

# A new 5 Flavour LO Analysis of Parton Distributions

and

## the Parton Parametrization of the Real Photon

hep-ph/0212160, submitted to Phys. Rev. D

<http://www.fuw.edu.pl/~pjank/para.html>

<http://www-zeuthen.desy.de/alorca/id4.html>

F. Cornet, Granada U.

P. Jankowski and M. Krawczyk Warsaw U.

A. Lorca, DESY-Zeuthen

PHOTON 2003,

April 2003, Frascati, Italy

CJKL

## How well do we know the PHOTON?

• a realistic spectra of partons inside a photon

• We present (at PHOTON 2001 - a promise):

• a new LO analysis of the 'structure' of an **unpolarised, real photon**

• all available  $F_2(x, Q^2)$  data are used, including new LEP data at high  $Q^2$  (up to 2001)

• a special treatment of the heavy quarks threshold a'la **ACOT <sub>$\chi$</sub>**

S. Kretzer, C. Schmidt, W. Tung – J. Phys. G28, 983 (2002)

## Stage I: two fits, one parametrization (analytic) CJKL

- Radiatively generated  $u, d, s, c, b$  and gluon densities in real  $\gamma$ .
- Global 3 parameter fit, based on LO DGLAP evolution equations (Klasen et al.' 02 NNO fit to extract  $\alpha_s$ )
- All available data for the structure function  $F_2^\gamma(x, Q^2)$
- A new theoretical approach ACOT $_\chi$ , originally introduced for the proton to deal with heavy quark thresholds.
- $\Rightarrow$  CJKL model  $\Rightarrow$  FFNS $_{c j k l}$  model (for a comparison)
- a good description of the experimental data on  $F_2^\gamma(x, Q^2)$   
CJKL better than FFNS $_{c j k l}$
- A simple analytic parametrization of the resulting parton densities in the CJKL model

Stage II (new): three fits, two parametrizations (grid) CJKL 1, CJKL 2

## Parton Model for the photon $\rightarrow$ Bethe-Heitler process

Deep Inelastic Scattering on a real photon (target)

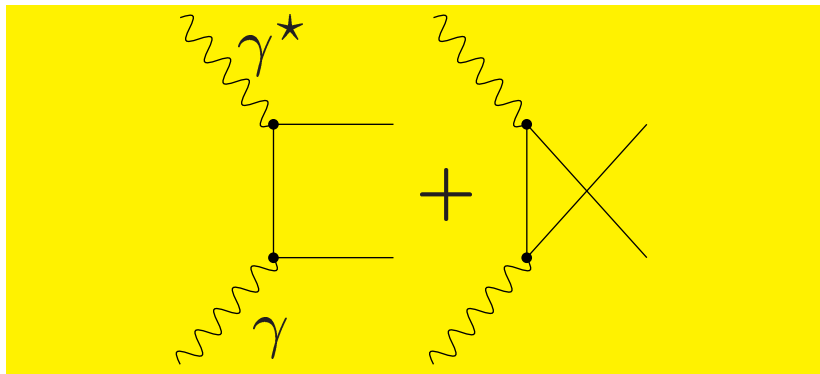
Structure Functions for  $\gamma$ , eg  $F_2^\gamma \Rightarrow$  sum over  $q^\gamma(x, Q^2)$

large at large  $x$

$\log Q^2$  - dependence in PM !

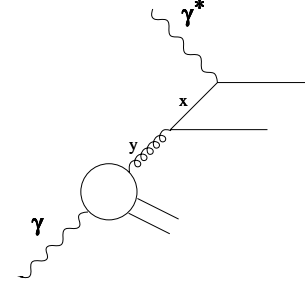
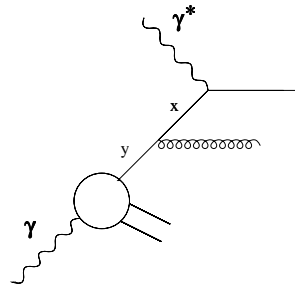
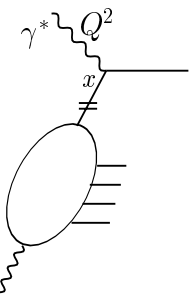
For energy  $W$  larger than  $2 m_Q \Rightarrow$  production of two quarks is possible

$W \rightarrow$



QCD modifies this 'PM'  $\log Q^2$  behaviour: collinear configuration of emission of gluons leads to corrections:  $\alpha_s \log Q^2$

$\Rightarrow$  DGLAP evolution equation



$$\frac{dq_i^\gamma(x, Q^2)}{d \ln Q^2} = \frac{\alpha}{2\pi} e_i^2 k(x) + \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[ P_{qq}\left(\frac{x}{y}\right) q_i^\gamma(y, Q^2) + P_{qG}\left(\frac{x}{y}\right) G^\gamma(y, Q^2) \right] \quad (1)$$

$$\frac{dG^\gamma(x, Q^2)}{d \ln Q^2} = 0 + \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[ P_{Gq}\left(\frac{x}{y}\right) \sum_{i=1}^{2 \times N_f} q_i^\gamma(y, Q^2) + P_{GG}\left(\frac{x}{y}\right) G^\gamma(y, Q^2) \right]$$

The  $k(x)$  term is from the Bethe-Heitler process, describes a photon into quark splitting function,  $P_{q\gamma}$  (for 3 colours)

$$k(x) = 3 \left[ x^2 + (1-x)^2 \right]$$

$P_i(x)$  - are the LO splitting functions,

$$P_{qq}(x) = \frac{4}{3} \left[ \frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right]$$

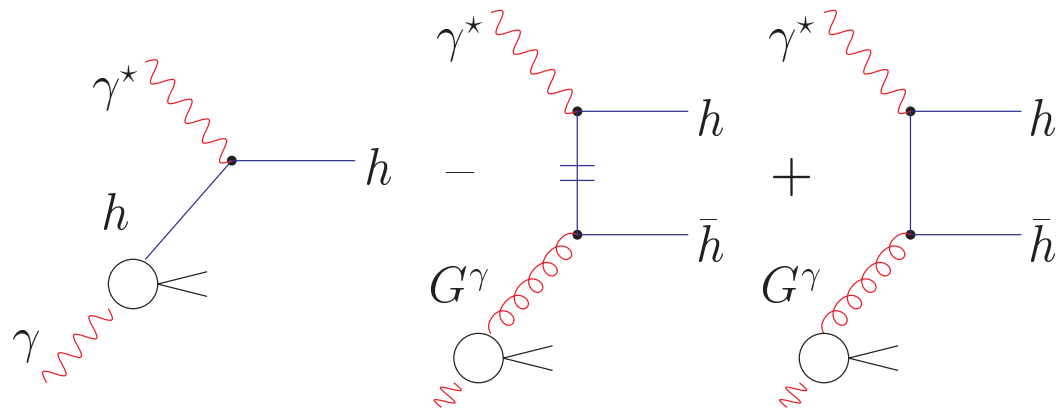
$$P_{qG}(x) = \frac{1}{2} [x^2 + (1-x)^2]$$

$$P_{Gq}(x) = \frac{4}{3} \frac{1+(1-x)^2}{x}$$

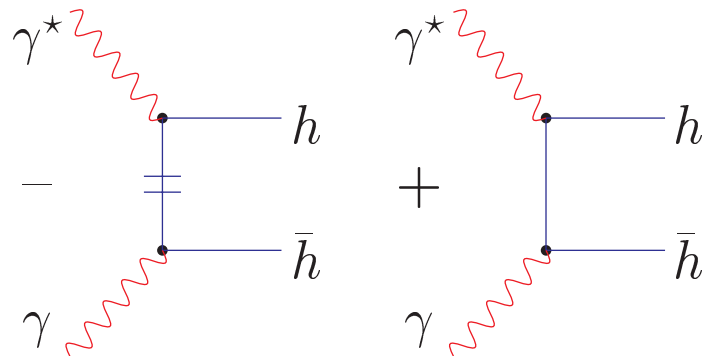
$$P_{GG}(x) = 6 \left[ \frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \left[ \frac{11}{2} - \frac{N_f}{3} \right] \delta(1-x)$$

# ACOT<sub>x</sub> approach for a photon– heavy quarks h

in DGLAP evolution eqs. heavy quarks as partons and/or Bether-Heitler production of heavy quarks (FFNS=B-H)



There is a double counting here, one should subtract some terms: logarithmic contribution in diagrams with “=”





## Threshold for heavy quark $h$

Bethe-Heitler process leads to:

$$\frac{1}{x} F_{2,h}^{\gamma}(x, Q^2)|_{direct} = 3 \frac{\alpha}{\pi} e_h^4 \omega(x, Q^2)$$

with

$$\omega(x, Q^2) = \left[ \beta \left( -1 + 8x(1-x) - x(1-x) \frac{4m_h^2}{Q^2} \right) + \left( x^2 + (1-x)^2 + x(1-3x) \frac{4m_h^2}{Q^2} - x^2 \frac{8m_h^4}{Q^4} \right) \ln \left( \frac{1+\beta}{1-\beta} \right) \right],$$

$$\beta = \sqrt{1 - \frac{4m_h^2 x}{(1-x)Q^2}} = \sqrt{1 - \frac{4m_h^2}{W^2}}.$$

for large  $Q^2$ :  $\ln \left( \frac{1+\beta}{1-\beta} \right) \rightarrow \ln Q^2 / m_h^2$

Threshold:  $1 - \frac{4m_h^2}{W^2} > 0$  - In the evolution  $Q^2$  is a natural variable -  
 setting a threshold condition difficult

$\chi$  variable

Another heavy quark contribution to  $F_2^\gamma$ :

$$\gamma^* G^\gamma \rightarrow h \bar{h},$$

with a gluonic parton of the initial real photon

$$F_{2,h}^\gamma(x, Q^2)|_{resolved} = \frac{\alpha_s(Q^2)}{2\pi} e_h^2 \int_\chi^1 \frac{x}{z} \omega\left(\frac{x}{z}, Q^2\right) G^\gamma(z, Q^2) dz$$

$$\chi \equiv x(1 + 4m_h^2/Q^2)$$

Kinematics OK:  $\chi$  - lower limit of integration over  $z$  -  
the (relative) momentum of gluon in a photon

$\chi \rightarrow 1$  contribution  $\rightarrow 0!$

Note, that  $\chi$  approaches  $x$  for a large  $Q^2$

This analysis **CJKL** (DIS02)

We follow ACOT<sub>χ</sub> approach, and introduce the χ variable instead of x to deal with the heavy quark threshold

Finally, we have

$$\begin{aligned}
 F_2^\gamma(x, Q^2) = & \sum_{i=1}^{2 \times 3} x e_i^2 q_i^\gamma(x, Q^2) + \sum_{h=c,b}^{2 \times 2} x e_h^2 q_h^\gamma(\chi, Q^2) \\
 & + \sum_{h=c,b}^{2 \times 2} \left[ F_{2,h}^\gamma(x, Q^2)|_{direct} + F_{2,h}^\gamma(x, Q^2)|_{resolved} \right] \\
 & - \chi \sum_{h=c,b}^{2 \times 2} \ln \frac{Q^2}{m_h^2} \left[ e_h^4 \frac{\alpha}{2\pi} k(\chi) + e_h^2 \frac{\alpha_s}{2\pi} \int_\chi^1 \frac{dy}{y} P_{qG} \left( \frac{\chi}{y} \right) G^\gamma(y, Q^2) \right].
 \end{aligned}$$

Note, that the direct contribution does not disappear at  $\chi \rightarrow 1$

stage II.

## Solving DGLAP evolution eqs.

In the solving the DGLAP evolution eqs. we use Mellin technique.

=> point-like (PL) and hadronic part for each parton density.

PL can be obtained without any input, hadronic parts need input at  $Q_0^2$  scale. We take  $Q_0^2 = 0.25$ .

Input: VMD model at low scale

Following GRV our input scale is low :  $Q_0^2 = 0.25$ .

For hadronic part we have to model

$$f_{had}^\gamma(x, Q_0^2) = \sum_V \frac{4\pi\alpha}{\hat{f}_V^2} f^V(x, Q_0^2)$$

$$f_{had}^\gamma(x, Q_0^2) = \kappa \frac{4\pi\alpha}{\hat{f}_\rho^2} f^\rho(x, Q_0^2).$$

$$xv^\rho(x, Q_0^2) = N_v x^\alpha (1-x)^\beta,$$

$$xG^\rho(x, Q_0^2) = N_g xv^\rho(x, Q_0^2)$$

$$v^\rho = (u^{\rho^+} + \bar{u}^{\rho^-} + d^{\rho^-} + \bar{d}^{\rho^+})/4$$

with normalization to 1.

## Constraints

We impose two constraints on the parameters of both models

$$\int_0^1 2v^\rho(x, Q_0^2) dx = 2$$

$$\int_0^1 x(2v^\rho(x, Q_0^2) + G^\rho(x, Q_0^2)) dx = 1$$

Using them  $\Rightarrow$  the normalization factors  $N_v, N_g$

We have  $\alpha, \beta$  and  $\kappa$  as free parameters

We fix them by fitting the  $F_2^\gamma$  experimental data.

The fits: stage I

$N_f=3$  (FFNS),  $N_f=5$  (CJKL)  
 $\alpha_s(Q^2)$

the direct contribution does not disappears for  $\chi \rightarrow 1$  (large  $x$ )

no error analysis

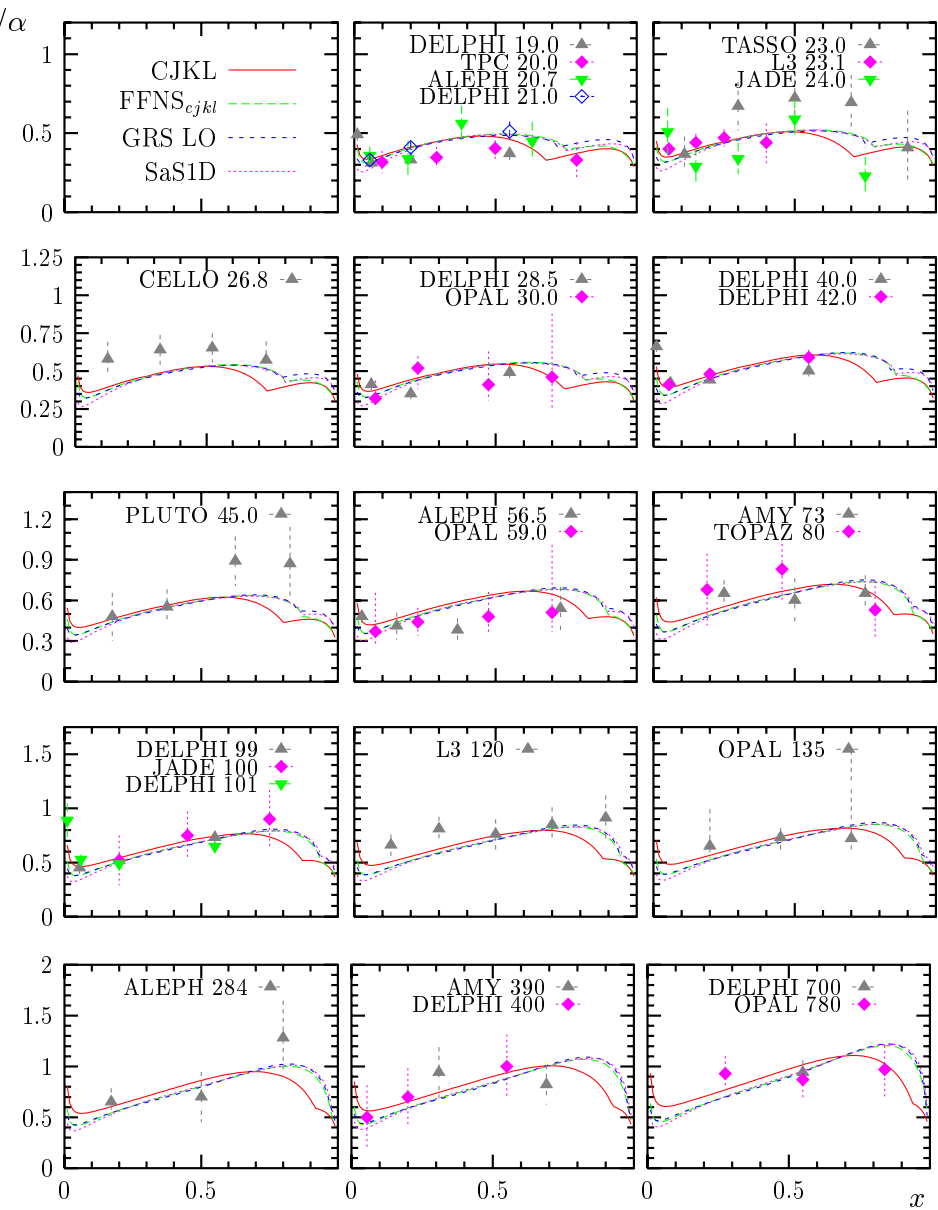
model	$\chi^2$ (208 pts)	$\chi^2/DOF$	$\kappa$	$\alpha$	$\beta$	$N_v$	$N_{gl}$
FFNS <sub><i>cjkl</i></sub>	471	2.30	1.726	0.465	0.127	0.504	1.384
CJKL	431	2.10	1.125	0.843	2.359	2.435	2.982

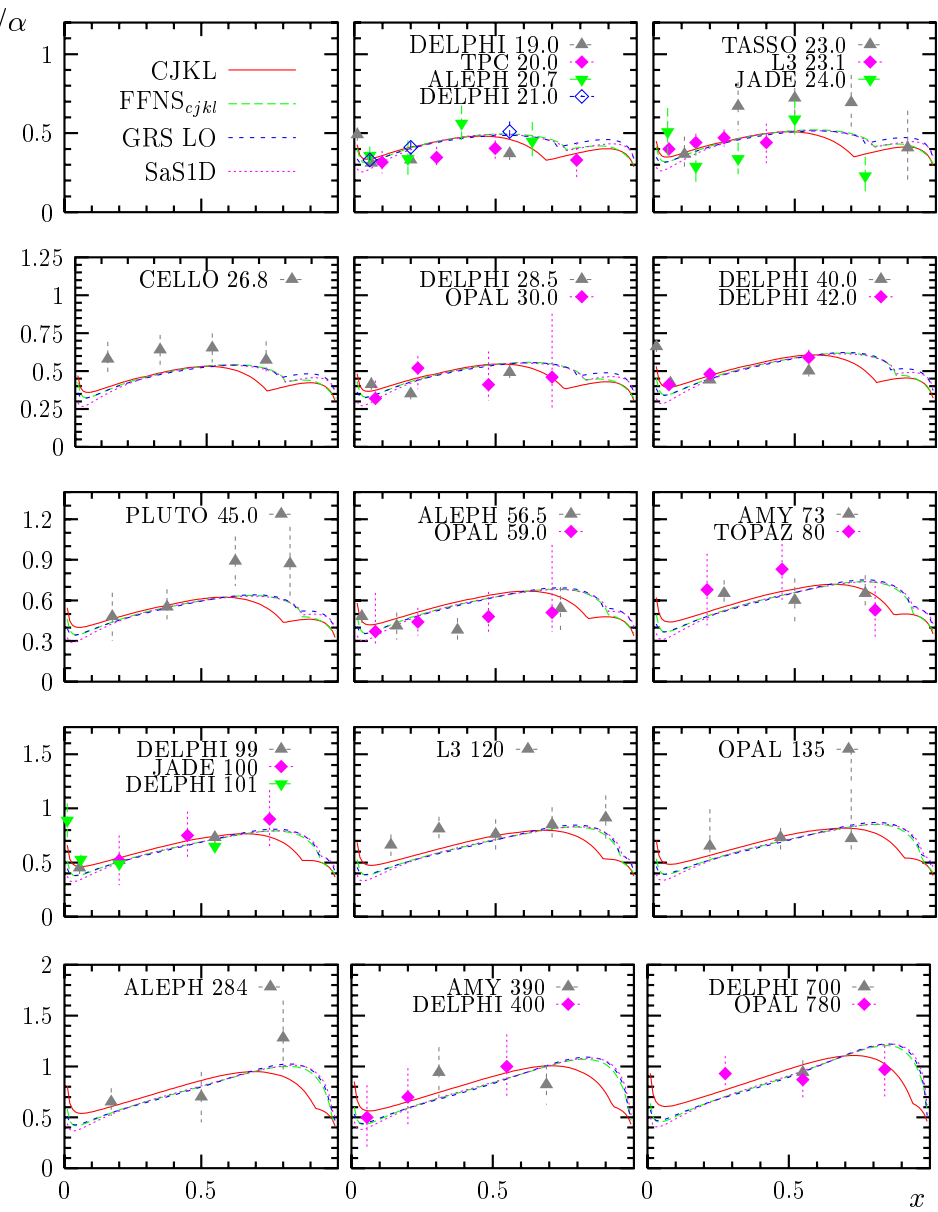
Comparison of CJKL and FFNS<sub>*cjkl*</sub> fits with other parametrizations

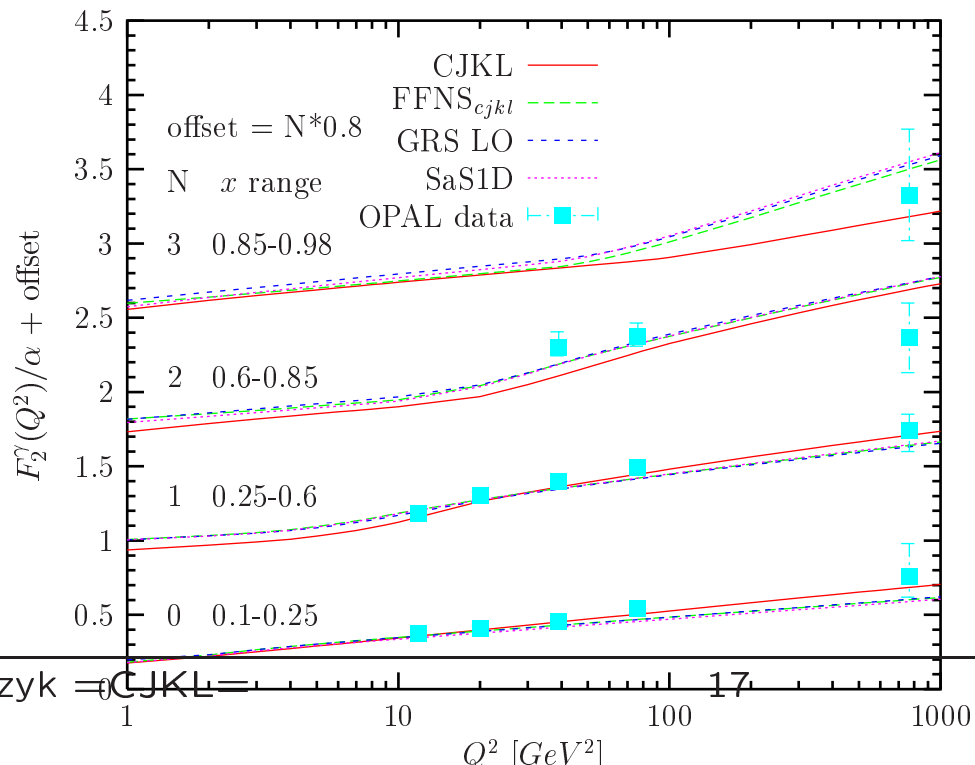
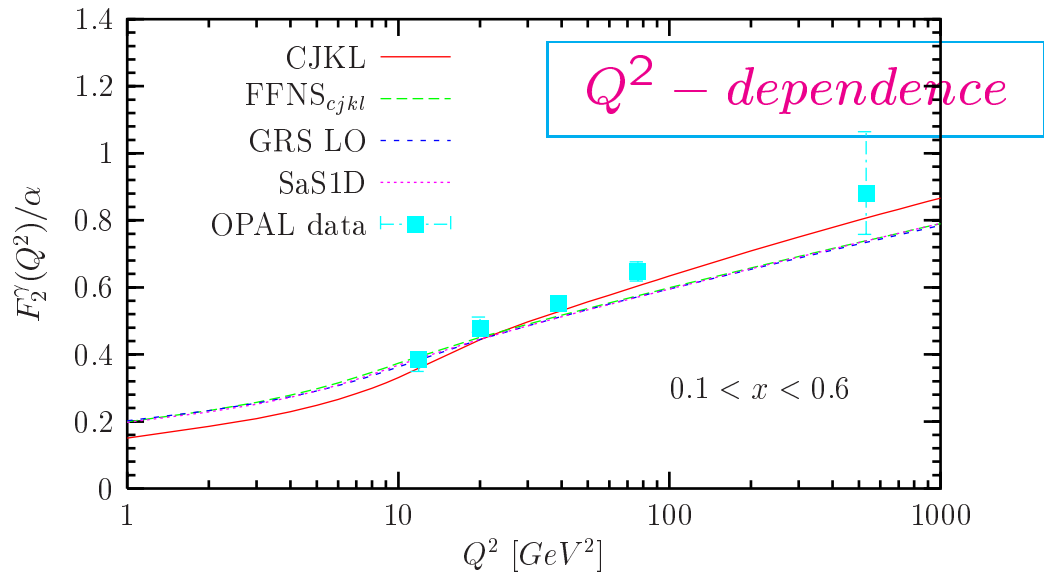
model	# of par.	$\chi^2$ (205 pts)	$\chi^2/DOF$	$\chi^2$ (182 pts*)	$\chi^2/DOF$
SaS1D	6	657	3.30	611	3.47
GRS LO	0	499	2.43	366	2.01
FFNS <sub><i>cjkl</i></sub>	3	442	2.19	357	1.99
CJKL	3	406	2.01	320	1.79

\* - no TPC data

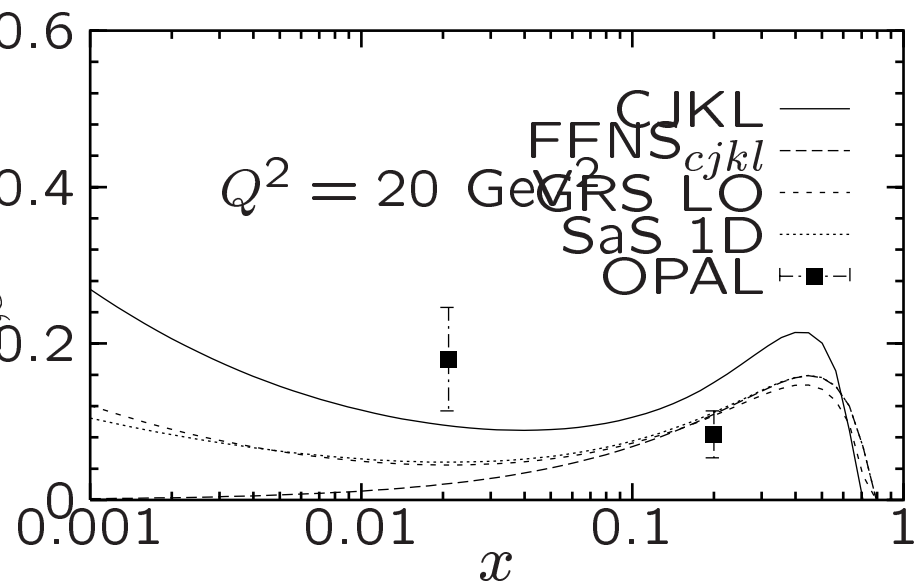








Comparison with  $F_{2c}^\gamma$  data



## Stage II

New models vs old models - changes introduced.

### 1. Subtraction terms

$$\text{Direct: } \frac{dq_h^\gamma(x, Q^2)}{d \ln Q^2} = \frac{\alpha}{2\pi} e_h^2 k(x)$$

$$\text{Before } F_{2,h}^\gamma|_{dir,subtr} \sim k(x) \ln \frac{Q^2}{m_h^2} \quad \text{Now } \sim k(x) \ln \frac{Q^2}{Q_0^2}$$

### 2. $\alpha_s$ running

$$\alpha_s(Q^2)^{(N_l)} = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda_{QCD}^{(N_l)})} \quad \text{with } \beta_0 = 11 - \frac{2}{3}N_l$$

$$\text{Before } N_l = 3 \text{ in FFNS}_{CJKL} \quad N_l = 5 \text{ in CJKL}$$

$$\text{Now } N_l = N_l(Q^2) \text{ in all models}$$

### 3. $\Lambda_{QCD}^3$ is a free parameter

Former value was too high and caused high  $\chi^2$  of the fits

### 4. High- $x$ $F_2^\gamma$ behaviour

The 'by-hand' imposed condition in CJKL  $F_{2,h}^\gamma \geq 0$

At high  $x$   $F_{2,h}^\gamma < F_{2,h}^\gamma(x, Q^2)|_{direct} + F_{2,h}^\gamma(x, Q^2)|_{resolved}$

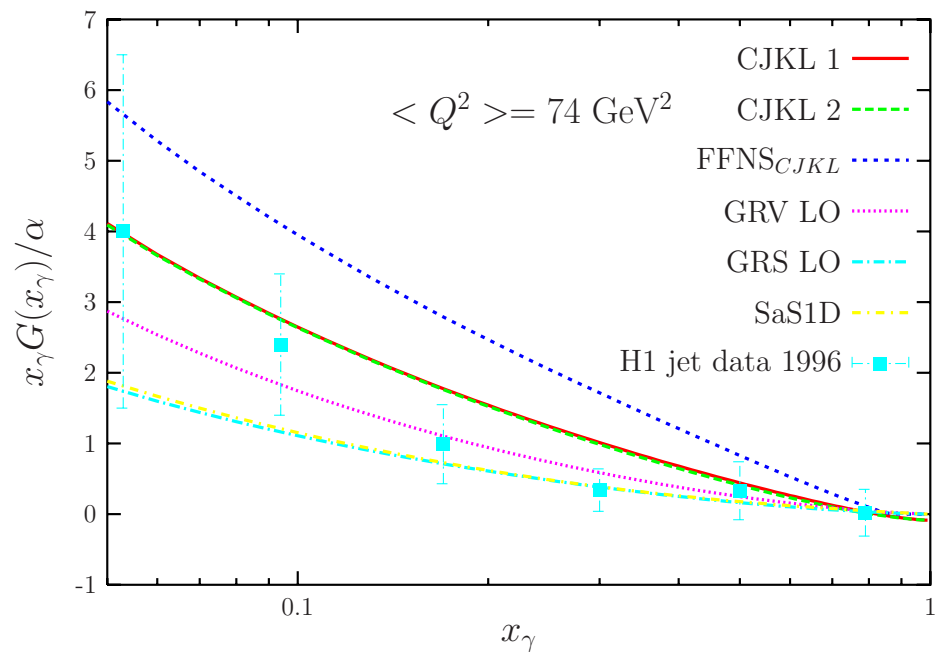
CJKL 1 - old CJKL condition

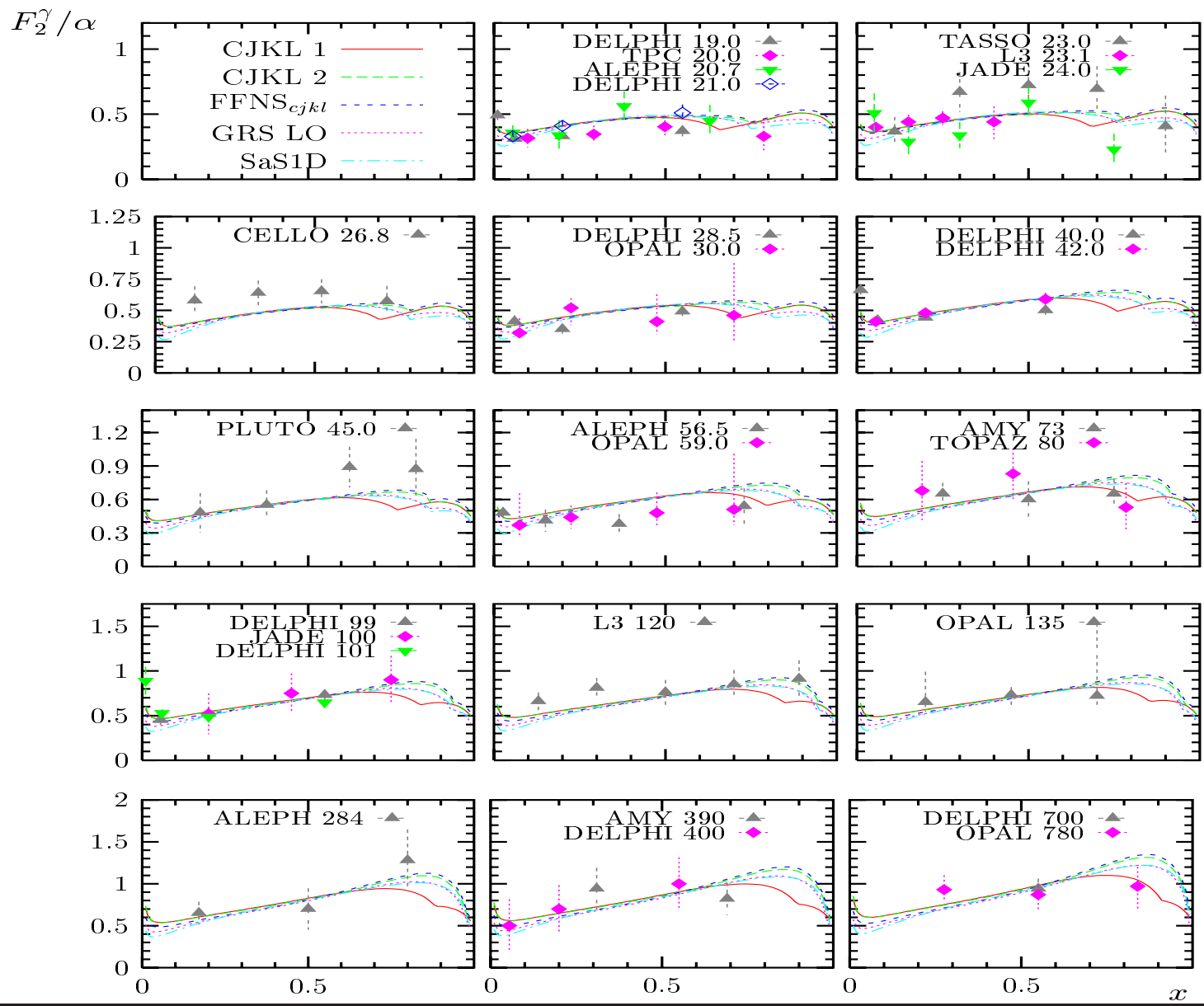
CJKL 2 - new condition  $F_{2,h}^\gamma \geq F_{2,h}^\gamma(x, Q^2)|_{direct} + F_{2,h}^\gamma(x, Q^2)|_{resolved}$

## 5. Vector Meson Dominance model

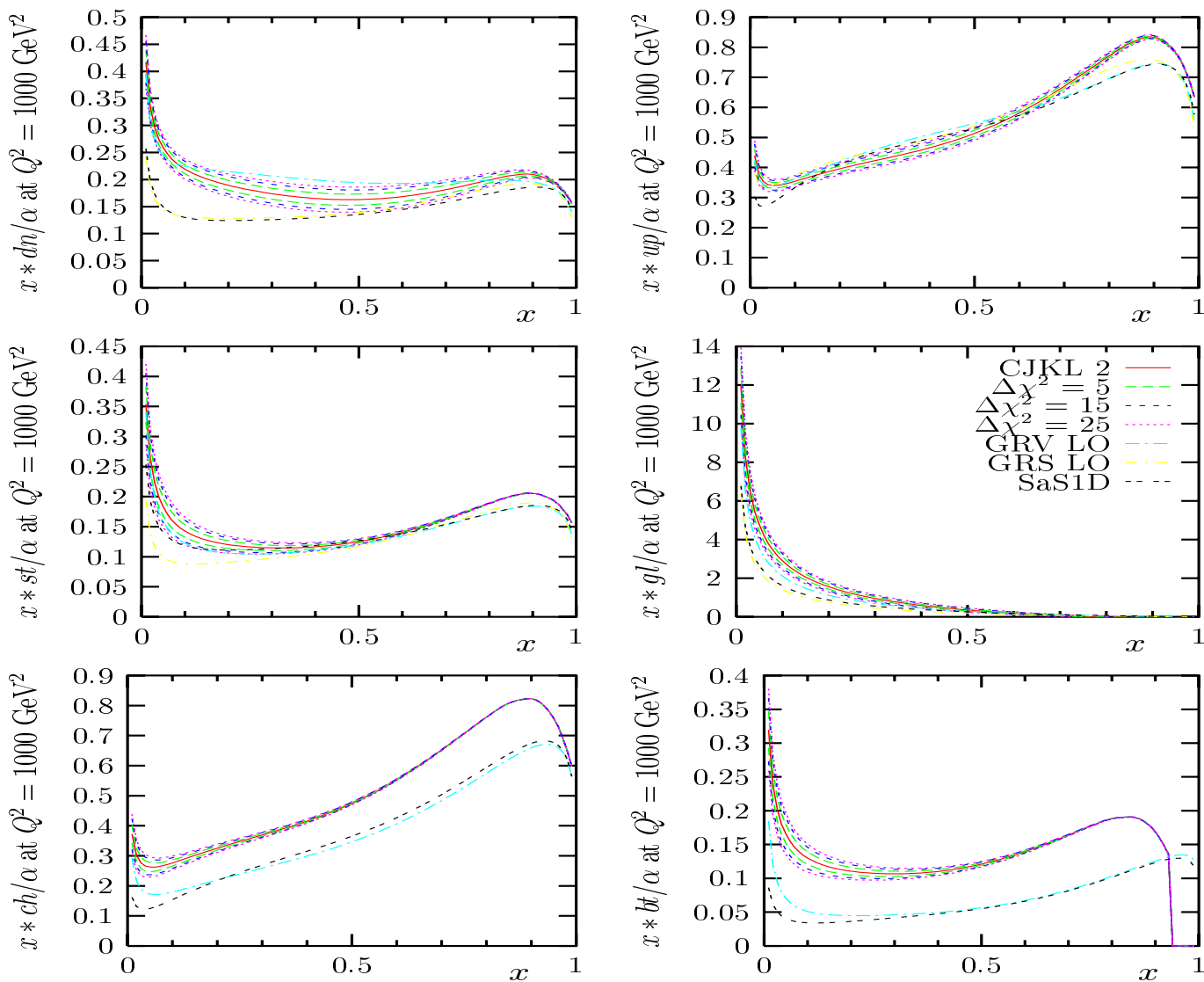
$$\begin{aligned}
 &= 2.00 \quad \text{THEORY} \\
 \int_0^1 2v^\rho(x, Q_0^2) dx &= 0.91 \quad \text{FFNS}_{CJKL} \\
 &= 1.99 \quad \text{CJKL 1} \\
 &= 1.91 \quad \text{CJKL 2}
 \end{aligned}$$

$$\int_0^1 (2v^\rho + G^\rho)(x, Q_0^2) dx = 1 \quad \text{FIXED!}$$

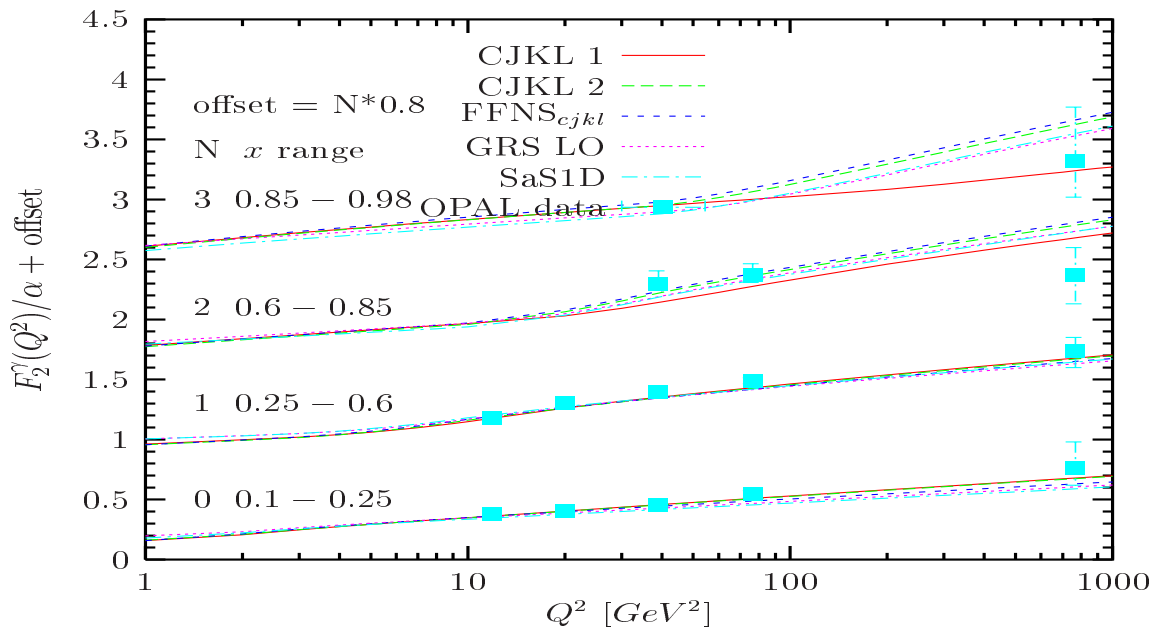
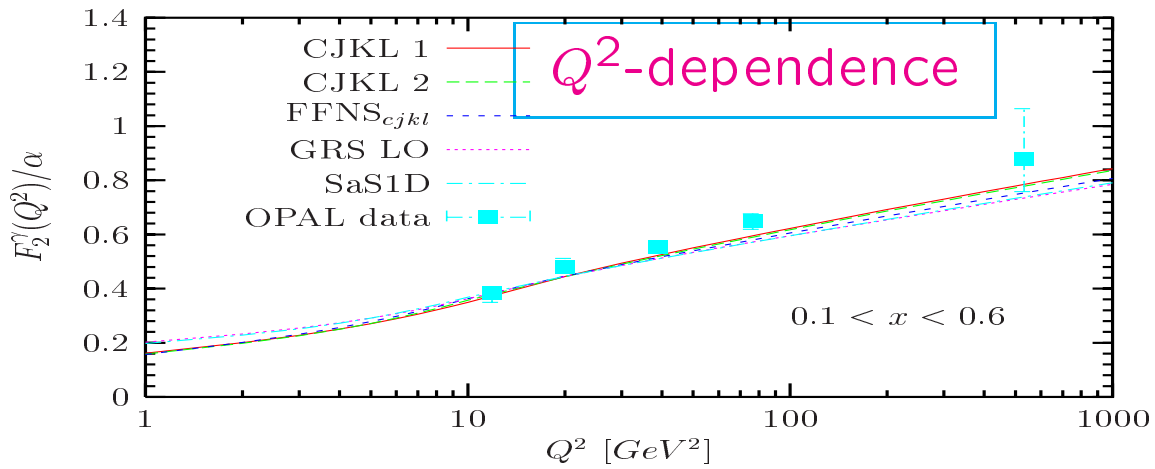




## Parton distributions with uncertainties







# Conclusion

LO analysis of  $F_2^\gamma$  based on existing data

Fix Flavor Number scheme and ACOT( $\chi$ ) scheme

CJKL model based on ACOT( $\chi$ )

→ LO parton parametrizations CJKL, CJKL 1, CJKL 2

NLO analysis in progress