Status of BabaYaga@NLO and its theoretical accuracy

Carlo M. Carloni Calame

INFN & University of Southampton

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Outline

- Introduction
- Theoretical framework of the old and new **BabaYaga** (**v.3.5** and **BabaYaga@NLO**)
- Estimate of the Bhabha theoretical accuracy
  - comparison with independent generators
  - comparison with two loop calculations
  - vacuum polarization uncertainties
  - other 2-loop (order α^2) uncertainties

⋆ **BabaYaga@NLO** for $e^+e^- \to \gamma\gamma$ and $e^+e^- \to \mu^+\mu^-$

- Conclusions

Physics motivations

- The program of low energy e^+e^- colliders requires a precise determination of the luminosity.
  e.g. R measurement $\rightarrow$ $(g-2)_\mu$ and $\alpha_{EM}(M_Z)$
- precise theoretical calculations of well known processes are mandatory

$$\mathcal{L} = \frac{N^X_{obs}}{\sigma^X_{theory}}$$

$$\frac{\delta\mathcal{L}}{\mathcal{L}} = \frac{\delta N^X_{obs}}{N^X_{obs}} \oplus \frac{\delta\sigma^X_{theory}}{\sigma^X_{theory}}$$

- QED processes are the best choice
  - ⋆ $e^+e^- \rightarrow e^+e^-$
  - ⋆ $e^+e^- \rightarrow \gamma\gamma$
  - ⋆ $e^+e^- \rightarrow \mu^+\mu^-$

The original BabaYaga (v.3.5)

- it is a MCEG for $e^+e^- \to e^+e^-, \gamma\gamma, \mu^+\mu^-, \pi^+\pi^-$ at flavour factories, developed for luminosity measurement

C.M.C.C. et al., **NPB** 584 (2000)

C.M.C.C., **PLB** 520 (2001)

- the QED RC corrections were included with an (original) QED Parton Shower (PS), allowing for
  1. an *exact* solution of the QED DGLAP equation
  2. fully exclusive multi-photon generation (up to ∞ photons)
  3. natural inclusion of $\mathcal{O}(\alpha)$ and higher order QED photonic corrections in leading-log (LL) approximation
- theoretical error due to missing $\mathcal{O}(\alpha)$ non-log terms, not naturally reproduced by the PS.
  Estimated accuracies:
  - 0.5% for Bhabha
  - $\simeq \mathcal{O}(1\%)$ for $\gamma\gamma$ and $\mu^+\mu^-$

PS and exact $\mathcal{O}(\alpha)$ matrix elements (`BabaYaga@NLO`)

G. Balossini et al., **NPB** 758 (2006) 227, `hep-ph/0607181`

Parton Shower (LL) and exact $\mathcal{O}(\alpha)$ (QED NLO) matrix elements must be combined and matched. How?

- $d\sigma^{\infty}_{LL} = \Pi(Q^2,\varepsilon) \; \sum_{n=0}^{\infty} \frac{1}{n!} \; |\mathcal{M}_{n,LL}|^2 \; d\Phi_n$
- $d\sigma^{\alpha}_{LL} = [1 + C_{\alpha,LL}] \, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1 \equiv d\sigma_{SV}(\varepsilon) + d\sigma_H(\varepsilon)$
- $d\sigma^{\alpha}_{exact} = [1 + C_{\alpha}] \, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_1|^2 d\Phi_1$
- $F_{SV} = 1 + (C_\alpha - C_{\alpha,LL}) \qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,LL}|^2}{|\mathcal{M}_{1,LL}|^2}$
- $d\sigma^{\alpha}_{exact} \overset{\text{at } \mathcal{O}(\alpha)}{=} F_{SV}(1 + C_{\alpha,LL})|\mathcal{M}_0|^2 d\Phi_0 + F_H |\mathcal{M}_{1,LL}|^2 d\Phi_1$

$$d\sigma^{\infty}_{matched} = F_{SV} \; \Pi(Q^2,\varepsilon) \; \sum_{n=0}^{\infty} \frac{1}{n!} \; \left(\prod_{i=0}^{n} F_{H,i}\right) \; |\mathcal{M}_{n,LL}|^2 \; d\Phi_n$$

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$$d\sigma^{\infty}_{matched} = F_{SV}\ \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ \left(\prod_{i=0}^{n} F_{H,i}\right)\ |\mathcal{M}_{n,LL}|^2\ d\Phi_n$$

Contents of the *matched* formula

- F_{SV} and $F_{H,i}$ are infrared safe and account for missing $\mathcal{O}(\alpha)$ non-logs, avoiding double counting of LL
- $\left[\sigma^{\infty}_{matched}\right]_{\mathcal{O}(\alpha)} = \sigma^{\alpha}_{exact}$
- resummation of higher orders LL contributions preserved
- the cross section is still fully differential in the momenta of the final state particles (e^+, e^- and $n\gamma$)
- as a by-product, part of photonic $\alpha^2 L$ included by means of terms of the type $F_{SV \mid H,i} \times LL$

G. Montagna et al., **PLB** 385 (1996)

- the error is shifted to $\mathcal{O}(\alpha^2)$ (NNLO, 2 loop) not infrared terms: very naively and roughly (for photonic corrections at 1 GeV)

$$\frac{1}{2}\alpha^2 L \equiv \frac{1}{2}\alpha^2 \log\frac{s}{m^2} \sim 0.5 \times 10^{-4}$$

Vacuum Polarization

- $\alpha \to \alpha(q^2) \equiv \frac{\alpha}{1-\Delta\alpha(q^2)}$ $\quad \Delta\alpha = \Delta\alpha_{e,\mu,\tau,\text{top}} + \Delta\alpha^{(5)}_{\text{had}}$
- $\Delta\alpha^{(5)}_{\text{had}}$ is a non-perturbative contribution. Evaluated with `HADR5N` by F. Jegerlehner. It returns also an error.

  S. Eidelman and F. Jegerlehner, **Z. Phys. C** 67 (1995)

  F. Jegerlehner, **NPB Proc. Supp.** 131 (2004)
- VP included both in lowest order and (at best) in one-loop diagrams $\Rightarrow$ part of the 2 loop factorizable corrections are included
- Z exchange included at lowest order.
  Its effect can be $\mathcal{O}(0.1\%)$ at 10 GeV

Effects of RC corrections

set up	(a)	(b)	(c)	(d)
δ_{VP}	1.76	2.49	4.81	6.41
δ_{α}	-11.61	-14.72	-16.03	-19.57
δ_{HO}	0.39	0.82	0.73	1.44
δ_{HO}^{PS}	0.35	0.74	0.68	1.34
$\delta_{\alpha^2 L}$	0.04	0.08	0.05	0.10
$\delta_{\alpha}^{\text{non-log}}$	-0.34	-0.56	-0.34	-0.56

Table: Relative corrections (in per cent) to the Bhabha cross section

(a) $\sqrt{s} = 1.02 \text{ GeV}, \; E_{min}^{\pm} = 0.408 \text{ GeV}, \; 20^\circ < \theta_{\pm} < 160^\circ, \; \xi_{max} = 10^\circ$

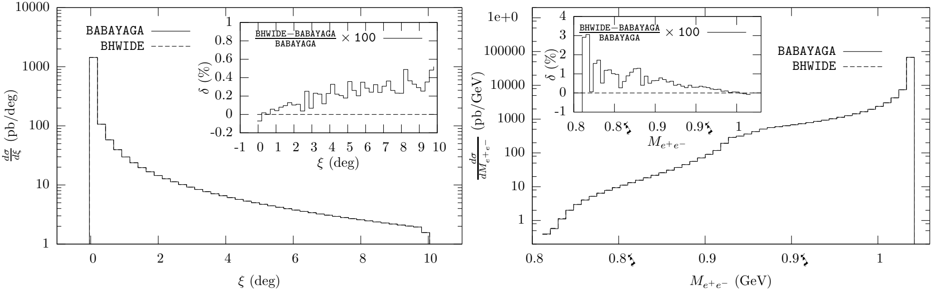
(b) $\sqrt{s} = 1.02 \text{ GeV}, \; E_{min}^{\pm} = 0.408 \text{ GeV}, \; 55^\circ < \theta_{\pm} < 125^\circ, \; \xi_{max} = 10^\circ$

(c) $\sqrt{s} = 10 \text{ GeV}, \; E_{min}^{\pm} = 4 \text{ GeV}, \; 20^\circ < \theta_{\pm} < 160^\circ, \; \xi_{max} = 10^\circ$

(d) $\sqrt{s} = 10 \text{ GeV}, \; E_{min}^{\pm} = 4 \text{ GeV}, \; 55^\circ < \theta_{\pm} < 125^\circ, \; \xi_{max} = 10^\circ$

Estimate of the theoretical accuracy

- switching off VP, tuned comparisons with independent calculations/approaches (**Labspv**, **Bhwide**)
  - ⋆ $\Delta\sigma/\sigma < 0.03\%$ on cross sections
  - ⋆ up-to-0.5% differences between **BabaYaga** and **Bhwide** only in distribution tails



- this is also a test of the technical accuracy

Estimate of the theoretical accuracy

- comparison with existing perturbative 2-loop calculations
  1. Penin: complete virtual 2-loop photonic corrections (for $Q^2 \gg m_e^2$) plus real soft radiation

     A.A. Penin, **NPB** 734 (2006), **PLB** 95 (2005)
  2. Bonciani et al.: virtual $N_F = 1$ [only electron in the loops] fermionic contributions plus real soft radiation

     R. Bonciani and A. Ferroglia, **PRD** 72 (2005)

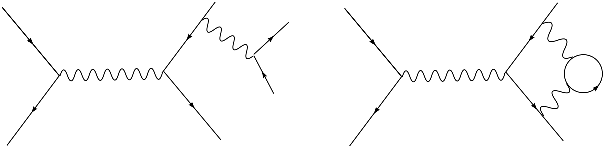
     R. Bonciani et al., **NPB** 716 (2005), **NPB** 701 (2004), **NPB** 690 (2004), **NPB** 681 (2004), **NPB** 702 (2004), **NPB** 676 (2004), **NPB** 661 (2003), **NPB** 702 (2004)

⋆ the photonic and $N_F = 1$ $\mathcal{O}(\alpha^2)$ content of the S+V part in the **BabaYaga** matched formula can be easily extracted. The terms to be directely compared to 1. and 2. can be read out!

⋆ the impact of the missing $\mathcal{O}(\alpha^2)$ S+V corrections can be quantified within realistic setup

Light-pair corrections (real & virtual)

- They contribute at $\mathcal{O}(\alpha^2)$, VPC (part of 2-loop $N_F = 1$) and RPC largely cancel. Not included in **BabaYaga**.

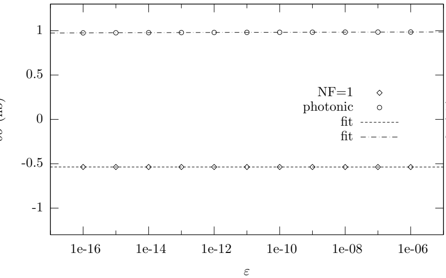


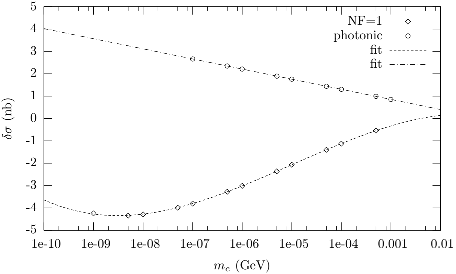
- To estimate the impact, VPC evaluated as in Jadach et al. ('97); Kniehl ('90); Burgers ('85); Barbieri et al. ('72); RPC evaluated in soft approximation as in Arbuzov et al. ('97)
- the correction does not exceed 0.05% in LABS [1] and VLABS [2] at 1 and 10 GeV (see Balossini et al., **NPB** (2006))

[1] $20^\circ < \vartheta_\pm < 160^\circ$
[2] $55^\circ < \vartheta_\pm < 125^\circ$

Differences from Penin & Bonciani et al.

- diff. between Penin and Bonciani et al. and the corresponding **BabaYaga** content, as $f(\varepsilon)$ and $g(\log(m_e))$. E.g. LABS at 1 GeV





- ⋆ differences are infrared safe
- ⋆ $\delta\sigma(phot.)/\sigma_0 \quad \propto \alpha^2 L$ from photonic α^2 corrections
  $\delta\sigma(N_F = 1)/\sigma_0 \propto \alpha^2 L^2$ from fermionic α^2 corrections
- ⋆ Numerically, in LABS and VLABS,

$$\delta\sigma(phot.) + \delta\sigma(N_F = 1) < 0.015\% \times \sigma_0$$

$\Delta\alpha^{(5)}_{\text{had}}$ and other $\mathcal{O}(\alpha^2)$ uncertainties

- $\Delta\alpha^{(5)}_{\text{hadr}}$ is affected by an error, returned by **HADR5N**
  - ⋆ the error induced on Bhabha cross section is negligible around the Φ and $< 0.05\%$ at 10 GeV
  - ⋆ it is larger (0.5%) around J/Ψ resonances, becoming here a limiting factor
- the 1-loop virtual corrections to the 1-photon real emission are not completely known for Bhabha (even if feasible)
  - ⋆ relying on the LEP experience and being the error at the $\alpha^2 L$ level, the missing corrections are $\leq 0.05\%$
- the double real bremsstrahlung contribution is in principle approximated
  - ⋆ observed really negligible differences with the exact matrix elements, calculated with the **ALPHA** (Caravaglios and Moretti ('95)) algorithm/routine

Summary of theoretical errors

- for Bhabha cross section, within realistic setup for luminometry, the theoretical errors of **BabaYaga@NLO** are summarized

$\|\delta^{err}\|$ (%)	(a)	(b)	(c)	(d)
$\|\delta^{err}_{VP}\|$	0.01	0.00	0.02	0.04
$\|\delta^{err}_{pairs}\|$	0.02	0.03	0.03	0.04
$\|\delta^{err}_{H,H}\|$	0.00	0.00	0.00	0.00
$\|\delta^{err}_{phot+N_f=1}\|$	0.01	0.01	0.00	0.01
$\|\delta^{err}_{SV,H}\|$	0.05	0.05	0.05	0.05
$\|\delta^{err}_{total}\|$	0.09	0.09	0.10	0.14

Table: LABS (a) (c), VLABS (b) (d), 1.02 GeV (a) (b), 10 GeV (c) (d)

Resummation beyond α^2

- ⋆ with a complete 2-loop generator at hand, (leading-log) resummation beyond α^2 can be neglected?

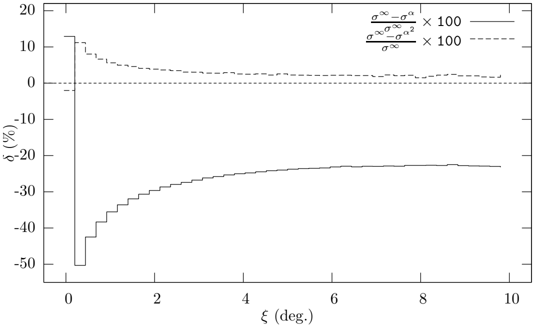


Figure: Impact of α^2 (solid line) and resummation of higher order ($\geq \alpha^3$) (dotted) corrections on the acollinearity distribution

- ⋆ resummation beyond α^2 still important

G. Balossini et al., `arXiv:0801.3360 [hep-ph]`, accepted by **PLB**

- ⋆ the matching is now applied also to $e^+e^- \to \gamma\gamma$, relying on the 1-loop formulae in Berends and Kleiss **NPB** 186 (1981) and Berends et al. **NPB** 202 (1981)
- ⋆ double counting, affecting **`v.3.5`**, is avoided in the new approach
- e.g., E_{cms} = 1 GeV, at least 2 photons with $20^o < \vartheta_\gamma < 160^o$, $E_\gamma >$ 0.3 GeV and varying the acollinearity cut

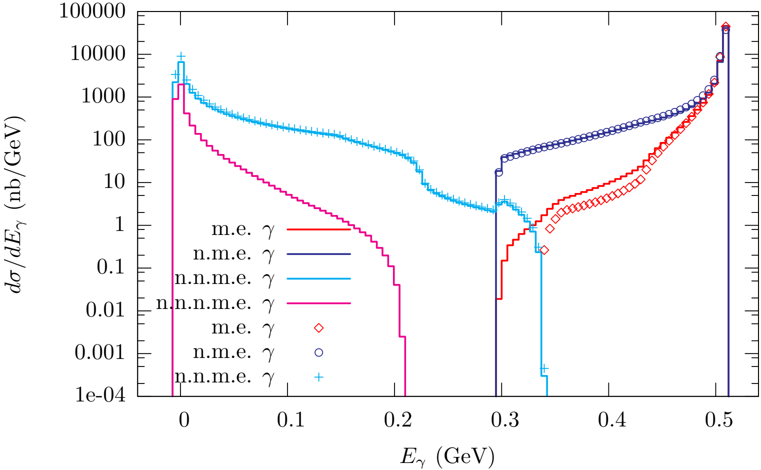
$\xi_{\gamma\gamma}$ (^{o})	σ_0 (nb)	$O(\alpha)_{PS}$	$O(\infty)_{PS}$	$O(\alpha)_{ex}$	$O(\infty)_{matched}$
5	329.8	302.5	304.0	304.4	305.6
10	329.8	314.3	314.8	316.3	316.6
15	329.8	320.2	320.4	322.2	322.2
20	329.8	323.6	323.6	325.6	325.4

- $\mathcal{O}(\alpha)$ non-log $\simeq 0.7\%$, now included
- ⋆ estimated theoretical error $\leq 0.1\%$ (VP error is not present here)

$e^+e^- \to \gamma\gamma$ distributions

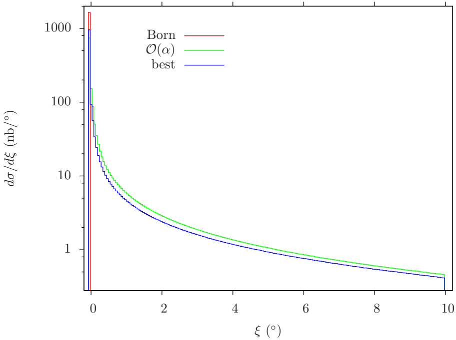
- photons' energy

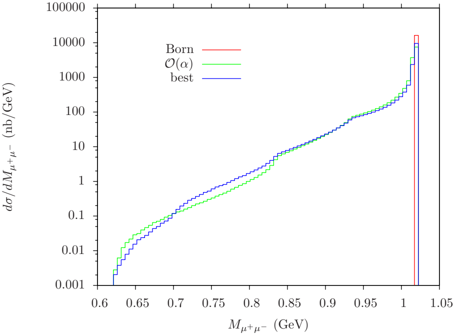
  markers = $\mathcal{O}(\alpha)$, histograms = $\mathcal{O}(\infty)$



$e^+e^- \to \mu^+\mu^-$

- now the *matching* algorithm is implemented also for $\mu^+\mu^-$
- the accuracy has still to be studied in detail
  - ⋆ probably, VP error has a larger impact here





- on integrated cross sections (setup (a) with $E^{\pm}_{min} = 0.35$ GeV), including VP effect in LO,

$$\delta_\alpha = -8.50\% \qquad \delta_{h.o.} = 0.64\%$$

Conclusions

- **BabaYaga** web site

http://www.pv.infn.it/hepcomplex/babayaga.html

- in the current release exact $\mathcal{O}(\alpha)$ corrections are matched with h.o. in a PS approach. The *matching* allows inclusion of some α^2 corrections for free
- the matching is now applied to $e^+e^- \to e^+e^-$, $\to \gamma\gamma$ and $\to \mu^+\mu^-$
- the Bhabha theoretical error is at 2-loop order and estimated $\leq 0.15\%$, by considering
  - ⋆ missing $\mathcal{O}(\alpha^2)$ corrections (pairs, photonic 2-loops, $N_F = 1$,...)
  - ⋆ vacuum polarization uncertainties
  - ⋆ VP induces larger errors around the charmonium resonances (J/Ψ)
- $e^+e^- \to \gamma\gamma$ is not affected by VP uncertainty, the theoretical error is estimated $\leq 0.1\%$
- $e^+e^- \to \mu^+\mu^-$ recently implemented, accuracy not estimated yet