Pseudoscalar meson transition form factors

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There is a single FF for each $\gamma^* \rightarrow \gamma P$ transition to be defined by

\[
\langle P(p)\gamma(k)|J_{\mu}^{\text{e.m.}}|o\rangle = \epsilon_{\mu\nu\alpha\beta}p^\nu \epsilon^\alpha k^\beta F_{\alpha P}(q^2)
\]

(1)

\[
J_{\mu}^{\text{e.m.}} = 2/3\bar{u}\gamma_\mu u - 1/3\bar{d}\gamma_\mu d - 1/3\bar{s}\gamma_\mu s
\]

$\epsilon^\alpha$ is the polarization vector of $\gamma$

$q^2$ squared momentum to be transferred by $\gamma^*$

$\epsilon_{\mu\nu\alpha\beta}$ appears as only the pseudoscalar meson belongs to the abnormal spin-parity series.

A straightforward calculation of $F_{\gamma P}(Q^2)$ in QCD is not possible!


\[
\lim_{Q^2 \rightarrow \infty} Q^2 F_{\gamma P}(Q^2) = 2f_P
\]

(2)
where \( f_P \) is the meson weak decay constant and \( Q^2 = -q^2 \equiv t \).

The behaviour of \( F_{\gamma P}(Q^2) \) for \( Q^2 \to 0 \) can be determined from the axial anomaly in the chiral limit of QCD

\[
\lim_{Q^2 \to 0} Q^2 F_{\gamma P}(Q^2) = \frac{1}{4\pi^2 f_P} \equiv F_{\gamma P}(0)
\]  

(3)

In order to describe the soft nonperturbative region of \( Q^2 \) a simple interpolation formula has been proposed by Brodsky, Lepage

\[
F_{\gamma P}(Q^2) = \frac{1}{4\pi^2 f_P} \cdot \frac{1}{1 + (Q^2/8\pi^2 f_P^2)} = \frac{1}{4\pi^2 f_P} \cdot \frac{8\pi^2 f_P^2}{(8\pi^2 f_P^2 + Q^2)}
\]  

(4)

which however

**does not describe even the space-like data well.**

Then: in space like region in \( t < 0 \), one usually fits the observed \( t = q^2 = -Q^2 \) dependence of \( F_{\gamma P}(t) \) by a normalized phenomenological formula

\[
F_{\gamma P}(t) = F(\Lambda_P, t)/F(\Lambda_P, 0) = \frac{1}{(1 - t/\Lambda_P)},
\]  

(5)

where \( 1/\Lambda_P = \langle r_P^2 \rangle /6 \) is related to the size \( \langle r_P^2 \rangle \) of the pseudoscalar meson P.
And in the time-like region $t > 0$ of the well known resonances one is left with Breit-Wigner extension of the VMD model

$$F_{\gamma P}(Q^2) = \sum_v \frac{f_v P_\gamma}{f_v} \cdot \frac{m_v^2}{m_v^2 - t - im_v \Gamma_v}$$

(6)

which, however, is justified only at the region of resonances and generally violates the unitarity.

Recently [Phys.Rev.D57 No.1 (1998) 33] plenty of $F_{\gamma P}$ data at large momentum transfer appeared and on an account of that it is necessary to achieve a description of all existing data in both, $t < 0$ and $t > 0$, regions in the framework of one more sophisticated approach.

**Recently** [Phys. rev. D57 No. 1 (1998) 33] plenty of $F_{\gamma P}$ data at large momentum transfer appeared:

- for $\pi$: experimental data — 33 points
- for $\eta$: experimental data — 52 points
- for $\eta'$: experimental data — 59 points
- in time-like region $\Phi$ peak for $\pi^0$, $\eta$
there is necessary to achieve a description of all existing data in both, $t < 0$ and $t > 0$, regions in the framework of one more sophisticated approach.

Our intention is:

- To achieve a description of all $t < 0$, $t > 0$ data by one analytic function from $-\infty \to +\infty$, respecting all known properties of $F_{\gamma P}(t)$ in the framework of the unitary and analytic model (U& A)

I. Summary of the properties of $F_{\gamma P}(t)$

- asymptotic behaviour: $F_{\gamma P}(t)|t| \to \infty \equiv \frac{2f_P}{Q^2} \sim t^{-1}$

- normalization: $F_{\gamma P}(0) = \frac{1}{4\pi^2 f_P} = \frac{2}{\alpha m_P} \sqrt{\frac{\Gamma(P \to \gamma\gamma)}{\pi m_P}}$

However, in order to take into account the fact that $f_\eta$ and $f_{\eta'}$ are not directly measurable quantities, employing the relation for the two-photon partial width

$$\Gamma(P \to \gamma\gamma) = \frac{\alpha^2}{64\pi^3 f_P^2 m_P^3} \tag{7}$$
of the pseudoscalar meson P, one comes to a redefinition of
the norm
\[
F_{\gamma P}(0) = \frac{2}{\alpha m_P} \sqrt{\frac{\Gamma(P \to \gamma\gamma)}{\pi m_P}}
\]  
(8)

analytic properties:

The \( F_{\gamma P}(t) \) is analytic in the whole complex \( t \)-plane besides
the cut on the positive real axis from \( t = m_{\pi}^2 \) up to \(+\infty\), as
there is the intermediate state \( \pi^0\gamma \) allowed, which generates the
lowest branch point at the corresponding FF’s.

**Construction of U&A model of \( F_{\gamma P}(t) \)**

Because \( F_{\gamma P}(t) \) has the asymptotic behaviour like VMD, then
in the VMD parametrization of \( F_{\gamma P}(t) \), besides \( \rho, \omega, \phi \), another
isoscalar vector-meson can be taken into account in order to
achieve the normalization automatically. We consider it to be
\( \omega' \).

Then one obtains
\[
F_{\gamma P}(t) = \sum_{i=\rho,\omega,\phi,\omega'} \frac{m_i^2}{m_i^2 - t} (f_i \gamma_P / f_i^e),
\]  
(9)

where \( f_i^e \) is the universal vector-meson coupling constant of the
\[ \gamma^* \rightarrow V \] transition and numerically given by

the leptonic decay width \( \Gamma(V \rightarrow l^+l^-) \) of the corresponding vector meson.

Requirement of the normalization (8) leads to

\[
F_{\gamma P}(0) = \sum_{i=\rho,\omega,\phi,\omega'} (f_{i\gamma P}/f_{i\ell}^\epsilon).
\]

If we express \( (f_{\omega'\gamma P}/f_{\ell\omega'}^\epsilon) \) through other coupling constant ratios one obtains the suitable for a construction of U & A model VMD parametrization

\[
F_{\gamma P}(t) = F_{\gamma P}(0) \frac{m_{\omega'}^2}{m_{\omega'}^2 - t} + \frac{m_\rho^2}{m_\rho^2 - t} - \frac{m_{\omega'}^2}{m_{\omega'}^2 - t} \left[ a_\rho + \frac{m_\rho^2}{m_\rho^2 - t} - \frac{m_{\omega'}^2}{m_{\omega'}^2 - t} \right] a_\omega + \frac{m_\phi^2}{m_\phi^2 - t} - \frac{m_{\omega'}^2}{m_{\omega'}^2 - t} \left[ a_\phi + \frac{m_\phi^2}{m_\phi^2 - t} - \frac{m_{\omega'}^2}{m_{\omega'}^2 - t} \right]
\]

to be automatically normalized. In order to obtain from (11) the U & A model, we incorporate a two-cut approximation of the correct FF analytic properties by an application of the non-linear transformation

\[
t = t_0 - \frac{4(t_{in} - t_0)}{[1/V - V]^2}
\]

and subsequently non-zero values of vector-meson widths

\( \Gamma \neq 0 \) are incorporated.
As a result all VMD terms first give the **factorization form**

\[
\frac{m_i^2}{(m_i^2 - t)} = \left( \frac{1 - V^2}{1 - V_N^2} \right)^2.
\]

\[
\frac{(V_N - V_{i0})(V_N + V_{i0})(V_N - 1/V_{i0})(V_N + 1/V_{i0})}{(V - V_{i0})(V + V_{i0})(V - 1/V_{i0})(V + 1/V_{i0})}; \quad i = \rho, \omega, \omega' \phi
\]

a) on the pure asymptotic term \((1 - V^2)/(1 - V_N^2)^2\), independent on the
flavour of vector mesons under consideration and carrying just the asymptotic behaviour \(|t| \to \infty \sim t^{-1}\) of the VMD model

b) and the so-called finite-energy term (the second one in (13)),
describing the resonant structure of VMD terms, which, however, for \(|t| \to \infty\) are going out on the real constants and so, they do not contribute to the asymptotic behaviour of the VMD model.

The subindex 0 in (13) means that still \(\Gamma = 0\) of all vector mesons is considered.
Note:

The following change of the exponent in the asymptotic term

\[
\left( \frac{1 - V^2}{1 - V_N^2} \right)^2 \to \left( \frac{1 - V^2}{1 - V_N^2} \right)^{2n}, \quad n = 1, 2, 3... \quad (14)
\]

leads to the change of the asymptotic behaviour

\[
|\alpha| \to \infty \sim t^{-1} \to |\alpha| \to \infty \sim t^{-n} \quad (15)
\]

of the corresponding FF.

One can utilize in (13) relations between complex conjugated values of the corresponding zero-width VMD model pole positions in V-plane

\[
V_{i0} = -V_{i0}^* \text{ for } i = \rho, \omega, \phi \text{ and } V_{\omega_0'} = 1/V_{\omega_0'}^* \quad (16)
\]

following from the assumption that in a fitting procedure of data on \( F_{\gamma P}(t) \) such \( t_{in} \) will be found that

\[
(m_i^2 - \Gamma_i^2/4) < t_{in}, \quad i = \rho, \omega, \phi \quad \text{and} \quad (m_{\omega'}^2 - \Gamma_{\omega'}^2/4) > t_{in}. \quad (17)
\]

Finally incorporating \( \Gamma \neq 0 \) by a substitution

\[
m_v^2 \to (m_v - i\Gamma_v/2)^2 \quad (18)
\]
one comes to the U& A model of $F_{\gamma P}(t)$

\[
F_{\gamma P}[V(t)] = \left( \frac{1 - V^2}{1 - V_N^2} \right)^2 
\]

\[
\cdot \frac{F_{\gamma P}(0)}{(V - V_{\omega'}) (V - V_{\omega}^*) (V + V_{\omega'}) (V + V_{\omega}^*)} \left\{ \begin{array}{l}
\frac{(V_N - V_{\rho})(V_N - V_{\rho}^*)(V_N - 1/V_{\rho})(V_N - 1/V_{\rho}^*)}{(V - V_{\omega})(V - V_{\omega}^*)(V - 1/V_{\omega})(V - 1/V_{\omega}^*)} a_{\rho} \\
\frac{(V_N - V_{\omega})(V_N - V_{\omega}^*)(V_N - 1/V_{\omega})(V_N - 1/V_{\omega}^*)}{(V - V_{\omega})(V - V_{\omega}^*)(V - 1/V_{\omega})(V - 1/V_{\omega}^*)} a_{\omega} \\
\frac{(V_N - V_{\phi})(V_N - V_{\phi}^*)(V_N - 1/V_{\phi})(V_N - 1/V_{\phi}^*)}{(V - V_{\omega})(V - V_{\omega}^*)(V - 1/V_{\omega})(V - 1/V_{\omega}^*)} a_{\phi}
\end{array} \right\} + 
\]

where

\[
V(t) = i \frac{\sqrt{q_{\text{in}} + q} - \sqrt{q_{\text{in}} - q}}{\sqrt{q_{\text{in}} + q} + \sqrt{q_{\text{in}} - q}}
\]

\[
q = \{(t - t_0)/t_0\}^{1/2}; \quad q_{\text{in}} = \{(t_{\text{in}} - t_0)/t_0\}^{1/2},
\]

$V_N = V(t)|_{t_0}$ and $V_i$ ($i=\rho, \omega, \phi, \omega'$) are positions of vector-meson poles in $V$-plane.

9
Now, (19) is applied for a description on existing data on $\pi^0$, $\eta$, $\eta'$ transition form factors.

$\pi^0$: The best description (see Fig 1) is achieved with $\chi = 18.5$ i.e. $\chi/ndf = 0.66$

and following values of parameters:

$$t^v_{in} = 0.5909 GeV^2;\ t^z_{in} = 0.9714 GeV^2$$

$$\left(\frac{f_{\rho\gamma\pi^0}}{f_{r \rho}}\right) = 0.0045 \pm 0.0009$$

$$\left(\frac{f_{\omega\gamma\pi^0}}{f_{\omega\pi}}\right) = 0.025 \pm 0.0009$$

$$\left(\frac{f_{\phi\gamma\pi^0}}{f_{\phi\pi}}\right) = -0.004 \pm 0.0001$$

$\eta$: The best description (see Fig 2) is achieved with $\chi = 76.7.5$

i.e. $\chi/ndf = 1.63$

and following values of parameters:

$$t^v_{in} = 2.0712 GeV^2;\ t^z_{in} = 1.0091 GeV^2$$

$$\left(\frac{f_{\rho\gamma\pi^0}}{f_{r \rho}}\right) = -0.0077 \pm 0.0106$$

$$\left(\frac{f_{\omega\gamma\pi^0}}{f_{\omega\pi}}\right) = 0.0542 \pm 0.0113$$

$$\left(\frac{f_{\phi\gamma\pi^0}}{f_{\phi\pi}}\right) = -0.00035 \pm 0.0004$$
$\eta'$: The best description (see Fig 3) is achieved with $\chi = 76.5$

i.e. $\chi/ndf = 1.40$

and following values of parameters:

$t_{in}^v = 1.8860 GeV^2; \ t_{in}^s = 0.6089 GeV^2$

$(f_{\rho \gamma \pi^0}/f_{\text{rho}}) = -.2211 \pm 0.0190$

$(f_{\omega \gamma \pi^0}/f_{\text{omega}}) = 0.3599 \pm 0.0289$

$(f_{\phi \gamma \pi^0}/f_{\text{phi}}) = -.0583 \pm 0.0485$
Conclusion

• We have predicted behaviour of the transition pseudoscalar-meson form factors in the space-like and time-like regions simultaneously

• The U&A model of pseudoscalar-meson transition form factors possesses all known form factor properties

• The obtained results can be applied to:
  i) prediction of various cross-sections and decay rates in which transition form factors are appearing
  ii) for the first time one can evaluate contributions of $\sigma(e^+e^- \rightarrow \gamma P)$ into the muon anomalous magnetic moment
  iii) we are able to predict the strange pseudoscalar transition form factors
Figure 1: $\pi^0$ transition form factor

Figure 2: $\eta$ transition form factor
Figure 3: $\eta \eta'$ transition form factor