Test of non-commutative QED

in $e^+e^- \rightarrow \gamma\gamma$ at LEP

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- What is NCQED?
- NCQED in $e^+e^- \rightarrow \gamma\gamma$
- What we (don’t) see in the OPAL data
Non-commutative geometry

\[ [x_\mu, x_\nu] = i \theta_{\mu \nu} \]

- \( \theta_{\mu \nu} \) : antisymmetric, constant, frame independent \( \sim \text{length}^2 = 1/\text{energy}^2 \sim 1/ \Lambda^2 \).
  Analog of Planck constant \( h \) in ordinary Quantum Mechanics. \( [x_\mu, p_\nu] = i \hbar \delta_{\mu \nu} \)

- In string theory, noncommutative geometry may arise through quantisation of string in the presence of background fields. (Connes, Douglas, Schwarz, Seiberg, Witten,..)

- \( \Lambda \) is perhaps \( \sim \) the Planck scale.

- \( \Lambda \) might be at TeV scale \( \Leftarrow \) large extra dimension, D-brane, ... : TeV scale gravity

- Possible experimental signatures ?
Non-commutative QED

Non-commutative quantum field theory is not well known.

Non-commutative QED exists (NCQED). Renormalizable, U(1) gauge symmetry, ...

- **eeγ vertex contains a kinematic phase:** $e^{\frac{i}{2}p^\mu \theta_{\mu \nu} p^\nu}$
  Dependence on momenta and $\theta_{\mu \nu}$.
  $\rightarrow$ Unique direction.
  Violation of Lorentz invariance.

- Nonabelian-like $3\gamma, 4\gamma$ self couplings,
  also dependent on $p^\mu \theta_{\mu \nu} p^\nu$

- Relevant high energy processes:
  $\gamma e \rightarrow \gamma e, \gamma \gamma \rightarrow \gamma \gamma, \ e^+ e^- \rightarrow \gamma \gamma, .....

- Low energy experiments:
  limits from Lamb shift, Aharonov-Bohm effect, clock comparisons, ...
$e^+e^- \rightarrow \gamma\gamma$ in NCQED

e$^+e^-$ → $\gamma\gamma$ is sensitive only to $\theta_{0i} = \frac{1}{\Lambda_{NC}^2} c_{0i} \ (i = 1, 3)$

space-time noncommutativity.

c$^{0i}$ : Unit space vector pointing to the ‘unique direction’

$\Lambda_{NC}$ : Energy scale of non-commutativity

Differential cross-section for $e^+e^- \rightarrow \gamma\gamma$ (in the c.m. frame of $e^+e^-$ collision) :

$$\frac{d\sigma}{d\cos \theta d\phi} = \frac{\alpha^2}{s} \frac{1 + \cos^2 \theta}{1 - \cos^2 \theta} (1 - \sin^2 \theta \sin^2 \Delta_{NC})$$

$$\Delta_{NC} = \frac{s}{4\Lambda_{NC}^2} (c_{01} \sin \theta \cos \phi + c_{02} \sin \theta \sin \phi + c_{03} \cos \theta)$$

Dependence on $\phi$ as well as $\theta$. 

The unit vector $c_E$ is perhaps sitting on some large scale structure in space e.g. the rest frame of the cosmic microwave background.

Experiment on the earth is changing its orientation as the earth rotates.
Transformation of coordinate system

\[
c_E = \begin{pmatrix}
  s_\alpha s_\delta \\
  c_\delta \\
  -c_\alpha s_\delta
\end{pmatrix}
s_\eta \cdot \cos(\zeta - \xi)
\]

+ \begin{pmatrix}
  c_\alpha \\
  0 \\
  s_\alpha
\end{pmatrix}
s_\eta \cdot \sin(\zeta - \xi)

+ \begin{pmatrix}
  -s_\alpha c_\delta \\
  s_\delta \\
  c_\alpha c_\delta
\end{pmatrix}
c_\eta

\zeta = \omega t, \quad \omega = \frac{2\pi}{T_{sd}} \quad (T_{sd} = \text{sidereal day})

Two components of \(c_E\): constant term and terms varying with time.

when \(\eta=0\) (parallel to the rotation axis of the earth), \(c_E\) is stationary.
Location and orientation of OPAL experiment

\[ \delta = 46.29^\circ \]
\[ \alpha = 33.60^\circ \]
OPAL data

**OPAL** $e^+e^- \rightarrow \gamma\gamma$ sample

- $\sqrt{s} = 181 - 209$ GeV
- Collected in 1997-2000
- $\mathcal{L} = 672$ pb$^{-1}$
- 5235 events $|\cos \theta| < 0.93$

\[
\frac{d\sigma}{d \cos \theta d\phi}(\Lambda, \eta, \xi; \zeta = \omega t)
\]

Consider 3 cross-sections

- $\frac{d\sigma}{d \cos \theta}$: $\phi$ integrated, time averaged
- $\frac{d\sigma}{d\phi}$: $\cos \theta$ integrated, time averaged
- $\sigma(t)$: $\cos \theta$ integrated (0.0-0.6), $\phi$ integrated
Time averaged, $\phi$ integrated $\cos \theta$ distribution

Accidentally, for the OPAL orientation, Very small $\eta$ dependence
Depends $\approx$ only on $\Lambda_{NC}$

$\sqrt{s} = 200$ GeV, $\Lambda_{NC} = 200$ GeV

$\Lambda_{NC} = 141$ GeV

$\Lambda_{NC} > 141$ GeV at 95% CL
Independent of $\eta$
Time averaged, $\cos \theta$ integrated $\phi$ distribution

$\phi$ structure, depending on $\eta$
Amplitude depends on $\Lambda$

$\eta = 0^\circ$, $\Lambda_{NC} = 167$ GeV
$\eta = 90^\circ$, $\Lambda_{NC} = 152$ GeV
$\eta = 55^\circ$, $\Lambda_{NC} = 120$ GeV

$e^+ e^- \rightarrow \gamma \gamma$, $\cos \theta < 0.6$

$e^+ e^- \rightarrow \gamma \gamma$ excluded
Sidereal daily structure of cross-section

If observed ⇒ infer $\xi$

Observed cross-section vs $(\zeta - \xi)$ (time):

$\chi^2$/dof = 39.9/30
for constant QED cross-section
Conclusion

- Studied NCQED in $e^+e^- \rightarrow \gamma\gamma$ with OPAL data at LEP
- Taking into account earth rotation. The unique direction specified by $\eta$ and $\xi$.
- Limit on $\Lambda_{NC} > 141$ GeV at 95% CL, independent of $\eta$ from $\frac{d\sigma}{d\cos\theta}$.
- $\eta$ dependent limit on $\Lambda_{NC}$ from $\frac{d\sigma}{d\phi}$. $\Lambda_{NC} > 167$ GeV at $\eta = 0^\circ$.
- The first limit on NCQED from $e^+e^-$ collider. (CERN-EP-2003-010, hep-ex/0303035)
- Study at higher energy linear collider $\Rightarrow \Lambda_{NC}$ to TeV.