Conversion decays of light vector mesons and QED process $e^+e^- \rightarrow e^+e^-\gamma\gamma$
in the energy region $2E \sim 1$ GeV

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Conversion decays and double bremsstrahlung: what do they have in common?
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   - Equal final state in case of $\gamma^* \rightarrow e^+e^-$ conversion
   - Background for each other
Conversion decays of light vector mesons

Conversion decay \((l = e, \mu)\)

\[4m_i^2 \leq q^2 \leq (m_V - m_P)^2\]

Radiative decay

\[q^2 = 0, \quad \Gamma \sim |f(0)|^2\]

\[
\frac{d}{dq^2} \frac{\Gamma(V \to Pl^+l^-)}{\Gamma(V \to P\gamma)} = \frac{\alpha}{3\pi} \frac{|F(q^2)|^2}{q^2} \left(1 + \frac{2m_i^2}{q^2}\right) \sqrt{1 - \frac{4m_i^2}{q^2}} \times \\
\times \left(\left(1 + \frac{q^2}{m_V^2 - m_P^2}\right)^2 - \frac{4q^2m_V^2}{(m_V^2 - m_P^2)^2}\right)^{\frac{3}{2}}, \text{ where } F(q^2) = \frac{f(q^2)}{f(0)}
\]

In pole approximation, \(F(q^2) = \frac{1}{1 - q^2/\Lambda^2}\)
### Branching fractions: experimental results

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Upper Limit</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^0 \rightarrow \pi^0 e^+ e^-$</td>
<td>$&lt; 1.2 \cdot 10^{-5}$</td>
<td>SND08</td>
</tr>
<tr>
<td>$\rho^0 \rightarrow \eta e^+ e^-$</td>
<td>$&lt; 7 \cdot 10^{-6}$</td>
<td>CMD05</td>
</tr>
<tr>
<td>$\omega \rightarrow \pi^0 e^+ e^-$</td>
<td>$(7.7 \pm 0.6) \cdot 10^{-4}$</td>
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<tr>
<td>$\omega \rightarrow \eta e^+ e^-$</td>
<td>$&lt; 1.1 \cdot 10^{-5}$</td>
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<tr>
<td>$\phi \rightarrow \pi^0 e^+ e^-$</td>
<td>$(1.12 \pm 0.28) \cdot 10^{-5}$</td>
<td>PDG06</td>
</tr>
<tr>
<td>$\phi \rightarrow \eta e^+ e^-$</td>
<td>$(1.15 \pm 0.10) \cdot 10^{-4}$</td>
<td>PDG06</td>
</tr>
<tr>
<td>$\phi \rightarrow \eta \mu^+ \mu^-$</td>
<td>$&lt; 9.4 \cdot 10^{-6}$</td>
<td>CMD01</td>
</tr>
</tbody>
</table>

### Statistical errors
- best $\sim 7\%$
- typical $\sim 10\%$

### Systematic errors
- best $\sim 5\%$
- typical $\lesssim 10\%$

In many measurements, $\sigma_{sys} \ll \sigma_{stat}$
## Branching fractions: comparison with theory

<table>
<thead>
<tr>
<th>Process</th>
<th>Branching Fraction</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho^0 \to \pi^0 e^+ e^- )</td>
<td>(&lt; 1.2 \cdot 10^{-5} )</td>
<td>Exp.</td>
</tr>
<tr>
<td>(</td>
<td>\rho^0</td>
<td>= 5.1 \cdot 10^{-6} )</td>
</tr>
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<td>(</td>
<td>\rho^0</td>
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</tr>
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<tr>
<td>(</td>
<td>\omega</td>
<td>= 3.5 \cdot 10^{-6} )</td>
</tr>
<tr>
<td>(</td>
<td>\omega</td>
<td>= 6 \cdot 10^{-6} )</td>
</tr>
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<td>(</td>
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<tr>
<td>(</td>
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<td>= 2.1 \cdot 10^{-6} )</td>
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<td>Exp.</td>
</tr>
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<td>\phi</td>
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**Abbreviations:**
- **PLP**: Point-like particles (\( F=1 \))
- **VDM**: Vector dominance model
- **HLS**: Hidden local symmetry
- **LQCD**: Lattice QCD

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Conversion decays and double bremsstrahlung
Transition form factors: experimental results

\[ \omega \rightarrow \pi^0 \gamma^* \text{ transition} \]

\[ \phi \rightarrow \eta \gamma^* \text{ transition} \]

\[ |F_{\omega\pi}(q)|^2 \]

\[ |F_{\phi\eta}(q)|^2 \]

- **SERP81** \((\omega \rightarrow \pi^0 \mu^+ \mu^-)\)
- **CMD05** \((\omega \rightarrow \pi^0 e^+ e^-)\)
- **SND08** \((\omega \rightarrow \pi^0 e^+ e^-)\)

- **SND01** \((\phi \rightarrow \eta e^+ e^-)\)
Transition form factors: comparison with theory

\[ |F_{\pi\gamma^*}(q)|^2 \]

- Pole approximation (\( \Lambda = 653 \pm 15 \text{ MeV} \))
- DSE (Maris - Tandy)
- VDM with one \( \rho \) pole
- VDM with \( \rho \) and \( \rho' \) poles

\[ |F_{\eta\gamma^*}(q)|^2 \]

- Pole approximation (\( \Lambda = 650 \pm 270 \text{ MeV} \))
- VDM with one \( \phi \) pole

**\( \omega \to \pi^0\gamma^* \) transition**

Not described by VDM with \( \rho \) and extended VDM with \( \rho \) and \( \rho' \)

**\( \phi \to \eta\gamma^* \) transition**

No preferable model due to low accuracy

New experimental data are required, especially at large \( q \)
## Conversion decays: further prospects

### VEPP-2M

\[ \mathcal{L} \sim 10^{30} \text{ cm}^{-2}\text{s}^{-1} (2E = m_\phi) \]

Detectors: ND, SND, CMD–2

### VEPP-2000

\[ \mathcal{L} \sim 10^{31} \text{ cm}^{-2}\text{s}^{-1} (2E = m_\phi) \]

\[ \mathcal{L} \sim 10^{32} \text{ cm}^{-2}\text{s}^{-1} (2E = 2 \text{ GeV}) \]

Detectors: SND, CMD–3

### Typical accuracy

- Statistical \( \sim 10\% \), systematic \( \sim 10\% \)

### Systematic error determined by

- Photon conversion on material before tracking system
- Close \( e^+ e^- \) tracks separation

### Estimated accuracy

- \( Br \sim 10^{-4} - 10^{-5} \): statistical \( \sim 3 - 5\% \), systematic \( \approx 5\% \)
- \( Br \sim 10^{-6} \): statistical \( \sim 30\% \), systematic \( \approx 5\% \)

### Improvement of systematic error

- Upgrade of tracking systems

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Conversion decays of \( \phi \) meson also can be studied with KLOE at DAFNE

Alexandr E. Obrazovsky
Conversion decays and double bremsstrahlung
High-order QED processes: double bremsstrahlung $e^+e^- \rightarrow e^+e^-\gamma\gamma$
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Tests of QED:
High-order QED processes: double bremsstrahlung $e^+e^- \rightarrow e^+e^-\gamma\gamma$

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  - Small $q^2$: $(g_e - 2)/2$, accuracy of $3 \cdot 10^{-9}$;
High-order QED processes:
double bremsstrahlung $e^+e^- \rightarrow e^+e^-\gamma\gamma$

Tests of QED:
- Small $q^2$: $(g_e - 2)/2$, accuracy of $3 \cdot 10^{-9}$;
- Large $q^2$:
  - most of experimental data were obtained at high energies ($\gtrsim 10$ GeV);
High-order QED processes: double bremsstrahlung $e^+ e^- \rightarrow e^+ e^- \gamma \gamma$

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Background for $e^+ e^- \rightarrow$ hadrons, e.g. $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$ and conversion decays, e.g. $\omega \rightarrow \pi^0 e^+ e^-$;
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Encouragement for development of calculational techniques and precise MC generators;
High-order QED processes:
double bremsstrahlung \( e^+e^- \rightarrow e^+e^-\gamma\gamma \)

1. **Tests of QED:**
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     - most of experimental data were obtained at high energies (\( \gtrsim 10 \) GeV);
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2. **Background for** \( e^+e^- \rightarrow \) hadrons, e.g. \( e^+e^- \rightarrow \pi^+\pi^-\pi^0 \) and conversion decays, e.g. \( \omega \rightarrow \pi^0 e^+e^- \);

3. **Encouragement** for development of calculational techniques and precise MC generators;

4. **Search** for exotics (heavy leptons, axions etc.).
$e^+e^- \rightarrow e^+e^-\gamma\gamma$: QED tests

<table>
<thead>
<tr>
<th>Date</th>
<th>Detector</th>
<th>Collider</th>
<th>$2E$, GeV</th>
<th>$N_{\text{evt}}$</th>
<th>$\sigma_{\text{exp}}/\sigma_{\text{th}}$</th>
</tr>
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<tbody>
<tr>
<td>1987</td>
<td>JADE</td>
<td>PETRA</td>
<td>34.4</td>
<td>176</td>
<td>$1.035 \pm 0.078 \pm 0.030$</td>
</tr>
<tr>
<td>1989</td>
<td>ASP</td>
<td>PEP</td>
<td>29</td>
<td>931</td>
<td>$0.94 \pm 0.03 \pm 0.03$</td>
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<tr>
<td>1989</td>
<td>MARK II</td>
<td>PEP</td>
<td>29</td>
<td>41</td>
<td>$0.995 \pm 0.075 \pm 0.154$</td>
</tr>
<tr>
<td>2000</td>
<td>SND</td>
<td>VEPP-2M</td>
<td>0.98 – 1.04</td>
<td>649</td>
<td>$0.998 \pm 0.085 \pm 0.061$</td>
</tr>
</tbody>
</table>

$e^+e^- \rightarrow e^+e^-\gamma\gamma$: QED tests and search for exotics

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<thead>
<tr>
<th>Date</th>
<th>Detector</th>
<th>Collider</th>
<th>$2E$, GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1988</td>
<td>AMY</td>
<td>TRISTAN</td>
<td>50 – 56</td>
</tr>
<tr>
<td>1991</td>
<td>ND</td>
<td>VEPP-2M</td>
<td>0.6 – 1.4</td>
</tr>
<tr>
<td>1993</td>
<td>ALEPH</td>
<td>LEP</td>
<td>91.2</td>
</tr>
<tr>
<td>1993</td>
<td>L3</td>
<td>LEP</td>
<td>91.2</td>
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<tr>
<td>1993</td>
<td>OPAL</td>
<td>LEP</td>
<td>91.2</td>
</tr>
<tr>
<td>2008</td>
<td>SND</td>
<td>VEPP-2M</td>
<td>0.36 – 1.38</td>
</tr>
</tbody>
</table>
$e^+ e^- \to e^+ e^- \gamma \gamma$: SND results

**SND00**

$36^\circ \leq \theta_i \leq 144^\circ$, $\Delta \varphi \geq 5^\circ$, $\alpha_{ij} \geq 20^\circ$, $E_{min} = 10$ MeV, $\omega_{min} = 20$ MeV

Statistical error: 9%
Systematic error: 6%

**SND08 preliminary**

$20^\circ \leq \theta_i \leq 160^\circ$, $\Delta \varphi \geq 9^\circ$, $\alpha_{ij} \geq 9^\circ$, $E_{min} = 10$ MeV, $\omega_{min} = 10$ MeV

Statistical error: 8%
Systematic error: to be determined
# High-order QED: further prospects

## VEPP-2M
\[ \mathcal{L} \sim 10^{30} \text{ cm}^{-2}\text{s}^{-1}(2E = m_\phi) \]
**Detectors:** ND, SND

## Typical accuracy
- Statistical \( \lesssim 10\% \)
- Systematic \( \lesssim 10\% \)

## Systematic error determined by
**Normalization uncertainty**

## VEPP-2000
\[ \mathcal{L} \sim 10^{31} \text{ cm}^{-2}\text{s}^{-1}(2E = m_\phi) \]
\[ \mathcal{L} \sim 10^{32} \text{ cm}^{-2}\text{s}^{-1} (2E = 2 \text{ GeV}) \]
**Detectors:** SND, CMD-3

## Estimated accuracy
- 4-th order: Statistical \( \sim 3 - 5\% \), systematic \( \sim 5\% \)
- 5-th order: Statistical \( \sim 30\% \), systematic \( \sim 10\% \)

## Improvement of systematic error
- Upgrade of tracking systems
- Precise simulation of high-order QED

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*Alexander E. Obrazovsky*  
*Conversion decays and double bremsstrahlung*
Conclusions: conversion decays

1. 8 conversion decay modes of light vector mesons $\rho, \omega, \phi$ has been studied experimentally. Typical accuracy of recent measurements is $\sim 10\%$ (both for statistical and systematic errors). Experimental data on branching fractions are in good agreement with theoretical predictions.

2. $\omega \rightarrow \pi^0 \gamma^*$ and $\phi \rightarrow \eta \gamma^*$ transition form factors were measured. Experimental data on $\omega \rightarrow \pi^0 \gamma^*$ disagree with VDM and extended VDM at large $q^2$. Accuracy of $\phi \rightarrow \eta \gamma^*$ measurement is insufficient to make choice between theoretical models. New form factor measurements are required, especially at large $q^2$.

3. Estimates of conversion decay accuracy in future experiments at VEPP-2000 collider: $3 - 5\%$ for statistical error and $5\%$ for systematic error ($Br \sim 10^{-4} - 10^{-5}$), $30\%$ for statistical error and $10\%$ for systematic error ($Br \sim 10^{-6}$). Conversion decays of $\phi$ meson can also be studied with KLOE at DAΦNE.
Conclusions: $e^+e^- \rightarrow e^+e^-\gamma\gamma$

1. Double bremsstrahlung process $e^+e^- \rightarrow e^+e^-\gamma\gamma$ has been studied in 7 experiments at high energies ($2E \gtrsim 10$ GeV) and 3 experiments at intermediate energies ($2E \sim 1$ GeV). Typical accuracy at intermediate energies is $\sim 10\%$ (both for statistical and systematic errors).

2. No deviations from QED is found within limits of measurement errors.

3. Estimates of high-order QED accuracy in future experiments at VEPP-2000 collider: $3 - 5\%$ for statistical error and $5\%$ for systematic error (4-th order QED), $30\%$ for statistical error and $10\%$ for systematic error (5-th order QED).
Appendix: spectrum of $l^+l^-$ invariant masses
Appendix: CMD-2 detector

**Drift chamber**

\[ \sigma_{R\phi} = 250 \, \mu m \]

**Barrel CsI calorimeter**

\[ \sigma_E/E = 9 \% \]

**End-cap BGO calorimeter**

\[ \sigma_E/E = 4-9 \% \]

**Solid angle**

\[ \Omega/4\pi = 0.92 \]

1–vacuum chamber; 2–drift chamber; 3– Z–chamber; 4–main solenoid; 5–compensating solenoid; 6–BGO endcap calorimeter; 7–CsI barrel calorimeter; 8–muon range system; 9–iron yoke; 10–storage ring lenses
Appendix: SND detector

Tracking system: 2 — drift chambers, 3 — cylindrical scintillation counter. 
Calorimeter: 6 — NaI(Tl) crystals, 7 — vacuum phototriods. 
Muon system: 9 — streamer tubes, 11 — scintillation counters.

<table>
<thead>
<tr>
<th>Tracking system</th>
<th>NaI(Tl) calorimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_\theta = 0.5^\circ$, $\sigma_\varphi = 2^\circ$</td>
<td>$\sigma_E/E = 4.2% / \sqrt[4]{E(\text{GeV})}$, $\sigma_\varphi = 1.5^\circ$</td>
</tr>
<tr>
<td>$\Omega/4\pi = 0.95$</td>
<td>$\Omega/4\pi = 0.9$</td>
</tr>
</tbody>
</table>
### Theoretical works on conversion decays

#### Calculation of branching fractions in vector dominance model

#### Hidden local symmetry model with isospin/SU(3) breaking terms

#### Lattice QCD calculation of branching fractions

#### Form factor calculation using Dyson-Schwinger equations