Precise quark masses from R(s)

Christian Sturm

Physics Department
Brookhaven National Laboratory
High Energy Theory Group
Upton, New York

based on Nucl. Phys. B 778, 192 (2007)
In collaboration with: J. H. Kühn and M. Steinhauser

I. Motivation
II. Introduction & Method
III. Analysis & Results
IV. Summary & Conclusion

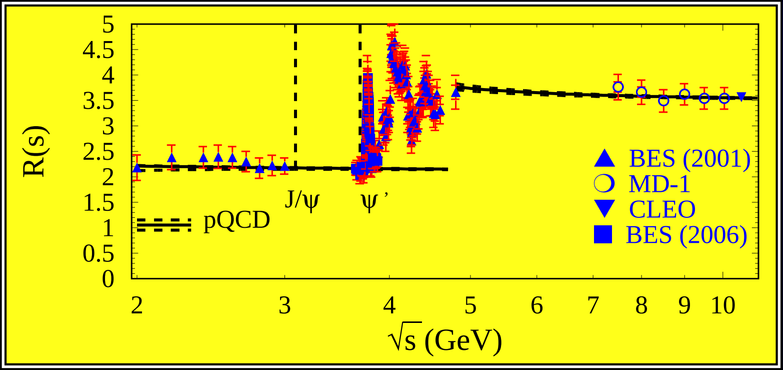
Introduction
Motivation

Precise determination of the charm- and bottom-quark masses

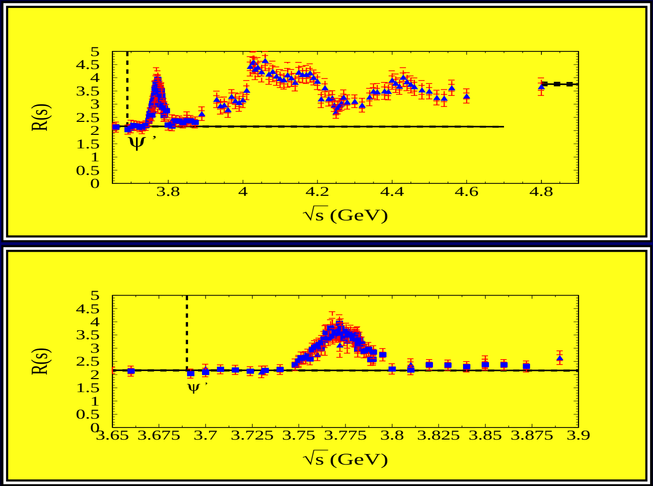
- Quark masses are fundamental parameters of the Standard Model
- Play an important role in Higgs physics:
  Higgs decay: $\Gamma(H->b\bar{b}) = \frac{G_f M_h}{4\sqrt{2}\pi} m_b^2 \left(1 + \mathcal{O}(\alpha_s) + \ldots\right)$
- Flavor physics
- Comparison with other methods

Introduction
Experiment

$$R(s) = \frac{\sigma(e^+e^- \to \text{Hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



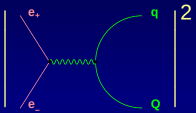
Introduction
Experiment

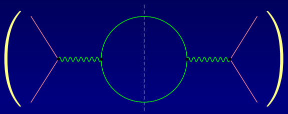


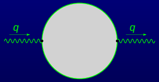
Introduction

Relation: experiment $\Longleftrightarrow$ theory

- $R(s) = \dfrac{\sigma(e^+e^- \to \text{Hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 12\pi \,\mathrm{Im}\left[\Pi(q^2 = s + i\varepsilon)\right]$

$\int d\Pi$ 

$= 2\ \mathrm{Im}$ 

$i(-g^{\mu\nu} + q^\mu q^\nu / q^2)\Pi(q^2) =$ 

- With the help of dispersion-relations:

$$\boxed{\Pi(q^2) = \Pi(q^2 = 0) + \frac{q^2}{12\pi^2} \int ds \frac{R(s)}{s(s - q^2)}}$$

- Exp. moments are related to derivatives of $\Pi(q^2)$ at $q^2 = 0$

Introduction

Relation: theory $\Longleftrightarrow$ experiment

- Exp. moments are related to derivatives of $\Pi(q^2)$ at $q^2 = 0$:

$$\frac{12\pi}{n!}\left(\frac{d}{dq^2}\right)^n \Pi(q^2)\Big|_{q^2=0} = \boxed{\mathcal{M}_n^{\exp} = \int \frac{ds}{s^{n+1}} R^{\exp}(s)}$$

- In terms of expansion coefficients:

$$\Pi(q^2) = \frac{3Q_f^2}{16\pi^2}\sum_n \overline{C}_n\left(\frac{q^2}{4m^2}\right)^n,$$

Q_f: charge of quark

$m = m(\mu)$: $\overline{\text{MS}}$ mass

$\overline{C}_n$ can be calculated perturbatively

- First and higher derivatives of $\Pi(q^2)$ allow direct determination of the $\overline{\text{MS}}$ charm- and bottom-quark mass:

$$\boxed{m(\mu) = \frac{1}{2}\left(Q_f^2 \frac{9}{4}\frac{\overline{C}_n}{\mathcal{M}_n^{\exp}}\right)^{1/(2n)}}$$

← Theory

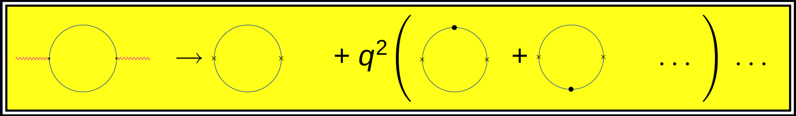
← Experiment

c-quarks: Novikov, et al. '78; b-quarks: Reinders, et al. '85

$\overline{C}_n$ depend on the quark mass through $\log(m(\mu)^2/\mu^2)$

Computation
Theory

Expansion diagrammatically:



$\hookrightarrow$ One-scale multi-loop integrals in pQCD

- 4-loop moment:
  Methods:
  – Reduction to master integrals S. Laporta,E. Remiddi'96; S. Laporta'01
  $\rightsquigarrow$ several million of equations; several GB of solutions
  K. G. Chetyrkin, J. H. Kühn, C.S.'06; R. Boughezal, M. Czakon, T. Schutzmeier'06
  – Computation of master integrals
  different methods: Y.Schröder, A.Vuorinen'05; K.G.Chetyrkin, M.Faisst, C.S., M.Tentyukov'06

Theory result

The charm quark case

$$\begin{aligned}\overline{C}_n = \overline{C}_n^{(0)} &+ \left(\frac{\alpha_s}{\pi}\right)\left(\overline{C}_n^{(10)} + \overline{C}_n^{(11)} l_{m_c}\right)\\ &+ \left(\frac{\alpha_s}{\pi}\right)^2\left(\overline{C}_n^{(20)} + \overline{C}_n^{(21)} l_{m_c} + \overline{C}_n^{(22)} l_{m_c}^2\right)\\ &+ \left(\frac{\alpha_s}{\pi}\right)^3\left(\overline{C}_n^{(30)} + \overline{C}_n^{(31)} l_{m_c} + \overline{C}_n^{(32)} l_{m_c}^2 + \overline{C}_n^{(33)} l_{m_c}^3\right)\\ &+ \ldots, \text{with } l_{m_c} = \log(m_c^2/\mu^2)\end{aligned}$$

$n_f = 4$:

	1-loop	2-loop		3-loop			4-loop			
n	$\overline{C}_n^{(0)}$	$\overline{C}_n^{(10)}$	$\overline{C}_n^{(11)}$	$\overline{C}_n^{(20)}$	$\overline{C}_n^{(21)}$	$\overline{C}_n^{(22)}$	$\overline{C}_n^{(30)}$	$\overline{C}_n^{(31)}$	$\overline{C}_n^{(32)}$	$\overline{C}_n^{(33)}$
1	1.0667	2.5547	2.1333	2.4967	3.3130	-0.0889	-7.7624	-0.0599	1.5851	-0.0543
2	0.4571	1.1096	1.8286	3.2319	5.0798	1.9048	–	4.0100	7.2551	0.1058
3	0.2709	0.5194	1.6254	2.0677	4.5815	3.3185	–	5.6496	13.4967	2.3967
4	0.1847	0.2031	1.4776	1.2204	3.4726	4.4945	–	3.9381	17.2292	6.2423

Result also available completely analytically

Extraction of the experimental moments

The charm quark case

Determine: $\mathcal{M}_n^{exp} = \int \frac{ds}{s^{n+1}} R^{\exp}(s) = \mathcal{M}_n^{res} + \mathcal{M}_n^{thr} + \mathcal{M}_n^{cont}$

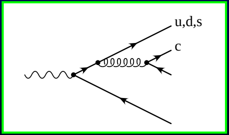
For charm quarks:

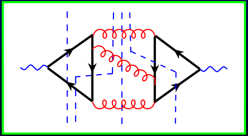
$\mathcal{M}_n^{res}$: Contains: $J/\Psi, \Psi(2S)$ treated in narrow width approximation

$$R^{res}(s) = \frac{9\pi M_R \Gamma_{ee}}{\alpha^2}\left(\frac{\alpha}{\alpha(s)}\right)^2 \delta(s - M_R^2)$$

	J/Ψ	$\Psi(2S)$
M_Ψ(GeV)	3.096916(11)	3.686093(34)
Γ_{ee}(keV)	5.55(14)	2.48(6)
$(\alpha/\alpha(M_\Psi))^2$	0.957785	0.95554

$\mathcal{M}_n^{thr}$: BES data ($\sqrt{s} \geq 3.73$ GeV) subtract background from R_{uds},





$\bar{R}$ from data below 3.73 GeV, $\sqrt{s}$-dependence from theory

Extraction of the experimental moments
The charm quark case

$\mathcal{M}_n^{cont}$: pQCD above $\sqrt{s} \geq 4.8$ GeV ,

no data,

$R(s)$ with full quark mass dependence rhad: R. Harlander, M. Steinhauser '02

$\mathcal{M}_n^{exp}$:

n	$\mathcal{M}_n^{\rm res}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm thresh}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm cont}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm exp}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm np}$ $\times 10^{(n-1)}$
1	0.1201(25)	0.0318(15)	0.0646(11)	0.2166(31)	$-0.0001(2)$
2	0.1176(25)	0.0178(8)	0.0144(3)	0.1497(27)	0.0000(0)
3	0.1169(26)	0.0101(5)	0.0042(1)	0.1312(27)	0.0007(14)
4	0.1177(27)	0.0058(3)	0.0014(0)	0.1249(27)	0.0027(54)

Determination of the charm quark mass

$$\mathcal{M}_n^{th} + \mathcal{M}_n^{np} = \mathcal{M}_n^{exp} \quad \text{with} \quad \mathcal{M}_n^{th} = \frac{9}{4} Q_q^2 \left(\frac{1}{4m_c^2}\right)^n \overline{C}_n$$

$$m(\mu) = \frac{1}{2}\left(Q_f^2 \frac{9}{4} \frac{\overline{C}_n}{\mathcal{M}_n^{exp} - \mathcal{M}_n^{np}}\right)^{1/(2n)}$$

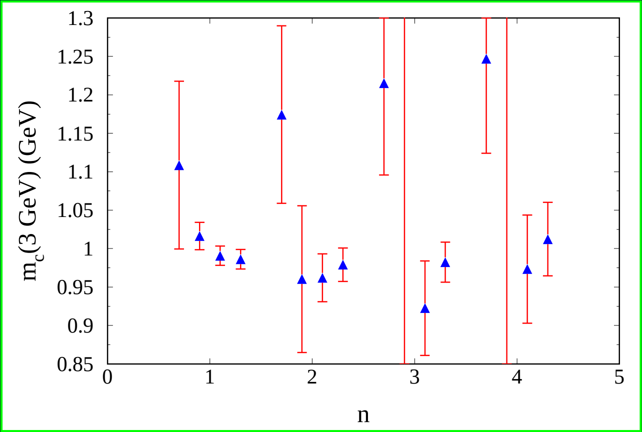
$\mu = (3 \pm 1)\text{GeV} \qquad \alpha_s(M_Z) = 0.1189 \pm 0.002$

n	$m_c(3\text{ GeV})$	exp	α_s	μ	np	total	$\delta\bar{C}_n^{(30)}$	$m_c(m_c)$
1	0.986	0.009	0.009	0.002	0.001	0.013	—	1.286
2	0.979	0.006	0.014	0.005	0.000	0.016	0.006	1.280
3	0.982	0.005	0.014	0.007	0.002	0.016	0.010	1.282
4	1.012	0.003	0.008	0.030	0.007	0.032	0.016	1.309

$$-6.0 \leq \overline{C}_2^{(30)} \leq 7.0,\ -6.0 \leq \overline{C}_3^{(30)} \leq 5.2,\ -6.0 \leq \overline{C}_4^{(30)} \leq 3.1$$

Determination of the charm quark mass

Charm-quarks



$m_c(3\text{GeV}) = 0.986(13)\text{GeV}$

Extraction of the experimental moments

The bottom quark case

$\mathcal{M}_n^{th}$: analog to charm case, only $n_f = 5$

$\mathcal{M}_n^{np}$: negligible

$\mathcal{M}_n^{res}$: $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, $\Upsilon(4S)$

$\mathcal{M}_n^{thr.}$: CLEO data up to 11.24 GeV

$\mathcal{M}_n^{cont}$: pQCD above 11.24 GeV

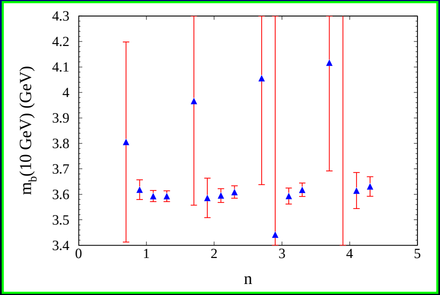
$\mathcal{M}_n^{exp}$:

n	$\mathcal{M}_n^{\rm res}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm thresh}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm cont}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm exp}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm np}$ $\times 10^{(n-1)}$
1	0.1201(25)	0.0318(15)	0.0646(11)	0.2166(31)	$-0.0001(2)$
2	0.1176(25)	0.0178(8)	0.0144(3)	0.1497(27)	0.0000(0)
3	0.1169(26)	0.0101(5)	0.0042(1)	0.1312(27)	0.0007(14)
4	0.1177(27)	0.0058(3)	0.0014(0)	0.1249(27)	0.0027(54)

Determination of the bottom quark mass

n	$m_b(10\text{ GeV})$	exp	α_s	μ	total	$\delta\bar{C}_n^{(30)}$	$m_b(m_b)$
1	3.593	0.020	0.007	0.002	0.021	—	4.149
2	3.609	0.014	0.012	0.003	0.019	0.006	4.164
3	3.618	0.010	0.014	0.006	0.019	0.008	4.173
4	3.631	0.008	0.015	0.021	0.027	0.012	4.185

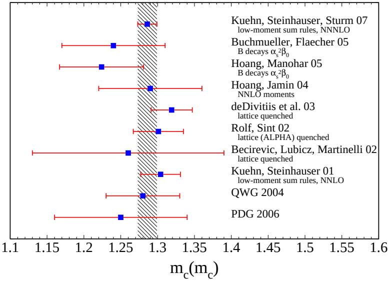
$$\mu = (10 \pm 5)\text{GeV}; \; -6.0 \leq \overline{C}_2^{(30)} \leq 7.0, \; -6.0 \leq \overline{C}_3^{(30)} \leq 5.2, \; -6.0 \leq \overline{C}_4^{(30)} \leq 3.1$$



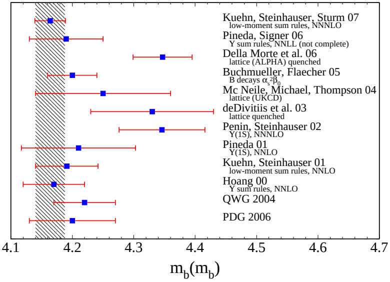
$$m_b(10\text{GeV}) = 3.609(25)\text{GeV}$$

Comparison with other methods

charm-quarks



bottom-quarks



Summary

- $\overline{\text{MS}}$ quark masses can be extracted from the measured R-ratio and computable expansion coefficients of the vacuum polarization function in combination with dispersion relations
- Analysis performed for charm- and bottom-quark masses including moments up to NNNLO in pQCD
- Quark masses:
  - Charm-mass: $m_c(m_c) = 1.286(13)$ GeV
  - Bottom-mass: $m_b(m_b) = 4.164(25)$ GeV