = E_\gamma$(lab), in GeV



$\gamma + p \rightarrow \gamma + p$, using real analytic amplitudes

Vector Dominance, using γp Cross Sections, in $\sigma_{\gamma \mathrm{p}}(s) = \frac{2}{3} P^{\gamma}_{\mathrm{had}} \sigma_{\mathrm{nn}}(s)$

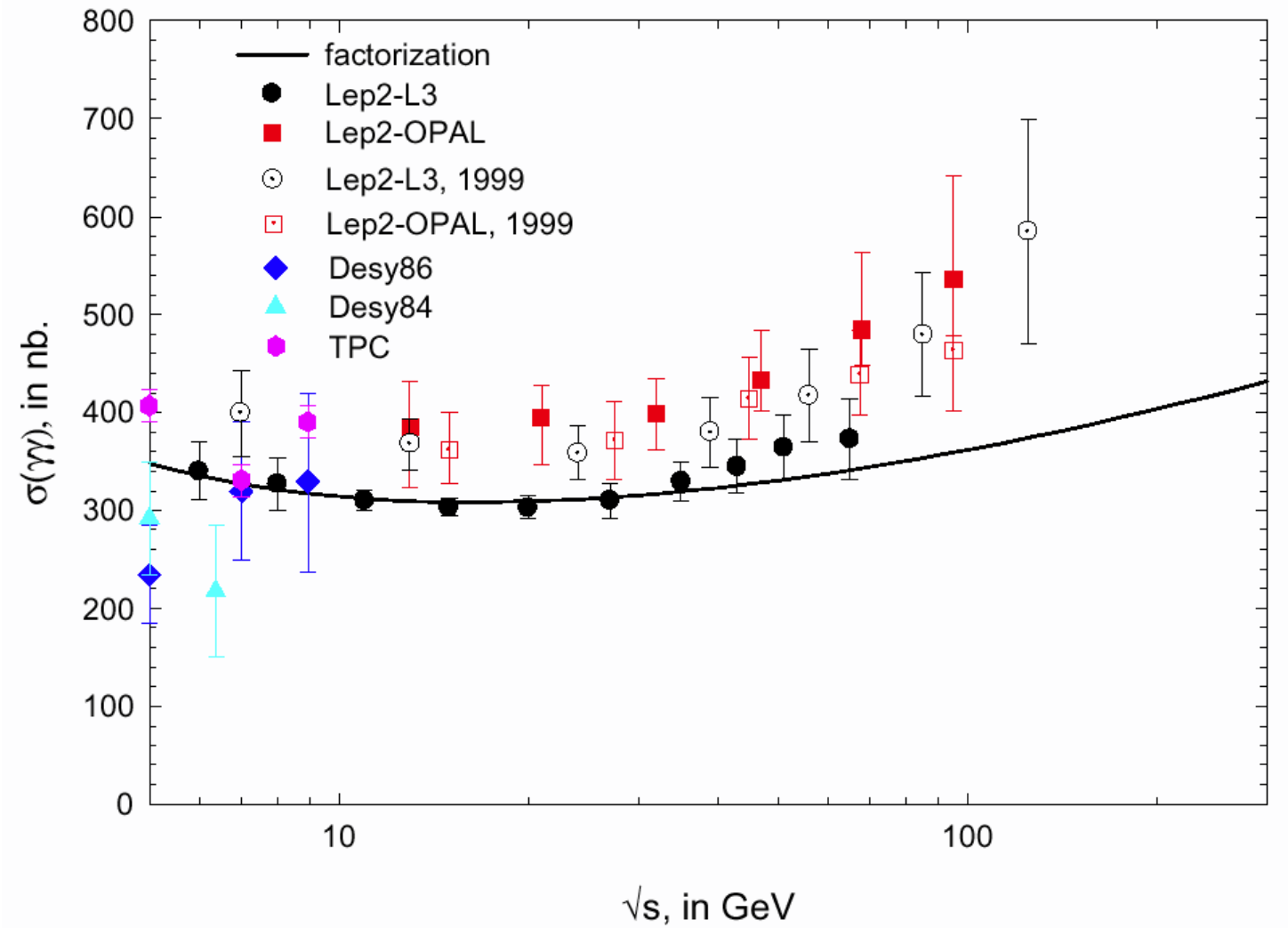
Using $f_\rho^2/4\pi = 2.2$, $f_\omega^2/4\pi = 23.6$ and $f_\phi^2/4\pi = 18.4$, we find $\Sigma_V(4\pi\alpha/f_V^2) = 1/249$, where $V = \rho, \omega, \phi$.



We find $P^{\gamma}_{\text{had}} = 1/240$ normalizing to the low energy cross sections $\sigma_{\gamma p}$. This value is approximately 4% greater than that derived from vector dominance, 1/249.

Verification of Factorization using $\gamma\gamma$ Scattering :

$$\sigma_{\gamma\gamma}(s) = \frac{2}{3}P^{\gamma}_{\rm had}\,\sigma_{\gamma p} = \left(\frac{2}{3}P^{\gamma}_{\rm had}\right)^2 \sigma_{nn}(s), \quad P^{\gamma}_{\rm had} = 1/240$$



Does factorization depend on using a dipole-dipole form factor?

$$W_{\rm nn}(b;\mu) \quad = \quad \frac{\mu^2}{96\pi}(\mu b)^3 K_3(\mu b)$$

Could we also use a monopole-monopole for γγ, with a monopole-dipole for γ-p?

$$W_{\rm nn}(b;\mu) = \frac{1}{(2\pi)^2}\int \frac{\mu^8}{(q^2+\mu^2)^4} e^{i\vec{q}\cdot\vec{b}} d^2\vec{q},$$

dipole-dipole

$$W_{\gamma p}(b;\mu,\nu) = \frac{1}{(2\pi)^2}\int \frac{\mu^4\nu^2}{(q^2+\mu^2)^2(q^2+\nu^2)} e^{i\vec{q}\cdot\vec{b}} d^2\vec{q},$$

dipole-monopole

$$W_{\gamma\gamma}(b;\nu) = \frac{1}{(2\pi)^2}\int \frac{\nu^4}{(q^2+\nu^2)^2} e^{i\vec{q}\cdot\vec{b}} d^2\vec{q}$$

monopole-monopole

Fourier transforms

$$W_{\rm nn}(b;\mu) = \frac{\mu^2}{96\pi}(\mu b)^3 K_3(\mu b)$$

$$W_{\gamma p}(b;\mu,\nu) = \frac{\mu^2\nu^2}{4\pi(\mu^2-\nu^2)}\left(\frac{2\mu^2}{\mu^2-\nu^2}(K_0(\nu b)-K_0(\mu b)) - (\mu b)K_1(\mu b)\right)$$

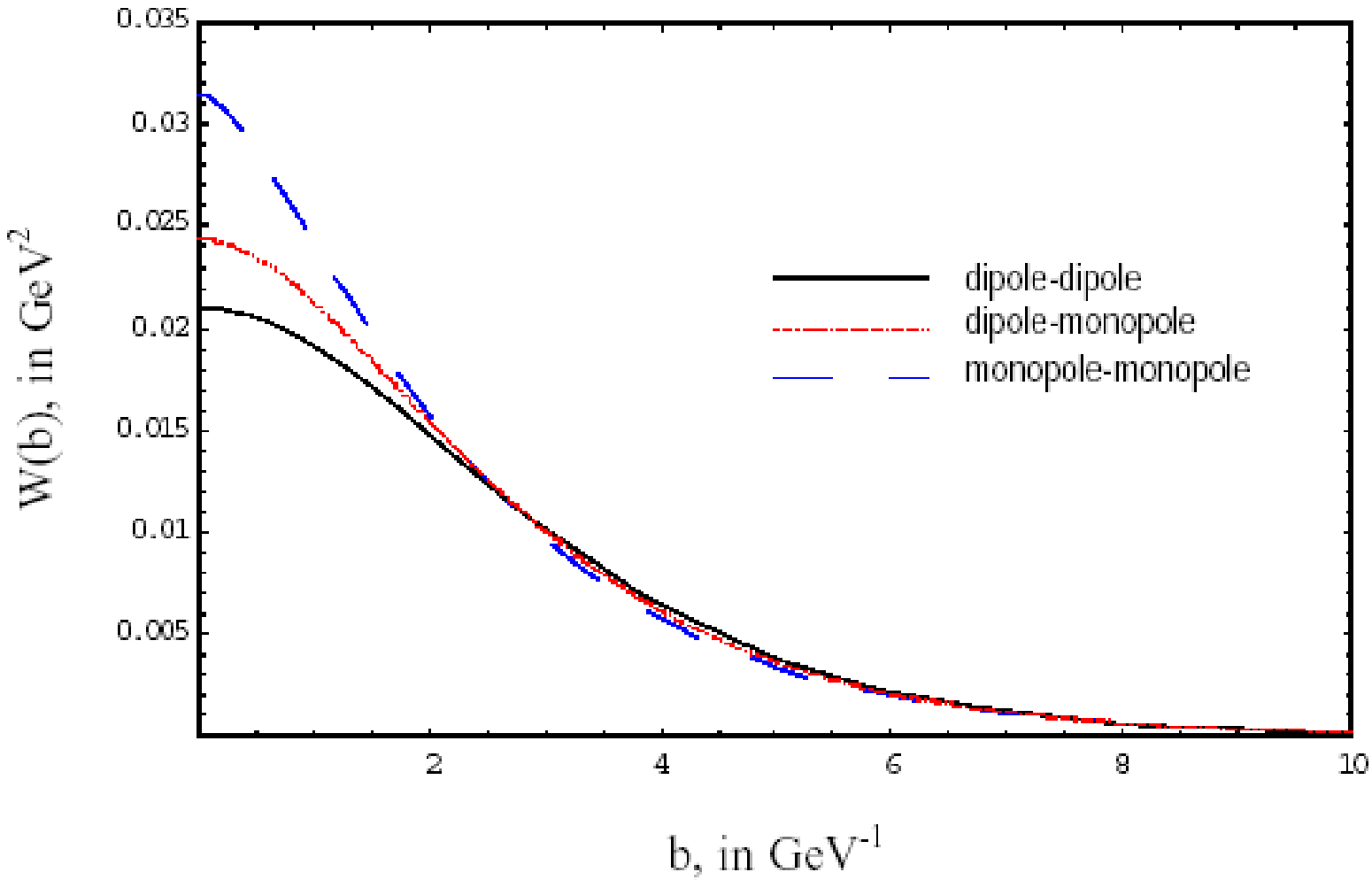
$$W_{\gamma\gamma}(b;\nu) = \frac{\nu^2}{4\pi}(\nu b)K_1(\nu b),$$

$$< b^2 >= \int b^2 W(b)\, d^2\vec{b} = \frac{16}{\mu^2} = \frac{8}{\nu^2}$$

set $\langle b^2\rangle_{\rm nn} = \langle b^2\rangle_{\gamma\gamma}$

yields energy –independent relation: $\nu = \mu/\sqrt{2}$

W(b), dipole-dipole, dipole-monopole, monopole-monopole



Total cross section and the ratio of elastic to total cross section

$$\sigma_{\rm tot} = 2\int \left[1 - e^{-\sigma_0 W(b)}\right] d^2\vec{b}$$

$$\frac{\sigma_{\rm el}}{\sigma_{\rm tot}} = \frac{\int \left[1 - e^{-\sigma_0 W(b)}\right]^2 d^2\vec{b}}{\sigma_{\rm tot}}.$$

nn uses dipole-dipole, γp uses monopole-dipole, γγ uses monopole-monopole

	$\sqrt{s} = 1500$ GeV		$\sqrt{s} = 400$ GeV		$\sqrt{s} = 25$ GeV	
Reaction	$\sigma_{\rm el}/\sigma_{\rm tot}$	$\sigma_{\rm tot}$, in mb	$\sigma_{\rm el}/\sigma_{\rm tot}$	$\sigma_{\rm tot}$, in mb	$\sigma_{\rm el}/\sigma_{\rm tot}$	$\sigma_{\rm tot}$, in mb
nn	0.272	73.00	0.236	57.4	0.184	40.06
γp	0.268	71.89	0.235	56.54	0.186	39.56
$\gamma\gamma$	0.263	70.68	0.233	55.54	0.188	38.93
$\sigma_{\rm tot}^{\rm nn}/\sigma_{\rm tot}^{\gamma p}$	1.015		1.015		1.013	
$\sigma_{\rm tot}^{\gamma p}/\sigma_{\rm tot}^{\gamma\gamma}$	1.017		1.018		1.016	

To test factorization, we will utilize *real analytical amplitudes*

Part VI: Experimental evidence for factorization, using real analytical amplitudes, for n-n, γp and $\gamma\gamma$ collisions

M. M. Block and K. Kang

Kyungsik Kang,

In

Paris

Real Analytic Amplitudes

even: $\frac{4\pi}{p} f^{+}(s) = i\left\{A + \beta[\ln(s/s_0) - i\pi/2]^2 + c s^{\mu-1} e^{i\pi(1-\mu)/2}\right\}$

odd: $\frac{4\pi}{p} f^{-}(s) = -D s^{\alpha-1} e^{i\pi(1-\alpha)/2}$

where A, β, c, s_0, D, μ and α are real constants.

$$\sigma_{\bar{p}p}(s) = A + \beta\left[\ln^2 s/s_0 - \frac{\pi^2}{4}\right] + c\,\sin(\pi\mu/2)s^{\mu-1} - D\cos(\pi\alpha/2)s^{\alpha-1}$$

$$\sigma_{pp}(s) = A + \beta\left[\ln^2 s/s_0 - \frac{\pi^2}{4}\right] + c\,\sin(\pi\mu/2)s^{\mu-1} + D\cos(\pi\alpha/2)s^{\alpha-1}$$

$$\sigma_{nn}(s) = A + \beta\left[\ln^2 s/s_0 - \frac{\pi^2}{4}\right] + c\,\sin(\pi\mu/2)s^{\mu-1}$$

$$\rho_{\bar{p}p}(s) = \frac{\beta\,\pi\ln s/s_0 - c\,\cos(\pi\mu/2)s^{\mu-1} - D\sin(\pi\alpha/2)s^{\alpha-1}}{\sigma_{\bar{p}p}}$$

$$\rho_{pp}(s) = \frac{\beta\,\pi\ln s/s_0 - c\,\cos(\pi\mu/2)s^{\mu-1} + D\sin(\pi\alpha/2)s^{\alpha-1}}{\sigma_{pp}}$$

$$\rho_{nn}(s) = \frac{\beta\,\pi\ln s/s_0 - c\,\cos(\pi\mu/2)s^{\mu-1}}{\sigma_{nn}}.$$

Real analytic amplitudes for γp and γγ

$$\frac{4\pi}{p} f_{\gamma p}(s) = iN\left\{A + \beta[\ln(s/s_0) - i\pi/2]^2 + cs^{\mu-1}e^{i\pi(1-\mu)/2}\right\}$$

$$\frac{4\pi}{p} f_{\gamma\gamma}(s) = iN^2\left\{A + \beta[\ln(s/s_0) - i\pi/2]^2 + cs^{\mu-1}e^{i\pi(1-\mu)/2}\right\}$$

where N is the proportionality constant in the factorization relation $\frac{\sigma_{nn}(s)}{\sigma\gamma p(s)} = \frac{\sigma_{\gamma p}(s)}{\sigma\gamma\gamma(s)} = N$.

Using the additive quark model and vector dominance,

$$N = \frac{2}{3}P^{\gamma}_{\text{had}}, \quad \text{where } P^{\gamma}_{\text{had}} \approx \Sigma_V \frac{4\pi\alpha}{f_V^2} = 1/249$$

$$\sigma_{\gamma p}(s) = \frac{2}{3}P^{\gamma}_{\text{had}}\left(A + \beta\left[\ln^2 s/s_0 - \frac{\pi^2}{4}\right] + c\sin(\pi\mu/2)s^{\mu-1}\right)$$

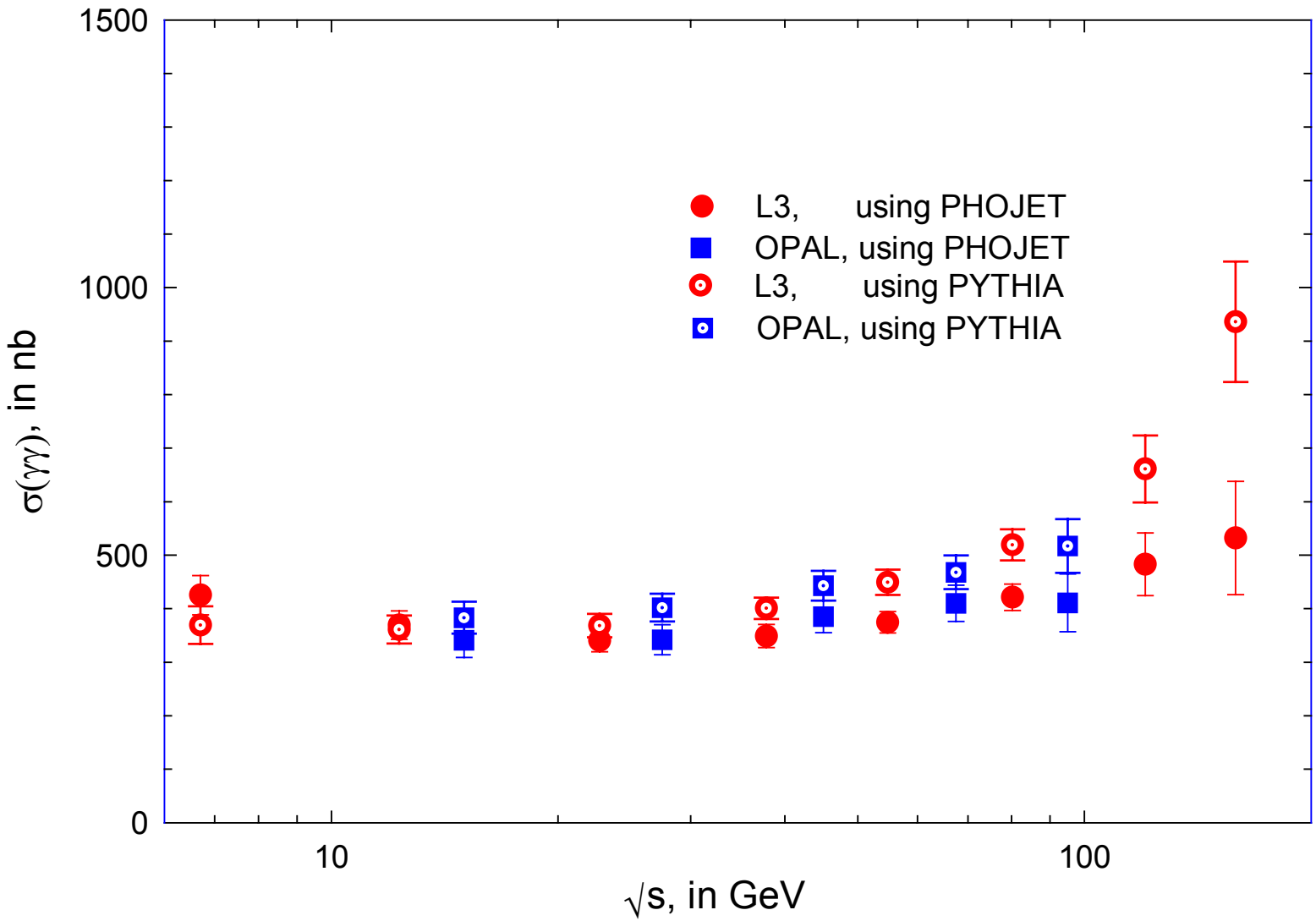
$$\sigma_{\gamma\gamma}(s) = \left(\frac{2}{3}P^{\gamma}_{\text{had}}\right)^2\left(A + \beta\left[\ln^2 s/s_0 - \frac{\pi^2}{4}\right] + c\sin(\pi\mu/2)s^{\mu-1}\right)$$

with the real constants A, β, s_0, c, D and P^{γ}_{had} (assuming $\alpha = \mu = 1/2$) being fitted by experiment.

$\sigma_{\gamma\gamma}$, using *either* PHOJET or PYTHIA--data are from L3 and OPAL

M. Acciari et al, Phys. Lett. B519, 33 (2001),

G. Abbiendi et al, Eur. Phys. J. C14 ,199 (2000).



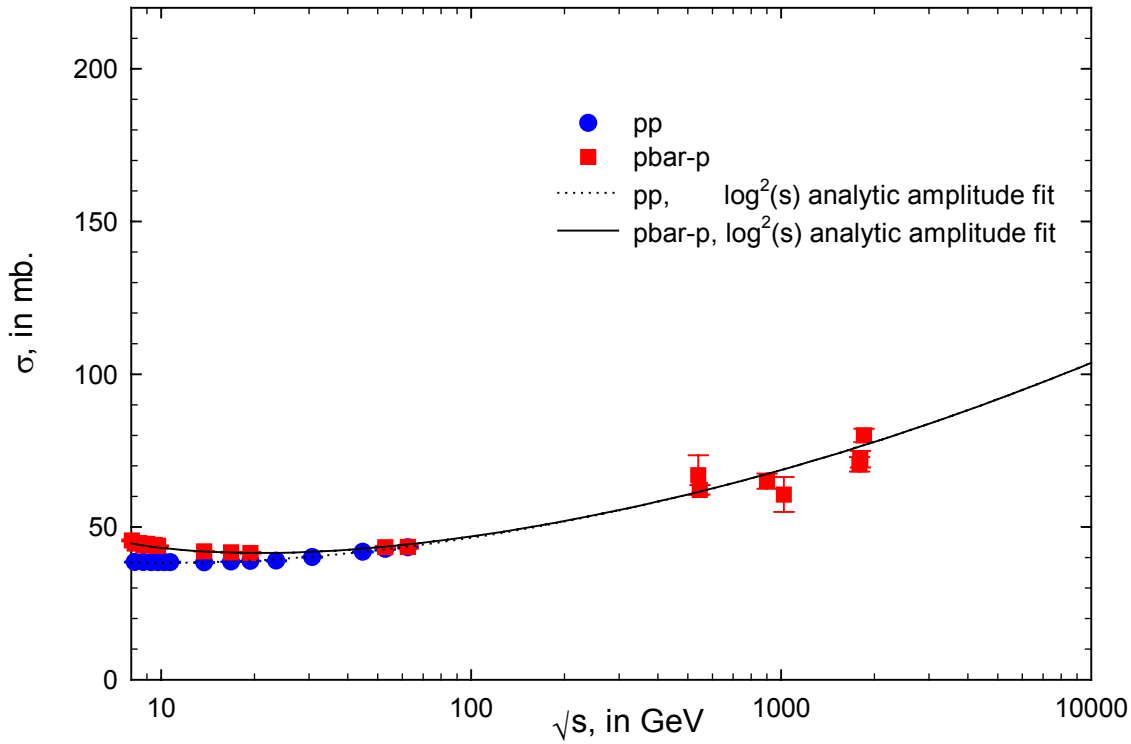
GLOBAL FIT

Parameters	$\sigma_{tot} \sim \ln^2(s/s_0)$		
	Fit 1	Fit 2	Fit 3
	no $\sigma_{\gamma\gamma}$	$\sigma_{\gamma\gamma}$, PHOJET	$\sigma_{\gamma\gamma}$, PYTHIA
A(mb)	37.2 ± 0.81	37.1 ± 0.87	37.3 ± 0.77
β(mb)	0.304 ± 0.023	0.302 ± 0.024	0.307 ± 0.022
$s_0((\mathrm{GeV})^2)$	34.3 ± 14	32.6 ± 16	35.1 ± 14
$D(\mathrm{mb}(\mathrm{GeV})^{2(1-\alpha)})$	-35.1 ± 0.83	-35.1 ± 0.85	-35.4 ± 0.84
α	0.5	0.5	0.5
$c(\mathrm{mb}(\mathrm{GeV})^{2(1-\mu)})$	55.0 ± 7.5	55.9 ± 8.1	54.6 ± 7.3
μ	0.5	0.5	0.5
$P^{\gamma}_{\mathrm{had}}$	$1/(233.1 \pm 0.6)$	$1/(233.1 \pm 0.6)$	$1/(233.1 \pm 0.6)$
N_{OPAL}	———	0.929 ± 0.037	0.861 ± 0.050
N_{L3}	———	0.929 ± 0.025	0.808 ± 0.020
χ^2/d.f.	1.62	1.49	1.87
d.f.	68	78	78
total χ^2	110.5	115.9	146.0

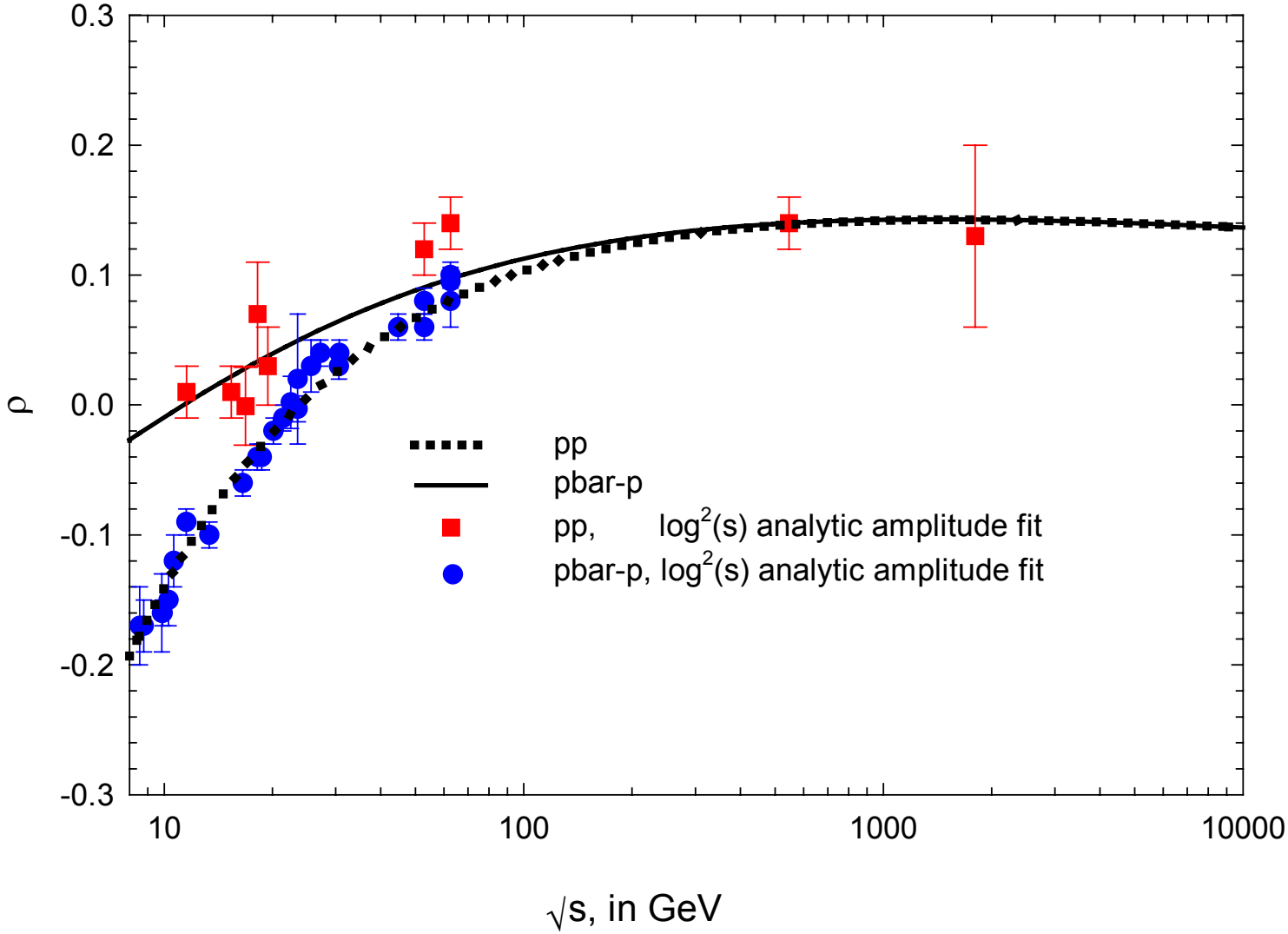
GOOD

BAD

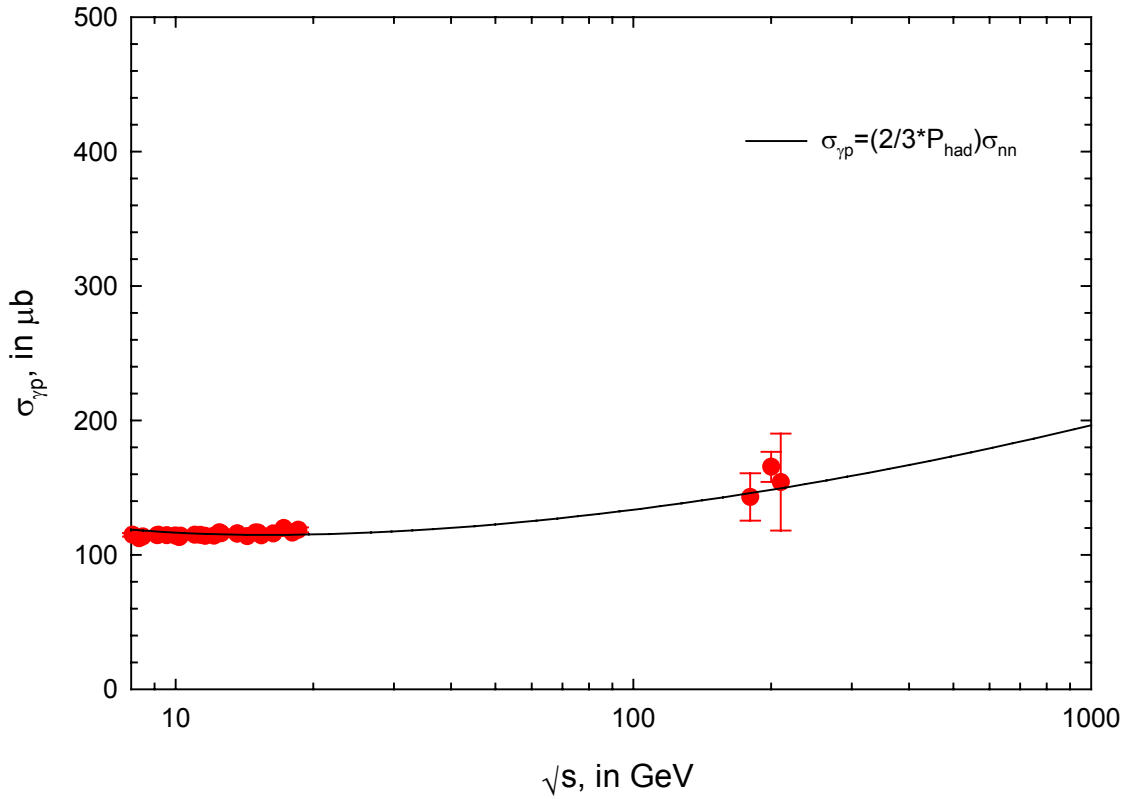
σ_{pp} , $\sigma_{pbar\text{-}p}$ *vs.* $\sqrt{s}$



ρ_{pp} , $\rho_{pbar\text{-}p}$ *vs.* $\sqrt{s}$

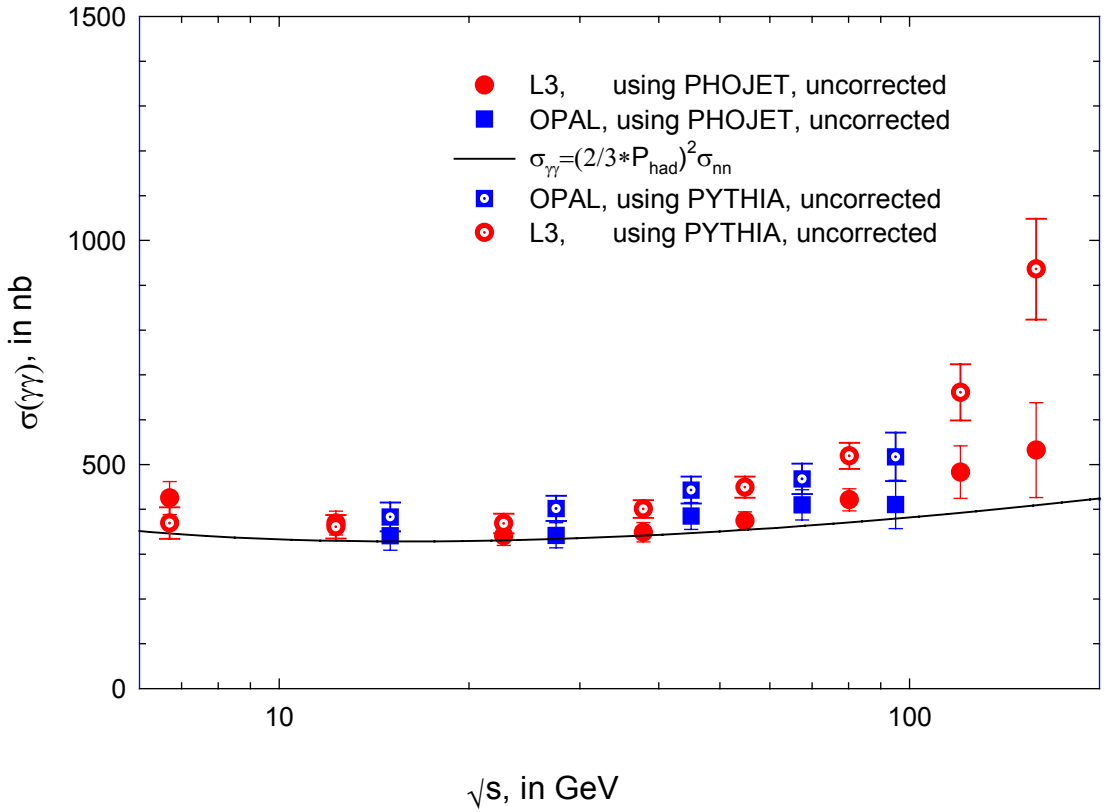


$\sigma_{\gamma\text{-}p} = (2/3 * P_{had})\sigma_{nn}$

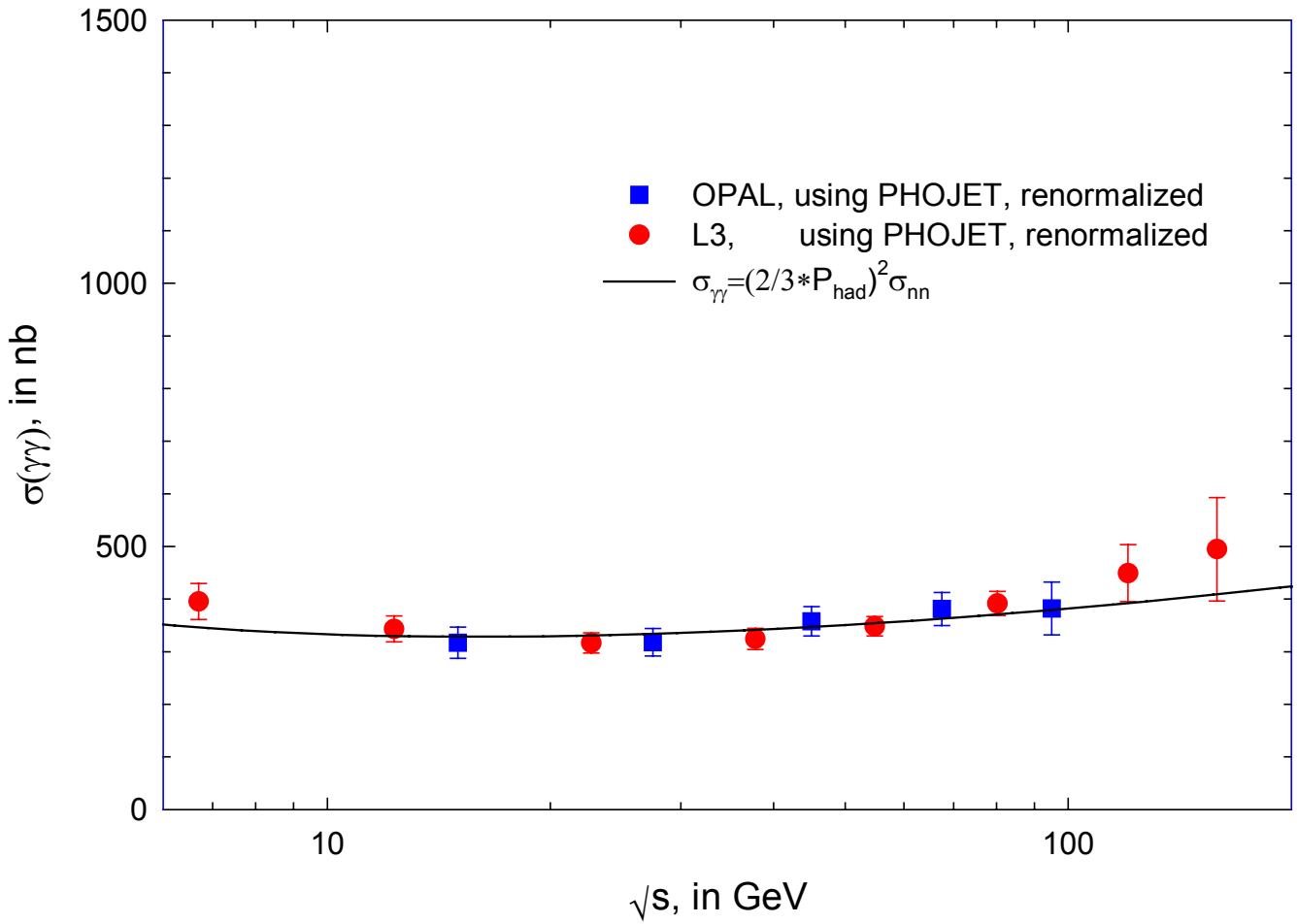


$\sigma_{\gamma\gamma} = (2/3 * P_{had})^2 \sigma_{nn}$,

L3 and OPAL data, uncorrected with PYTHIA and PHOJET

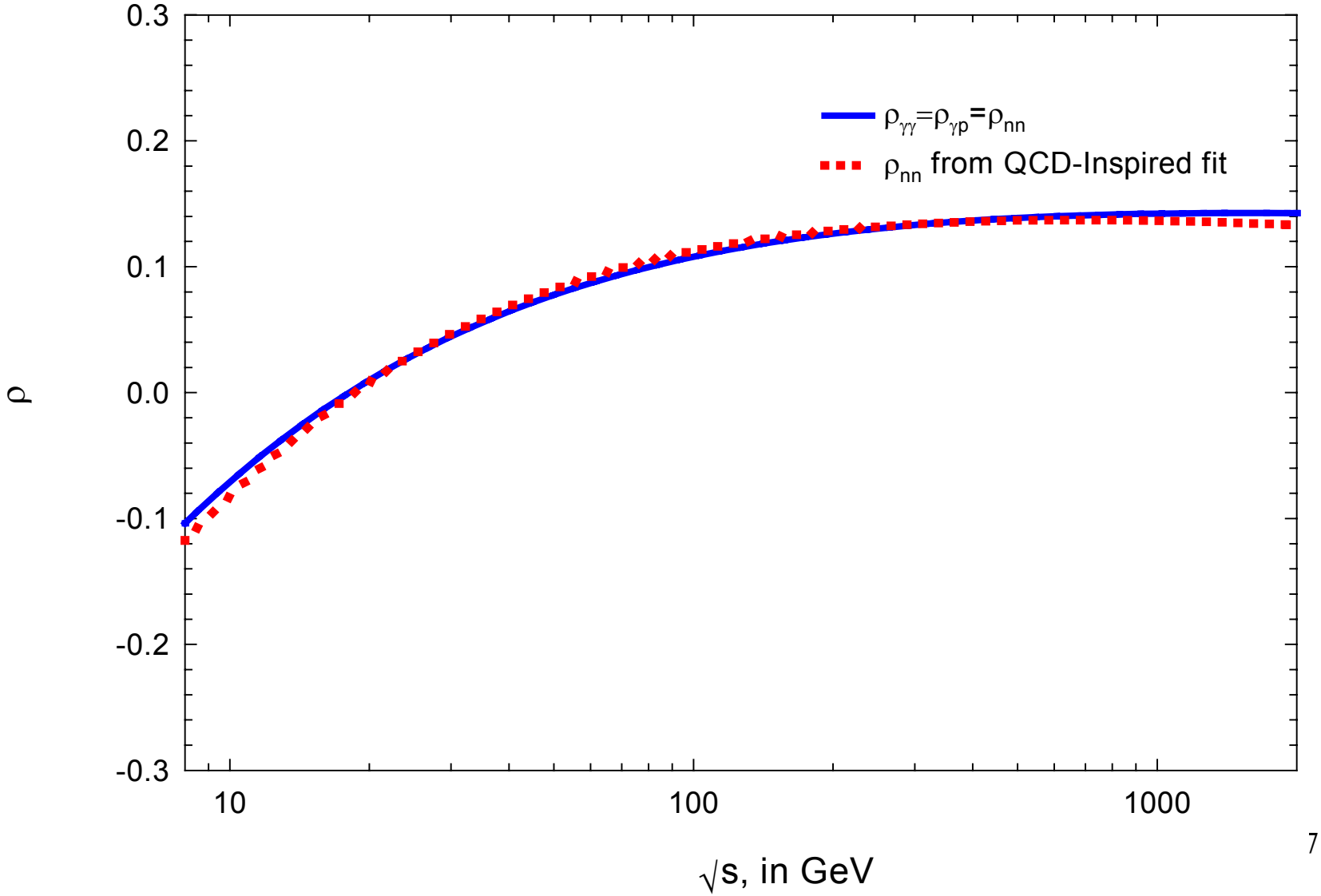


curve: $\sigma_{\gamma\gamma} = (2/3 * P_{had})^2 \sigma_{nn}$
data: L3 and OPAL, renormalized by factor 0.929, using PHOJET



solid line: $\rho_{nn}=\rho_{\gamma p}=\rho_{\gamma\gamma}$

dotted line: ρ_{nn} from QCD-Inspired Fit (Aspen Model)



Conclusions

The available data on nn, γp and $\gamma\gamma$ reactions lend strong experimental support to the factorization hypotheses of

- the well-known cross section factorization theorem:

$$\sigma_{\mathrm{nn}}/\sigma_{\gamma\mathrm{p}} = \sigma_{\gamma\mathrm{p}}/\sigma_{\gamma\gamma}$$

- the lessor-known nuclear slope factorization theorem:

$$B_{\mathrm{nn}}/B_{\gamma\mathrm{p}} = B_{\gamma\mathrm{p}}/B_{\gamma\gamma}$$

- the relatively obscure requirement:

$$\rho_{\mathrm{nn}} = \rho_{\gamma\mathrm{p}} = \rho_{\gamma\gamma}$$

- verifying the additive quark model:

  by measuring $\kappa = 0.661 \pm 0.008$, a result within $\approx 1\%$ of the value of 2/3 expected for the quark model, using $B_{\gamma p}$ measurements over a wide span of energies, $3 \leq \sqrt{s} \leq 200$ GeV.

- confirming vector dominance:

  using σ_{nn}, $\sigma_{\gamma p}$ and $\sigma_{\gamma\gamma}$ over an energy region of $3 \leq \sqrt{s} \leq 100$ GeV and a cross section factor of over 10^5.

Factorization Theorem for Differential Elastic Scattering

We note that:

$$\frac{d\sigma^{\mathrm{nn}}}{dt}(s,t) = \frac{1}{4\pi}\left|\int J_0(qb)\left(1 - e^{i\chi^{\mathrm{even}}(b,s)}\right) d^2\vec{b}\right|^2 , \quad t = -q^2$$

It is straightforward to prove the factorization theorem for differential elastic scattering:

$$\frac{d\sigma^{\mathrm{nn}}}{dt}(s,t) \Big/ \frac{d\sigma^{\gamma p}_{Vp}}{dt}(s,\tfrac{3}{2}t) = \frac{d\sigma^{\gamma p}_{Vp}}{dt}(s,\tfrac{3}{2}t) \Big/ \frac{d\sigma^{\gamma\gamma}_{VV}}{dt}(s,\tfrac{9}{4}t)$$

Differential Elastic Scattering

The differential cross section for the "elastic" scattering reaction is

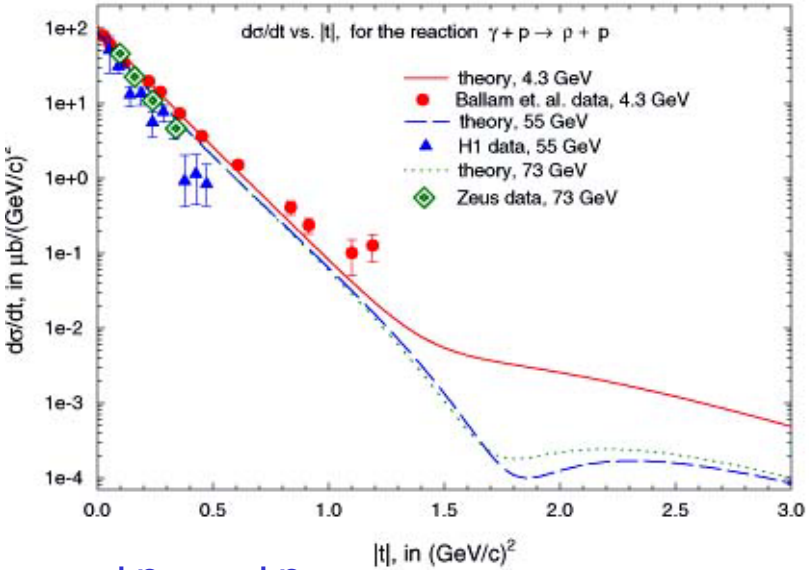
$$\frac{d\sigma_{V_i}^{\gamma p}}{dt}(s,t) = \frac{4}{9P_{V_i}^{\gamma}}\frac{d\sigma^{\mathrm{nn}}}{dt}(s,\frac{2}{3}t), \quad \gamma + p \to V_i + p, \quad V_i = \rho,\ \omega \text{ or } \phi.$$

For the "elastic" $\gamma\gamma$ scattering reaction

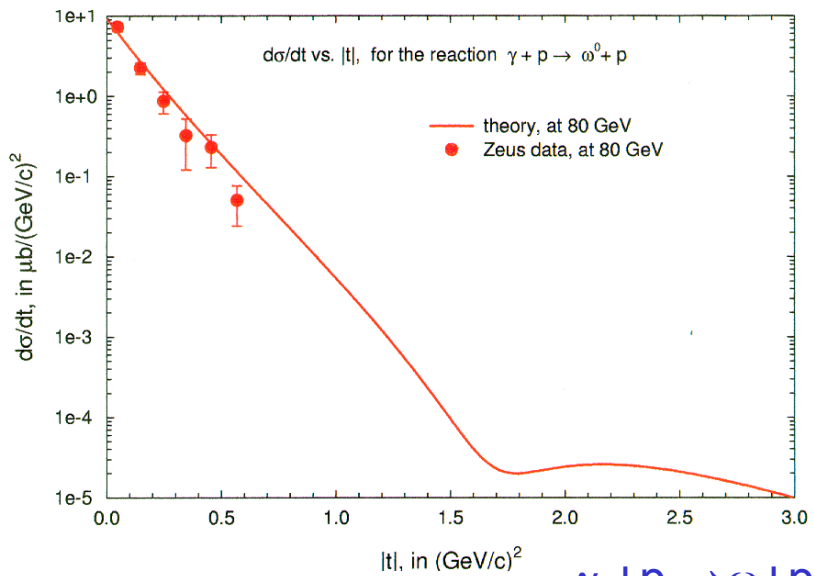
$$\frac{d\sigma_{V_iV_i}^{\gamma\gamma}}{dt}(s,t) = \left(\frac{4}{9P_{V_i}^{\gamma}}\right)^2\frac{d\sigma^{\mathrm{nn}}}{dt}(s,\frac{4}{9}t), \quad \gamma + \gamma \to V_i + V_i.$$

Thus, a knowledge of $\frac{d\sigma^{\mathrm{nn}}}{dt}(s,t)$ for elastic nucleon-nucleon scattering determines the differential "elastic" scattering cross sections for the reactions $\gamma + p \to V_i + p$ and $\gamma + \gamma \to V_i + V_i$.

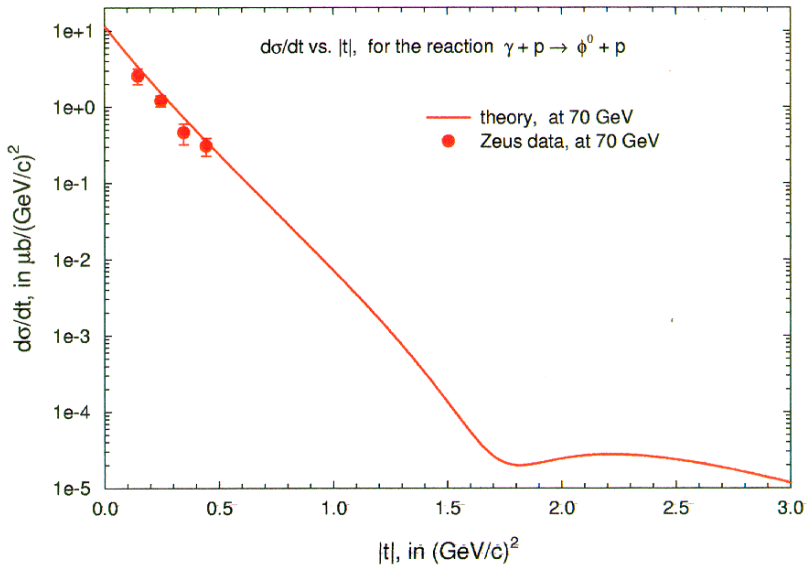
dσ/dt, for "Elastic" Scattering



$\gamma + p \rightarrow \rho + p$



$\gamma + p \rightarrow \omega + p$

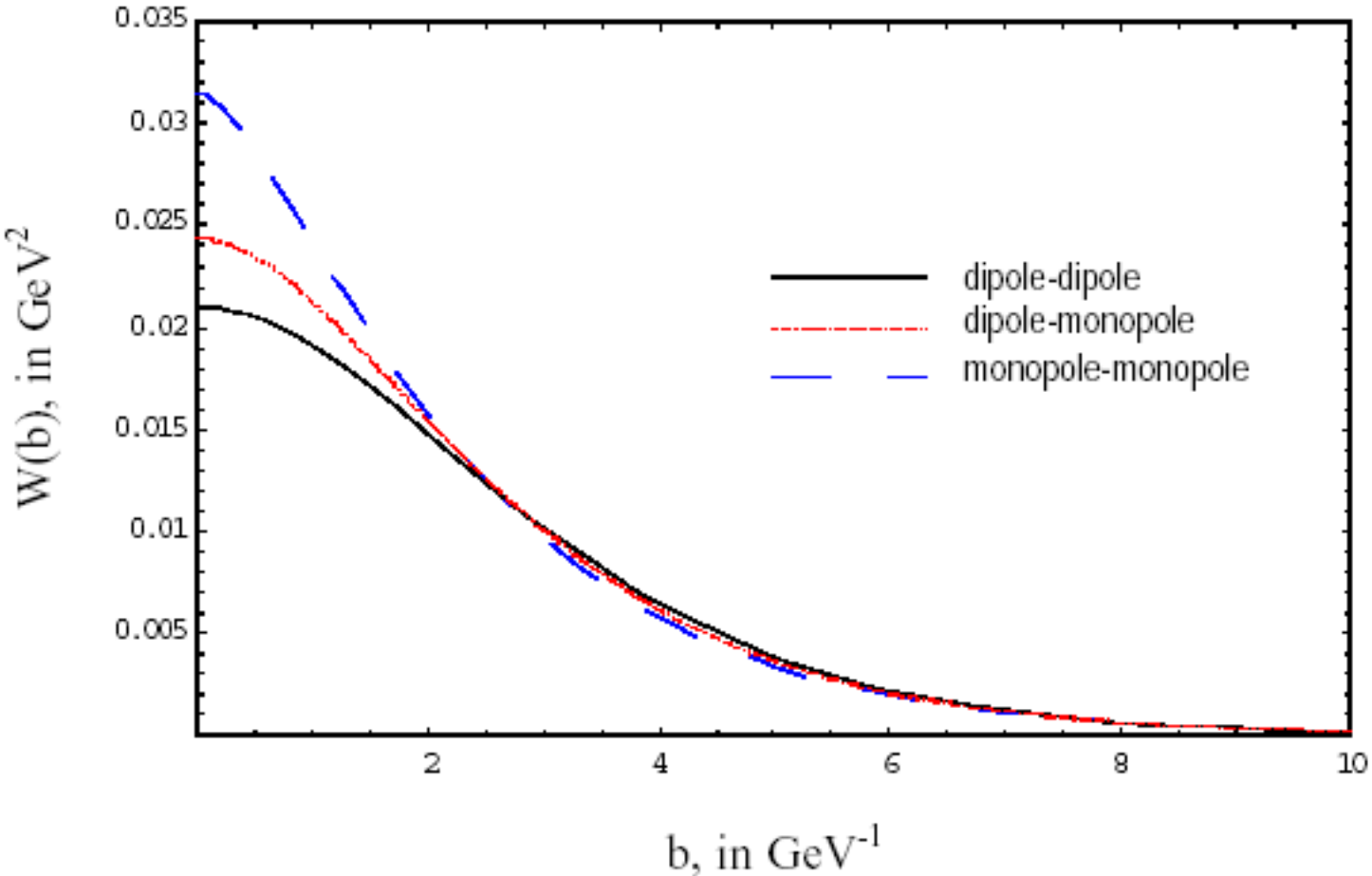


$\gamma + p \rightarrow \phi + p$

Regge Model

We note that in the Regge approach the functions $W_i(b,\mu)$ depend on s, since the radius of interaction ($R \sim 1/\mu$) for the Pomeron exchange which determines an eikonal grows with energy as $R^2 = R_0^2 + \alpha'_P ln(s/s_0)$. Thus, in the Regge eikonal model, factorization breaks down in general. However, the Pomeron slope α'_P is very small ($\approx$ 0.2 GeV^{-2}) and the R_0 for different processes are approximately in the same relation as in the Additive Quark model. As a result, the factorization relations are valid with good accuracy even in this model, in a broad region of energies.

Form Factors W(b), for same <b²>



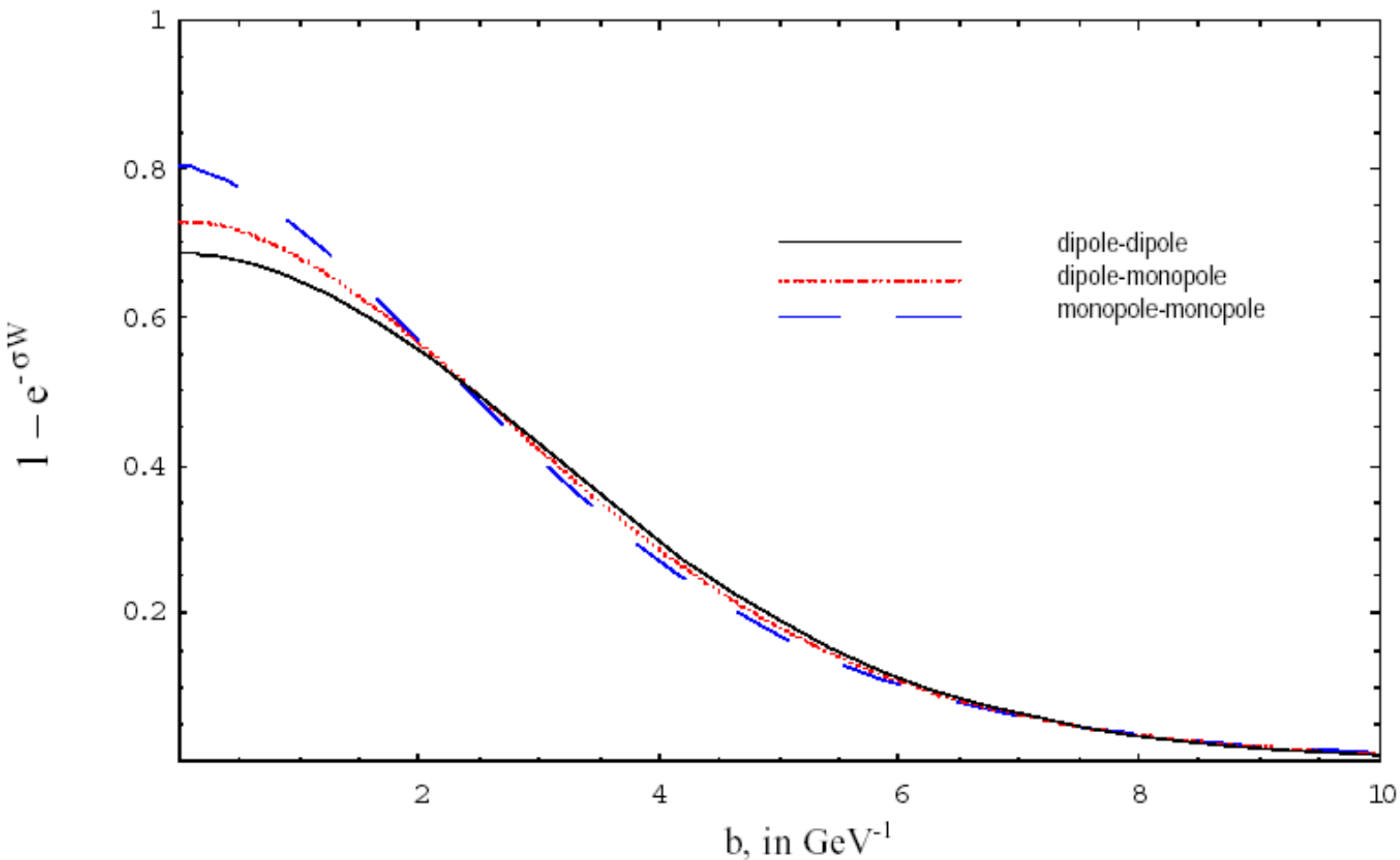
Total cross sections σ_{tot} and σ_{el}/σ_{tot}, for dipole-dipole (n-n), dipole-monopole (γ-p) and monopole-monopole (γ–γ) form factors, for same $\langle b^2 \rangle$

	$\sqrt{s} = 1500$ GeV		$\sqrt{s} = 400$ GeV		$\sqrt{s} = 25$ GeV	
Reaction	σ_{el}/σ_{tot}	σ_{tot}, in mb	σ_{el}/σ_{tot}	σ_{tot}, in mb	σ_{el}/σ_{tot}	σ_{tot}, in mb
nn	0.272	73.00	0.236	57.4	0.184	40.06
γp	0.268	71.89	0.235	56.54	0.186	39.56
$\gamma\gamma$	0.263	70.68	0.233	55.54	0.188	38.93
$\sigma_{tot}^{nn}/\sigma_{tot}^{\gamma p}$	1.015		1.015		1.013	
$\sigma_{tot}^{\gamma p}/\sigma_{tot}^{\gamma\gamma}$	1.017		1.018		1.016	

Table 1: The energy dependence of σ_{el}/σ_{tot} and σ_{tot} for the three reactions nn, γp and $\gamma\gamma$, and the energy dependence of the ratios $\sigma_{tot}^{nn}/\sigma_{tot}^{\gamma p}$ and $\sigma_{tot}^{\gamma p}/\sigma_{tot}^{\gamma\gamma}$.

Toy eikonal $\chi=\sigma W$, for n-n, γ–p and γ–γ scattering

where $\sigma_{\rm tot} \;=\; 2\int \left[1 - e^{-\sigma_0 W(b)}\right] d^2\vec{b}$



Survival Probability of Large Rapidity Gaps in nucleon-nucleon, γp and $\gamma\gamma$ Collisions

M.M. Block, Northwestern University F. Halzen, Wisconsin University

Francis, at the table

EIKONAL FORMULATION

$$a(b,s) = \frac{i}{2}\left(1 - e^{i\chi(b,s)}\right)$$
$$= \frac{i}{2}\left(1 - e^{-\chi_{\rm I}(b,s)+i\chi_{\rm R}(b,s)}\right),$$

$$\sigma_{\rm tot}(s) = 2\int \left[1 - e^{-\chi_{\rm I}(b,s)}\cos(\chi_{\rm R}(b,s))\right] d^2\vec{b},$$

$$\sigma_{\rm elastic}(s) = \int \left|1 - e^{-\chi_{\rm I}(b,s)+i\chi_{\rm R}(b,s)}\right|^2 d^2\vec{b},$$

$$\sigma_{\rm inelastic}(s) = \sigma_{\rm tot}(s) - \sigma_{\rm elastic}(s),$$

So,

$$\boxed{\sigma_{\rm inelastic}(s) = \int \left[1 - e^{-2\chi_{\rm I}(b,s)}\right] d^2\vec{b}.}$$

Hence, $e^{-2\chi_{\rm I}(b,s)}$ is the probability in impact parameter space of *no* inelastic interaction.

Survival Probability, $< |S|^2 >$

$W(b, \mu_{qq})$, the normalized probability distribution in impact parameter space b for quarks, is the Fourier transform of a dipole form factor and is given by

$$W(b, \mu_{qq}) = \frac{\mu_{qq}^2}{96\pi}(\mu_{qq} b)^3 \, K_3(\mu_{qq} b),$$

where $\mu_{qq} = 0.901 \pm 0.005$ GeV.

Thus, $< |S|^2 >$, the survival probability of a large rapidity gap, is given by

$$< |S|^2 > = \int W(b\,; \mu_{qq}) e^{-2\chi_I(s,b)} d^2\vec{b}.$$

Note that the energy dependence of the survival probability $< |S|^2 >$ is through the energy dependence of the eikonal χ_I.

To Prove:
Survival is process-independent, i.e.,
$\langle|S|^2\rangle_{nn} = \langle|S|^2\rangle_{\gamma p} = \langle|S|^2\rangle_{\gamma\gamma}$

$$\chi^{\text{even}}(s,b) = i\left[\sigma_{gg}(s)W(b;\mu_{gg}) + \sigma_{qg}(s)W(b;\mu_{qg}) + \sigma_{qq}(s)W(b;\mu_{qq})\right]$$

$$\chi^{\gamma p}(s,b) = i\left[\tfrac{2}{3}\sigma_{gg}(s)W(b;\sqrt{\tfrac{3}{2}}\mu_{gg}) + \tfrac{2}{3}\sigma_{qg}(s)W(b;\sqrt{\tfrac{3}{2}}\mu_{qg}) + \tfrac{2}{3}\sigma_{qq}(s)W(b;\sqrt{\tfrac{3}{2}}\mu_{qq})\right],$$

$$\chi^{\gamma\gamma}(s,b) = i\left[\tfrac{4}{9}\sigma_{gg}(s)W(b;\tfrac{3}{2}\mu_{gg}) + \tfrac{4}{9}\sigma_{qg}(s)W(b;\tfrac{3}{2}\mu_{qg}) + \tfrac{4}{9}\sigma_{qq}(s)W(b;\tfrac{3}{2}\mu_{qq})\right],$$

$$
\begin{aligned}
< |S^{nn}|^2 > \;=\; & \frac{1}{96\pi}\int x_n^3 K_3(x_n)\times \\
& \exp - \Big[\frac{1}{48\pi}\sigma_{gg}\mu_{gg}^2\left(\frac{\mu_{gg}}{\mu_{qq}}x_n\right)^3 K_3\left(\frac{\mu_{gg}}{\mu_{qq}}x_n\right) \\
& \quad + \sigma_{qg}\mu_{qg}^2\left(\frac{\mu_{qg}}{\mu_{qq}}x_n\right)^3 K_3\left(\frac{\mu_{qg}}{\mu_{qq}}x_n\right) \\
& \quad + \sigma_{qq}\mu_{qq}^2 (x_n)^3 K_3(x_n)\Big]\, d^2\vec{x_n}, \qquad x_n = \mu_{qq} b
\end{aligned}
$$

$$
\begin{aligned}
< |S^{\gamma p}|^2 > \;=\; & \frac{1}{96\pi}\int x_q^3 K_3(x_q)\times \\
& \exp - \Big[\frac{1}{48\pi}\sigma_{gg}\mu_{gg}^2\left(\frac{\mu_{gg}}{\mu_{qq}}x_q\right)^3 K_3\left(\frac{\mu_{gg}}{\mu_{qq}}x_q\right) \\
& \quad + \sigma_{qg}\mu_{qg}^2\left(\frac{\mu_{qg}}{\mu_{qq}}x_q\right)^3 K_3\left(\frac{\mu_{qg}}{\mu_{qq}}x_q\right) \\
& \quad + \sigma_{qq}\mu_{qq}^2 (x_q)^3 K_3(x_q)\Big]\, d^2\vec{x_q}, \quad x_q = \sqrt{\tfrac{3}{2}}\mu_{qq} b
\end{aligned}
$$

$$
\begin{aligned}
< |S^{\gamma\gamma}|^2 > \;=\; & \frac{1}{96\pi}\int x_g^3 K_3(x_g)\times \\
& \exp - \Big[\frac{1}{48\pi}\sigma_{gg}\mu_{gg}^2\left(\frac{\mu_{gg}}{\mu_{qq}}x_g\right)^3 K_3\left(\frac{\mu_{gg}}{\mu_{qq}}x_g\right) \\
& \quad + \sigma_{qg}\mu_{qg}^2\left(\frac{\mu_{qg}}{\mu_{qq}}x_g\right)^3 K_3\left(\frac{\mu_{qg}}{\mu_{qq}}x_g\right) \\
& \quad + \sigma_{qq}\mu_{qq}^2 (x_g)^3 K_3(x_g)\Big]\, d^2\vec{x_g}, \quad x_g = \tfrac{3}{2}\mu_{qq} b
\end{aligned}
$$

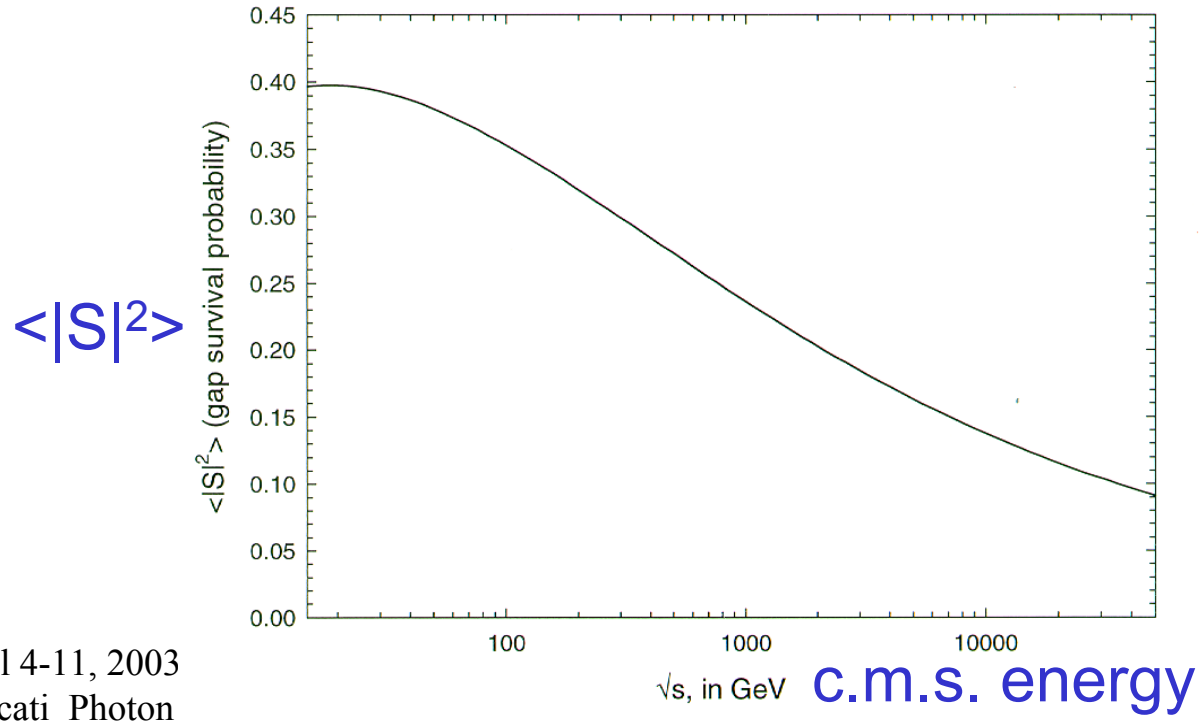
Survival is process-independent !

$\therefore <|S|^2>_{nn} = <|S|^2>_{\gamma p} = <|S|^2>_{\gamma\gamma}$

Gap Survival Probability for pbar-p and pp Collisions

C.M.S. Energy (GeV)	Survival Probability($\bar{p}p$), in %	Survival Probability(pp), in %
63	37.0 ± 0.9	37.5 ± 0.9
546	26.7 ± 0.5	26.8 ± 0.5
630	26.0 ± 0.5	26.0 ± 0.5
1800	20.8 ± 0.3	20.8 ± 0.3
14000	12.6 ± 0.06	12.6 ± 0.06
40000	9.7 ± 0.07	9.7 ± 0.07

Gap Survival (%) for nucleon-nucleon, γp and $\gamma\gamma$ Collisions



Consequences of the Factorization Hypothesis in nucleon-nucleon, γ-p and γ–γ Collisions,

M. M. Block, Northwestern University

A. B. Kaidalov, Moscow University

Survival Probability of Large Rapidity Gaps in nucleon-nucleon, γp and $\gamma\gamma$ Collisions

M.M. Block, Northwestern University

F. Halzen, Wisconsin University

Francis, at the table