HADRONIC CONTRIBUTIONS TO $\pi^0(\not{Z} - \theta)$ and $\alpha_{\text{QED}}$ (Faser)
Hadron effects in

– theoretical uncertainties

Outline of Talk:

Introduction

Evaluation of \( \frac{\pi}{\eta} (\pi - \eta) \equiv \eta_\sigma \)

Evaluation of \((Z_W)^o\)

What pseudo-observable do we need?

Iso-spin breaking corrections in \( \eta \) vs. \( \eta + \pi \)

Status and Outlook
Hadronic effects in electroweak effects (leptons etc.) calculable in perturbation theory

Electroweak effects (leptons etc.) calculable in perturbation theory

Dispersion integrals over $\vec{p} + \vec{q}$

Errors of data in theoretical uncertainties

The art of getting precise results from non-precision measurements!

New challenge for precision experiments on non-perturbative hadron effects on $\mu + e$.

Data encoded in the theoretical uncertainties of data.

\[ \frac{\int \delta \sum_{\text{hadrons}}}{\int \delta \sum_{\text{hadrons}}} \equiv (s) H \]

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Error of data in theoretical uncertainties.

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Error of data in theoretical uncertainties.
In general:

Normalization: mostly by Bhabha [or itself]

\[ \frac{d \sigma}{d \Omega} = \left( s \right)^0 \frac{N_{\text{norm}}}{N_{\text{had}}} \left( \frac{\mathcal{B} \sigma + 1}{N_{\text{had}}} \right) \]

Experimential:

Hadronic effects in \( Z \) and \( \alpha \) – theoretical uncertainties
Hadronic effects in $(g-2)^\nu$, and $\omega$ - theoretical uncertainties

Recent progress (and a step back?):

- **2002**: Final CMD-2 data: syst error 1.4% (Akhmetshin et al.)

  \[ \frac{\Gamma}{\Gamma_{\text{had}}}(s) \sim \frac{10^{-10}}{10^{10}} \times \frac{11.75(8.25)}{\epsilon \, \epsilon_\pi} \]

  Previously unaccounted contributions: $e^+e^- - \text{data}$ at 10% level

  New situation: $T-$data not compatible with $e^+e^- - \text{data}$ at 10% level

- **2001**: BES-II data: syst error 20% (Bai et al. 2000, 2002)

  Region 2.0 GeV to 5.0 GeV

  **2000**: $T-$data

- **2000**: CMD-2, SND data at 1.4 GeV

  New VP subtraction!

  Some other CMD-2, SND data at 1.4 GeV

  $\omega$ - syst error 1.4% at 0.6% i

  Previous results in $(g-2)^\nu$ and $\omega$ - theoretical uncertainties
substantially lower than old $e^+ e^- - data$.
Hadronic effects in $\sigma - \sigma$ and $\pi^0$ [Hagiwara et al.] 02.

Most problematic $\pm e_\text{region now 1.4-2.0 GeV}$

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Special effort in interpretation of these data [Hagiwara et al.] 02.

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F. Jegerlehner: Photon 2003 – April 7-11, 2003 –

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Hadronic effects in $\sigma - \sigma$ and $\pi^0$ and $\eta$ – Theoretical uncertainties.
Non-perturbative hadronic contributions can be evaluated in terms of

\[ \left( s - s^0 \right) \frac{dP_{\text{had}}}{ds} \int_{s^0}^{s} + \int_{s^0}^{\infty} \left( s - s^0 \right) \frac{dP_{\text{data}}}{ds} \int_{s^0}^{s} \left( s - s^0 \right) \frac{dP_{\text{had}}}{ds} \]
Hadronic effects in $\left(ZW\right)$ and $\left(\pi - \pi\right)$ -- theoretical uncertainties

Evaluation F1 2002 update:

Evaluation F1 2003 -- Photon 2003 -- April 7-11, 2003 --

$\begin{align*}
128.936 \pm 0.046 &= (ZW)_{\text{BP01}}^0 \\
128.923 \pm 0.046 &= (ZW)_{\text{BP01}}^0 \\
0.0276 \pm 0.0036 &= (\Delta)_{\text{BP01}}^0 \\
0.0277 \pm 0.0034 &= (\Delta)_{\text{BP01}}^0
\end{align*}$

and for the high energy tail above 13 GeV

perturbative QCD from 5.0 to 9.6 GeV

and for J/Ψ resonances region between 9.6 and 13 GeV

$H(s)$ data up to $\sqrt{s}$ = 7 GeV

$e^+e^-\rightarrow ZW$ at $\sqrt{s} = 91.19$ GeV
Hadronic effects in (g − 2) and theoretical uncertainties

Evaluations of $a_T$ before 2000

All values have been rescaled to $M_W = 91.187$ GeV. The small top quark and the $W$ contributions are not included.
Recent evaluations of $\Delta (\not{Z}_W)$

Hadronic effects in $\delta - \rho$ and $\Delta (\not{Z}_W)$ – theoretical uncertainties

Jegerlehner (98), Hohler et al. (97), DCET, ...

Jegerlehner (01), \( R \)-integral

Jegerlehner (01), Euclidean

Buchardt & Pietrzyk (01)

Martini et al. (00), "inclusive"

Martini et al. (00), "exclusive"

Jegerlehner (98)
Hadronic effects in precision physics

Uncertainties of hadronic contributions to effective $\alpha$ are a problem for electroweak precision physics:

**Precision physics limitations:**

$\alpha, G_\mu, M_Z$ most precise input parameters

partially non-perturbative precision predictions

$\alpha(M_Z), G_\mu, M_Z$ best effective input parameters for VB physics (Z,W) etc.

\[
\begin{align*}
\frac{\delta \alpha}{\alpha} & \sim 3.6 \times 10^{-9} \\
\frac{\delta G_\mu}{G_\mu} & \sim 8.6 \times 10^{-6} \\
\frac{\delta M_Z}{M_Z} & \sim 2.4 \times 10^{-5} \\
\frac{\delta \alpha(M_Z)}{\alpha(M_Z)} & \sim 1.6 \div 6.8 \times 10^{-4} \quad \text{(present)} \\
\frac{\delta \alpha(M_Z)}{\alpha(M_Z)} & \sim 5.3 \times 10^{-5} \quad \text{(TESLA requirement)}
\end{align*}
\]

**LEP/SLD:**

\[
\begin{align*}
\sin^2 \Theta_{\text{eff}} = (1 - g_V l / g_A l) / 4 = 0.23148 \pm 0.00017 \\
\delta \Delta \alpha(M_Z) = 0.00036 \quad \Rightarrow \quad \delta \sin^2 \Theta_{\text{eff}} = 0.00013
\end{align*}
\]

affects Higgs mass bounds !!!
Hadronization effects in $\mu^+\mu^-$ and $\pi^0$ decay rate measurements

$$\sin^2 \theta^* = \frac{1 - 9 \lambda^2}{4}$$

**Indirect Higgs boson mass measurement**

Direct lower bound:
$$m_H > 193 \text{ GeV}$$

Indirect upper bound:
$$m_H < 114 \text{ GeV}$$

Higgs boson mass measurement

Hadronic effects in $(\pi^0 - Z\gamma)$ and $\Delta \sin^2 \theta^*$ - theorectical uncertainties
Hadronic effects in $\eta' = (7 - \delta)\eta_d$ and $\eta_d$ - theoretical uncertainties
Hadronic effects in $\Delta (g-2)$ and $\eta (g-2)$ – Theoretical uncertainties

Evaluation of hadronic vacuum polarization diagram is included.

Only the leading hadronic vacuum polarization diagram is included.
Hadronic effects in $(g - 2)_\mu$ and $\alpha(M_Z)$ – theoretical uncertainties

**Recent evaluations of $\alpha^\text{had}_\mu$**

- **Eidelman & Jegerlehner (95)**
- **Alemany et al. (97) ($e^+e^-$)**
- **Alemany et al. (97) ($e^+e^-,\tau$)**
- **Davier & Hoecker (97) pQCD+...**
- **Davier & Hoecker (98) SR+...**
- **Eidelman & Jegerlehner (98)**
- **Erler & Luo (01)**
- **Narison (01)**
- **Troconiz & Yndurain (01)**
- **Jegerlehner (01) CMD–2,BESII, Euclidean**
- **Jegerlehner (01) CMD–2,BESII, R-integral**
Hadronic effectsin

and •

Theoretical uncertainties

Higher order hadronic contributions

\[ \sum_{\eta \text{new physics}} \eta p + (\text{had})^{(\text{weak})} + (\text{had})^{(\text{weak})} + (\text{had})^{(\text{weak})} + (\text{had})^{(\text{weak})} + (\text{had})^{(\text{weak})} + (\text{had})^{(\text{weak})} = \eta p \]
**$\alpha_\mu$ Summary: experiment vs. theory**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>CERN (79)</td>
<td></td>
</tr>
<tr>
<td>KNY (85)</td>
<td></td>
</tr>
<tr>
<td>E821 (98)</td>
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<tr>
<td>E821 (99)</td>
<td>$\mu^+$</td>
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<td>E821 (00)</td>
<td>$\mu^+$</td>
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<tr>
<td>World</td>
<td></td>
</tr>
<tr>
<td>E821 (01)</td>
<td>$\mu^-$</td>
</tr>
<tr>
<td>E821 goal</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>EJ 95 ($e^+e^-$)</td>
<td>3.0 $\sigma$</td>
</tr>
<tr>
<td>181.3 $\pm$ 16.0</td>
<td>Theory ($e^+e^-$)</td>
</tr>
<tr>
<td>DEHZ 03 ($e^+e^-$)</td>
<td>0.9 $\sigma$</td>
</tr>
<tr>
<td>169.3 $\pm$ 07.8</td>
<td>Theory ($\tau$)</td>
</tr>
<tr>
<td>DEHZ02 ($\tau$)</td>
<td></td>
</tr>
<tr>
<td>193.6 $\pm$ 06.9</td>
<td></td>
</tr>
<tr>
<td>FJ 02 ($e^+e^-$)</td>
<td></td>
</tr>
<tr>
<td>168.3 $\pm$ 09.3</td>
<td></td>
</tr>
<tr>
<td>HMNT02 ($e^+e^-$ incl.)</td>
<td>166.9 $\pm$ 07.4</td>
</tr>
<tr>
<td>NAR03 ($e^+e^- + S\gamma$)</td>
<td></td>
</tr>
<tr>
<td>179.9 $\pm$ 11.1</td>
<td></td>
</tr>
<tr>
<td>NAR03 ($\tau + S\gamma$)</td>
<td></td>
</tr>
<tr>
<td>202.3 $\pm$ 11.3</td>
<td></td>
</tr>
</tbody>
</table>

\[
(a_\mu - 11659000) \times 10^{-10}
\]

Given theory results only differ by $a_\mu^{\text{had}(1)}$!
Hadronic effects in $Z_W$ and $\phi$ - theoretical uncertainties

Type and size of contributions
Hadronic effects in ...

What pseudo-observable do we need?

\[ \left( (0), \Pi - (s), \Pi \right) \text{Re} - = \varphi \nabla \]

\[ \frac{\varphi \nabla - 1}{\varphi} = (s) \varphi \]

What do we need for calculating \( \varphi \) and \( (s) \varphi \)?

---

Strong, electromagnetic, or weak interaction. At low energies:

\[ \text{ hadron } \leftrightarrow \text{ pseudo-observable } \]

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\[ \text{ hadron } \leftrightarrow \text{ pseudo-observable } \]
Hadronic effects in physical quantities must include photonic corrections to the hadronic 1PI:

\[ \zeta((s)^D)^{\nu}/(s)^{\nu}\eta = (s)^{\nu}\eta \]

(see also: Mehnikov, Trocinski&Yndurain 2001)

\[ \times \left( \frac{38^\circ}{1^\circ} \right) = \left( \frac{n^\circ}{1^\circ} \right) \]

corresponding to the anomalous magnetic moment of the muon is a known correction factor (Schwinger 1989). The corresponding theoretical prediction for FS radiation (including full photon phase space):

\[ \cdots + \text{had} + \text{had} \]

Add theoretical prediction for FS radiation (including full photon phase space):

\[ \zeta((s)^D)^{\nu}/(s)^{\nu}\eta = (s)^{\nu}\eta \]

\[ \Rightarrow \]
Hadronic effects in
\( (g - \gamma) (Z W) \)
and formal theoretical uncertainties

KLN theorem at work

theoretical uncertainties

\( \text{model ambiguity (QCD vs. QED)} \)

\( \text{adding missing H part using the same model (as used for subtracting V+S).} \) \( 1\% \)

\( \text{model ambiguity (QCD vs. QED)} \)

\( \text{cutting out all hard photons and subtracting V+S using QCD.} \) \( 5\% \)

\( \text{All logs cancel in sum (inclusive)?} \)

\( \text{V+S log's:} \)

\( \text{H: log's the same as V+S but precisely of opposite sign!} \)

\( \text{complementary hard photon part (summing to full phase space):} \)

\( \text{small positive correction typically} \sim \cdot 2\% \)

\( \text{exclusively qualitative incl. large Log's on photon energy cut and collinear logs!} \)

\( \text{section:} \)

\( \text{fully inclusive = including virtual (V) plus soft (S) plus all hard (H) photons cross (} \text{Kinoshita, Lee  \\& Nauenberg)} \)
Hadronic effects in (\(\not p\)) and \(\not p(\not p - \theta)\) \(\not p(\not p - \theta)\) theoretical uncertainties

\[ |(s)^\mu H((s), II - I)| \leq |(s)^\mu H| \]

Allows us to get substantially improved low energy contributions.

\[ \zeta(s_H) = \sum |(s)^\mu H((s), II - I)| \leq |(s)^\mu H| \]

Work in progress: Colangelo, Casser, Leutwyler, F.L.

Omnès–Muskhelishvili Theorem:

\[ \zeta(s_H) \]

How to get phase of \(H\)

\[ \zeta(s)^\mu H \]

Analyticity + Unitarity + Chiral limit

Relates: Space–like data, \(s\)–scattering phase shifts and time–like data

Severe theoretical constraints!

\(\zeta\)–space

(see also: Geshkenbein ... 89, Yndurain ... 01)

(see also: Geshkenbein ... 89, Yndurain ... 01)
In terms of the $\gamma$ spectral function $\nu_1$.

\[ \nu \gtrsim \sqrt{s} \quad \therefore \quad X^2 \frac{s}{4m^2} = 0 \quad \gamma^\nu I = I \nu \]

The cross-section is then given by

The isovector part of $e^- p = \pi^0$ and the hadronic states related by iso-spin rotation.

The isovector part of $e^- p$ may be calculated by iso-spin rotation from $\pi^0$ via $\gamma$-decay.

Hadronic effects in $Z_W$ and $\pi(\pi - \delta)$ - theoretical uncertainties
All kind of isospin breaking effects have to be taken into account!!

\[
\begin{align*}
0.0005 \leftarrow 0.0067 & : \pi^0 \\
10.2 \times 10^{-10} \leftarrow 15.6 \times 10^{-10} & : \eta_c \\
\end{align*}
\]

ADH 95:

\[\text{Additional} \ \pi^0 \text{ data cannot replace } e^+ e^- \text{ data}\]

\[\text{which is dominating in } \phi_{\text{had}} \ (72\%)\]

\[\text{mainly improves the knowledge of the } \phi^+ \pi^- \text{ channel (}\phi\text{ resonance contribution)}\]

ALPHA, OPA, CLEO (ADH96, DEHZ02)

Additional "e^+ e^- data + CC + data + \text{hadronic effects in } (\phi - \eta)\text{ and } M_W\)}
Hadronic effects in (ZW) and o
\( \eta(2) - \delta \) theoretical uncertainties

- systematic deviations largely can be understood!
After correction no systematic deviations between CLEO/ALEPH and DM1/OLYA/CDM1/CDM2 data beyond fluctuations of $e^+e^-$ data, while OPAL data show substantial deviations.

$e$ determined by fit to the data: $e = 0.00172$

$\zeta(\frac{m - \zeta}{\gamma} - m_W) = m_s$

With

$\zeta \left( \frac{(s - m_s)}{s^2} + 1 \right) \left/ \left( s - m_{data} \right) \right| = \zeta \left| (s) \Delta \right|$

$\Delta_{data} = data_{corrected}$ for iso-spin violations:

$Z(W)_{\zeta}(\zeta - 6) + \pi(\zeta - 6) - \zeta \left( \Delta \right)$
Hadronic effects in

\[ (S) \frac{\nu}{A} \]

\[ \text{data} \]

\[ \text{data analytized} + \text{unitarized} + \text{chiral limit: solid line vs. data} \]

\[ \text{NA7+CMD-2 data analytized} + \text{unitarized} + \text{chiral limit: solid line vs. data} \]

\[ \text{F.Jegerlehner Photon 2003 – April 7-11, 2003} \]
QED corrections are obviously not related by a iso-spin rotation and must be subtracted before CVC arguments can be applied.

FSR correction in $\tau$-decay: Inclusive approach.

Iso-spin breaking in $\tau$ vs. $\nu$: ☕️

Hadronic effects in $f(2Z, \theta)$ and $t(\pi - \theta)$ - Theoretical uncertainties.
Hadron effects in $\Lambda_f$ and $\gamma$ decay. $\gamma$ and $\delta$ theoretical uncertainties.
<table>
<thead>
<tr>
<th></th>
<th>$\eta$</th>
<th>FF</th>
<th>EM</th>
<th>KIN</th>
<th>$S_{EW}$</th>
<th>$\eta_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{IB}$</td>
<td>-120</td>
<td>61 $\pm$ 26 $\pm$ 3</td>
<td>-10</td>
<td>61 $\pm$ 26 $\pm$ 3</td>
<td>-11</td>
<td>-119 $\pm$ 26 $\pm$ 3</td>
</tr>
<tr>
<td>$\eta$</td>
<td>-120</td>
<td>61 $\pm$ 26 $\pm$ 3</td>
<td>-10</td>
<td>61 $\pm$ 26 $\pm$ 3</td>
<td>-11</td>
<td>-120 $\pm$ 26 $\pm$ 3</td>
</tr>
</tbody>
</table>

Summary of results:

Contributions to $\frac{\eta_{max}^{\eta_{\eta \eta}}}{\text{max}}$ (in units of GeV²)

For different values of $\eta_{max}$ (in units of GeV²).

From various sources of iso-spin violation (in units of $10^{-11}$).

Hadronic effects in $(\delta - \eta) \text{ and } \eta_{\gamma \gamma}$ and theoretical uncertainties.
Hadron effects in 

- Surprisingly, after corrections at larger energies, 10% deviations!

Isospin breaking effects: (Cirigliano et al.)

Hadrronic effects in $\left(\frac{Z_{\mu}}{\delta}\right)$ and $\eta(\tau - \delta)$ - theoretical uncertainties.
Hadronic effects in $(g-2)\,\mu$ and $\alpha$ and theoretical uncertainties

\textit{cannot be understood within SM} \textit{likely an experimental problem}

The plots above the region $\theta \geq \theta_{\text{arg}}$ (Feynman $I$ GEP)

The plots below the region $\theta \geq \theta_{\text{arg}}$ (HLO2)
Hadronic effects in $\mathcal{Z}_{\mu\nu}$ and $\mathcal{O}(\mathcal{Z} - \delta)$ - Theoretical uncertainties

Comparison of $\tau$-data:

ALEPH vs. OPAL no good agreement.

$\tau$-data may be not so easy; DELPHI, L3 could not measure $\tau$-spectral functions;
Hadronic effects in $(Z_J, \phi)$ and $\pi^0(\gamma - \theta)$ - Theoretical uncertainties

The measured branching ratios for $B(\tau^- \to \mu^- \nu \bar{\nu})$ compared to the prediction from the CVC after applying the isospin breaking correction.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Branching Ratio (in %)</th>
<th>CLEO</th>
<th>OPAL</th>
<th>ALEPH</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>25.42 ± 0.42</td>
<td>25.44 ± 0.34</td>
<td>25.47 ± 0.13</td>
<td>25.46 ± 0.12</td>
</tr>
</tbody>
</table>

The measured branching ratios for $B(\tau^- \to \mu^- \nu \bar{\nu})$ compared to the prediction from the CVC after applying the isospin breaking correction.
Hadronic effects in $\gamma \gamma$ 2 GeV

Requires non-trivial upgrade of DAΦNE such that energy scan is possible

Energy scan

hadrons

\[
\left[ \frac{\mathcal{E}_{\text{beam}}}{\gamma \mathcal{E}} = \gamma \right] \quad (\gamma - 1) s = s \quad \Phi_{\gamma} = s
\]

Photon tagging

hard

Precise measurement of $\phi$ at DAΦNE

Aim: Reduce hadronic uncertainty in future of precision physics

In progress by KLOE at DAΦNE

Precise measurement of $\phi$ at DAΦNE

Hadronic effects in $Z \gamma$ and $\gamma \gamma$ - theoretical uncertainties
Hadronic effects in 

\[ \frac{M}{2} \geq 2 \text{ GeV} \]

from

\[ \eta_{\text{had}} \geq 26 \times 10^{-11} \]

\[ \eta_{\text{had}} \]

\[ \uparrow \]

In the Table is in progress.

At the level of precision as indicated to calculate radiative corrections theoretical work with the aim

From exclusive channels

\[ \eta_{\text{had}} \]

\[ \times \eta_{\text{had}} \]

\[ \frac{\eta_{\text{had}}}{10} \]

Contributions to

Hadronic effects in \((Z/M)\alpha\) and \(\eta(\gamma - \phi)\) theoretical uncertainties.
Experimentially: acceptance cuts, efficiencies etc. In addition

(only soft photon part known)

unfolding problem to get $(s)^0 o$.

Here additional problem: model-dependent

\[ \frac{\pi}{\sqrt{s}} \int_{s}^{2m} (s)^0 o \] +

\[ \frac{\pi}{\sqrt{s}} \int_{s}^{2m} (s)^0 o \]

\[ \left[ (\gamma V)^{\mu\nu} \phi + (\gamma V)^{\mu\nu} \phi + 1 \right] (s)^0 o = (s)^0 o \]

Interference:

Normally „observed“ cross section $\gamma V$-invariant cuts, one-loop no initial-final state

Photon-tagging measurements: KLOE/Frascati, BABAR/SLAC, BELLE/KEK

Radiative Return: Inclusive method

Hadronic effects in $Z^\nu o$ and $\pi(o - \nu)$ theoretical uncertainties
the observed experimental pion–pair spectral function.

Photon energy is determined. The point cross sections are assumed to be given by theory and is a given $s$ and a given $\Theta$.

This is a remarkable equation since it tells us that the inclusive pion–pair invariant mass spectrum allows

\[
\left( \frac{d^2p}{dp}\right)_{\text{cut}} \frac{d^2m}{dm} \left| \left( \frac{d^2p}{dp}\right)_{\text{cut}} \right| = \left| \left( \frac{d^2p}{dp}\right)_{\text{cut}} \right|
\]

and hence we may resolve for the pion form factor as

\[
\left( \frac{d^2p}{dp}\right)_{\text{cut}} \frac{d^2m}{dm} \left| \left( \frac{d^2p}{dp}\right)_{\text{cut}} \right| + \left( \frac{d^2p}{dp}\right)_{\text{cut}} \frac{d^2m}{dm} \left| \left( \frac{d^2p}{dp}\right)_{\text{cut}} \right| = \left( \frac{d^2p}{dp}\right)_{\text{cut}} \frac{d^2m}{dm} \left| \left( \frac{d^2p}{dp}\right)_{\text{cut}} \right|
\]

Factor ansatz:

anything (photon) $s$ fixed missing energy fixed, $\Theta$ "automatically" unfolding. Pion form factor measurement: look at $\frac{d^2p}{dp}$ and $\frac{d^2m}{dm}$ plus $s$ invariant mass distribution. Relative return measurement: look at $\frac{d^2p}{dp}$ and $\frac{d^2m}{dm}$ and $s$ theoretical uncertainties.
Hadronic effects in single \( (\ell - \ell') \) in addition range \( 1.4 \text{ GeV to 2 GeV} \).

Special effort by Karlsruhe group (Kühn et al.) to advance calculations.

KLOE, BABAR...; radiative corrections very crucial to get a precise answer. Theory: theoretical uncertainties.

Key role now for radiative return experiments on low energy hadronic cross sections.

Experimental groups have reconsidered older data to reduce errors (CMD-2); new data from BES (20\% ± 7\%) (2 GeV to 5 GeV); and data from ALEPH, OPAL, CLEO. The latter disagree with data in some regions at the 10\% level and essentially lead to two "incompatible" predictions for \( \eta_{\text{had}} \dagger \). What is double counted? etc.

All kinds of attempts to squeeze out of the old data more precise results; theory only partially can help. What is the appropriate "pseudo observable"? What is missing?

Experimenters need to settle this: region and in addition range 1.4 GeV to 2 GeV.

High precision experiments in theoretical uncertainties

\( \Theta \) \( (\pi W) \)\( \propto \eta (\ell - \ell') \)
Hadronic effects in $(\eta - \phi)$ theoretical uncertainties

Future precision physics requires dedicated effort on $\eta_{\text{had}}$ experimentally as well as theoretically (radiative corrections, final state radiation from hadrons etc.)

Maximum confusion! Likely only new experiments can settle the problems

At present would allow to get better Higgs boson mass limits.

Needs for linear collider (like TESLA): requires $\eta_{\text{had}}$ at 1% level up to the...
Hadronic effects in $(g - 2)_\mu$ and $\alpha$ — theoretical uncertainties

Future:

- **KLOE (DAΦNE Frascati):** upgrade to $2 \text{ GeV}$
  - upgrade up to $2 \text{ GeV}$
  - CMD-2, SND (VEPP-2M Novosibirsk): CMD-2 final data analysis, $0.6\%$ systematics.
  - Big attention for new input for $g - 2$ now!
  - Eagerly awaiting for first release of KLOE data.
  - Photon radiation from hadrons?
  - HC to photon tagging?
  - Large angle Bhabha?
  - Still a lot of theory efforts necessary!

- **Novosibirsk measurements:**
  - Very important new measurement with different systematics (cross check at $\sim 0.5\%$ theory.
  - $\sim 2\%$ theory.
  - Inverse mass spectrum $\ell^+\ell^-$, photon tagging method $\gamma < \text{GeV}^1$, $\gamma < \text{GeV}^1$.

- **CMD-2, SND (VEPP-2M Novosibirsk):** CMD-2 final data analysis, $0.6\%$ systematics.

- **BES (BEPCE Belle):** upgrade, higher luminosity, improved detector $2$ to $5 \text{ GeV}$ region.

- **CMD-2, SND (VEPP-2M Novosibirsk):** CMD-2 final data analysis, $0.6\%$ systematics.

- **CMD-2, SND (VEPP-2M Novosibirsk):** CMD-2 final data analysis, $0.6\%$ systematics.

- **CMD-2, SND (VEPP-2M Novosibirsk):** CMD-2 final data analysis, $0.6\%$ systematics.
Hadronic effects in $\zeta \phi (\zeta - \theta)$ and $\phi W$ (BABAR/SLAC, BELLE/KEK).

Also: Radiative return in mass spectrum at B factories (BABAR/SLAC).

$\phi f W > \phi W > \phi W$ measurement in range $R(s)$.
Long term: need $\sigma(e^+e^- \rightarrow hadrons) \lesssim 1\%$ up to 3.6 GeV both for $a_\mu^{\text{had}}$ and for $\Delta\alpha^{\text{had}}$.

Distribution of hadronic contributions to $\Delta\alpha^{\text{had}}$

“Adler function based” approach (EJKV99,FJ00):

$\begin{array}{c}
1.0\text{ GeV} \\
3.6\text{ GeV} \\
12.\text{ GeV}
\end{array}$

Shaded area: $10 \times$ error (scaled up)

- $\tau$–charm factory
  able to perform an energy scan between 2 and 3.6 GeV
  would satisfy requirements of future precision experiments
  g-2, GigaZ,...