On two-photon production of two $\rho$-mesons

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in collaboration with

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The new kinds of partonic distributions (GPD) and distribution amplitudes (GDA) are arisen from

\[ h = \{ N, \pi, \rho, \ldots \} ; \quad \mu \nu \Rightarrow Q^2 \to \infty ; \quad \mu \nu \Rightarrow q^2 \to 0 \]

I) \( h = N \Rightarrow \) a good subject from experimental point of view, the theoretical investigations are related to more complicated calculations, than for meson case, due to spin \( \frac{1}{2} \)

II) \( h = \pi, \rho, \ldots \Rightarrow \) the theoretical part is attracted thanks to relative simplicity. But the problems of experimental character certainly exist owing to the absence of \( \pi \)-target.

1) The probing the meson structure is available from both exper and theor. points of view.

- exp. \( \gamma^* \to \pi \pi \) CLEO Coll., J. Gershon et al., PRD 57, 33 (1998)
- theor. \( \gamma^* \to \pi \pi \) twist-2 and twist-3 have been considered in M. Diehl et al., PRL 81, 1782 (1998)
  N. Kivel & L. Mankiewicz, PRD 63, 054017
New (prelim.) experimental data have been obtained by L3 Collab. at LEP for $e^+e^- \rightarrow e^+e^- \pi^+\pi^0$ process (to be published in Phys. Lett. B through talk of S. Nesterov).

We focus on $Q^2$-dependence of $d\sigma_{e\bar{e}}/dQ^2$:

- exp.: $\frac{d\sigma_{e\bar{e}}}{dQ^2} \sim \frac{1}{Q^6}$

- theory:

\[ d\sigma_{e\bar{e}} \sim \frac{1}{Q^2} \left| T(\gamma^*\gamma \rightarrow 2\pi^0) \right|^2 \]

\[ \left| T(\gamma^*\gamma \rightarrow 2\pi^0) \right|^2 \Rightarrow Q^0 (tw \sim 2) \]

Possible treatment: the data probe the partonic picture of the composite structure of hadrons — scaling.
The reaction which we study is

\[ e(k) + e(l) \rightarrow e(k') + e(l') + p^0(p_1) + p^0(p_2) \]

and the subprocess of this reaction is

\[ e(k) + \gamma(q') \rightarrow e(k') + p^0(p_1) + p^0(p_2) \]

where

\[ q^2 = -Q^2 \text{ is very large} \]

\[ q^2 \approx 0 \]
Within the $\gamma^*\gamma$ center of mass frame we write

$$q = (q_0, \vec{q}_r, q_\perp), \quad p_1 = (p_1^0, \vec{p}_1 \sin \theta, 0, \vec{p}_1 \cos \theta)$$

and $k_0 = E_1, \ p_0 = E_2$ for the initial lepton energies; the Mandelstam variables are written as

$$S_{ee} = (k+e)^2 \approx 2kE, \quad S_{e\gamma} = (k+q')^2 \approx 2kq' = x_2 S_{ee}$$

where the fraction $x_2$ is defined as $q'_0 = x_2 p_0$ is introduced.
Basis of light-cone vectors

\[ p^2 = \ell^2 = 0, \quad \ell^+ = p^+ = 0, \quad \ell n = 1 \]

\[ \text{dim}_M [\ell] = +1, \quad \text{dim}_M [n] = -1 \]

With the help of this basis, the meson momenta are presented as:

\[ p_1^\mu = \frac{1+\xi}{2} p^\mu + \frac{1-\xi}{2} \frac{W^2}{\ell^2} \ell^\mu - \Delta_T^\mu \]

\[ p_2^\mu = \frac{1-\xi}{2} p^\mu + \frac{1+\xi}{2} \frac{W^2}{\ell^2} \ell^\mu + \Delta_T^\mu \]

As usually for the DA kinematics, we introduce

\[ \Delta^\mu = p_2^\mu - p_1^\mu, \quad \Delta_T^\mu = p_2^\mu + p_1^\mu \]

The skewness parameter \( \xi \) is defined by

\[ \Delta n = -\xi \ell n \]

or

\[ \frac{1+\xi}{2} = \frac{P_1^+}{P^+} = \frac{1+\beta \cos \Theta}{2}, \quad \beta = \sqrt{1 - \frac{4m^2}{W^2}} \]

Note also that the transverse momentum \( \Delta_T = (0, \vec{a}_T, 0) \) is given by

\[ \Delta_T^2 = -\vec{a}_T^2 = t + (W^2 - 4m^2) \cos^2 \Theta \]
Leading twist (twist-2) accuracy.
Parameterization of relevant matrix elements

Keeping the terms of leading twist-2 contributions, the vector and axial correlators can be written as:

$$\langle p_1, \lambda_1; p_2, \lambda_2 | \overline{\psi}(0) \gamma_\mu \psi(x) | 0 \rangle = \int_0^1 dy \ e^{-i y \lambda}$$

$$P_\mu \sum_i e_i^a e_i^b \ V_{a\beta}^{(i)}(p_1, p_2, \lambda) \ H_{i}^{PP, \nu}(y, z, W^2)$$

$$\langle p_1, \lambda_1; p_2, \lambda_2 | \overline{\psi}(0) \gamma_5 \gamma_\mu \psi(x) | 0 \rangle = \int_0^1 dy \ e^{-i y \lambda}$$

$$P_\mu \sum_i e_i^a e_i^b \ A_{a\beta}^{(i)}(p_1, p_2, \lambda) \ H_{i}^{PP, \nu}(y, z, W^2)$$

Due to parity invariance

$$V_{a\beta}^{(i)} \rightarrow 5 \text{ tensor structures}$$

$$A_{a\beta}^{(i)} \rightarrow 4 \text{ tensor structures}$$
\[ \sum_{i} e_{1}^{a} e_{2}^{b} \left< \omega_{x}^{(i)} \left( p_{1}, p_{2}, n \right) \right| H_{i}^{p_{1} p_{2} n} \left( y \right) = \]

\[ = -\left( e_{1} \cdot e_{2} \right) H_{1}^{p_{1} p_{2} n} \left( y \right) + \frac{\left( e_{1} \cdot n \right) \left( e_{2} \cdot p \right) + \left( e_{1} \cdot p \right) \left( e_{2} \cdot n \right)}{p \cdot n} \left( H_{2}^{p_{1} p_{2} n} \left( y \right) \right) - \]

\[ - \frac{\left( e_{1} \cdot p \right) \left( e_{2} \cdot p \right)}{2m^{2}} \left( H_{3}^{p_{1} p_{2} n} \left( y \right) \right) + \frac{\left( e_{1} \cdot n \right) \left( e_{2} \cdot p \right) - \left( e_{1} \cdot p \right) \left( e_{2} \cdot n \right)}{p \cdot n} \left( H_{4}^{p_{1} p_{2} n} \left( y \right) \right) + \]

\[ + \left\{ \frac{4m^{2} \left( e_{1} \cdot n \right) \left( e_{2} \cdot n \right)}{(p \cdot n)^{2}} + \frac{1}{3} \left( e_{1} \cdot e_{2} \right) \right\} \left( H_{5}^{p_{1} p_{2} n} \left( y \right) \right) \]

for the vector tensor structures, and

\[ \sum_{i} e_{1}^{a} e_{2}^{b} \left< \omega_{x}^{(i)} \left( p_{1}, p_{2}, n \right) \right| H_{i}^{p_{1} p_{2} n} \left( y \right) = \]

\[ = i \left( e_{1} \cdot e_{2} \right) \frac{\left( H_{1}^{p_{1} p_{2} n} \left( y \right) \right)}{(p \cdot n)} + i \left( e_{1} \cdot e_{2} \right) \frac{\left( H_{1}^{p_{1} p_{2} n} \left( y \right) \right)}{(p \cdot n)} + i \left( e_{1} \cdot e_{2} \right) \frac{\left( H_{1}^{p_{1} p_{2} n} \left( y \right) \right)}{(p \cdot n)} + i \left( e_{1} \cdot e_{2} \right) \frac{\left( H_{1}^{p_{1} p_{2} n} \left( y \right) \right)}{(p \cdot n)} \]

for the axial tensor structures.

Here, \[ E_{x}^{T} \equiv E_{x}^{\mu} p_{\mu} n_{\nu} \]
Amplitude of $\gamma^* p \rightarrow p^0 p^0$ subprocess

Diagrams:

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[Diagram 1]
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Amplitude:

$$T_{\mu\nu} = \int \frac{dy}{y(1-y)} \left[ g_{\mu\nu} (1-2y) V(y, \cos \theta, W^2) + \right.$$  

$$\left. + \epsilon_\mu^\nu A(y, \cos \theta, W^2) \right]$$

where scalar and pseudo-scalar functions $V$ and $A$ denote the following:

$$V(y, \cos \theta, W^2) = \sum_i e_i^a e_2^\beta V^{(a)}_{\alpha \beta} H_{i \alpha \beta}^{\mu \nu} (y, \mu \cos \theta, W^2)$$

$$A(y, \cos \theta, W^2) = \sum_i e_i^a e_2^\beta A^{(a)}_{\alpha \beta} H_{i \alpha \beta}^{\mu \nu} (y, \mu \cos \theta, W^2)$$
The squared and polarization averaged functions $|V|^2$ and $|\Delta|^2$ which will appear in the differential cross-section read:

$$
|V|^2 = \mathcal{P}^{d_1 d_2} (p_1) \mathcal{P}^{d_2 d_1} (p_2) V_{d_1 \beta_1}^{(i)} H^{\text{ss},V}_{i} V_{d_2 \beta_2}^{(j)} H^{\text{ss},V}_{j},
$$

$$
|\Delta|^2 = \mathcal{P}^{d_1 d_2} (p_1) \mathcal{P}^{d_2 d_1} (p_2) A_{d_1 \beta_1}^{(i)} H^{\text{ss},A}_{i} A_{d_2 \beta_2}^{(j)} H^{\text{ss},A}_{j},
$$

where

$$
P_{\alpha \beta} (p) \overset{\text{def}}{=} \sum_{\lambda} e_{\alpha}^{(\lambda)} e_{\beta}^{*(\lambda)} =
$$

$$
= -g_{\alpha \beta} + \frac{p_{\alpha} p_{\beta}}{m^2}
$$
Differential Cross-Section

Diagrams:

Amplitude:

\[ A(ee \rightarrow ee\gamma) = \sum_{i, j} \left[ J_\mu(e, e') \cdot \xi_\mu^{j(i)} \right] \frac{1}{q'^2} A_{ij} \cdot \frac{1}{q^2} \left[ \xi_\nu^{j(i)} \cdot J_\nu(k, k') \right] \]

Due to parity invariance there are only three independent helicity amplitudes:

\[ A(\rightarrow), \quad A(\rightarrow), \quad A(\rightarrow) \]

Square of modulus of amplitude reads:

\[ |A(ee \rightarrow ee\gamma)|^2 = |A(e\gamma \rightarrow e\gamma)|^2 \frac{1}{q'^4} |A(e \rightarrow e\gamma)|^2 \]
It is useful to introduce the integrated functions which take the forms:

\[
\begin{align*}
\mathcal{V}(\cos \theta, w^2) &= \int_0^1 dy \frac{1-2y}{y(1-y)} \mathcal{V}(y, \cos \theta, w^2) \\
\mathcal{A}(\cos \theta, w^2) &= \int_0^1 dy \frac{1}{y(1-y)} \mathcal{A}(y, \cos \theta, w^2)
\end{align*}
\]

In this case, the amplitude is expressed as

\[
T_{\mu \nu} = g_{\mu \nu} \mathcal{V}(\cos \theta, w^2) + \varepsilon_{\mu \nu}^{(i)} \mathcal{A}(\cos \theta, w^2)
\]

The next objects are the helicity amplitudes:

\[
A(i, j) = \frac{\varepsilon_{\mu}^{(i)}}{q_{\mu}} \frac{\varepsilon_{\nu}^{(j)}}{q_{\nu}} T_{\mu \nu}
\]

where

\[
\begin{align*}
\varepsilon_{\mu}^{(\pm)} &= \left( 0, \frac{\pm 1}{\sqrt{2}}, \frac{\pm i}{\sqrt{2}}, 0 \right) \\
\varepsilon_{\mu}^{(0)} &= \left( 0, \frac{1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}, 0 \right), \quad \varepsilon_{\mu}^{(\pm)} = \left( \frac{1}{\sqrt{q^2}}, 0, 0, \frac{q_{\mu}}{q^2} \right)
\end{align*}
\]

\[Q^2 = -q^2\]
Finally, we write for the cross-section:

\[
\frac{d\sigma(\text{ee+ee} \rightarrow \text{ee})}{dQ^2} = \frac{d^4}{16\pi^2} \int_{x_2} d^2x_2 \ F_{WW}(x_2) + \frac{1}{16x_2^2E_2^4Q^2} \int_{w_{\text{min}}}^{w_{\text{max}}} \ dw^2 \beta \ F_{(+,+)}(w^2) - \frac{1}{2x_2E_2^4Q^2} \int_{w_{\text{min}}}^{w_{\text{max}}} \ dw^2 \beta \ F_{(+,+)}(w^2) \frac{Q^2 + w^2}{Q^2 + Q^2 + w^2} + \frac{2}{Q^2} \int_{w_{\text{min}}}^{w_{\text{max}}} dw^2 \beta \ F_{(+,+)}(w^2) \frac{Q^2 + w^2}{(Q^2 + Q^2 + w^2)^2} \]

where \( F_{WW}(x_2) \) - Weizsacker-Williams distribution function, and

\[
F_{(+,+)}(w^2) = \int d(\cos\theta) 4 \left\{ |W(\cos\theta, w^2)|^2 + |A(\cos\theta, w^2)|^2 \right\}
\]
Figure 3: Cross-section $d\sigma/dQ^2[pb/GeV^2]$ in logarithmic scale. The theoretical cross-section is plotted for the best fitted parameters which are $C_1 = 1$, $W_1 = 2.9$ GeV and $W_2 = 1.2$ GeV

\[
\frac{d\sigma}{dQ^2}(ee\rightarrow ee2\pi^0) = \frac{\alpha^4}{16\pi^2} C_1 \int dx_2 F_{ww}(x_2)
\]

\[
\left\{ \frac{1}{16x_2 E_2^4 Q^2} - \frac{1}{2x_2 E_2^2 Q (Q^2 + <W_2^2>)} + \frac{2}{Q^2 (Q^2 + <W_2^2>)^2} \right\}
\]

The best fit correspond to

$C_1 = 1.0 \pm 0.12$ GeV$^2$ \hspace{0.5cm} $<W_2> = 1.2 \pm 0.08$ GeV

$<W_1> = 2.9 \pm 1.8$ GeV

$\chi^2$/d.o.f. = 0.25