New vistas of the meson structure in QCD from low to high energies

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- Pion's dilemma: To B or to D?

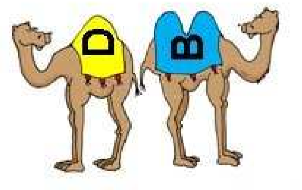


Figure: Camels and the π distribution amplitude.

- Left: asymptotic $\longleftrightarrow$ perturbative QCD
- Right: Chernyak-Zhitnitsky $\longleftrightarrow$ QCD sum rules

- **CLEO veto** (CLEO Collaboration, 1998)

- CLEO data on $F_{\pi\gamma^*\gamma}(Q^2, 0)$ *exclude* CZ DA
  Kroll, Raulfs (1996)
- Schmedding-Yakovlev CLEO-data analysis (2000) *definitely excludes* CZ DA and *likely excludes* asymptotic DA
- Bakulev-Mikhailov-Stefanis CLEO-data analysis (2003-2006) *excludes* CZ DA (at 4σ level) **and** Asy DA (at 3σ level)

- Other nonperturbative approaches, e.g., instanton models, (Petrov et al., 1999; Anikin, Dorokhov, Tomio, 2000; Praszalowicz, Rostworowski, 2001)
- QCD sum rules with nonlocal condensates (NLC) (Mikhailov, Radyushkin, 1986). NLC of higher dimensionality unknown. Resort to simple Gaussian model in Euclidean space:

$$\langle \bar{q}(0)q(z)\rangle = \langle \bar{q}(0)q(0)\rangle \mathrm{e}^{-|z^2|\lambda_q^2/8}$$

which decays with correlation length

$$\Lambda \sim 1/\lambda_q \simeq 0.35 - 0.28 \ \mathrm{fm}$$

$\lambda_q^2 = \langle k_q^2 \rangle$: finite vacuum quark virtuality

Estimates for λ_q^2:

$$\lambda_q^2 = \frac{\langle\bar{q}(0)\nabla^2 q(0)\rangle}{\langle\bar{q}(0)q(0)\rangle} = \langle k_q^2\rangle$$

$$\Rightarrow \frac{\langle\bar{q}(0)\left(ig\sigma_{\mu\nu}G^{\mu\nu}\right)q(0)\rangle}{2\langle\bar{q}(0)q(0)\rangle} \qquad \text{(in chiral limit)}$$

$$= \begin{cases} 0.4 \pm 0.1\ \mathrm{GeV}^2 & [\text{QCD SRs},\ 1987] \\ 0.5 \pm 0.05\ \mathrm{GeV}^2 & [\text{QCD SRs},\ 1991] \\ 0.4 - 0.5\ \mathrm{GeV}^2 & [\text{Lattice},\ 1999-2002] \\ 0.4 \pm 0.05\ \mathrm{GeV}^2 & [\text{CLEO data analysis},\ 2004] \end{cases}$$

Features of pion DAs

- Convexity vs. concavity of π distribution amplitude
  1. Asymptotic π DA: **Dromedary-like** - everywhere convex
  2. Chernyak-Zhitnitsky π DA [PR112 (1984) 173]: **Bactrian camel-like** convex at $x = 0, 1$; concave at $x = 1/2$
- Vacuum nonlocality provides endpoint suppression:
- Bakulev-Mikhailov-Stefanis DA [PLB508 (2001) 279]: **"Camelino"** concave at $x = 0, 1$; concave at $x = 1/2$

Important consequence:

- BMS DA approaches through evolution asymptotic DA $\varphi_{\rm BMS} \stackrel{Q^2\to\infty}{\longrightarrow} \varphi_{\rm asy} = 6x(1-x)$ from *below* $\Uparrow$
- CZ DA approaches asymptotic DA from *above* $\Downarrow$

The object of desire: π DA in QCD

- Matrix element of nonlocal axial current on light cone:

$$\langle 0|\bar{d}(z)\gamma^{\mu}\gamma_5\mathcal{C}(z,0)u(0)|\pi(P)\rangle|_{z^2=0} = if_{\pi}P^{\mu}\int_0^1 dx\mathrm{e}^{ix(z\cdot P)}\varphi_{\pi}(x,\mu_0^2)$$

- Path-ordered exponential (*connector*)

$$\mathcal{C}(z,0) = \mathcal{P}\exp\left[-ig\int_0^z dy^{\mu}t^aA_{\mu}^a(y)\right]$$

ensures gauge invariance

- Normalization

$$\int_0^1 dx\varphi_{\pi}(x,\mu_0^2) = 1$$

- QCD Sum Rule based on correlator of two axial currents:

$$f_\pi^2 \varphi_\pi(x) + f_{A_1}^2 \varphi_{A_1}(x) \exp\left(-\frac{m_{A_1}^2}{M^2}\right) = \int_0^{s_\pi^0} \rho_{\rm NLO}^{\rm pert}(x;s) e^{-s/M^2} ds$$
$$+ \frac{\langle \alpha_{\rm s} GG \rangle}{24\pi M^2} \Phi_{\rm G}\left(x; M^2; \lambda_q^2\right)$$
$$+ \frac{8\pi\alpha_{\rm s}\langle \bar{q}q \rangle^2}{81 M^4} \sum_{i={\rm S,V,T}_{1,2,3}} \Phi_i\left(x; M^2; \lambda_q^2\right) .$$

M^2: Borel parameter, s_π^0: duality interval in axial channel
i runs over S: scalar, V: vector, T: tensor condensates

- Endpoint contributions $x \to 0, 1$ **strongly suppressed** due to $\lambda_q^2 > 0$
- By virtue of finiteness of λ_q^2, one can extract values of moments

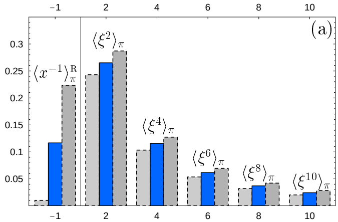
$$\langle \xi^N \rangle_\pi \equiv \int_0^1 dx (2x-1)^N \varphi_\pi(x) \, , \qquad \xi \equiv 2x - 1$$

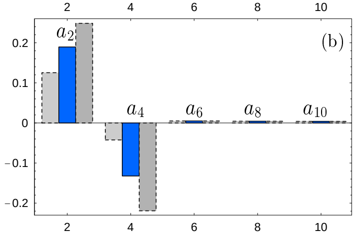
that decrease with increasing polynomial order to 0:

$$\langle \xi^N \rangle_\pi \to [3/(N+1)(N+3)]$$

As a result, one can reconstruct pion DA in terms of first two Gegenbauer polynomials, because higher ones are close to 0:

$$\varphi^{\rm BMS}(x;\mu_0^2) = \varphi^{\rm as}(x)\big[1 \quad + \quad a_2(\mu_0^2)\; C_2^{3/2}(2x-1) \\ \qquad\qquad\qquad\qquad + \quad a_4(\mu_0^2)\; C_4^{3/2}(2x-1)\big]$$

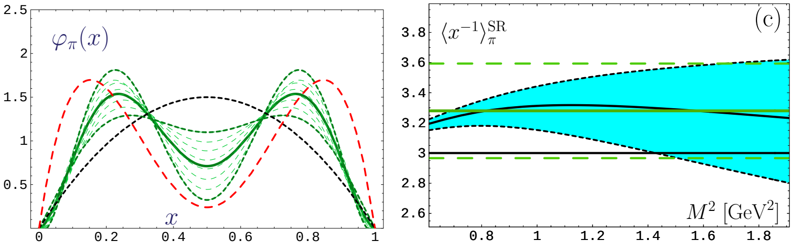




moments **Gegenbauer coefficients**

- First N=10 moments at $\mu_0^2 \simeq 1$ GeV^2, determined with nonlocal QCD sum rules. Uncertainties are shown as grey bars.
- $\langle x^{-1}\rangle_\pi^{\rm R} = (1/3)\langle x^{-1}\rangle_\pi - 1$.

QCD sum rules with nonlocal condensates yield "bunch" of π-DAs



$\lambda_q^2 = 0.4$ GeV^2 $\qquad$ $\mu^2 = 1.35$ GeV^2

- dotted line: asymptotic π-DA ($a_2 = a_4 = 0$)
- dashed line: CZ π-DA $(a_2 = 0.56,\ a_4 = 0)|_{\mu_0^2=1\ \rm GeV^2}$
- solid line inside "bunch": BMS π-DA ($a_2 = 0.2$, $a_4 = -0.14$)

Light-cone sum rules employ dispersion relation to yield $F^{\gamma^*\gamma\pi}$ [Khodjamirian, EPJC6 (1999) 477]:

$$F_{\mathrm{LCSR}}^{\gamma^*\gamma\pi}(Q^2, q^2 \to 0) = \frac{1}{\pi}\int_0^{s_0}\frac{ds}{m_\rho^2}\,\rho(Q^2, s; \mu^2)\mathrm{e}^{(m_\rho^2 - s)/M^2} + \frac{1}{\pi}\int_{s_0}^{\infty}\frac{ds}{s}\,\rho(Q^2, s; \mu^2)\,.$$

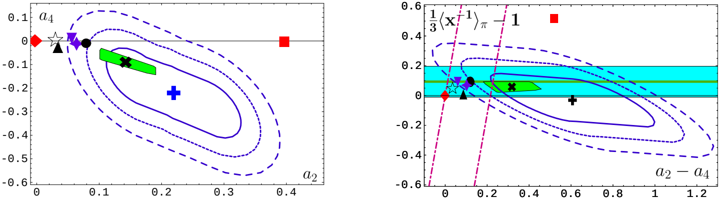
- $\rho(Q^2, s; \mu^2) \equiv \mathbf{Im}\left[F_{\mathrm{QCD}}^{\gamma^*\gamma^*\pi}(Q^2, q^2 = -s; \mu^2)\right]$
- $s_0 = 1.5$ GeV^2: Effective threshold in ρ-channel
- $M^2 \simeq 0.7$ GeV^2: Borel parameter; m_ρ: ρ-meson mass
- μ^2: Factorization scale fixed by Schmedding & Yakovlev, PRD62 (2000) 116002 to $\mu_{\mathrm{SY}}^2 = 5.76\ \mathrm{GeV}^2$.

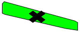
Bakulev-Mikhailov-Stefanis CLEO-data analysis

[PRD67(2003)074012, PLB578(2004)91, PRD73(2006)056002]

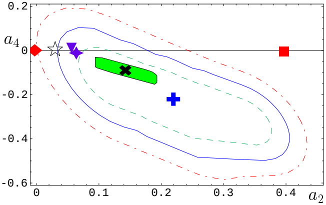
- Accurate NLO evolution of $\varphi\left(x, Q^2_{\rm exp}\right)$ and $\alpha_s\left(Q^2_{\rm exp}\right)$, including quark-mass thresholds
- Estimates of uncertainties from twist-four contributions via nonlocality scale $\boxed{\delta^2_{Tw-4} \simeq \lambda^2_q/2}$ to get $\delta^2_{Tw-4} = 0.19 \pm 0.02$ for $\lambda^2_q = 0.4$ GeV^2
- Determination of error ellipses of CLEO data by employing renormalon-based model to maximally include twist-four contributions
- Constraints on $\langle x^{-1}\rangle$ from CLEO data

NLO Light-cone SRs; $\delta^2_{\rm Tw-4} = 0.19(4)$ GeV^2; $\lambda^2_q = 0.4$ GeV^2; $\mu^2 = 5.76$ GeV^2 (left panel); $\mu^2 \simeq 1$ GeV^2 (right panel)



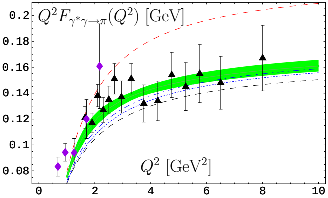
- ◆: asymptotic DA; ▼: transverse lattice; ●: Ball-Zwicky DA
- ☆; ✥; ▲: instanton-based DAs; ■: CZ DA
- ✚: BMS best-fit point; ✖: BMS DA;  : BMS bunch
- Green strip on RHS denotes **independent SR** for $\frac{1}{3}\langle x^{-1}\rangle_\pi - 1$

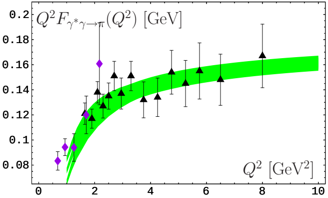
Higher-order perturbative uncertainties



- Complete NNLO coefficient yet **unknown**
- Partial results exist in conformal scheme [Melić, Müller, Passek-Kumerički, PRD68 (2003) 014013]
- Rough estimate obtained by varying reference scale $\mu^2 = \mu_{\rm R}^2 = \mu_{\rm F}^2 = Q^2$ in range $[Q^2/2, 2Q^2]$ entails large uncertainty $\Longrightarrow$ larger σ ellipses

Comparison between predictions for $\mathbf{F}_{\gamma^*\gamma\pi}$ and data



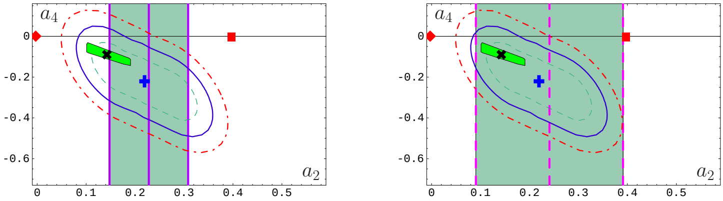


- **Left**: LCSR predictions for $Q^2 F_{\gamma^*\gamma\to\pi}(Q^2)$ vs CELLO (diamonds) and CLEO (triangles) data, for $\boxed{\delta^2_{\text{Tw}-4} = 0.19 \text{ GeV}^2}$ **Right**: $\boxed{\delta^2_{\text{Tw}-4} = 0.15 - 0.23 \text{ GeV}^2}$
- **Low-Q^2** CELLO data excludes As DA.
  High-Q^2 CLEO data excludes CZ DA

$\langle\xi^2\rangle_\pi$ from lattice

- Martinelli&Sachrajda, NPB306(1988)865
  $\langle\xi^2\rangle_\pi(2\mathrm{GeV}) = \mathbf{0.34} \pm \mathbf{0.17}$
- Del Debbio, Few Body Syst. 36(2005)77
  $\langle\xi^2\rangle_\pi(2.4\mathrm{GeV}) = \mathbf{0.27(6)}$
- QCDSF/UKQCD Collaboration, PRD74(2006)074501, hep-lat/0510089
  $\langle\xi^2\rangle_\pi(2\mathrm{GeV}) = \mathbf{0.269(39)}$
- UKQCD/RBC Collaboration, arXiv:0710.0869 [hep-lat]
  $\langle\xi^2\rangle_\pi(2\mathrm{GeV}) = \mathbf{0.28(3)}$

Comparison between CLEO-data analyses and lattice results



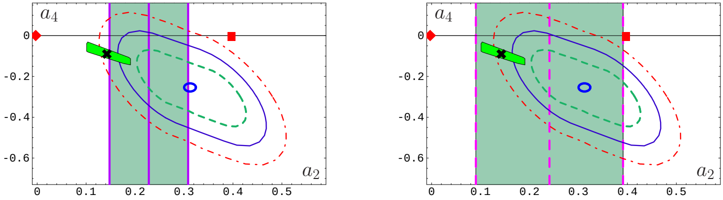
- **Left**: Strip shows QCDSF/UKQCD results [hep-lat/0510089]
- **Right**: Strip shows Del Debbio results

Error ellipses correspond to NLO Light-Cone SRs with

$$\delta^2_{\mathrm{Tw}-4} = 0.19 \pm 0.04\ \mathrm{GeV}^2, \qquad \lambda_q^2 = 0.4\ \mathrm{GeV}^2$$

Bakulev-Mikhailov-Stefanis, PLB578(2004)91

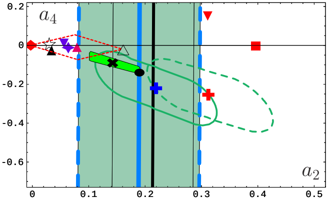
Twist-four estimates with renormalon model

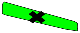


- **Left**: Strip shows QCDSF/UKQCD results [hep-lat/0510089]
- **Right**: Strip shows Del Debbio results

• Error ellipses correspond to NLO Light-Cone SRs with $\lambda_q^2 = 0.4$ GeV^2 and twist-four contributions modelled by renormalon approach [Bakulev-Mikhailov-Stefanis, PRD73(2006)056002]

Collage of π DA models, CLEO data, and lattice results



- All results evaluated at $\mu^2_{\rm SY} = 5.76$ GeV^2 after NLO evolution
- Only 1 σ ellipse of CLEO data shown. **solid contour**: twist-4 via $\delta^2_{\rm Tw-4}$. **broken contour**: twist-4 via renormalon model
-  : BMS bunch; □: Braun-Filyanov; ■: CZ DA
- Dashed rhombus contains Ball-Zwicky constraints
- **Latest** lattice results superimposed in form of vertical strip

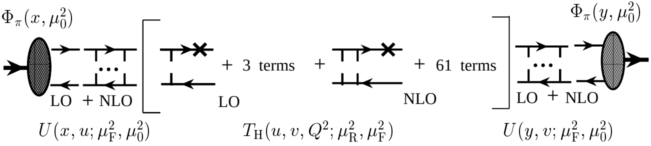
Definition of pion's electromagnetic form factor in QCD:

$$\langle \pi^+(P')|J_\mu(0)|\pi^+(P)\rangle = (P+P')_\mu F_\pi(Q^2)$$

Total form factor:

$$F_\pi = F_\pi^{\rm Fact}(Q^2) + F_\pi^{\rm non-Fact}(Q^2)$$

- $F_\pi^{\rm Fact}(Q^2)$ describes hard gluon exchanges in terms of **convolutions** with universal π DAs
  $(P'-P)^2 = q^2 \equiv -Q^2$: photon virtuality
- $F_\pi^{\rm non-Fact}(Q^2)$ originates from soft nonperturbative effects



- $F_{\pi}^{\rm Fact}(Q^2;\mu_{\rm R}^2) = F_{\pi}^{\rm LO}(Q^2;\mu_{\rm R}^2) + F_{\pi}^{\rm NLO}(Q^2;\mu_{\rm R}^2)$
- $F_{\pi}^{\rm LO}(Q^2;\mu_{\rm R}^2) = \boldsymbol{\alpha_{\rm s}(\mu_{\rm R}^2)}\ \mathcal{F}_{\pi}^{\rm LO}(Q^2)$; $\mu_{\rm R}$: renormalization scale

  $$Q^2\mathcal{F}_{\pi}^{\rm LO}(Q^2) = 8\,\pi\, f_{\pi}^2 \left[1 + \mathbf{a_2}^{\rm D,NLO}(Q^2) + \mathbf{a_4}^{\rm D,NLO}(Q^2)\right]^2$$

- $F_{\pi}^{\rm NLO}(Q^2;\mu_{\rm R}^2) =$
  $$\frac{\boldsymbol{\alpha_{\rm s}^2(\mu_{\rm R}^2)}}{\boldsymbol{\pi}}\left[\mathcal{F}_{\pi}^{\rm D,NLO}(Q^2;\mu_{\rm R}^2) + \mathcal{F}_{\pi}^{\rm ND,NLO}(Q^2;N_{\rm Max}=\infty)\right]$$
- $\mathbf{a_n}^{\rm D,NLO} \to \mathbf{a_n}^{\rm D,LO}$; $\mathbf{a_n}^{\rm ND,NLO} \to 0$.
  nonperturbative coefficients

• $N_{\rm Max}$ marks maximal number of Gegenbauer polynomials included
Full details provided in BPSS, PRD70(2004)033014

Theoretical treatment of $F_{\pi}^{\rm Fact}(Q^2;\mu_{\rm R}^2)$ at NLO

$F_{\pi}^{\rm Fact}(Q^2;\mu_{\rm R}^2)$ expressible in form of *convolution* at leading-twist two:

$$F_{\pi}^{\rm Fact}(Q^2;\mu_{\rm R}^2) = f_{\pi}^2 \int_0^1 [dx][dy]\varphi_{\pi}^*(x,\mu_{\rm F}^2)\, T_{\rm H}(x,y,Q^2;\mu_{\rm F}^2,\lambda_{\rm R}Q^2)\varphi_{\pi}(y,\mu_{\rm F}^2)$$

- $\mu_{\rm F}$ factorizes **long** from **short**-distance dynamics
- $\varphi_{\pi}(x,\mu_{\rm F}^2)$ encodes nonperturbative pion bound state
- $T_{\rm H}(x,y,Q^2;\mu_{\rm F}^2,\mu_{\rm R}^2)$ describes hard subprocesses in pQCD:

$$\begin{aligned} T_{\rm H}(x,y,Q^2;\mu_{\rm F}^2,\mu_{\rm R}^2) &= \alpha_s(\mu_{\rm R}^2)\, T_{\rm H}^{(0)}(x,y,Q^2) \\ &+ \frac{\alpha_s^2(\mu_{\rm R}^2)}{4\pi}\, T_{\rm H}^{(1)}(x,y,Q^2;\mu_{\rm F}^2,\mu_{\rm R}^2)\ \ldots . \end{aligned}$$

- $T_{\rm H}^{(0)}(x,y,Q^2) = \frac{N_{\rm T}}{Q^2}\frac{1}{\bar{x}\bar{y}}$; $N_{\rm T} = \frac{2\pi C_{\rm F}}{C_{\rm A}} = \frac{8\pi}{9}$; $C_{\rm F} = \frac{\left(N_{\rm c}^2-1\right)}{2N_{\rm c}} = \frac{4}{3}$;
  $C_{\rm A} = N_{\rm c} = 3$; $b_0 = \frac{11}{3}\,C_{\rm A} - \frac{4}{3}\,T_{\rm R}N_f$; $T_{\rm R} = \frac{1}{2}$; $\ln\frac{Q^2}{\mu_{\rm F}^2} = \ln\frac{\lambda_{\rm R}Q^2}{\Lambda^2} - \ln\frac{\lambda_{\rm R}\mu_{\rm F}^2}{\Lambda^2}$

$\mathbf{F}_\pi^{\rm Fact}$ in **Fractional Analytic Perturbation Theory**

• **Analytic** means no ghost singularities in strong running coupling by virtue of a dispersion relation with a spectral density based on perturbative QCD [Shirkov & Solovtsov, PRL79(1997)1209]:

pQCD | FAPT

$$\sum_\mu d_\mu a^\mu_{(l)}(Q^2) \quad \Rightarrow \quad \sum_\mu d_\mu \mathcal{A}^{(l)}_\mu(Q^2)\,,$$

μ: power | μ: index

• $a = b_0\alpha_s/4\pi$: "normalized" coupling $\Longrightarrow \mathcal{A}_1 \leq 1; \quad \mu \in \mathbb{R}$

$$\mathcal{A}^{(l)}_\nu(Q^2) \equiv \int_0^\infty \frac{\rho^{(l)}_\nu(\sigma)}{\sigma + Q^2}\, d\sigma\,; \qquad \rho^{(l)}_\nu(\sigma) \equiv \frac{1}{\pi}\,\mathbf{Im}\,[a^\nu_{(l)}(-\sigma)]$$

$$\mathcal{A}^{(1)}_1(Q^2) = \frac{1}{\ln(Q^2/\Lambda^2)} - \frac{1}{Q^2/\Lambda^2 - 1}$$

Corollary

Various Analytization Procedures

- *Naive* Analytization [Stefanis, Schroers, Kim, EPJC18(2000)137]

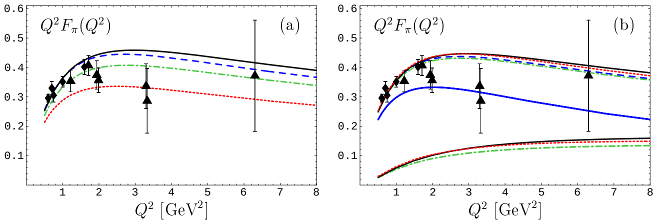
$$\left[Q^2 T_{\rm H}(x,y,Q^2;\mu_{\rm F}^2,\lambda_{\rm R}Q^2)\right]_{\rm Naive-An} = \mathcal{A}_1^{(2)}(\lambda_{\rm R}Q^2)\, t_{\rm H}^{(0)}(x,y) + \frac{\left[\mathcal{A}_1^{(2)}(\lambda_{\rm R}Q^2)\right]^2}{4\pi}\, t_{\rm H}^{(1)}(x,y;\lambda_{\rm R},\frac{\mu_{\rm F}^2}{Q^2})$$

- *Maximal* Analytization [Bakulev, Passek, Schroers, Stefanis, PRD 70 (2004) 033014]

$$\left[Q^2 T_{\rm H}(x,y,Q^2;\mu_{\rm F}^2,\lambda_{\rm R}Q^2)\right]_{\rm Max-An} = \mathcal{A}_1^{(2)}(\lambda_{\rm R}Q^2)\, t_{\rm H}^{(0)}(x,y) + \frac{\mathcal{A}_2^{(2)}(\lambda_{\rm R}Q^2)}{4\pi}\, t_{\rm H}^{(1)}(x,y;\lambda_{\rm R},\frac{\mu_{\rm F}^2}{Q^2})$$

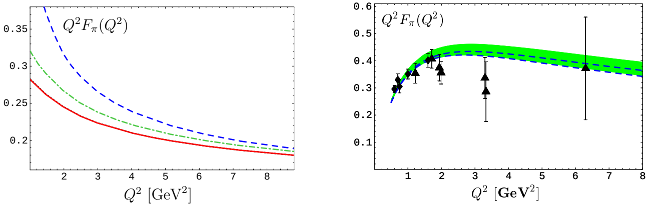
- Karanikas-Stefanis analytization [PLB 504 (2001) 225]. Includes all logs that can contribute to spectral density

Predictions for $Q^2 F_\pi$ using analytization and BMS π DA



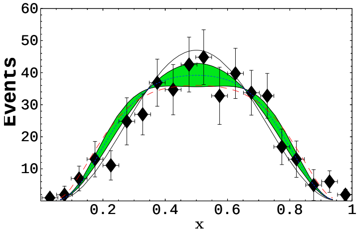
- **Left**: Naive Analytization [Stefanis, Schroers, Kim, PLB449(1999)299; EPJC18(2000)137]
- **Right**: Maximal Analytization [Bakulev, Passek, Schroers, Stefanis, PRD70(2004)033014]
- Solid line in right panel shows $Q^2 F_\pi^{\mathrm{non-Fact}}(Q^2)$ based on local duality

Predictions for $Q^2 F_\pi$ using Maximal Analytization and BMS π DAs bunch



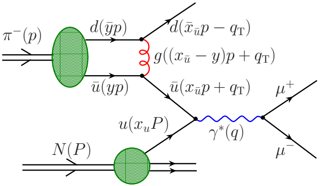
- **Left**: Dashed line denotes prediction in standard pQCD in $\overline{MS}$ scheme; dash-dotted line shows result with Naive Analytization; solid line represents result in Maximal and KS Analytization [Bakulev, Karanikas, Stefanis, PRD72(2005)074015]
- **Right**: Green strip shows predictions with Maximal Analytization for BMS bunch from QCD SRs with **nonlocal condensates** in comparison with existing experimental data
- Shaded area shows area corresponding to asymptotic π DA

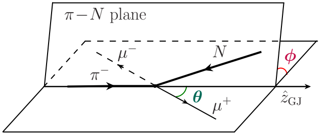
- First theoretical estimates by Frankfurt, Miller, Strikman, PLB304(1993)1
- Convolution approach to account for hard-gluon exchanges proposed by Braun et al., NPB638(2002)111
- Alternative approaches by Chernyak, Grozin, PLB517(2001)119; Nikolaev, Schäfer, Schwiele, PRD63(2001)014020
- Bakulev, Mikhailov, Stefanis, used convolution approach with results displayed below [PLB578(2004)91]



DA	χ^2
Asy	12.56
BMS bunch	10.96
CZ	14.15

- Drell-Yan process $\pi^- N \to \mu^+\mu^- X$ dominant mechanism for production of lepton pairs with large invariant mass Q^2

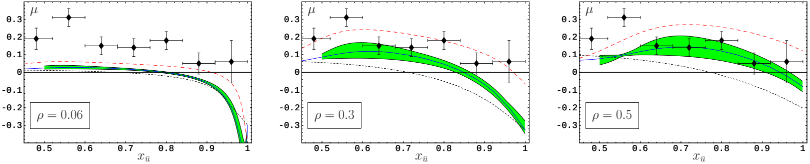




- FNAL experiment E615: $s = (p_\pi + P_N)^2 = 500$ GeV^2, $Q^2 = q^2 = 16 - 70$ GeV^2, $\rho \equiv Q_T/Q = 0 - 0.5$, $Q_T^2 = -q_T^2$
- As $x_{\bar{u}} \to 1$, $p_{\bar{u}}^2$ large and far spacelike $\Longrightarrow$ u-quark nearly free and on-shell, i.e., $x_u = x_N$ (no transverse momenta)

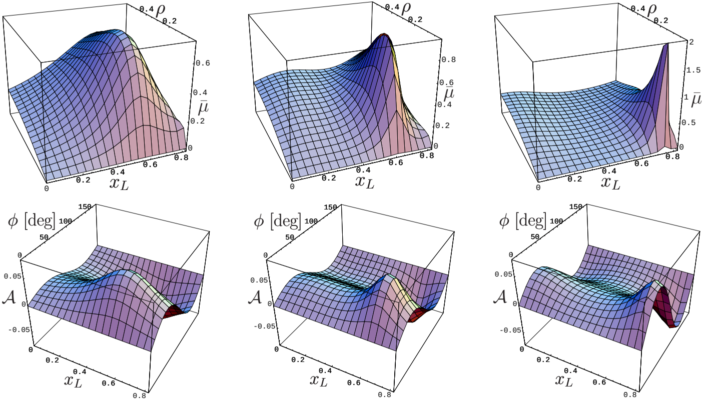
Angular distribution of μ^+ in pair rest frame of DY reaction with unpolarized target can be written in terms of kinematic variables λ, μ, ν [Brandenburg et al., PRL73(1994)939]:

$$\frac{d^5\sigma(\pi^- N \to \mu^+\mu^- X)}{dQ^2 dQ_T^2 dx_L \, d\cos\theta d\phi} \propto N(\tilde{x}, \rho) \;\; 1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi$$



[Bakulev, Stefanis, Teryaev, PRD76(2007)074032; Data from FNAL E615]

- Green strip corresponds to BMS bunch of π DAs
- Solid line shows result for asymptotic DA
- Dashed line denotes result for CZ π DA



- **Left**: Asy. π DA; **Center**: BMS DA; **Right**: CZ DA at $s = 100$ GeV^2
- **Upper row**: *Angular parameter* $\bar{\mu}(x_L, \rho)$ for polarized DY
  Lower row: *Azimuthal asymmetry* $\mathcal{A}(\phi, x_L, \rho = 0.3)$ at $q^2 = 16\ \mathrm{GeV}^2$ and $s = 100\ \mathrm{GeV}^2$

- BMS π DAs bunch yields best overall agreement with several sets of experimental data on various reactions from low to high momenta
- BMS CLEO-data LCSR analysis stable against both NNLO uncertainties and influence of twist-four contributions
- BMS π DAs bunch within 1σ error ellipse of CLEO data and in full compliance with most recent high-precision lattice results on first moment $\langle\xi^2\rangle_\pi$ (or Gegenbauer coefficient a_2)
- Asymptotic and CZ π DAs off CLEO data at 4σ and 3σ level, respectively
- Use of Maximal Analytization for electromagnetic pion form factor significantly reduces dependence on renormalization-setting procedure and renormalization scale
- Adopting Karanikas-Stefanis analytization provides insensitivity also to factorization scale in electromagnetic pion form factor, further reducing theoretical bias
- Drell-Yan process provides in terms of azimuthal asymmetries further possibilities to discriminate among various π DAs