

# Theoretical determination of $\eta_b$ 's electromagnetic decay width

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More recently – end of 2002 – **CDF in Run 1** has done a search for  $\eta_b$  in the process

$$\eta_b \rightarrow J/\psi J/\psi$$

with 7 events where 1.8 events are expected from background.

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**First order QCD** correction:

$$\Gamma(\eta_b \rightarrow \gamma\gamma) = \Gamma_B^P \left[ 1 + \frac{\alpha_s}{\pi} \left( \frac{\pi^2 - 20}{3} \right) \right] \approx \Gamma_B^P (1 - \alpha_s)$$

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Compare to the  $\Upsilon$  state: *Born level*

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Assuming the wavefunctions are the same in both cases  $\Rightarrow \mathcal{O}(\alpha_s/m_b^2)$  error.

# Relation to $\Upsilon$ width

Expanding in  $\alpha_s$ :

$$\frac{\Gamma(\eta_c \rightarrow \gamma\gamma)}{\Gamma(\Upsilon \rightarrow e^+e^-)} \approx \frac{1(1 - 3.38\alpha_s/\pi)}{3(1 - 5.34\alpha_s/\pi)} = \frac{1}{3} \left[ 1 + 1.96 \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right]$$

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Two-loop expression for  $\alpha_s$  :

$$\alpha_s^{(2)}(Q) = \frac{4\pi}{b_0 f(Q)} \left\{ 1 - \frac{b_1 \log[f(Q)]}{b_0^2 f(Q)} \right\}$$

$$b_0 = (33 - 2N_f)/3, b_1 = (306 - 38N_f)/3,$$

$$f(Q) = \log \left[ (Q/\Lambda_{\overline{MS}})^2 \right].$$

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Combining the formulæ  $\Rightarrow$

$$\Gamma(\eta_b \rightarrow \gamma\gamma) \pm \Delta\Gamma(\eta_b \rightarrow \gamma\gamma) = 489 \underbrace{\pm 19}_{\Upsilon \text{ error}} \overbrace{\pm 2}^{\alpha_s \text{ error}} \text{ eV}$$

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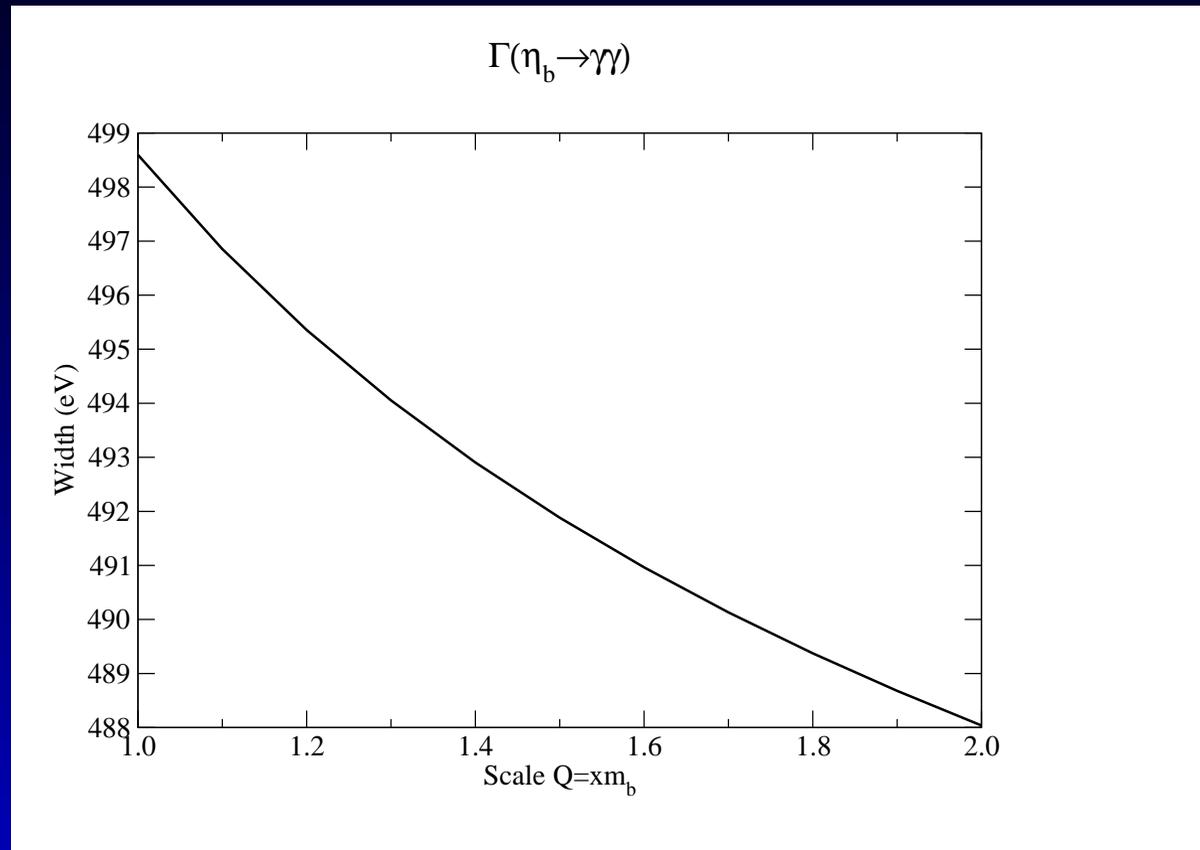
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Here we assumed the  $\alpha_s$  scale to be  $Q = 2m_b = 10.0 \text{ GeV}$ . In the following plot we shall see the effect of a different choice for the  $\alpha_s$  scale.

# $\eta_b \rightarrow \gamma\gamma$ width



*$\eta_b$  decay width to  $\gamma\gamma$  in eV, with respect to the scale chosen for  $\alpha_s$ . The fluctuation is of order 2%.*

# Potential Models

We present now results for  $\eta_b \rightarrow \gamma\gamma$  from the potential models. We have used:

- Cornell type potential:

$$V(r) = -\frac{k}{r} + \frac{r}{a^2}$$

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- Rosner's group potential:

$$V(r) = \frac{\lambda}{\alpha} \left[ \left( \frac{r}{r_0} \right)^\alpha - 1 \right] + C .$$

with  $\alpha = -0.14$ ,  $\lambda = 0.808$ ,  $C = -1.305 \text{ GeV}$ ,  $r_0 = 1 \text{ GeV}^{-1}$

# Potential Models

- Igi–Ono potential:

$$V_J(r) = V_{AR}(r) + d r e^{-gr} + ar, \quad V_{AR}(r) = -\frac{4}{3} \frac{\alpha_s^{(2)}(r)}{r}$$

$$\Lambda_{\overline{MS}} = 0.5 \text{ GeV and } \Lambda_{\overline{MS}} = 0.3 \text{ GeV}$$

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- Coulomb potential

$$V(r) = -\frac{4}{3} \frac{\alpha_s}{r}$$

where  $\alpha_s$  has been computed at a fixed scale.

# Potential Models

The potential of the Coulombic model can be written as:

$$V(r) = -\frac{16\pi}{3\beta_0 r \log 1/(\Lambda_{\overline{MS}} r)^2} \times \left[ 1 - \frac{\beta_1 \log \log 1/(\Lambda_{\overline{MS}} r)^2}{\beta_0^2 \log 1/(\Lambda_{\overline{MS}} r)^2} + \frac{(\frac{31}{3} - \frac{10}{9}n_f) / \beta_0 + 2\gamma_E}{\log 1/(\Lambda_{\overline{MS}} r)^2} \right]$$

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$\alpha_s$  is calculated at a fixed scale  $r_B$ , the Bohr radius:

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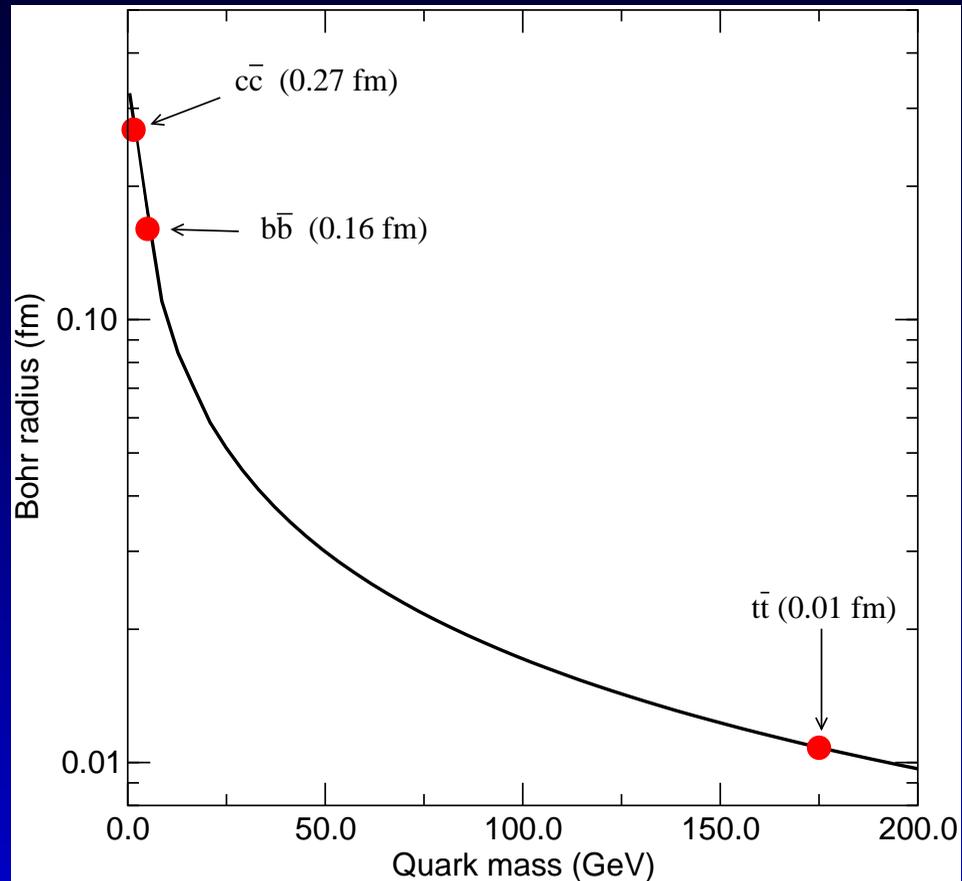
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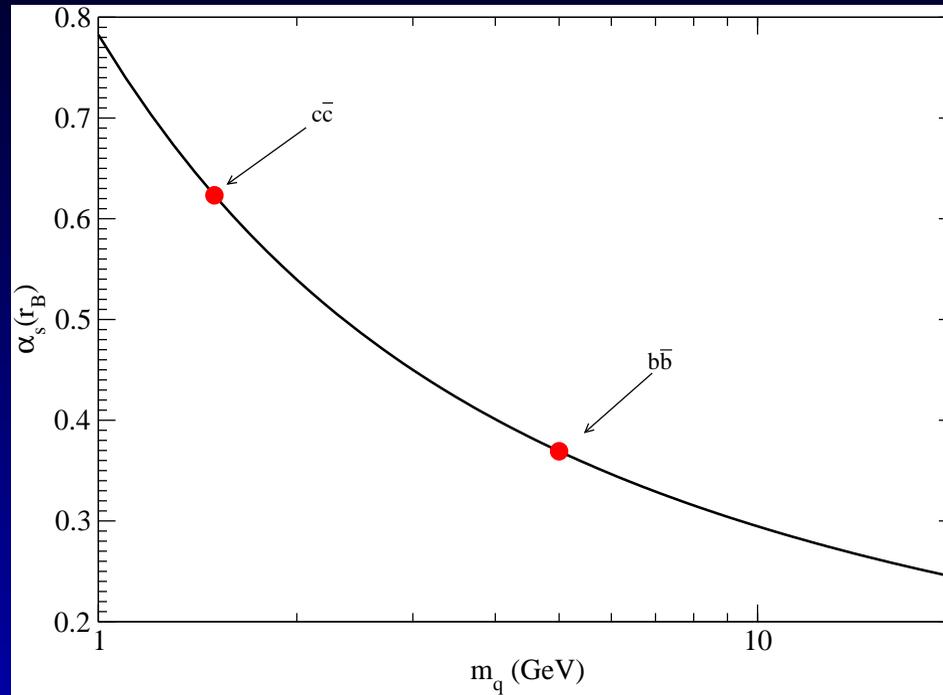
This model proved to be **very effective** for heavy quark systems, where  $r_B$  is small.

# $r_B$ as a function of $m_q$



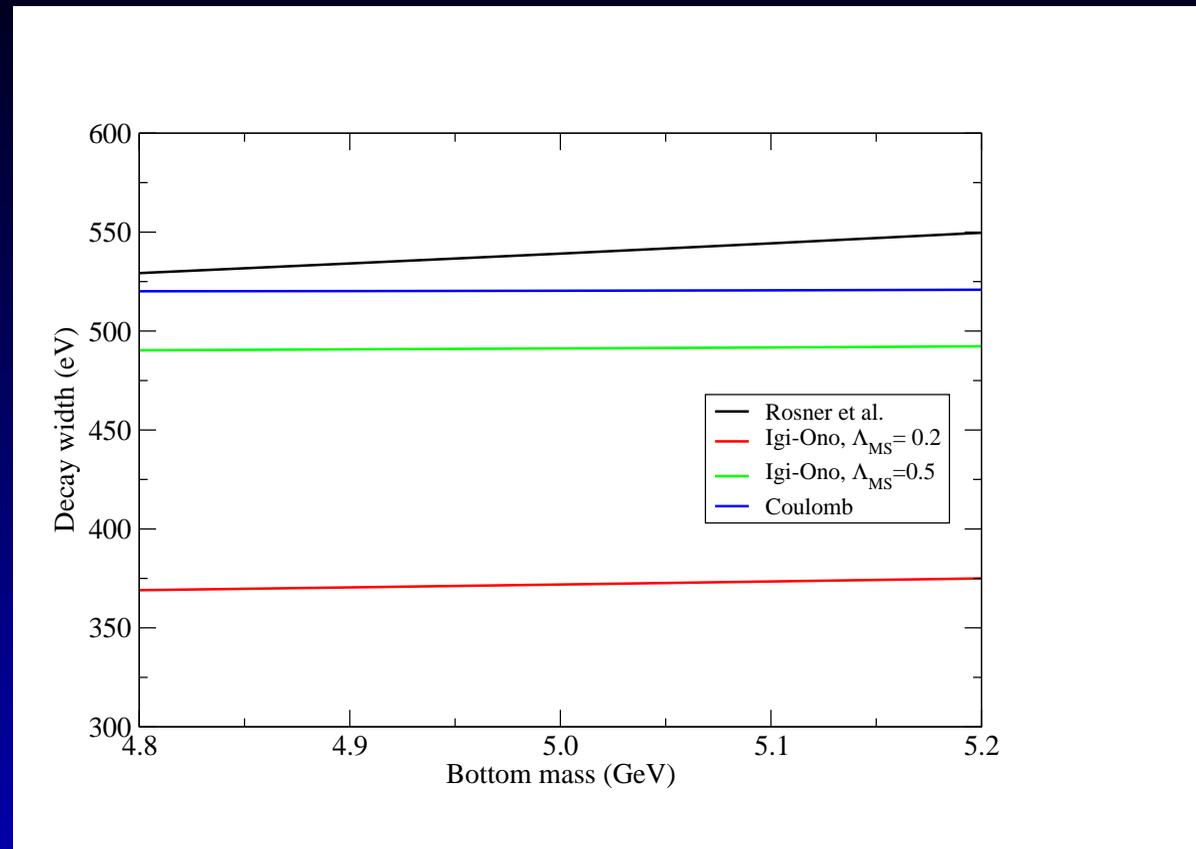
*Bohr radius as a function of the quark mass. The radii of the mesonic bound states are enhanced.*

# $\alpha_s(r_B)$ as a function of $m_q$



*Coulombic model potential with  $\alpha_s$  evaluated at Bohr radius as a function of the quark mass. We have enhanced the  $\alpha_s$  values of the mesonic bound states.*

# $\eta_b \rightarrow \gamma\gamma$ from different potential models



$\eta_b$  decay width to  $\gamma\gamma$  in eV, evaluated from potential models, as a function of the bottom mass. First order radiative correction in  $\alpha_s$  has been evaluated at  $Q = 2m_b$

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Error sources in calculation:

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$\eta_b \rightarrow \gamma\gamma$  potential models prediction gives a range of values:

$$\Gamma(\eta_b \rightarrow \gamma\gamma) = 466 \pm 101 \text{ eV}$$

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NRQCD has been used to separate the short distance scale of annihilation from the nonperturbative contributions of long distance scale.

This approach has been successfully used to explain the larger than expected  $J/\psi$  production at the Tevatron.

According to BBL, in the octet model for quarkonium, the decay widths of bottomonium states are given by:

# The BBL Procedure

$$\Gamma(\Upsilon \rightarrow LH) = \frac{2\langle \Upsilon | O_1(^3S_1) | \Upsilon \rangle}{m_b^2} \left( \frac{10}{243} \pi^2 - \frac{10}{27} \right) \alpha_s^3 \times [1 - (1.161n_f + 0.223) \frac{\alpha_s}{\pi}]$$

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$\Rightarrow$  **Only two unknown factors**,  $G_1$  and  $F_1$ .

We obtain a system for the  $\Upsilon$  decay widths:

$$\begin{cases} \Gamma(\Upsilon \rightarrow LH) &= G_1 h_1(\alpha_s, m_b) + F_1 h_2(\alpha_s, m_b) \\ \Gamma(\Upsilon \rightarrow e^+ e^-) &= G_1 h_3(\alpha_s, m_b) + F_1 h_4(\alpha_s, m_b) \end{cases}$$

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This result is used to compute the  $\eta_b$  decay widths.

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$$\Gamma(\eta_b \rightarrow \gamma\gamma) = 364 \underbrace{\pm 13}_{\gamma \text{ error}} \underbrace{\pm 8}_{\alpha_s \text{ error}} \text{ eV}$$

and

$$\Gamma(\eta_b \rightarrow LH) = 57.9 \underbrace{\pm 2.8}_{\gamma \text{ error}} \underbrace{\pm 4.6}_{\alpha_s \text{ error}} \text{ keV}$$

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We present also results from the **lattice calculation** of the long distance terms for the BBL approach:

$$\Gamma(\eta_b \rightarrow \gamma\gamma) = 364 \pm 104 \text{ eV}$$

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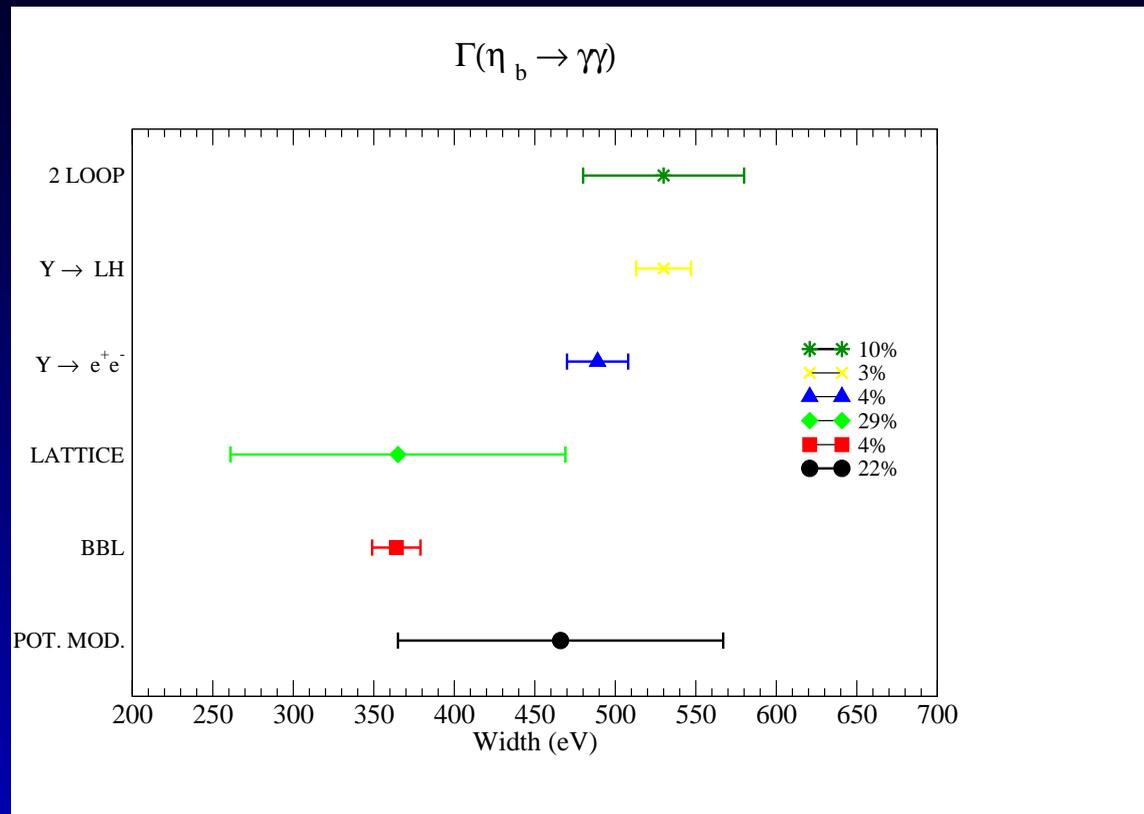
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# $\eta_b \rightarrow \gamma\gamma$ Decay Width



*Potential Models result; BBL approach with input from  $\Upsilon$  decay data; Lattice evaluation of  $G_1$  and  $F_1$  factors; Singlet picture with  $G_1$  obtained from  $\Upsilon \rightarrow e^+e^-$  and  $\Upsilon \rightarrow LH$  processes respectively. The last point refers to a  $\mathcal{O}(\alpha_s^2)$  correction to the decay rate.*

# Conclusions

- The  $\Gamma(\eta_b \rightarrow \gamma\gamma)$  decay width **prediction of potential models** considered gives the value:

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- This result is in **agreement with the naive estimate** from the  $\Upsilon$  decay
- Prediction of the **BBL procedure** are **consistent with the potential model results**, for both the long distance terms  $G_1$  and  $F_1$  extracted from the  $\Upsilon$  experimental decay widths and the one evaluated from lattice calculations.
- The results from the **singlet picture** are also **consistent** with the potential model results.