Theoretical precision in estimates of

HADRONIC CONTRIBUTIONS to $(g-2)_\mu$ and $\alpha_{\text{QED}}(M_Z)$

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Outline of Talk:

① **Introduction**

② **Evaluation of $\alpha(M_Z)$**

③ **Evaluation of $a_\mu \equiv (g-2)_\mu/2$**

④ **What pseudo-observable do we need?**

⑤ **e^+e^-–cross sections via τ–decay spectral functions**

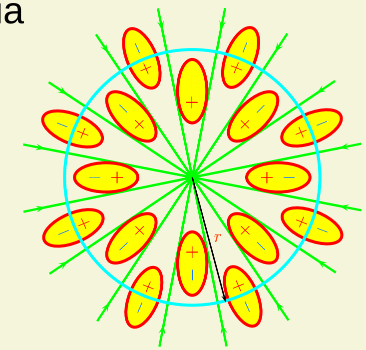
⑥ **Iso-spin breaking corrections in τ vs. e^+e^-**

⑦ **Status and Outlook**

① Introduction

Non-perturbative hadronic effects in electroweak precision observables, main effect via

effective finestructure “constant” $\alpha(E)$



(charge screening by vacuum polarization)

Of particular interest:

$$\alpha(M_Z) \text{ and } a_\mu \equiv (g-2)_\mu/2$$

- **electroweak effects (leptons etc.) calculable in perturbation theory**
- **strong interaction effects (hadrons/quarks etc.) perturbation theory fails**

$\Longrightarrow$ **Dispersion integrals over e^+e^-–data**

encoded in

$$R_\gamma(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

Errors of data $\Longrightarrow$ theoretical uncertainties !!!

The art of getting precise results from non-precision measurements !

New challenge for precision experiments on $\sigma(e^+e^- \to hadrons)$ KLOE, BABAR, ...

(S. Müller’s next talk)

Experimental:

$$R^{\exp}(s) = \frac{N_{\rm had}\,(1+\delta_{\rm RC})}{N_{\rm norm}\,\varepsilon}\,\frac{\sigma_{\rm norm}(s)}{\sigma_{\mu\mu,\,0}(s)}$$

$N_{\rm had}$	**number of observed hadronic events**
$N_{\rm norm}$	**number of observed normalizing events**
ε	**efficiency-acceptance product of hadronic events**
$\delta_{\rm RC}$	**radiative corrections to hadron production**
$\sigma_{\rm norm}(s)$	**physical cross section for normalizing events***
$\sigma_{\mu\mu,\,0}(s)$	$= 4\pi\alpha^2/3s$ **normalization**

* **including all radiative corrections integrated over the acceptance used for the luminosity measurement**

❊ **requires precise theory knowledge!**

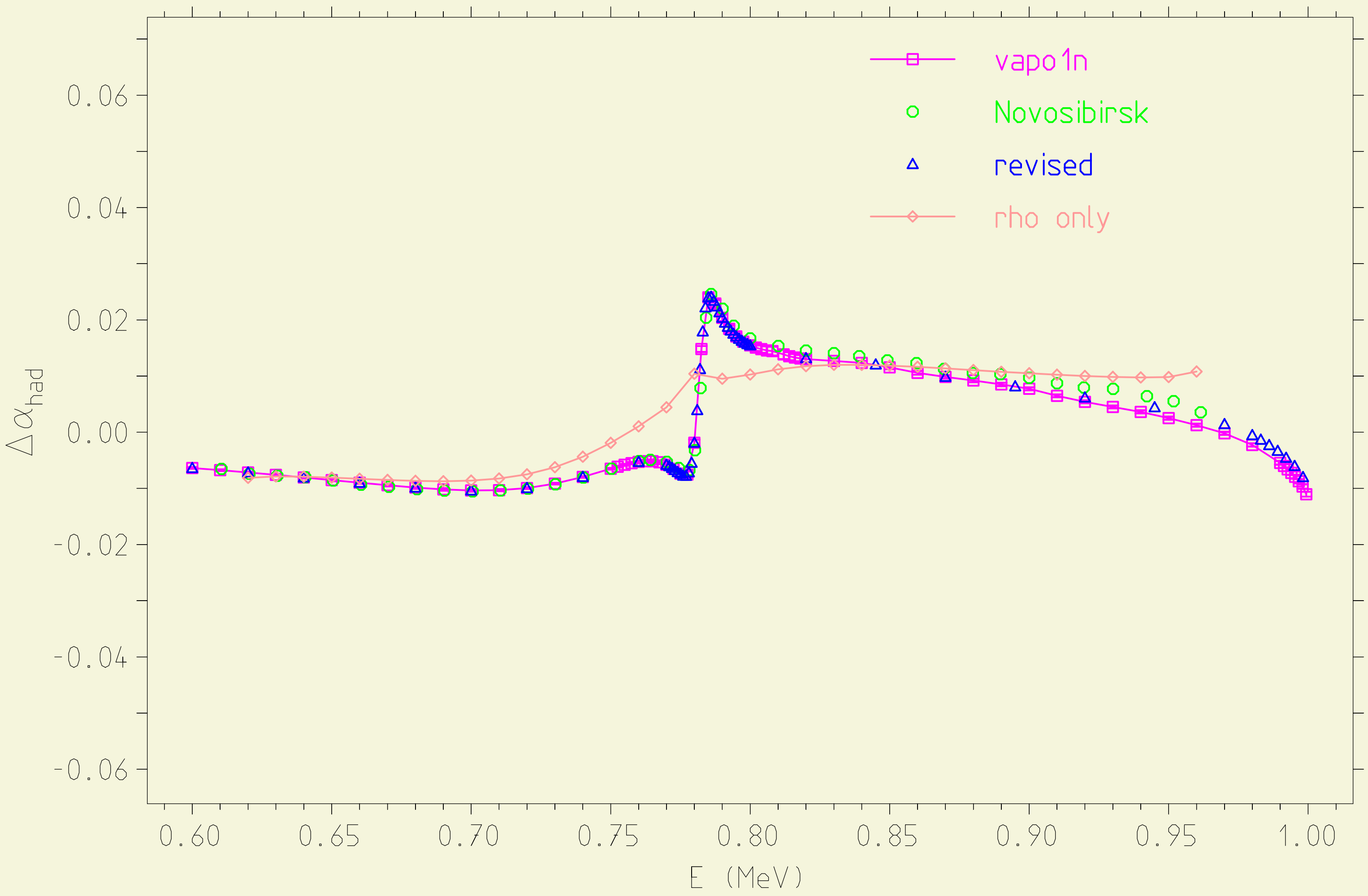
Normalization: **mostly by Bhabha [or $\mu\mu$ itself]**

In general: $\alpha(\mu)$ **enters with different scales μ in “had” and “norm”**

Recent progress (and a step back?) :

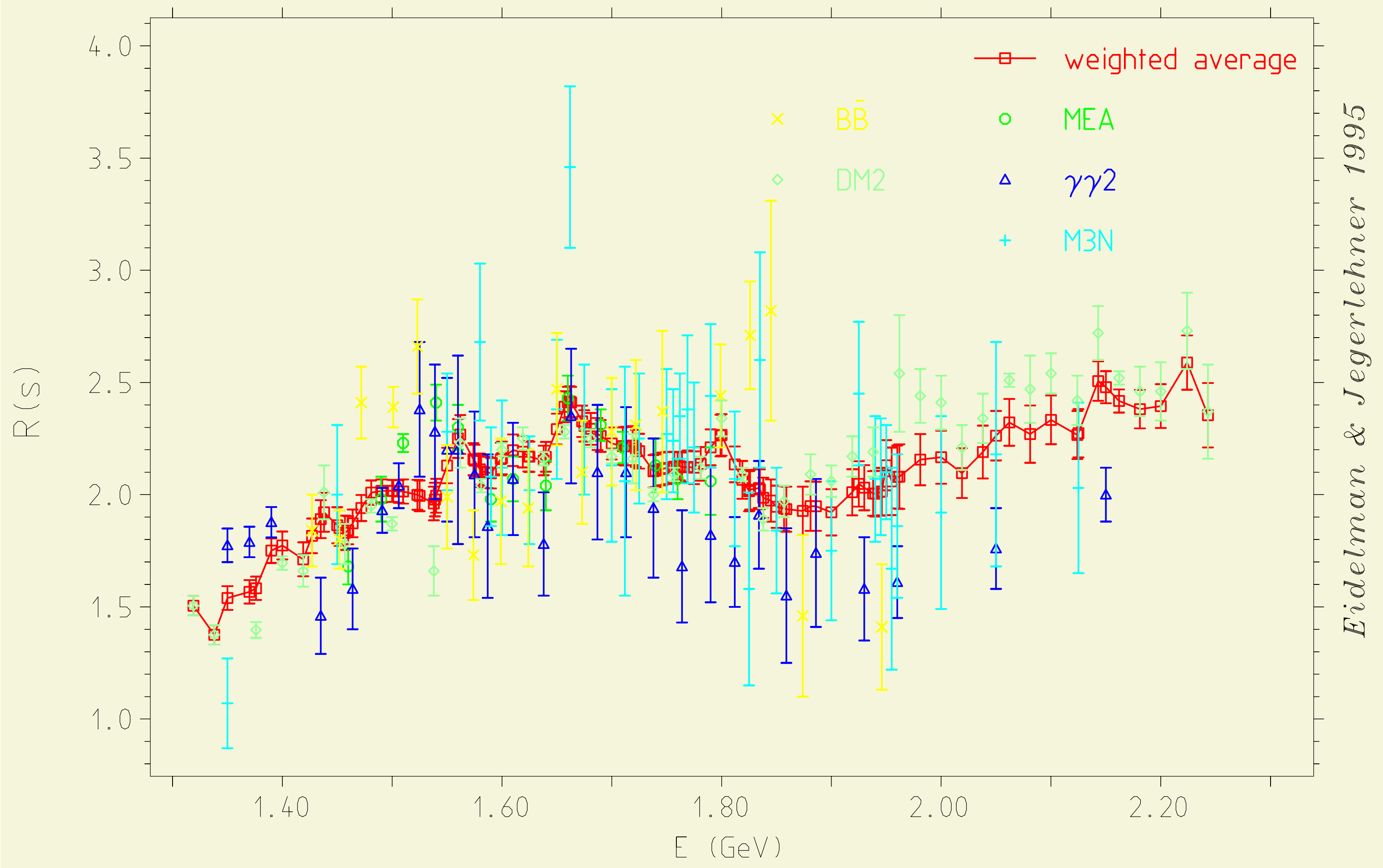
- **2002: Final CMD-2 $\pi\pi$–data: syst error 1.4% $\to$ 0.6 % !**
  some other CMD-2, SND data at $E < 1.4$ GeV **new VP subtraction!**
  (Akhmetshin et al.)
- **2001: BES-II R–data: syst error 20% $\to$ 7% !**
  region 2.0 GeV to 5.0 GeV
  (Bai et al. 2000, 2002)
- **2000: τ–data**
  (ALEPH, OPAL, CLEO Collab., 1996-2002)
- **Iso-spin breaking effects τ–data vs. e^+e^-–data**
  (Cirigliano et al. 2002)
- **New situation: τ–data not compatible with e^+e^-–data at 10% level**
- **Previously unaccounted contributions: $e^+e^- \to \sigma\gamma,\ f_0\gamma$ to $a_\mu^{\rm had}$ (data very poor)**
  $\Rightarrow \delta a_\mu^S = 11.75(8.25)\ \times 10^{-10}$, **i.e.,** $\sim +1\sigma$
  (Narison 2003)

- substantially lower than old e^+e^-–data !



checking VP subtraction by CMD-2 group !

- **Most problematic e^+e^- region now 1.4-2.0 GeV**

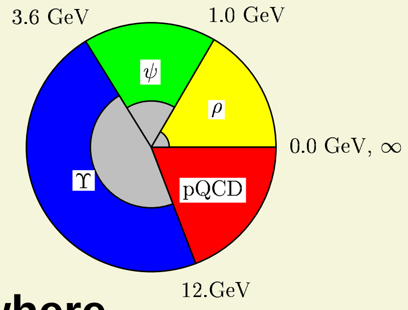


special effort in interpretation of these data ! **(Hagiwara et al. 02)**

② Evaluation of $\alpha(M_Z)$

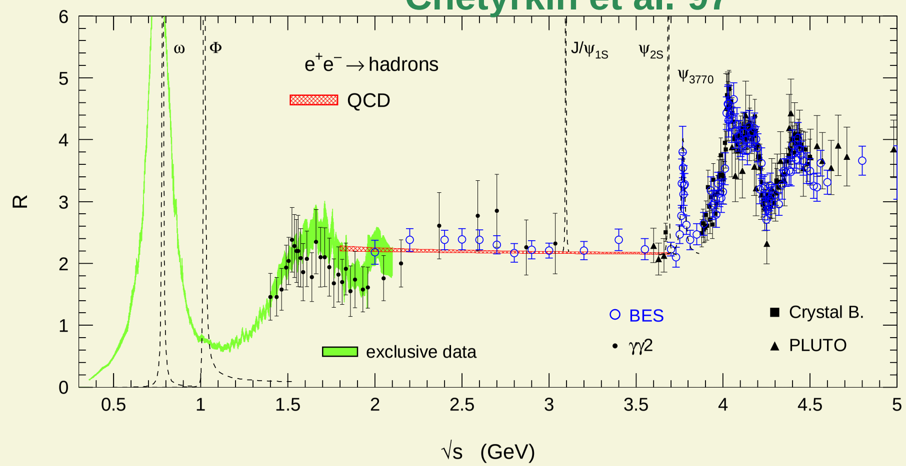
Non-perturbative hadronic contributions $\Delta\alpha^{(5)}_{\mathrm{had}}(s)$ can be evaluated in terms of $\sigma(e^+e^- \to \mathrm{hadrons})$ data via dispersion integral:

$$\Delta\alpha^{(5)}_{\mathrm{had}}(s) = -\frac{\alpha s}{3\pi}\left(\mathrm{P}\!\!\int_{4m_\pi^2}^{E^2_{\mathrm{cut}}} ds' \frac{R^{\mathrm{data}}_\gamma(s')}{s'(s'-s)} + \mathrm{P}\!\!\int_{E^2_{\mathrm{cut}}}^{\infty} ds' \frac{R^{\mathrm{pQCD}}_\gamma(s')}{s'(s'-s)} \right)$$



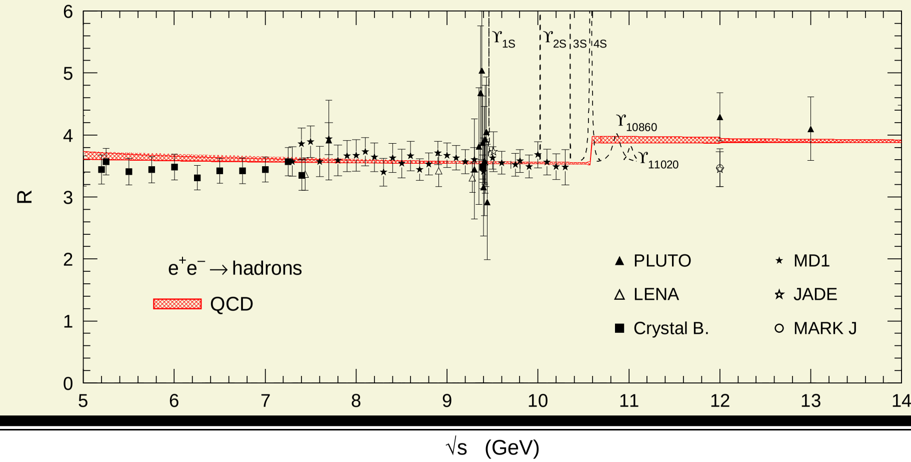
Compilation: Davier, Eidelman et al. 02

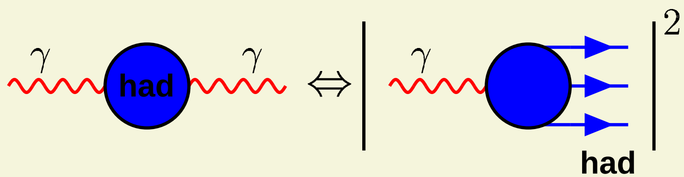
Theory = pQCD: Groshny et al. 91, Chetyrkin et al. 97



where

$$R_\gamma(s) \equiv \frac{\sigma^{(0)}(e^+e^-\to\gamma^*\to\mathrm{hadrons})}{\frac{4\pi\alpha^2}{3s}}$$





$\Pi'_\gamma(q^2)$ $\quad\sim \sigma^{\mathrm{had}}_{\mathrm{tot}}(q^2)$

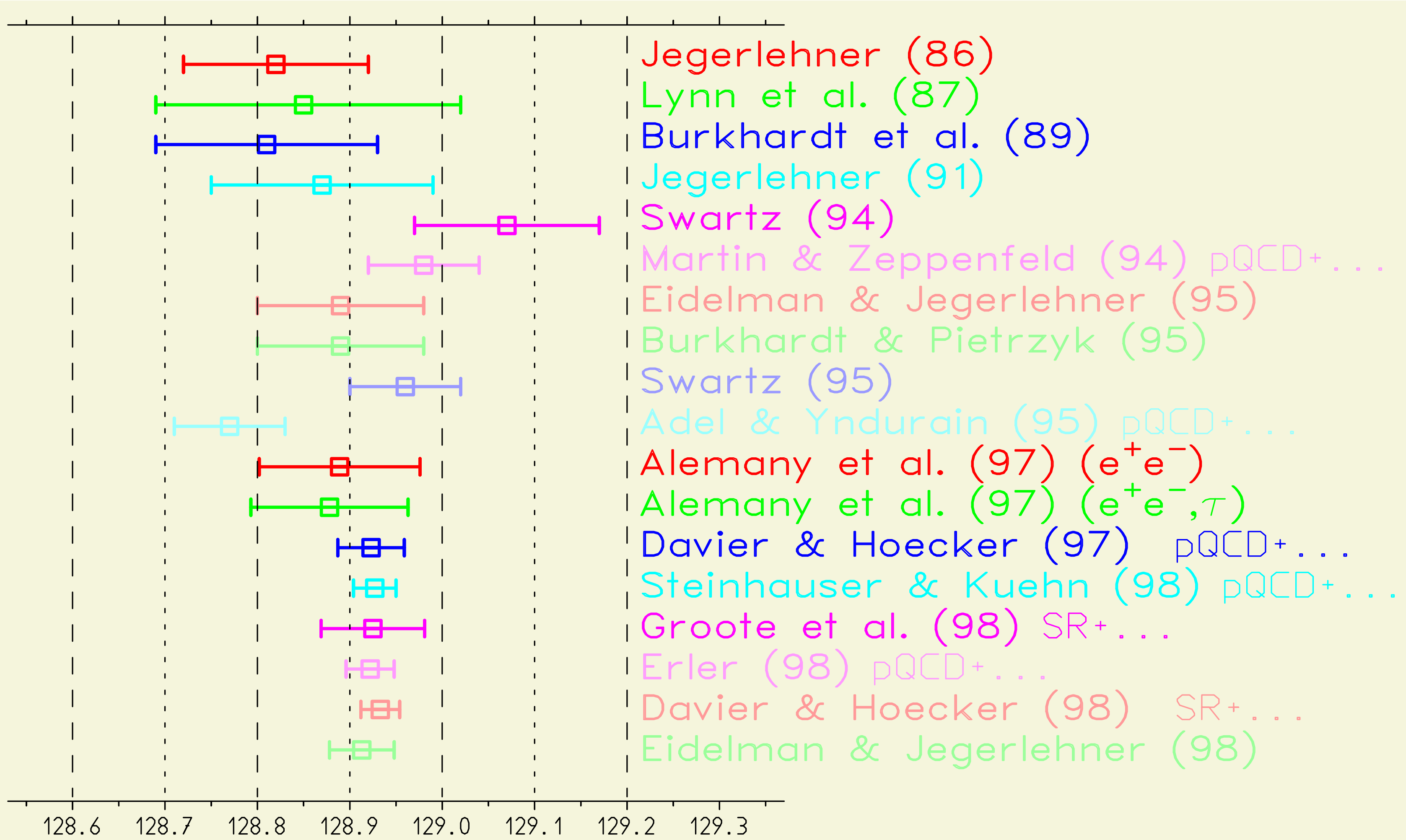
Evaluation FJ 2002 update: at $M_Z = 91.19$ GeV

- $R(s)$ data up to $\sqrt{s} = E_{cut} = 5$ GeV
  and for Υ resonances region between 9.6 and 13 GeV
- perturbative QCD from 5.0 to 9.6 GeV
  and for the high energy tail above 13 GeV

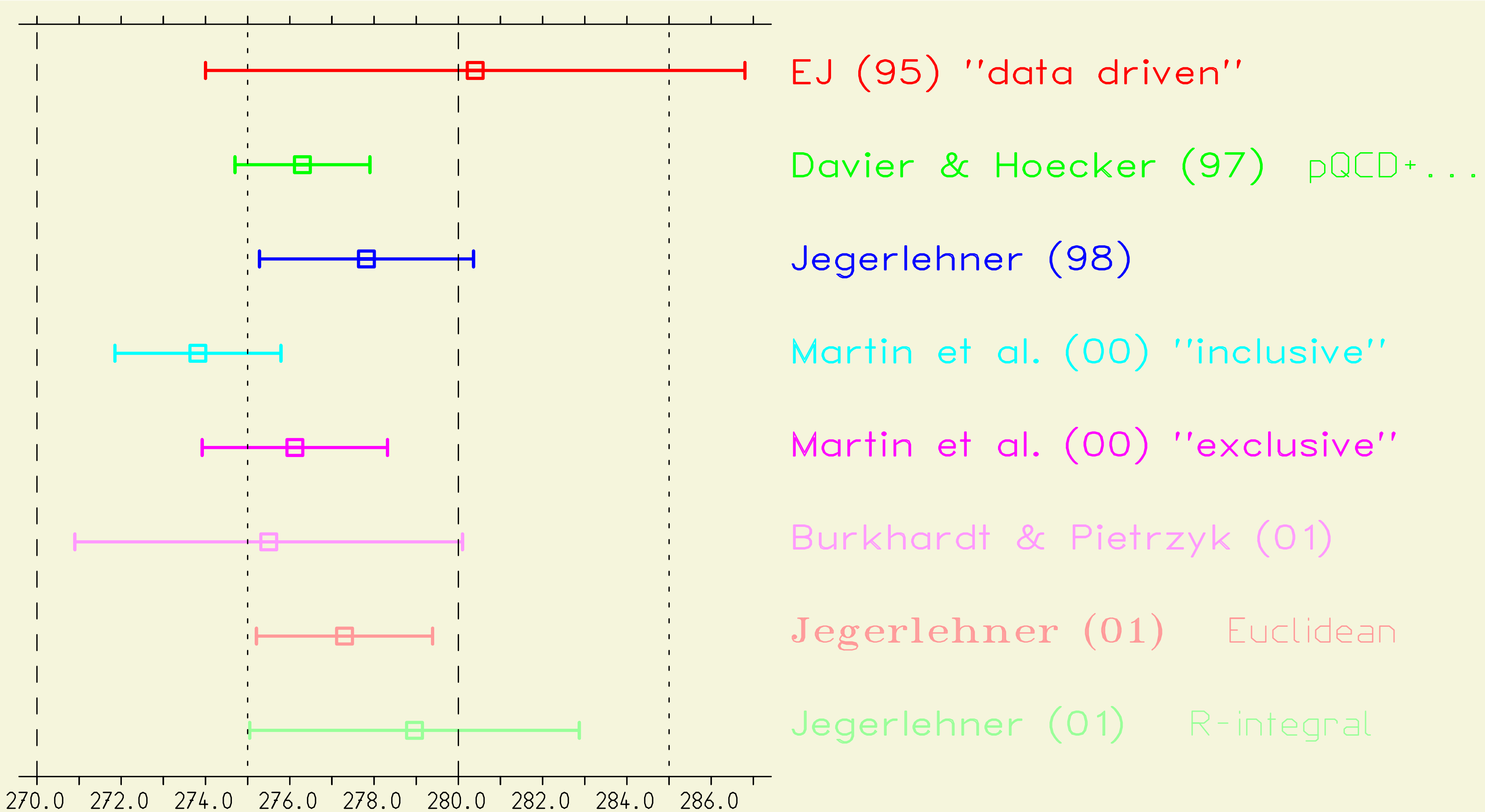
$$\begin{aligned} \Delta\alpha^{(5)}_{\mathrm{hadrons}}(M_Z^2) &= 0.02777 \pm 0.00034 \\ & 0.02761 \pm 0.00036 \quad \text{BP 01} \\ \alpha^{-1}(M_Z^2) &= 128.925 \pm 0.046 \\ & 128.936 \pm 0.046 \quad \text{BP 01} \end{aligned}$$

Evaluations of $\alpha(M_Z)$ by different authors (before 2000)

All values have been rescaled to $M_Z = 91.1887$ GeV. The small top quark and the W contributions are not included.



Recent evaluations of $\Delta\alpha(M_Z)$



❑ $\alpha(M_Z)$ in precision physics

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics:

Precision physics limitations:

α , G_μ, M_Z **most precise input parameters**

$\Updownarrow$

partially non-perturbative relationship

precision predictions

$\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

$\alpha(M_Z), G_\mu, M_Z$ **best effective input parameters for VB physics (Z,W) etc.**

$$\frac{\delta\alpha}{\alpha} \sim 3.6 \times 10^{-9}$$
$$\frac{\delta G_\mu}{G_\mu} \sim 8.6 \times 10^{-6}$$
$$\frac{\delta M_Z}{M_Z} \sim 2.4 \times 10^{-5}$$
$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 1.6 \div 6.8 \times 10^{-4} \quad \text{(present)}$$
$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 5.3 \times 10^{-5} \quad (\texttt{TESLA requirement})$$

LEP/SLD:

$$\sin^2\Theta_{\rm eff} = (1-g_{Vl}/g_{Al})/4 = 0.23148 \pm \boxed{0.00017}$$
$$\delta\Delta\alpha(M_Z) = 0.00036 \quad \Rightarrow \quad \delta\sin^2\Theta_{\rm eff} = \boxed{0.00013}$$

affects Higgs mass bounds !!!

Indirect

Higgs boson mass "measurement"

$m_H = 88^{+53}_{-35}$ **GeV**

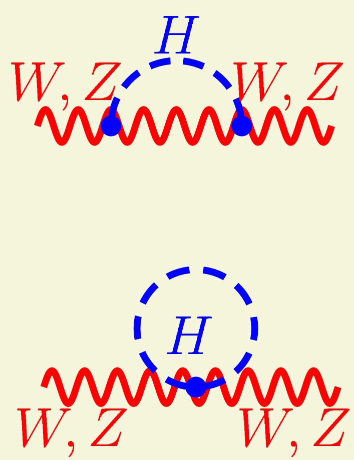
$e^+e^- \to \tau$: $\Rightarrow \delta m_H \sim -19$ **GeV**

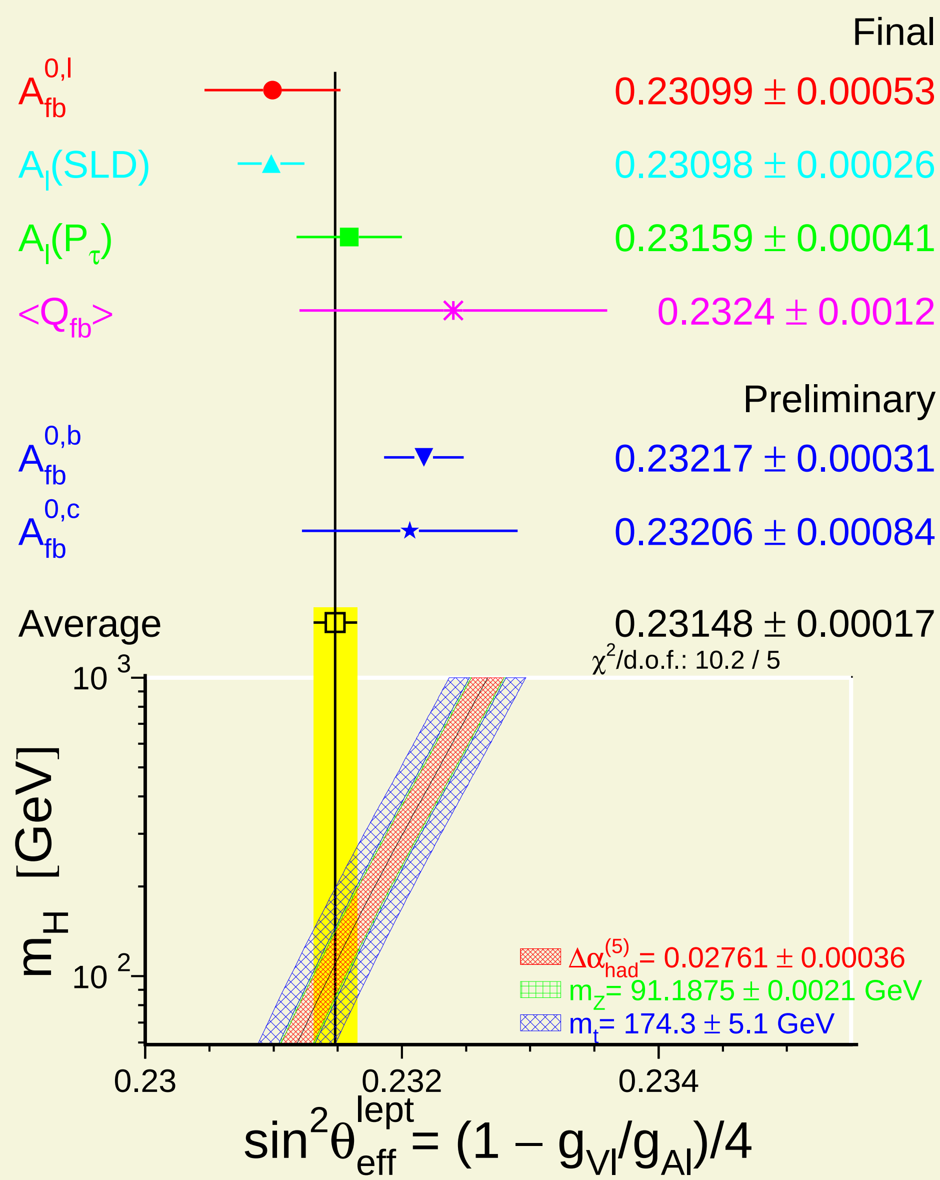
Direct lower bound:

$\boxed{m_H > 114}$ **GeV at 95% CL**

Indirect upper bound:

$\boxed{m_H < 193}$ **GeV at 95% CL**



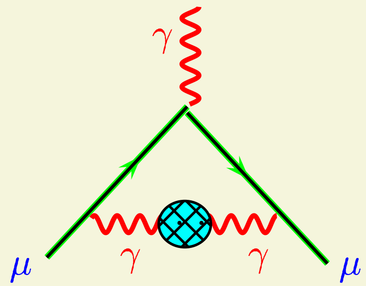


(LEP Electroweak Working Group: D. Abbaneo et al. 03)

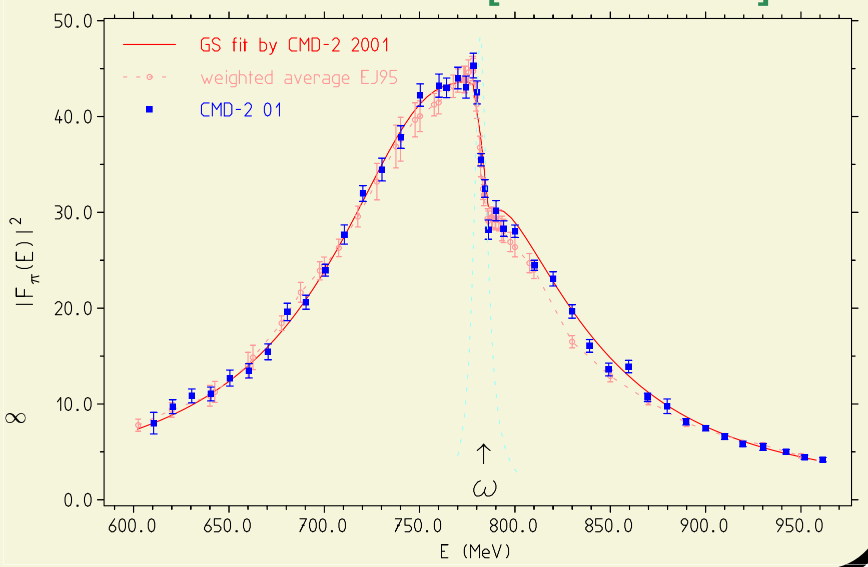
② Evaluation of $a_\mu \equiv (g-2)_\mu/2$

Leading non-perturbative hadronic contributions $a_\mu^{\rm had}$ can be obtained in terms of $R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}$ data via dispersion integral:

$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int\limits_{4m_\pi^2}^{E^2_{\rm cut}} ds\, \frac{R_\gamma^{\rm data}(s)\; \hat{K}(s)}{s^2} + \int\limits_{E^2_{\rm cut}}^{\infty} ds\, \frac{R_\gamma^{\rm pQCD}(s)\; \hat{K}(s)}{s^2} \right)$$

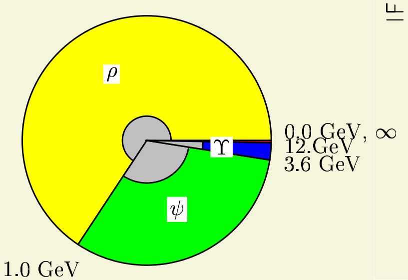


Data:
Akhmetshin et al. 02 [Collab. CMD-2]



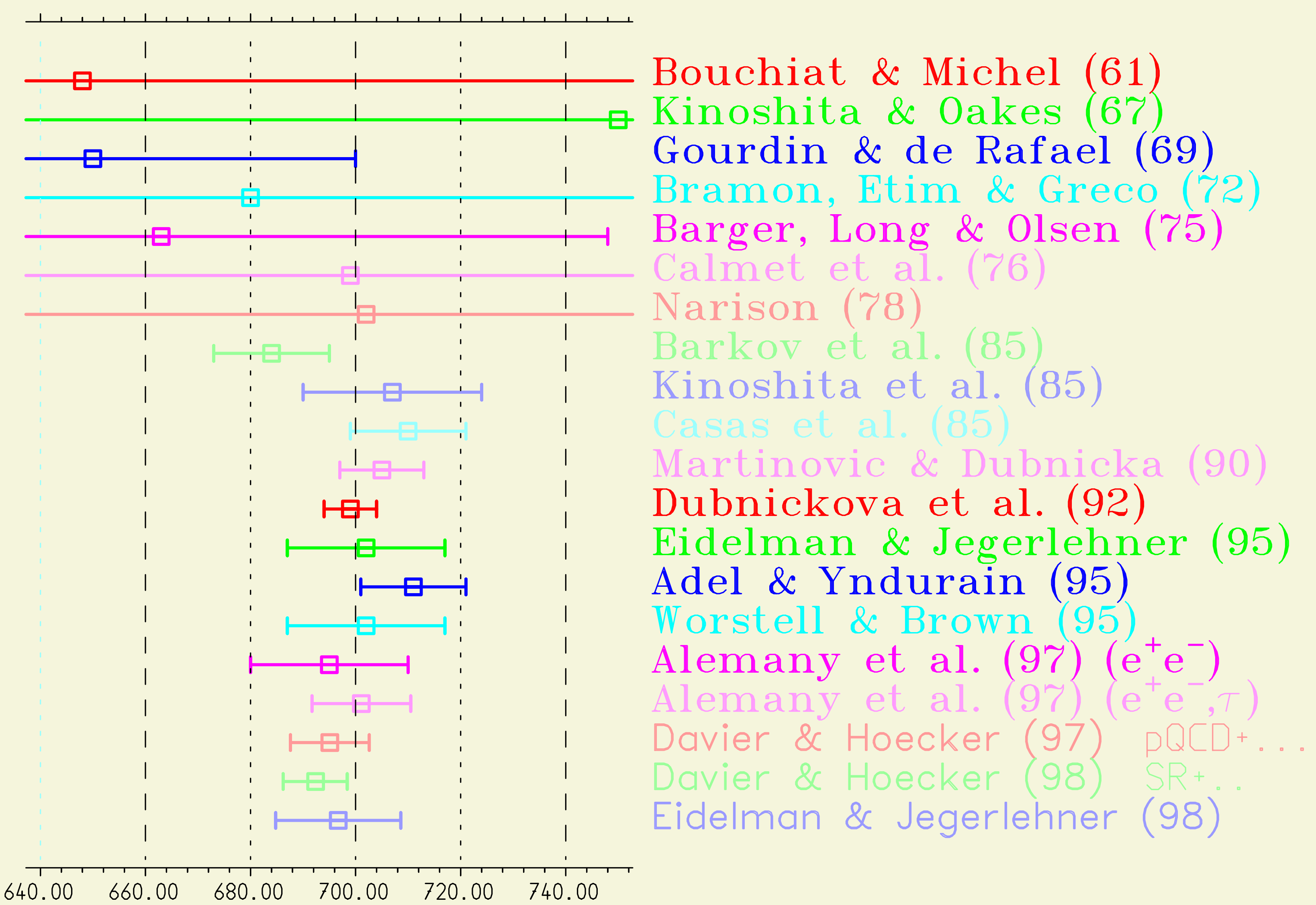
- **Experimental error implies theoretical uncertainty!**
- **Low energy contributions enhanced: $\sim 67\%$ of error on $a_\mu^{\rm had}$ comes from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$**

$$\boxed{a_\mu^{\rm had(1)} = (683.7 \pm 8.6)\; 10^{-10}}$$

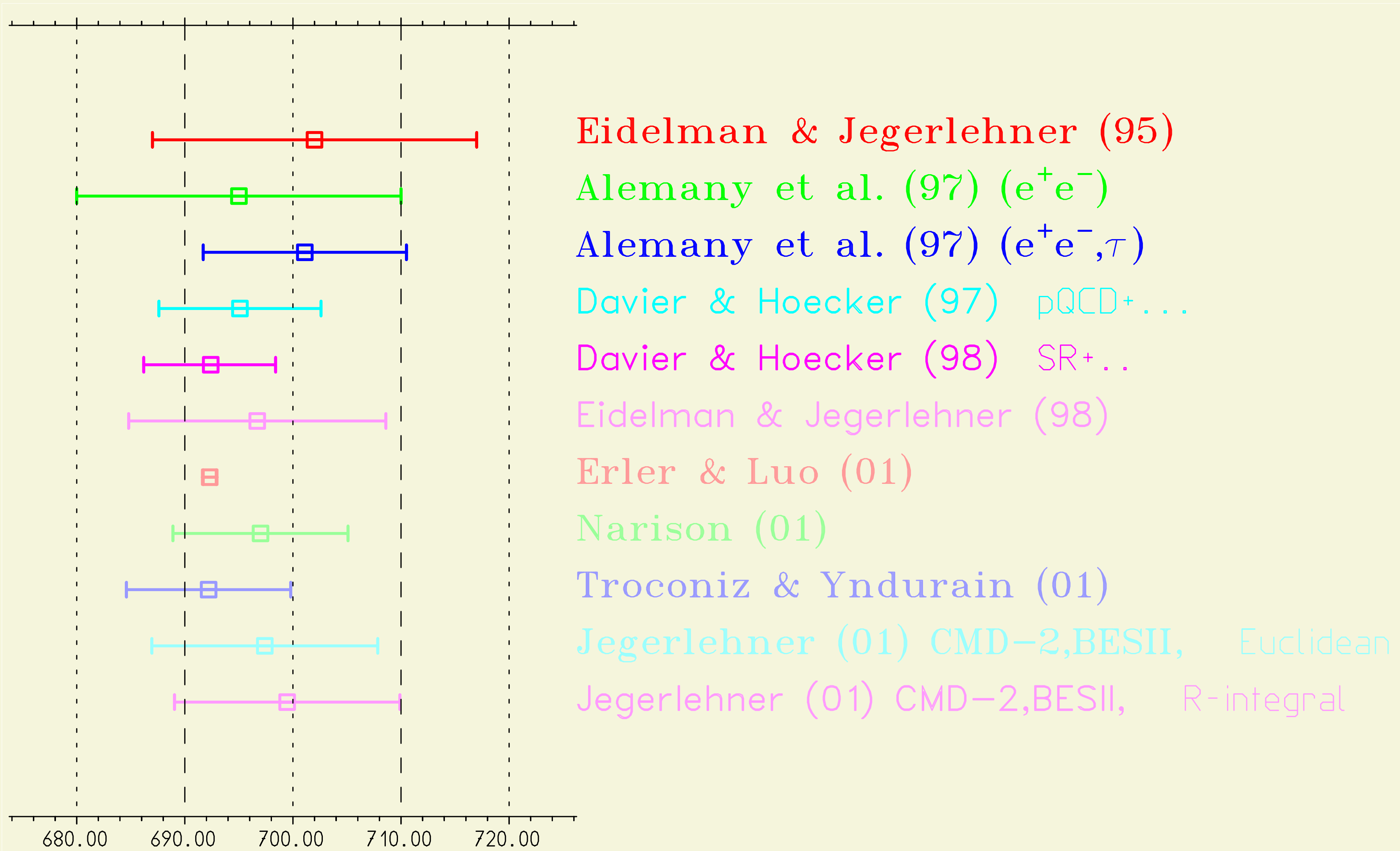


Evaluations of $a_\mu^{\mathrm{hadrons}} \times 10^{10}$ by different authors (before 2000)

Only the leading hadronic vacuum polarization diagram is included.

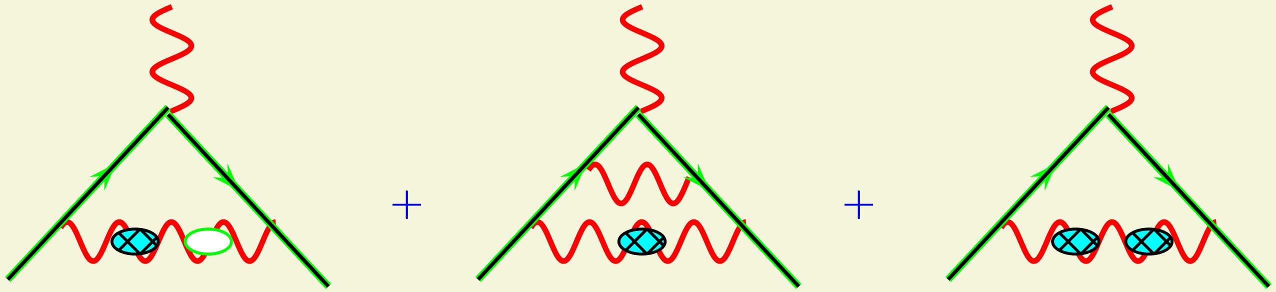


Recent evaluations of $a_\mu^{\rm had}$

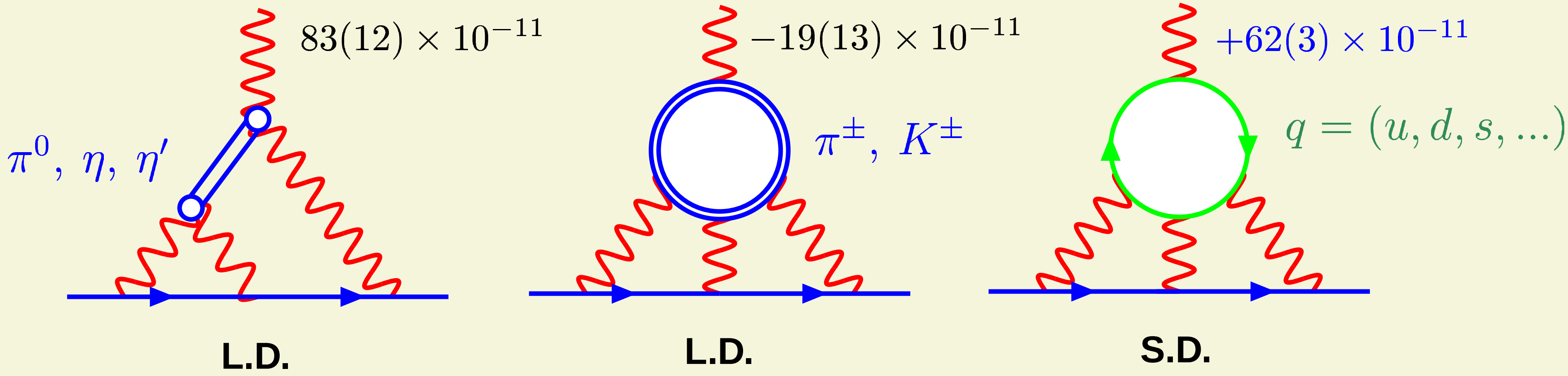


$$\boxed{a_\mu = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{had}(1)} + a_\mu^{\mathrm{had}(2)} + a_\mu^{\mathrm{weak}(1)} + a_\mu^{\mathrm{weak}(2)} + a_\mu^{\mathrm{lbl}} \ (+a_\mu^{\mathrm{new\ physics}})}$$

- **Higher order hadronic contributions** $a_\mu^{\mathrm{had}(2)} = -(100 \pm 6) \times 10^{-11}$ **(Krause 96)**

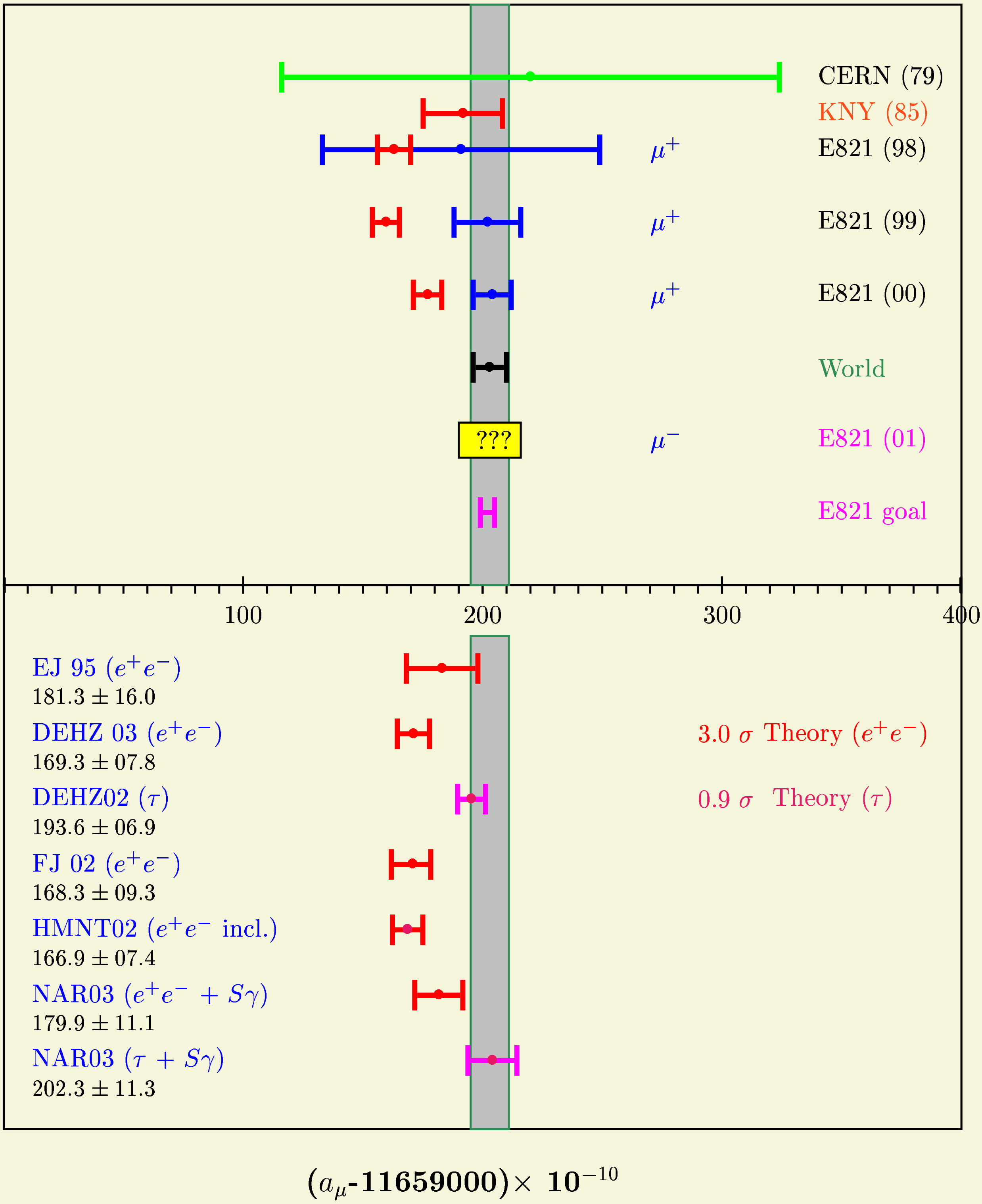


- **Hadronic light–by–light scattering** $a_\mu^{\mathrm{lbl}} = (80 \pm 40)\ \times 10^{-11}$ **(Nyffeler 02)**



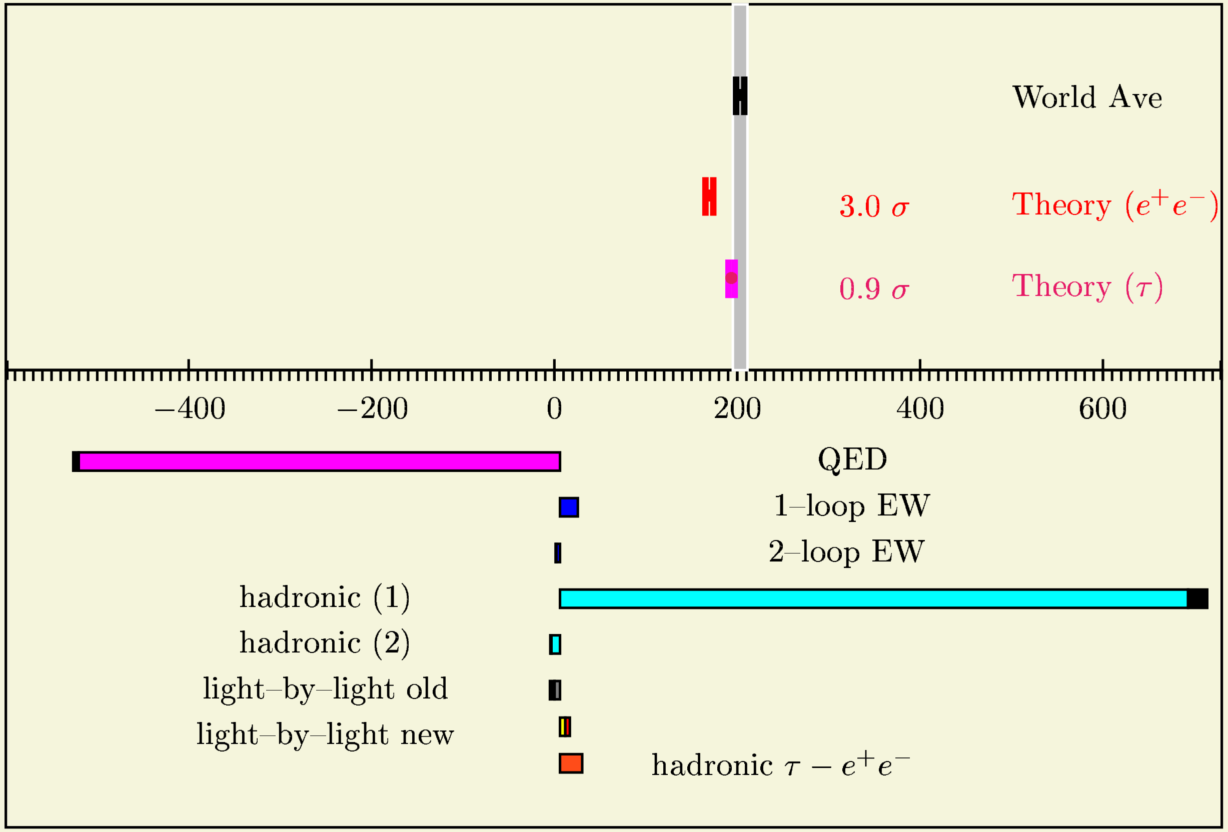
Low energy effective theory: e.g. ENJL

a_μ Summary: experiment vs. theory



Given theory results only differ by $a_\mu^{\mathrm{had}(1)}$!

a_μ: type and size of contributions



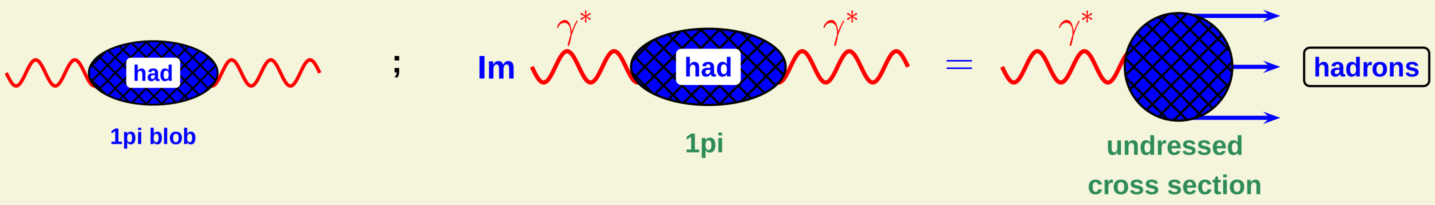
All kind of physics meets !

④ What pseudo-observable do we need?

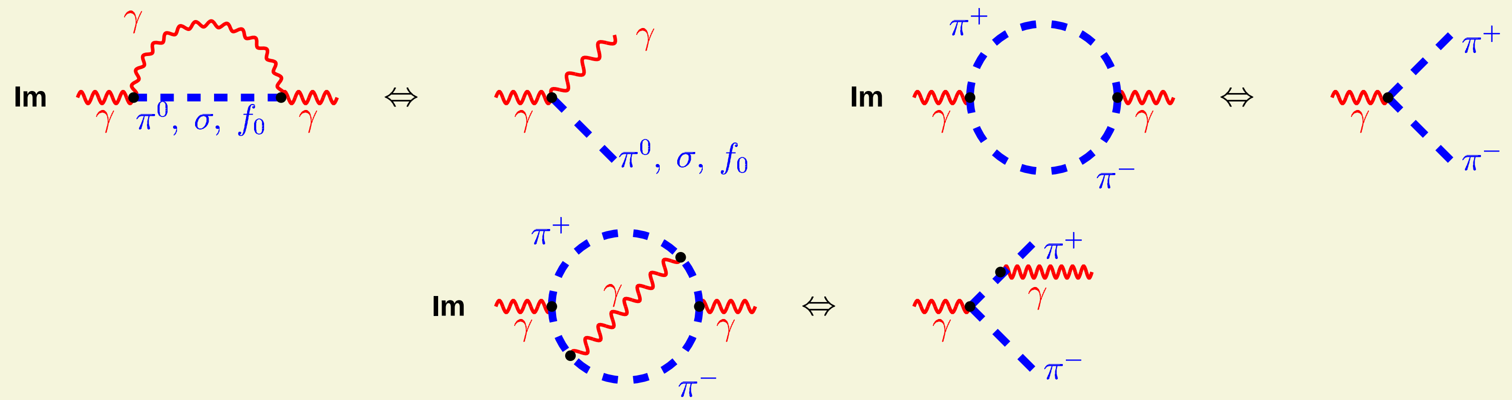
What do we need for calculating $\alpha(s)$ and $a_\mu^{\rm had}$?

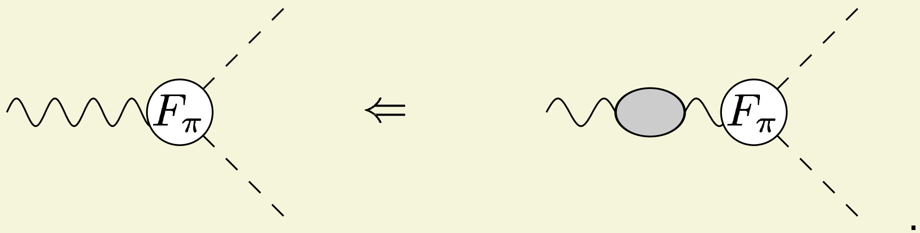
$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha} \quad ; \quad \Delta\alpha = -\mathrm{Re}\,\left(\Pi'(s) - \Pi'(0)\right)$$

$\Pi'_{\rm had}(q^2) =$



1pi blob: $\pi^0\gamma, \rho, \pi\pi, \pi\pi\gamma, 3\pi, 4\pi, \sigma\gamma\cdots, \pi\pi H, \cdots$ **at least one hadron plus any strong, electromagnetic or weak interaction. At low energies:**





$$|F_\pi^{(0)}(s)|^2 = |F_\pi(s)|^2 \, (\alpha/\alpha(s))^2$$

VP effects in physical quantities must include photonic corrections to the hadronic 1pi blob:

Add theoretical prediction for FS radiation (including full photon phase space):

$$|F_\pi^{(\gamma)}(s)|^2 = |F_\pi^{(0)}(s)|^2 \, \left(1+\eta(s)\frac{\alpha}{\pi}\right)$$

to order $O(\alpha)$, where $\eta(s)$ is a known correction factor (Schwinger 1989). The corresponding $O(\alpha)$ contribution to the anomalous magnetic moment of the muon is

$$\delta^\gamma a_\mu^{\rm had} = (38.6 \pm 1.0) \times 10^{-11}$$

(see also: Melnikov, Troconiz&Yndurain 2001)

KLN theorem at work (Kinoshita, Lee & Nauenberg)

fully inclusive = including virtual (V) plus soft (S) plus all hard (H) photons cross section: $\sigma_0\,(1+\frac{\alpha}{\pi}C)$ C a constant $O(1)$ like 3/4 for μ FSR, 3 for π FSR (in SQED)

$\Rightarrow$ small positive correction;typically $\sim$ .2%

exclusive quantities involve large log's on photon energy cut and collinear logs !

Bloch & Nordsieck: V+S cannot be separated (separately IR divergent)

V+S log's: $\frac{\alpha}{\pi}\,C_c\;\ln(s/m_\pi^2)$, $\frac{\alpha}{\pi}\,C_e\;\ln(\sqrt{s}/E_{\gamma\mathrm{cut}})$

complementary hard photon part (summing to full phase space)

H: log's the same as V+S but precisely of opposite sign!

All log's cancel in sum (inclusive)!

- cutting out all hard photons and subtracting V+S using sQED $\Rightarrow$ .5% model ambiguity (sQED vs. fQED)
- adding missing H part using the same model (as used for subtracting V+S) $\Rightarrow$ .1% model ambiguity (sQED vs. fQED)
- this guesstimate does not tell us the true model error!

How to get phase of $F_\pi(s)$?

Omnès–Muskhelishvili theorem:

(see also: Geshkenbein ... 89, Ynduràin ... 01)

analyticity + unitarity + chiral limit

relates: space–like data, $\pi\pi$–scattering phase shifts and time–like data

$\rightarrow$ severe theoretical constraints!

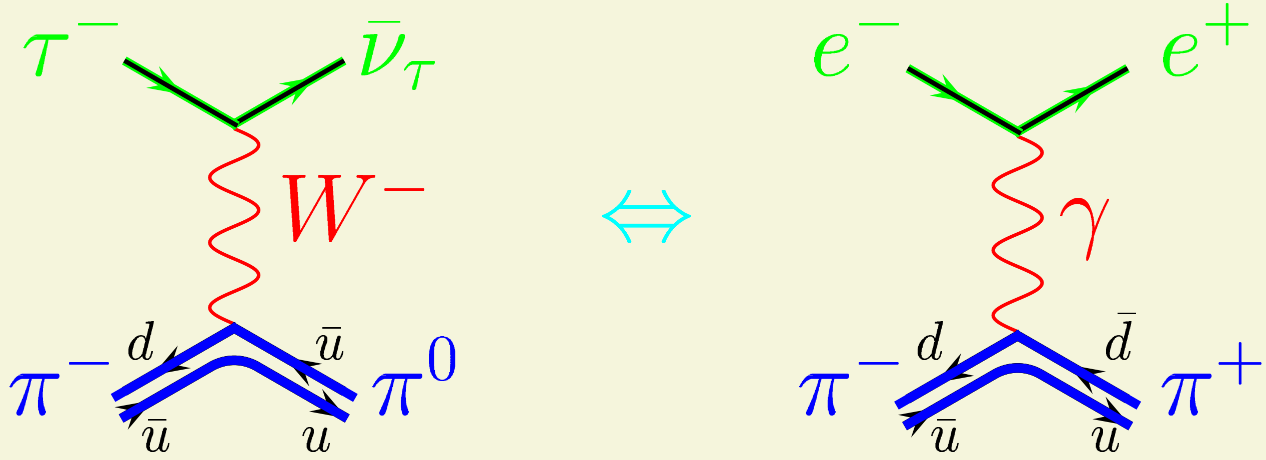
Work in progress: Colangelo, Gasser, Leutwyler, F.J.

$$\boxed{|F_\pi(s)|^2 \Rightarrow \ |(1-\Pi'(s))\ F_\pi(s)|^2}$$

Allows us to get substantially improved low energy contribution!

⑤ e^+e^-–cross sections via τ–decay spectral functions

The isovector part of $\sigma(e^+e^- \to \mathrm{hadrons})$ may be calculated by a isospin rotation from τ–decay spectra (to the extend that CVC is valid)



$\tau^- \to X^- \nu_\tau \qquad \leftrightarrow \qquad e^+e \to X^0$

X^- and X^0 are hadronic states related by iso-spin rotation.

The e^+e^- cross–section is then given by

$$\boxed{\sigma^{I=1}_{e^+e^- \to X^0} = \frac{4\pi\alpha^2}{s} v_{1,X^-} \quad , \qquad \sqrt{s} \leq M_\tau}$$

in terms of the τ spectral function v_1.

- **Additional "e^+e^-" data: τ–data + CVC** ALEPH, OPAL, CLEO (ADH96, DEHZ02)
- **mainly improves the knowledge of the $\pi^+\pi^-$ channel (ρ–resonance contribution)**
- **which is dominating in $a_\mu^{\rm had}$ (72%)**

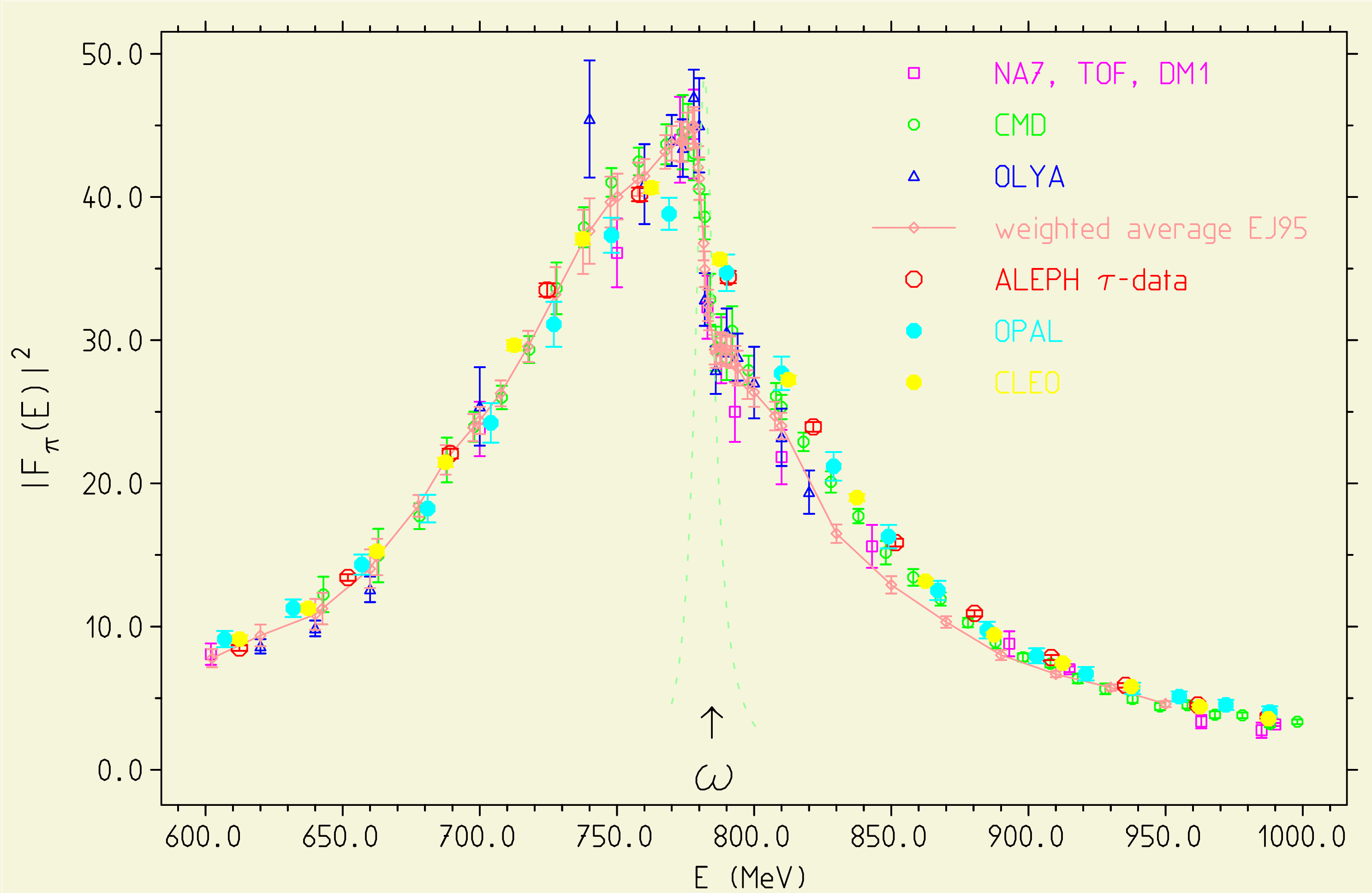
$I=1 \sim 75\%\,;\; I=0 \sim 25\% \Longrightarrow$ τ–data cannot replace e^+e^-–data

ADH 95:

$$\begin{array}{lcccc} \delta a_\mu & : & 15.6\times 10^{-10} & \rightarrow & 10.2\times 10^{-10} \\ \delta\Delta\alpha & : & 0.00067 & \rightarrow & 0.00065 \end{array}$$

All kind of isospin breaking effects have to be taken into account !!!

systematic deviations largely can be understood!



e^+e^-–data*= data corrected for iso-spin violations:

$$|F(s)|^2 = \left(|F(s)|^2\text{–data}\right) / \left|\left(1 + \frac{\epsilon s}{(s_\omega - s)}\right)\right|^2$$

with

$$s_\omega = (M_\omega - \tfrac{i}{2}\Gamma_\omega)^2$$

ϵ **determined by fit to the data:** $\epsilon = 0.00172$

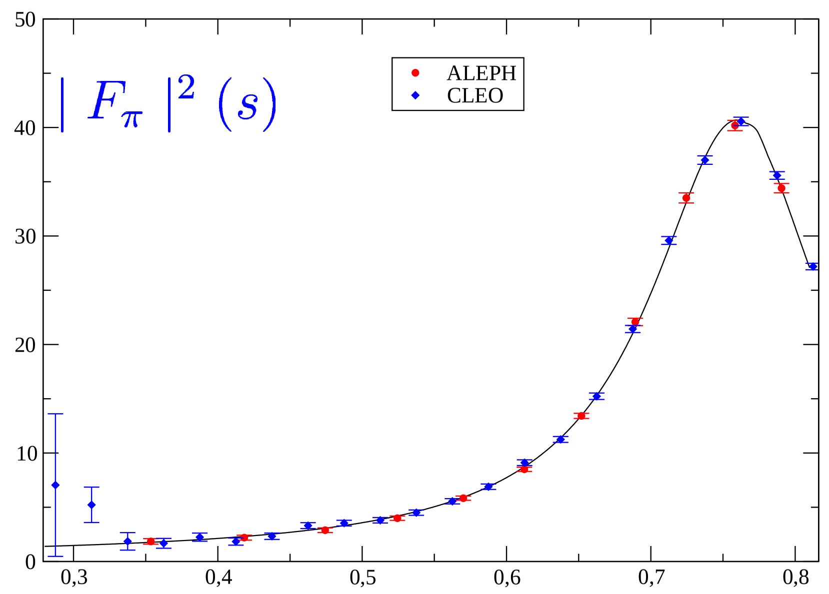
(Leutwyler 00)

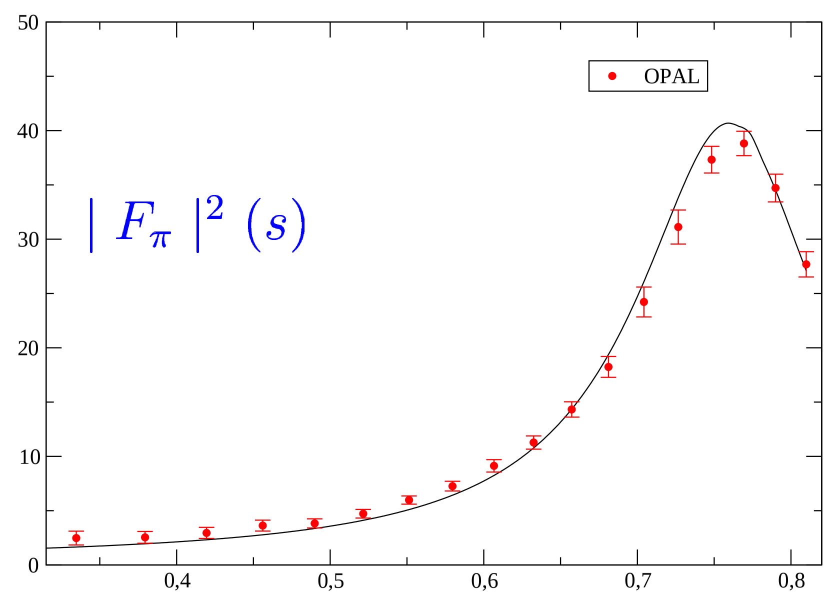
After correction no systematic deviations between CLEO/ALEPH and DM1/OLYA/CDM1/CDM2 data beyond fluctuations of e^+e^- data, while OPAL data show substantial deviations.

work in progress

G. Colangelo, J. Gasser, F. J.,H. Leutwyler

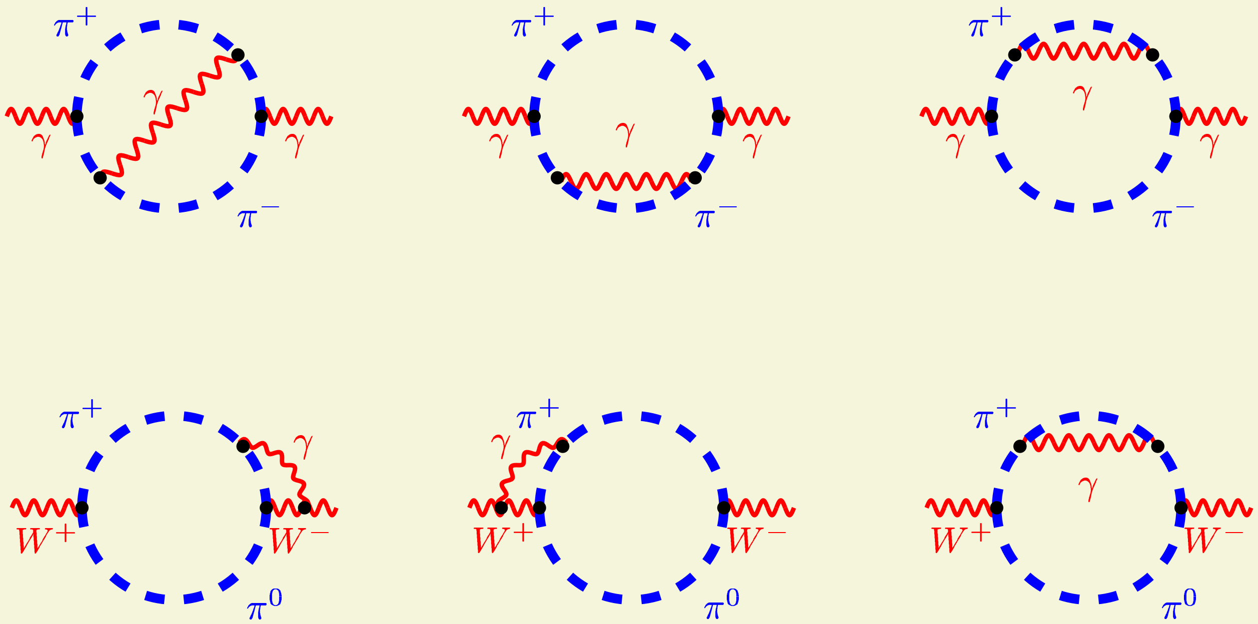
NA7+CMD-2 e^+e^-–data analytized+unitarized+chiral limit: solid line vs. τ –data





✥ Iso-spin breaking in τ vs. e^+e^-:

FSR correction in τ–decay: inclusive approach?



QED corrections are obviously not related by a iso-spin rotation and must be subtracted before CVC arguments can be applied

⑥ Iso-spin breaking corrections in τ vs. e^+e^-

Cirigliano ea al., hep-ph/0104267, hep-ph/0212386

$$a_\mu^{\mathrm{vacpol}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{-infty} ds\; K(s)\, \sigma^{(0)}_{e^+e^-\to \mathrm{hadrons}}(s)$$

$$\sigma^{(0)}_{\pi\pi} = \left[\frac{K_\sigma(s)}{K_\Gamma(s)}\right] \frac{d\Gamma_{\pi\pi[\gamma]}}{ds} \times \frac{R_{\mathrm{IB}}(s)}{S_{\mathrm{EW}}}$$

$$K_\sigma(s) = \frac{G_F^2\,|V_{ud}|^2\, m_\tau^3}{384\pi^3}\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+2\,\frac{s}{m_\tau^2}\right)$$

$$K_\sigma(s) = \frac{\pi\alpha^2}{3s}$$

Iso-spin breaking correction in

$$R_{\mathrm{IB}}(s) = \frac{1}{G_{\mathrm{EM}}(s)}\,\frac{\beta^3_{\pi^+\pi^-}}{\beta^3_{\pi^+\pi^0}}\,\left|\frac{F_V(s)}{f_+(s)}\right|^2$$

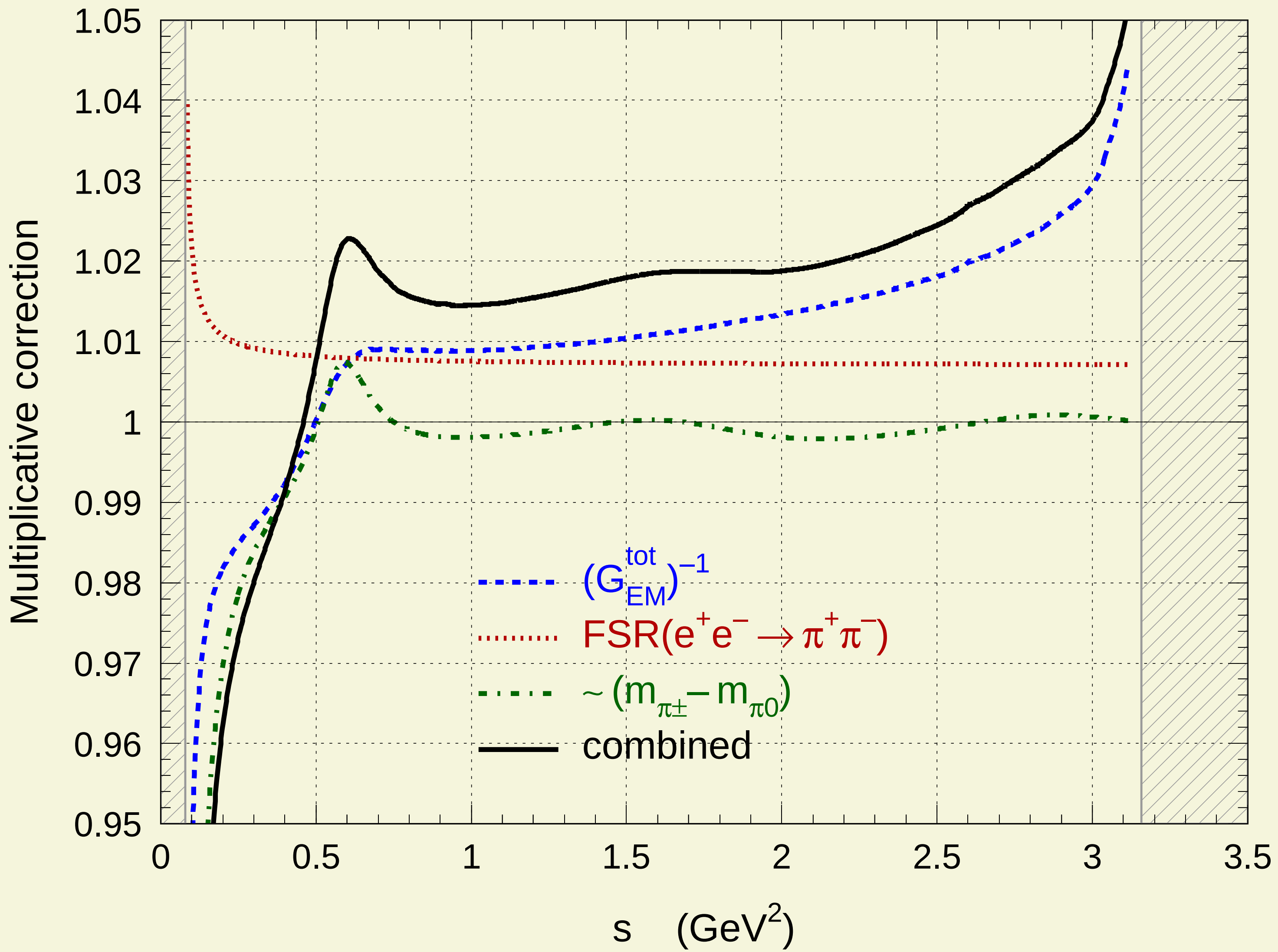
Summary of results:

Contributions to $\Delta a_\mu^{\mathrm{vacpol}}$ from various sources of iso-spin violation (in units of 10^{-11}) for different values of $t_{\max}$ (in units of GeV^2) .

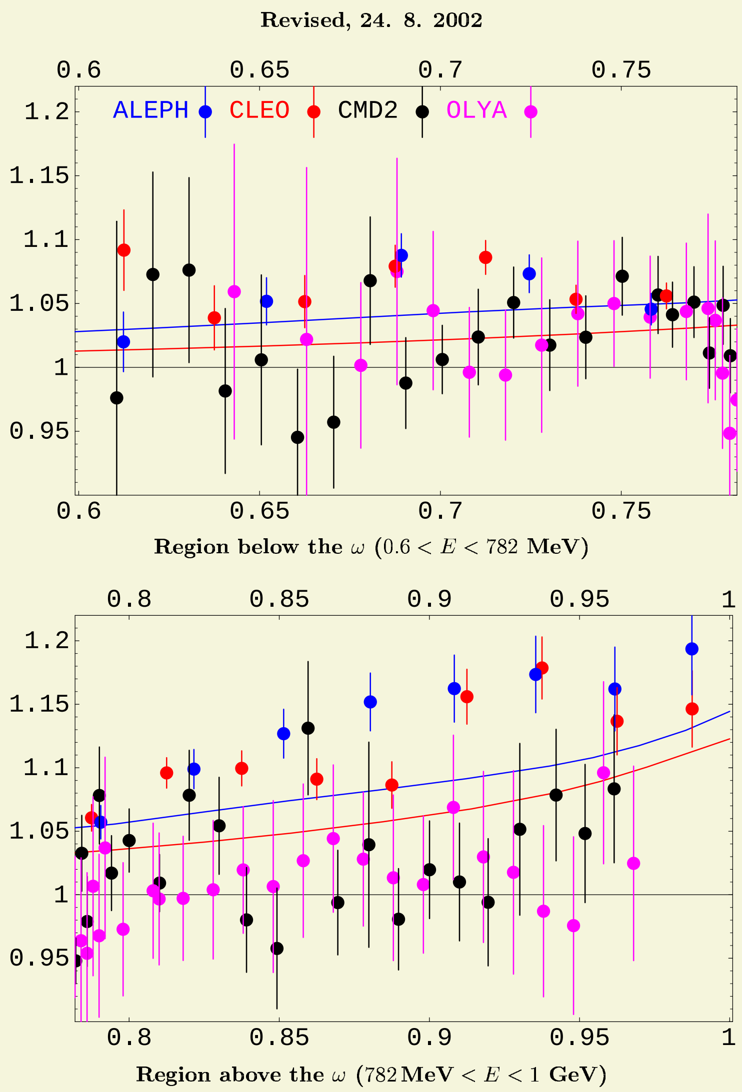
$t_{\max}$	S_{EW}	KIN	EM	FF	$\delta a_\mu^{\mathrm{IB}}$
1	- 95	- 75	- 11	$61 \pm 26 \pm 3$	- 119
2	- 97	- 75	- 10	$61 \pm 26 \pm 3$	- 120
3	- 97	- 75	- 10	$61 \pm 26 \pm 3$	- 120

Isospin breaking effects: (Cirigliano et al.)

Surprisingly, after corrections at larger energies 10% deviations !



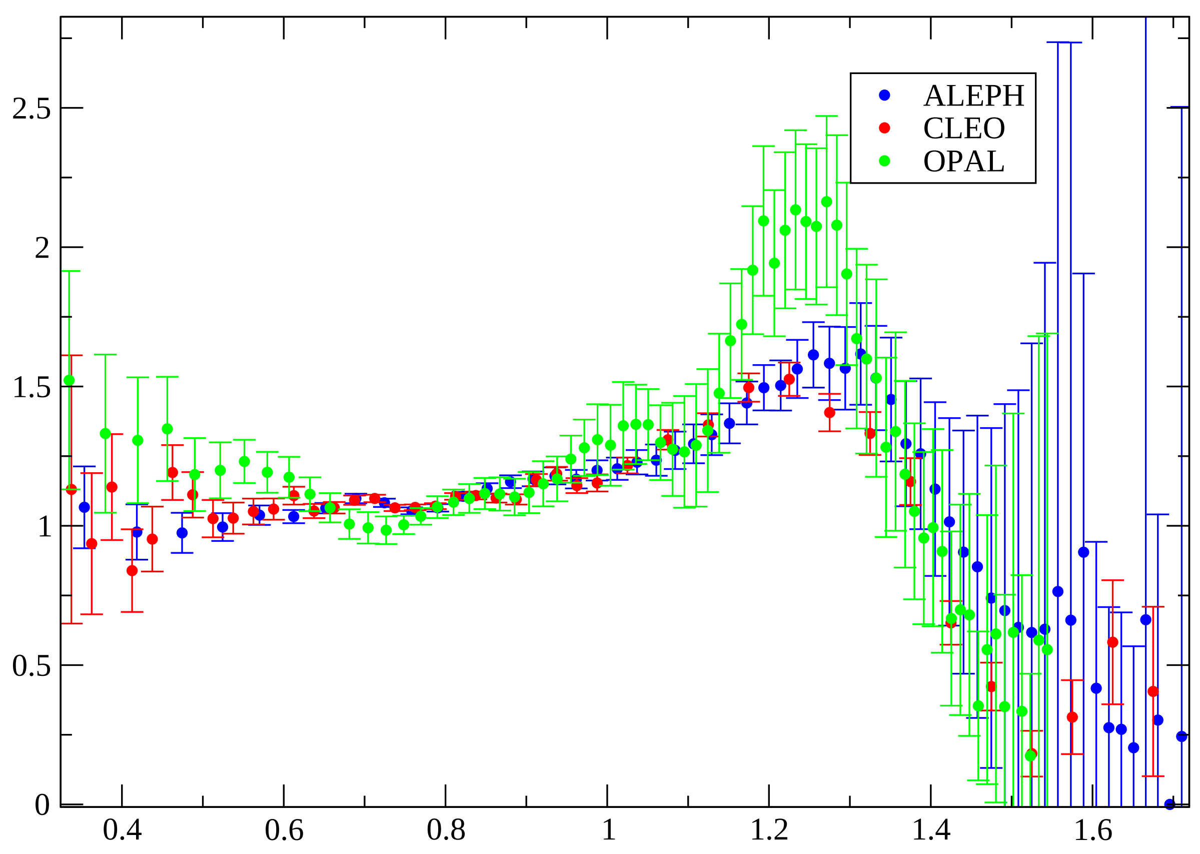
- τ **vs.** e^+e^- **(HL02)**



The plots show the square of the form factor divided by the Omnes factor. In the case of the e^+e^- data, the contributions from ω exchange and vacuum polarization are removed and only the statistical errors are shown. In the case of the τ data, the error bars represent the diagonal elements of the error matrix.

cannot be understood within SM ! likely an experimental problem

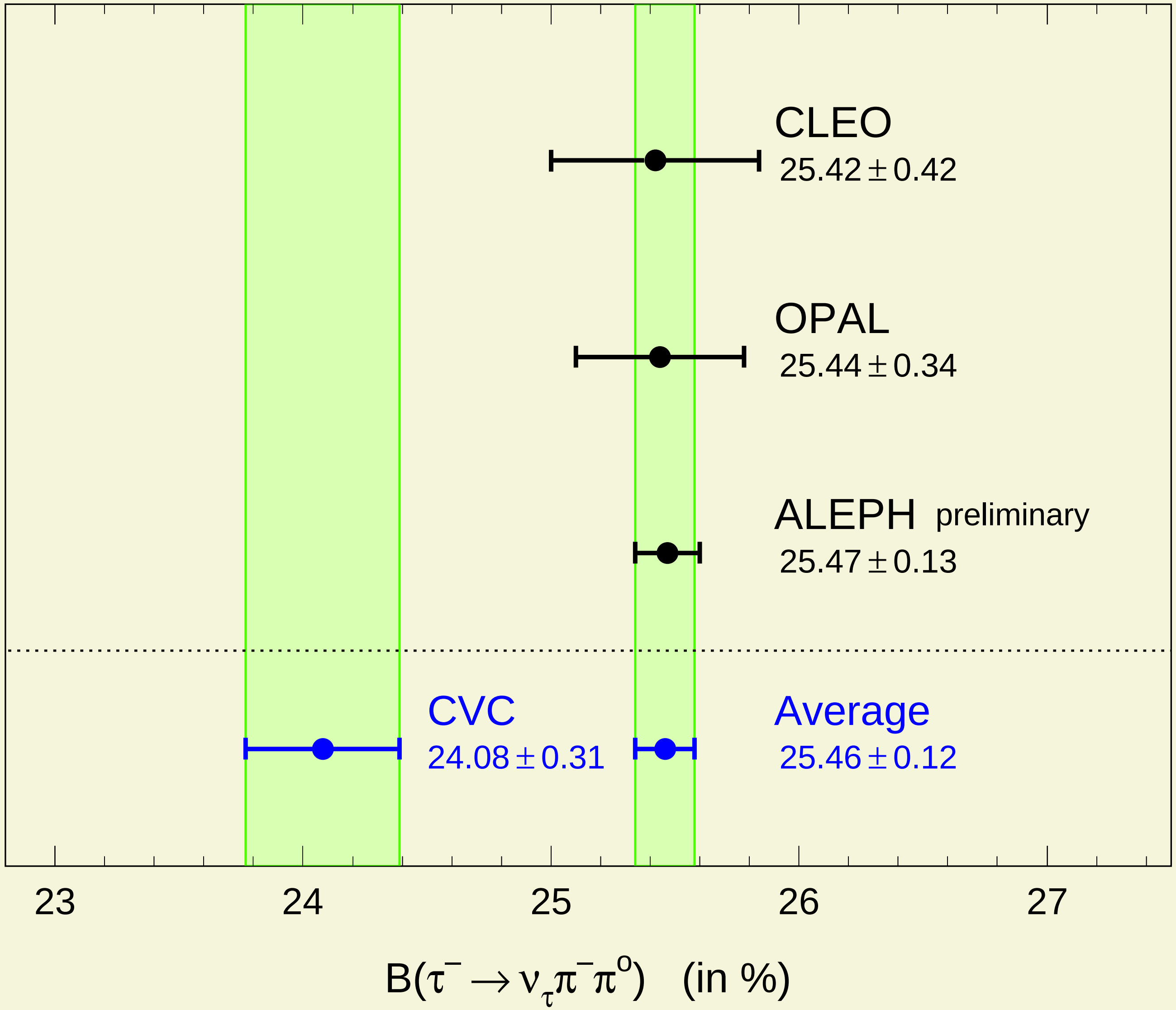
- **Comparison of τ–data:**



τ–data may be not so easy; DELPHI, L3 could not measure τ spectral–functions;

ALEPH vs. OPAL no good agreement.

The measured branching ratios for $\tau \to \nu_\tau \pi^- \pi^0$ compared to the prediction from the $e^+e^- \to \pi^+\pi^-$ after applying the isospin breaking correction



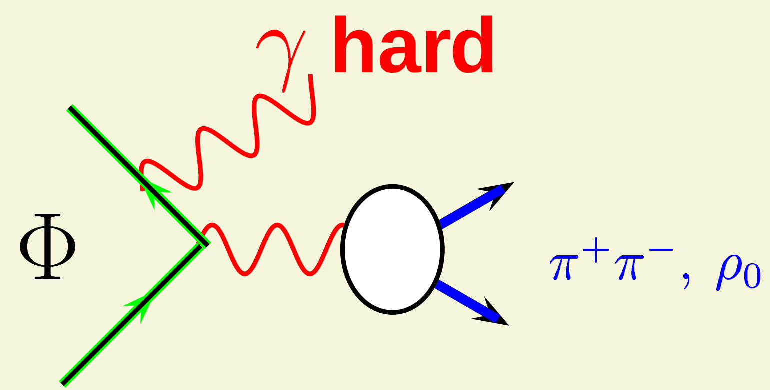
Precise measurement of $\sigma(e^+e^- \to hadrons)$ at DAΦNE

Aim: Reduce hadronic uncertainty in $(g-2)_\mu, \alpha(E), \cdots$ crucial for future of precision physics

In progress by $\boxed{\text{KLOE}}$ at ϕ–factory DAΦNE: $\sigma(e^+e^- \to hadrons)$ at $\sqrt{s} \lesssim 2$ GeV

❶ $F_\pi(s)$ $\sqrt{s} \lesssim 1$ GeV

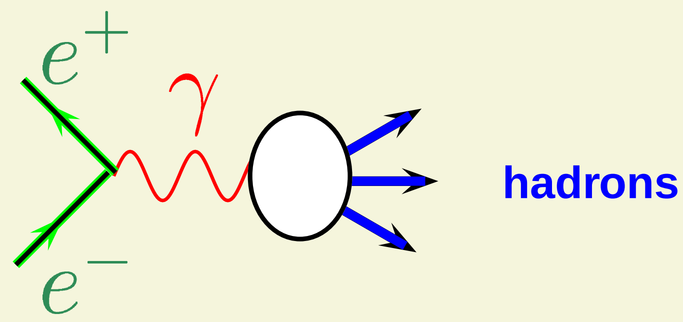
Photon tagging



$s = M_\Phi^2$ $\quad s' = s\,(1-k) \quad [k = E_\gamma/E_{\rm beam}]$

❷ $R(s)$ 1 GeV $\lesssim \sqrt{s} \lesssim$ 2 GeV

Energy scan



requires non trivial upgrade of DAΦNE such that energy scan is possible

$\to F_\pi(s)$, $R(s)$ direct $\sqrt{s} \lesssim 2$ GeV.

Contributions to

$$\tilde{a}_\mu^{\rm had} = a_\mu^{\rm had} \times 10^{10}$$

from exclusive channels

Theoretical work with the aim to calculate radiative corrections at the level of precision as indicated in the Table is in progress.

$$\Downarrow$$

$$\delta a_\mu^{\rm had} \lesssim 26 \times 10^{-11}$$

from

$\sqrt{s} \lesssim 2$ **GeV.**

channel	$\tilde{a}_\mu^{\rm had}$	acc.
$\rho,\omega \to \pi^+\pi^-$	506	0.3%
$\omega \to 3\pi$	47	$\sim$ 1%
ϕ	40	$\downarrow$
$\pi^+\pi^-\pi^0\pi^0$	24	·
$\pi^+\pi^-\pi^+\pi^-$	14	·
$\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$	5	10%
3π	4	$\downarrow$
K^+K^-	4	·
K_SK_L	1	·
$\pi^+\pi^-\pi^+\pi^-\pi^0$	1.8	·
$\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$	0.5	·
$p\bar{p}$	0.2	·
$2\,\mathrm{GeV} \le E \le M_{J/\psi}$	22	
$M_{J/\psi} \le E \le M_\Upsilon$	20	
$M_\Upsilon < E$	$\lesssim 5$	

Daphne

❑ Radiative return: inclusive method

Photon tagging measurements: KLOE/Frascati, BaBar/SLAC, Belle/KeK

Normally "observed" cross section (C-invariant cuts, one–loop $\to$ no initial-final state interference):

$$
\begin{aligned}
\sigma^{\mathrm{obs}}(s) &= \sigma_0(s)\;[1+\delta_{\mathrm{ini}}(\Lambda_{\mathrm{IR}})+\delta_{\mathrm{fin}}(\Lambda_{\mathrm{IR}})] \\
&+ \int_{4m_\pi^2}^{s-2\sqrt{s}\Lambda_{\mathrm{IR}}} ds'\; \sigma_0(s')\rho_{\mathrm{ini}}(s,s') \\
&+ \sigma_0(s)\int_{4m_\pi^2}^{s-2\sqrt{s}\Lambda_{\mathrm{IR}}} ds'\; \rho_{\mathrm{fin}}(s,s')\;,
\end{aligned}
$$

unfolding problem to get $\sigma_0(s)$. Here additional problem: $\rho_{\mathrm{fin}}(s,s')$ model-dependent (only soft photon part known)!

Experimentally: acceptance cuts, efficiencies etc. in addition

Radiative return measurement: look at $\pi^+\pi^-$ invariant mass s' distribution $\left(\frac{d\sigma}{ds'}\right)$ plus anything (photon). s' fixed $\to$ missing energy fixed $\to$ "automatic" unfolding. Pion form factor ansatz:

$$\left(\frac{d\sigma}{ds'}\right)_{\mathrm{sym-cut}} = |F_\pi(s')|^2 \left(\frac{d\sigma}{ds'}\right)^{\mathrm{point}}_{\mathrm{ini,\, sym-cut}} + |F_\pi(s)|^2 \left(\frac{d\sigma}{ds'}\right)^{\mathrm{point}}_{\mathrm{fin,\, sym-cut}}$$

and hence we may resolve for the pion form factor as

$$|F_\pi(s')|^2 = \frac{1}{\left(\frac{d\sigma}{ds'}\right)^{\mathrm{point}}_{\mathrm{ini,\, sym-cut}}} \left\{ \left(\frac{d\sigma}{ds'}\right)_{\mathrm{sym-cut}} - |F_\pi(s)|^2 \left(\frac{d\sigma}{ds'}\right)^{\mathrm{point}}_{\mathrm{fin,\, sym-cut}} \right\} .$$

This is a remarkable equation since it tells us that the inclusive pion–pair invariant mass spectrum allows us to get the pion form factor unfolded from photon radiation directly as for fixed s and a given s' the photon energy is determined. The point cross sections are assumed to be given by theory and $d\sigma/ds'$ is the observed experimental pion–pair spectral function.

⑦ Status and Outlook

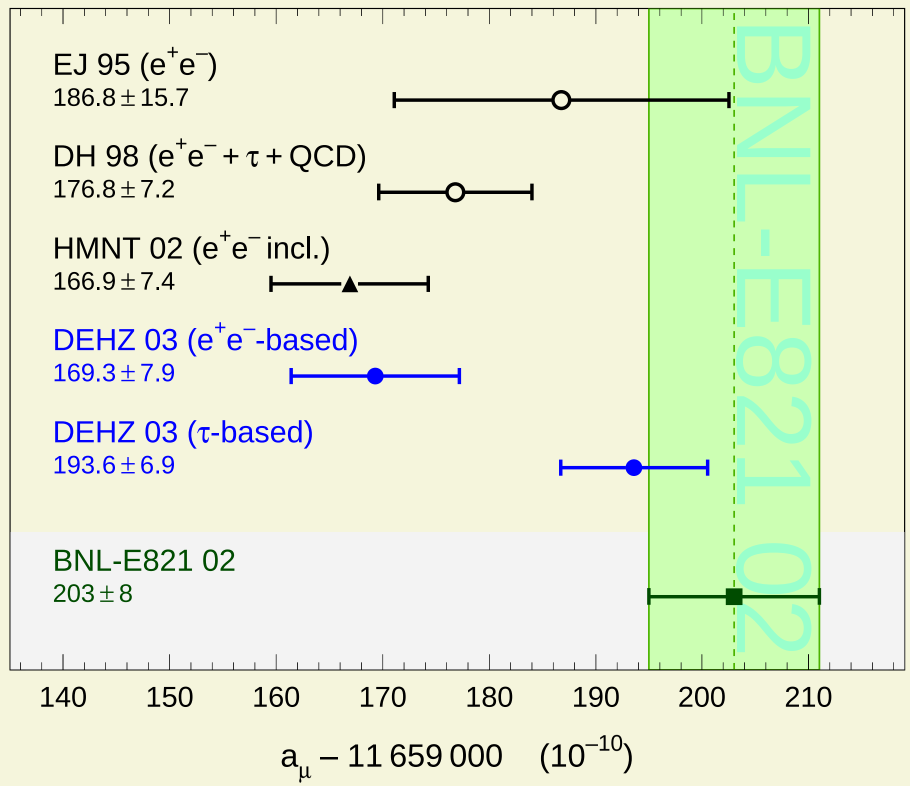
- High precision experiments on $a_\mu = (g-2)/2$ (BNL) and $\sin^2\Theta_{\rm eff}$, etc. (LEP/SLD) imposed a lot of pressure to theory to improve (or find errors in) their calculations and, in particular, to reduce hadronic uncertainties which mainly reflect the experimental errors of $R(s)^{\rm exp}_{\rm had}$
- Experimental groups have reconsider older data to reduce errors (CMD-2); new data from BES (20% $\to$ 7%) (2 GeV to 5 GeV), and τ data from ALEPH, OPAL, CLEO. The latter disagree with e^+e^- data in some regions at the 10% level and essentially lead to two “incompatible” prediction for $a_\mu^{\rm had}$.
- All kind of attempts to squeeze out of the old data more precise results; theory only partially can help. What is the appropriate “pseudo observable”?, What is missing (e.g., hard photon effects)?, What is double counted? Etc.
- Key role now for radiative return experiments on low energy hadronic cross sections: KLOE, BABAR,...; radiative corrections very crucial to get a precise answer. Theory: special effort by Karlsruhe group (Kühn et al.) to advance calculations.
- $(g-2)_\mu$: need settle ρ region and in addition range 1.4 GeV to 2 GeV.

- **Needs for linear collider (like TESLA): requires $\sigma_{\rm had}$ at 1% level up to the $\Upsilon \Rightarrow$ $\delta\alpha(M_Z)/\alpha(M_Z) \sim 5 \times 10^{-5}$. At present would allow to get better Higgs boson mass limits.**

- **Future precision physics requires dedicated effort on $\sigma_{\rm had}$ experimentally as well as theoretically (radiative corrections, final state radiation from hadrons etc.)**

$\boxed{a_\mu^{\rm had}}$ **Maximum confusion ! Likely only new experiments can settle the problems**

$\Rightarrow$ **Photon tagging experiments at KLOE, BARAR**



Future:

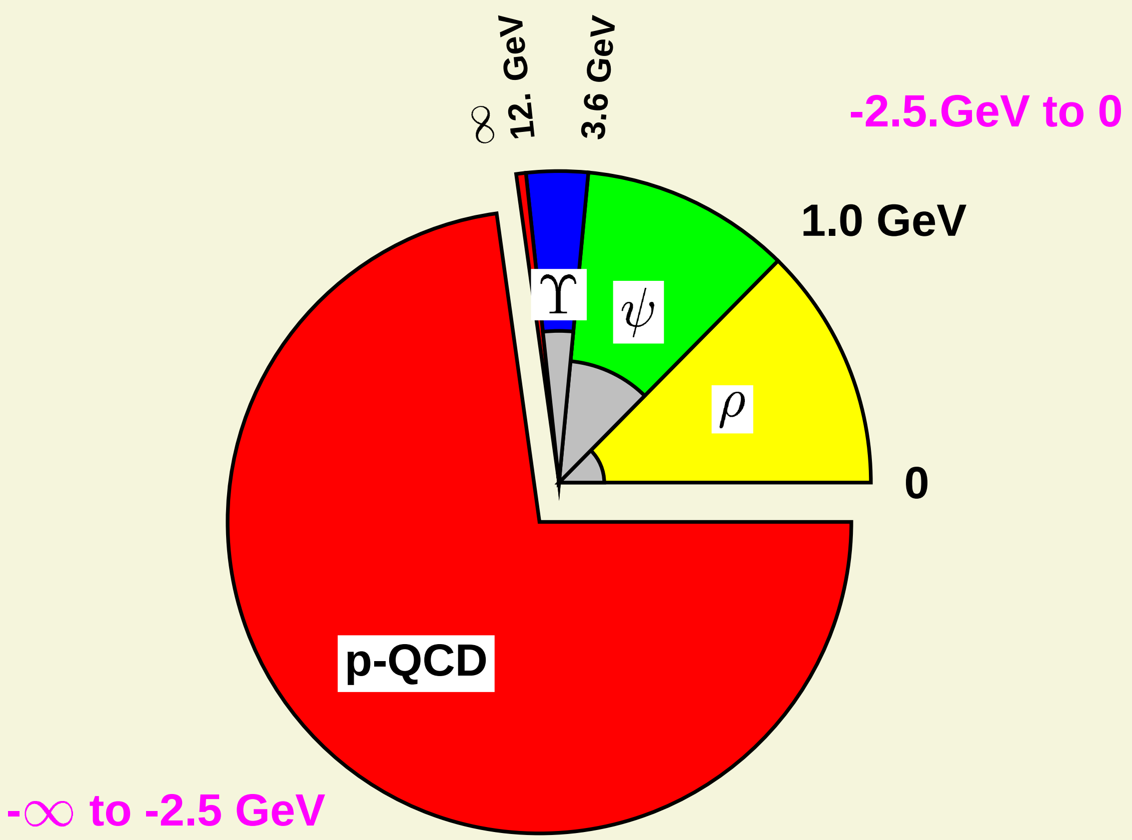
- KLOE (DAΦNE Frascati): expect a 1% measurement for $\sigma(e^+e^- \to hadrons)$ at $\sqrt{s} \lesssim$ 1 GeV
  - – photon tagging method — theory: $\sim$ 2%
  - – $\pi^+\pi^-$ inv. mass spectrum — theory: $\sim$ 0.5%
  - ✗ very important new measurement with different systematics (cross check of Novosibirsk measurements)
  - ✗ still a lot of theory efforts necessary !
    - large angle Bhabha ?
    - RC to photon tagging ?
    - photon radiation from hadrons ?
  - ✗ eagerly awaiting for fist release of KLOE data
    big attention for new input for $g-2$ now !
- CMD-2, SND (VEPP-2M Novosibirsk): CMD-2 final data analysis 0.6% systematics ?
  upgrade up to 2 GeV
- BES (BEPC Beijing): upgrade, higher luminosity, improved detector 2 to 5 GeV region

- **SLAC/BABAR:** $R(s)$ **measurement in range** $M_\Phi < E_{\mathrm{cm}} < M_{J/\psi}$
  Also: radiative return $\pi\pi$ **inv. mass spectrum at** B **factories (BABAR/SLAC, BELLE/KEK)**

Long term: need $\sigma(e^+e^- \to hadrons) \lesssim$ 1% up to 3.6 GeV both for $a_\mu^{\rm had}$ and for $\Delta\alpha^{\rm had}$:

Distribution of hadronic contributions to $\Delta\alpha^{\rm had}$

"Adler function based" approach (EJKV99,FJ 00):



Shaded area: 10 × error (scaled up)

- **τ–charm factory**

  able to perform an energy scan between 2 and 3.6 GeV

  would satisfy requirements of future precision experiments

  g-2, GigaZ,...