Pseudoscalar meson transition form factors

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There is a single FF for each $\gamma^* \to \gamma P$ transition to be de fined by

$$\langle P(p)\gamma(k)|J_\mu^{e.m.}|o\rangle = \epsilon_{\mu\nu\alpha\beta}p^\nu\epsilon^\alpha k^\beta F_{\alpha P}(q^2) \quad (1)$$

$$J_\mu^{e.m.} = 2/3\bar{u}\gamma_\mu u - 1/3\bar{d}\gamma_\mu d - 1/3\bar{s}\gamma_\mu s$$

ϵ^α the polarization vector of γ

q^2 squared momentum to be transferred by γ^*

$\epsilon_{\mu\nu\alpha\beta}$ appears as only the pseudoscalar meson belongs to the abnormal spin-parity series.

A straightforward calculation of $F_{\gamma P}(Q^2)$ in QCD is

not possible!

Brodsky, Lepage [Phys. Rev.D24 (1981)1808] employed PQCD to find the asymptotic behaviour

$$lim_{Q^2\to\infty}Q^2F_{\gamma P}(Q^2) = 2f_P \quad (2)$$

where f_P is the meson weak decay constant and $Q^2 = -q^2 \equiv t$.

The behaviour of $F_{\gamma P}(Q^2)$ for $Q^2 \to 0$ can be determined from the axial anomaly in the chiral limit of QCD

$$lim_{Q^2 \to 0} Q^2 F_{\gamma P}(Q^2) = \frac{1}{4\pi^2 f_P} \equiv F_{\gamma P}(0) \tag{3}$$

In order to describe the soft nonperturbative region of Q^2 a simple interpolation formula has been proposed by Brodsky, Lepage

$$F_{\gamma P}(Q^2) = \frac{1}{4\pi^2 f_P} \cdot \frac{1}{1 + (Q^2/8\pi^2 f_P^2)} = \frac{1}{4\pi^2 f_P} \cdot \frac{8\pi^2 f_P^2}{(8\pi^2 f_P^2 + Q^2)} \tag{4}$$

which however

does not describe even the space-like data well.

Then: in **space like region** in $t < 0$, one usually fits the observed $t = q^2 = -Q^2$ dependence of $F_{\gamma P}(t)$ by a normalized phenomenological formula

$$F_{\gamma P}(t) = F(\Lambda_P, t)/F(\Lambda_P, 0) = \frac{1}{(1 - t/\Lambda_P)}, \tag{5}$$

where $1/\Lambda_P = \langle r_P^2 \rangle /6$ is related to the size $\langle r_P^2 \rangle$ of the pseudoscalar meson P.

And in the **time-like region** $t > 0$ of the well known resonances one is left with Breit-Wigner extension of the VMD model

$$F_{\gamma P}(Q^2) = \sum_v \frac{f_{vP\gamma}}{f_v} \cdot \frac{m_v^2}{m_v^2 - t - i m_v \Gamma_v} \tag{6}$$

which, however, is justified only at the region of resonances and generally violates the unitarity.

Recently [Phys.Rev.D57 No.1 (1998) 33] plenty of $F_{\gamma P}$ data at large momentum transfer appeared and on an account of that it is necessary to achieve a description of all existing data in both, $t < 0$ and $t > 0$, regions in the framework of one more sophisticated approach.

Recently [Phys. rev. D57 No. 1 (1998) 33] plenty of $F_{\gamma P}$ data at large momentum transfer appeared: '

- for π : experimental data —33 points
- for η : experimental data —52 points
- for η' : experimental data —59 points
- in time-like region Φ peak for π^0, η

there is necessary ti achieve a description of all existing data in both, $t < 0$ and $t > 0$, regions in the framework of one more sophisticated approach.

Our intention is:

- To achieve a description of all $t < 0$, $t > 0$ data by **one analytic function** from $-\infty \to +\infty$, respecting all 'known properties of $F_{\gamma P}(t)$ in the framework of the unitary and analytic model (**U& A**)

I. Summary of the properties of $F_{\gamma P}(t)$

- asymptotic behaviour: $F_{\gamma P}(t)_{|t|\to\infty} \equiv \frac{2f_P}{Q^2} \sim t^{-1}$
- normalization: $F_{\gamma P}(0)=\frac{1}{4\pi^2 f_P} = \frac{2}{\alpha m_P}\sqrt{\frac{\Gamma(P\to\gamma\gamma)}{\pi m_P}}$

However, in order to take into account the fact that f_η and $f_{\eta'}$ are not directly measurable quantities, employing the relation for the two-photon partial width

$$\Gamma(P \to \gamma\gamma) = \frac{\alpha^2}{64\pi^3 f_P^2} m_P^3 \tag{7}$$

of the pseudoscalar meson P, one comes to a redefinition of

the norm

$$F_{\gamma P}(0) = \frac{2}{\alpha m_P}\sqrt{\frac{\Gamma(P \to \gamma\gamma)}{\pi m_P}} \tag{8}$$

analytic properties:

The $F_{\gamma P}(t)$ is analytic in the whole complex t-plane besides the cut on the positive real axis from $t = m^2_{\pi^0}$ up to $+\infty$, as there is the intermediate state $\pi^0\gamma$ allowed, which generates the lowest branch point at the corresponding FF's.

Construction of U&A model of $F_{\gamma P}(t)$

Because $F_{\gamma P}(t)$ has the asymptotic behaviour like VMD, then in the VMD parametrization of $F_{\gamma P}(t)$, besides ρ, ω, ϕ, another isoscalar vector-meson can be taken into account in order to achieve the normalization automatically. We consider it to be ω'.

Then one obtains

$$F_{\gamma P}(t) = \sum_{i=\rho,\omega,\phi,\omega'} \frac{m_i^2}{m_i^2 - t}(f_{i\gamma P}/f_i^e), \tag{9}$$

where f_i^e is the universal vector-meson coupling constant of the

$\gamma^* \to V$ transition and numerically given by

the leptonic decay width $\Gamma(V \to l^+l^-)$ of the corresponding vector meson.

Requirement of the normalization (8) leads to

$$F_{\gamma P}(0) = \sum_{i=\rho,\omega,\phi,\omega'} (f_{i\gamma P}/f_i^e). \tag{10}$$

If we express $(f_{\omega'\gamma P}/f_{\omega'}^e)$ through other coupling constant ratios one obtains the suitable for a construction of U& A model VMD parametrization

$$\begin{aligned} F_{\gamma P}(t) &= F_{\gamma P}(0)\frac{m_{\omega'}^2}{m_{\omega'}^2 - t} + \left[\frac{m_\rho^2}{m_\rho^2 - t} - \frac{m_{\omega'}^2}{m_{\omega'}^2 - t}\right] a_\rho + \\ &+ \left[\frac{m_\omega^2}{m_\omega^2 - t} - \frac{m_{\omega'}^2}{m_{\omega'}^2 - t}\right] a_\omega + \left[\frac{m_\phi^2}{m_\phi^2 - t} - \frac{m_{\omega'}^2}{m_{\omega'}^2 - t}\right] a_\phi \end{aligned} \tag{11}$$

to be automatically **normalized**. In order to obtain from (11) the U& A model, we incorporate a two-cut approximation of the correct FF analytic properties by an application of the non-linear transformation

$$t = t_0 - \frac{4(t_{in} - t_0)}{[1/V - V]^2} \tag{12}$$

and subsequently non-zero values of vector-meson widths $\Gamma \neq 0$ are incorporated.

As a result all VMD terms first give the **factorization form**

$$\frac{m_i^2}{(m_i^2 - t)} = \left(\frac{1 - V^2}{1 - V_N^2}\right)^2 \cdot \tag{13}$$
$$\frac{(V_N - V_{i_0})(V_N + V_{i_0})(V_N - 1/V_{i_0})(V_N + 1/V_{i_0})}{(V - V_{i_0})(V + V_{i_0})(V - 1/V_{i_0})(V + 1/V_{i_0})}; \ i = \rho, \omega, \omega' \phi$$

a) on the pure asymptotic term $(\frac{1-V^2}{1-V_N^2})^2$, independent on the flavour of vector mesons under consideration and carrying just the asymptotic behaviour $_{|t|\to\infty} \sim t^{-1}$ of the VMD model

b) and the so-called finite-energy term (the second one in (13)), describing the resonant structure of VMD terms, which, however, for $|t| \to \infty$ are going out on the real constants and so, they do not contribute to the asymptotic behaviour of the VMD model.

The subindex 0 in (13) means that still $\Gamma = 0$ of all vector mesons is considered.

Note:

The following change of the exponent in the asymptotic term

$$\left(\frac{1-V^2}{1-V_N^2}\right)^2 \rightarrow \left(\frac{1-V^2}{1-V_N^2}\right)^{2n}, \quad n = 1, 2, 3... \tag{14}$$

leads to the change of the asymptotic behaviour

$$_{|t|\rightarrow\infty} \sim t^{-1} \rightarrow_{|t|\rightarrow\infty} \sim t^{-n} \tag{15}$$

of the corresponding FF.

One can utilize in (13) relations between complex conjugated values of the corresponding zero-width VMD model pole positions in V-plane

$$V_{i_0} = -V_{i_0}^* \text{ for } i = \rho, \omega, \phi \text{ and } V_{\omega'_0} = 1/V_{\omega'_0}^* \tag{16}$$

following from the assumption that in a fitting procedure of data on $F_{\gamma P}(t)$ such t_{in} will be found that

$$(m_i^2 - \Gamma_i^2/4) < t_{in}, \quad i = \rho, \omega, \phi \text{ and} (m_{\omega'}^2 - \Gamma_{\omega'}^2/4) > t_{in}. \tag{17}$$

Finally incorporating $\Gamma \neq 0$ by a substitution

$$m_v^2 \rightarrow (m_v - i\Gamma_v/2)^2 \tag{18}$$

one comes to the U& A model of $F_{\gamma P}(t)$

$$
\begin{aligned}
F_{\gamma P}[V(t)] = & \left(\frac{1-V^2}{1-V_N^2}\right)^2 \cdot \qquad (19)\\
\cdot & \left[F_{\gamma P}(0)\frac{(V_N-V_{\omega'})(V_N-V_{\omega'}^*)(V_N+V_{\omega'})(V_N+V_{\omega'}^*)}{(V-V_{\omega'})(V-V_{\omega'}^*)(V+V_{\omega'})(V+V_{\omega'}^*)}+\right.\\
+ & \left\{\frac{(V_N-V_\rho)(V_N-V_\rho^*)(V_N-1/V_\rho)(V_N-1/V_{\rho^*})}{(V-V_\rho)(V-V_\rho^*)(V-1/V_\rho)(V-1/V_\rho^*)}-\right.\\
- & \left.\frac{(V_N-V_{\omega'})(V_N-V_{\omega'}^*)(V_N+V_{\omega'})(V_N+V_{\omega'}^*)}{(V-V_{\omega'})(V-V_{\omega'}^*)(V+V_{\omega'})(V+V_{\omega'}^*)}\right\}a_\rho+\\
+ & \left\{\frac{(V_N-V_\omega)(V_N-V_\omega^*)(V_N-1/V_\omega)(V_N-1/V_\omega^*)}{(V-V_\omega)(V-V_\omega^*)(V-1/V_\omega)(V-1/V_{\omega'}^*)}-\right.\\
- & \left.\frac{(V_N-V_{\omega'})(V_N-V_{\omega'}^*)(V_N+V_{\omega'})(V_N+V_{\omega'}^*)}{(V-V_{\omega'})(V-V_{\omega'}^*)(V+V_{\omega'})(V+V_{\omega'}^*)}\right\}a_\omega+\\
+ & \left\{\frac{(V_N-V_\phi)(V_N-V_\phi^*)(V_N-1/V_\phi)(V_N-1/V_\phi^*)}{(V-V_\phi)(V-V_\phi^*)(V-1/V_\phi)(V-1/V_\phi^*)}-\right.\\
- & \left.\left.\frac{(V_N-V_{\omega'})(V_N-V_{\omega'}^*)(V_N+V_{\omega'})(V_N+V_{\omega'}^*)}{(V-V_{\omega'})(V-V_{\omega'}^*)(V*+V_{\omega'})(V+V_{\omega'}^*)}\right\}a_\phi\right]
\end{aligned}
$$

where

$$V(t) = i\frac{\sqrt{q_{in}+q}-\sqrt{q_{in}-q}}{\sqrt{q_{in}+q}+\sqrt{q_{in}-q}}$$

$$q = \{(t-t_0)/t_0\}^{1/2}; \quad q_{in} = \{(t_{in}-t_0)/t_0\}^{1/2};$$

$V_N = V(t)_{|t=0}$ and V_i (i=ρ, ω, ϕ, ω') are positions of vector-meson poles in V-plane.

Now, (19) is applied for a description on existing data on π^0, η, η' transition form factors.

π^0: The best description (see Fig 1) is achieved with χ =18.5 i.e. χ/ndf =0.66

and following values of parameters :

$t_{in}^v = 0.5909 GeV^2$; $t_{in}^s = 0.9714 GeV^2$

$(f_{\rho\gamma\pi^0}/f_{rho})$=$0.0045 \pm 0.0009$

$(f_{\omega\gamma\pi^0}/f_{omega})$=$0.025 \pm 0.0009$

$(f_{\phi\gamma\pi^0}/f_{phi})$=$-0.0004 \pm 0.0001$

η: The best description (see Fig 2) is achieved with χ =76.7.5 i.e. χ/ndf =1.63

and following values of parameters :

$t_{in}^v = 2.0712 GeV^2$; $t_{in}^s = 1.0091 GeV^2$

$(f_{\rho\gamma\pi^0}/f_{rho})$=$-.0077 \pm 0.0106$

$(f_{\omega\gamma\pi^0}/f_{omega})$=$0.0542 \pm 0.0113$

$(f_{\phi\gamma\pi^0}/f_{phi})$=$-0.00035 \pm 0.0004$

η': The best description (see Fig 3) is achieved with χ =76.5 i.e. χ/ndf =1.40

and following values of parameters :

$t^v_{in} = 1.8860 GeV^2$; $t^s_{in} = 0.6089 GeV^2$

$(f_{\rho\gamma\pi^0}/f_{rho})$=$-.2211 \pm 0.0190$

$(f_{\omega\gamma\pi^0}/f_{omega})$=$0.3599 \pm 0.0289$

$(f_{\phi\gamma\pi^0}/f_{phi})$=$-.0583 \pm 0.0485$

Conclusion

- We have predicted behaviour of the transition pseudoscalar-meson form factors in the space-like and time-like regions simultaneously
- The U&A model of pseudoscalar-meson transition form factors posses all known form factor properties
- The obtained results can be applied to:

  i) prediction of various cross-sections and decay rates in which transition form factors are appearing

  ii) for the first time one can evaluate contributions of $\sigma(e^+e^- \to \gamma P)$ into the muon anomalous magnetic moment

  iii) we are able to predict the strange pseudoscalar transition form factors

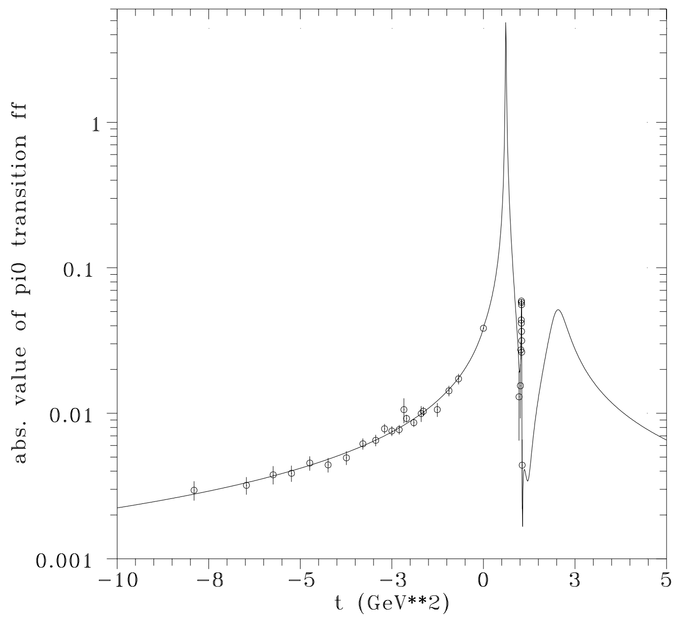

Figure 1: π^0 transition form factor

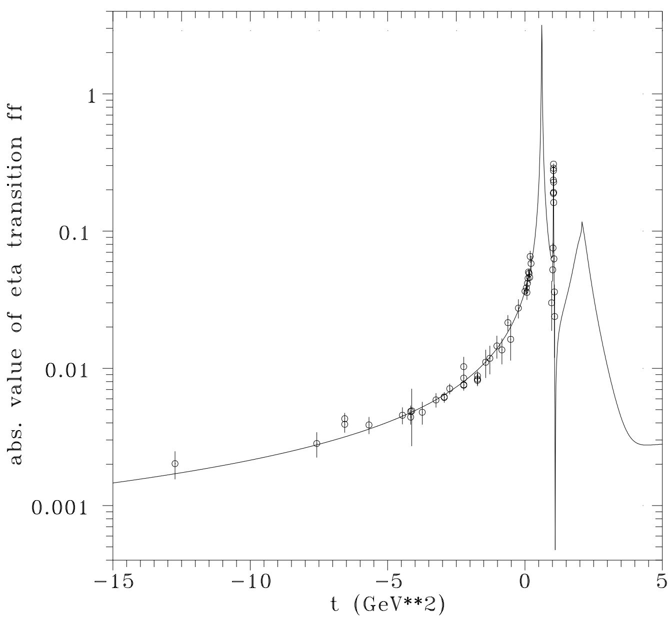

Figure 2: η transition form factor

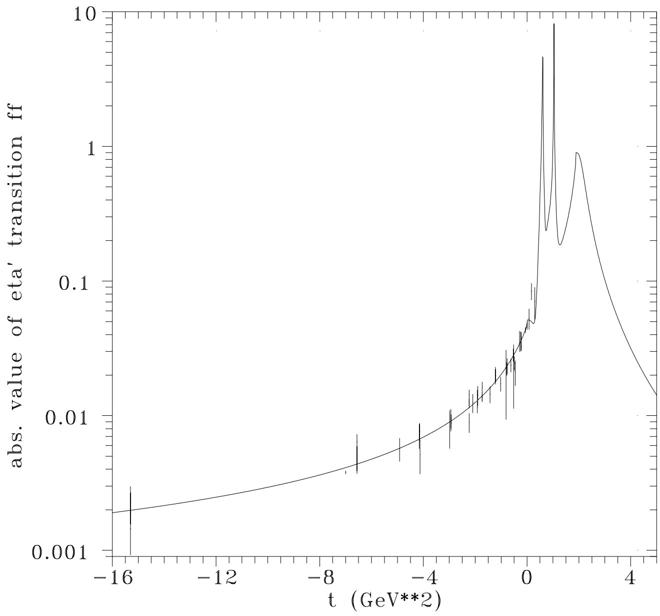

Figure 3: eta' transition form factor