$\alpha_{\rm em}(E)$

via the Adler function

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Outline of Talk:

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③ **Evaluation of** $\alpha(M_Z)$

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⑥ **Possible improvement by DAFNE-2**

① Introduction

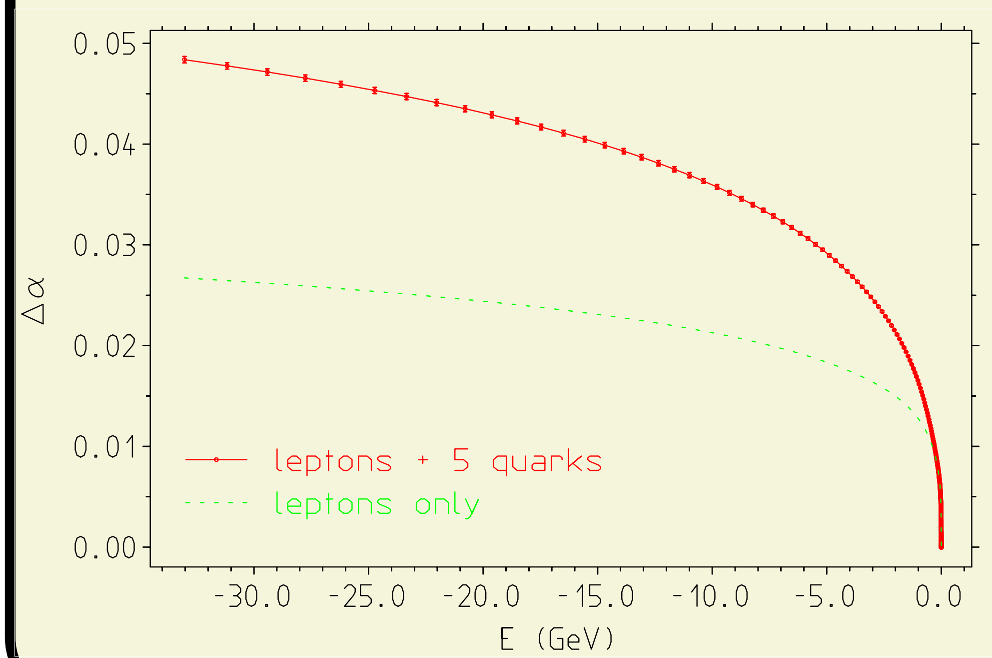
Non-perturbative hadronic effects in electroweak precision observables, main effect via

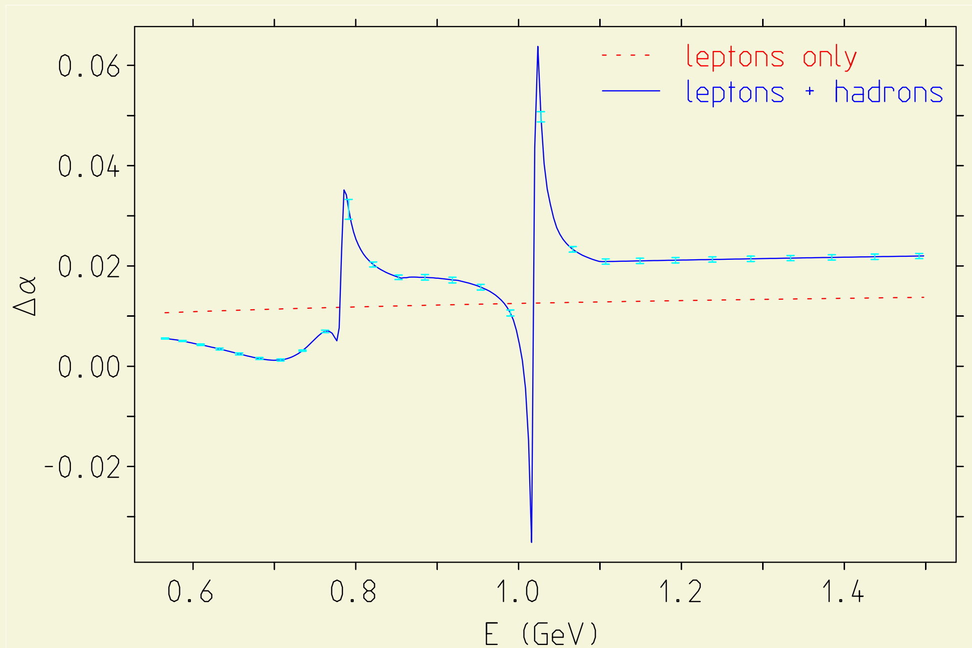
effective fine-structure "constant" $\alpha(E)$

(charge screening by vacuum polarization)

- **Need to know running of $\alpha_{\rm QED}$ very precisely.**

  Large corrections, steeply increasing at low E





② $\alpha(M_Z)$ in precision physics (precision physics limitations)

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics:

$\alpha\ ,\ G_\mu, M_Z$ most precise input parameters

50% non-perturbative relationship

$\Updownarrow$

precision predictions $\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

$\alpha(M_Z), G_\mu, M_Z$ best effective input parameters for VB physics (Z,W) etc.

$$\begin{array}{lclcl}
\frac{\delta\alpha}{\alpha} & \sim & 3.6 & \times & 10^{-9} \\
\frac{\delta G_\mu}{G_\mu} & \sim & 8.6 & \times & 10^{-6} \\
\frac{\delta M_Z}{M_Z} & \sim & 2.4 & \times & 10^{-5} \\
\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 1.6 \div 6.8 & \times & 10^{-4} \quad (\mathrm{present:\ lost\ 10^5\ in\ precision!}) \\
\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 5.3 & \times & 10^{-5} \quad (\mathtt{ILC\ requirement})
\end{array}$$

LEP/SLD: $\sin^2\Theta_{\rm eff} = (1 - g_{Vl}/g_{Al})/4 = 0.23148 \pm$ 0.00017

$$\delta\Delta\alpha(M_Z) = 0.00036 \quad \Rightarrow \quad \delta\sin^2\Theta_{\rm eff} = 0.00013$$

affects Higgs mass bounds, precision tests and new physics searches!!!

For perturbative QCD contributions very crucial: precise QCD parameters $\alpha_s,\ m_c,\ m_b,\ m_t \Rightarrow$ Lattice-QCD

Indirect

Higgs boson mass "measurement"

$m_H = 88^{+53}_{-35}$ **GeV**

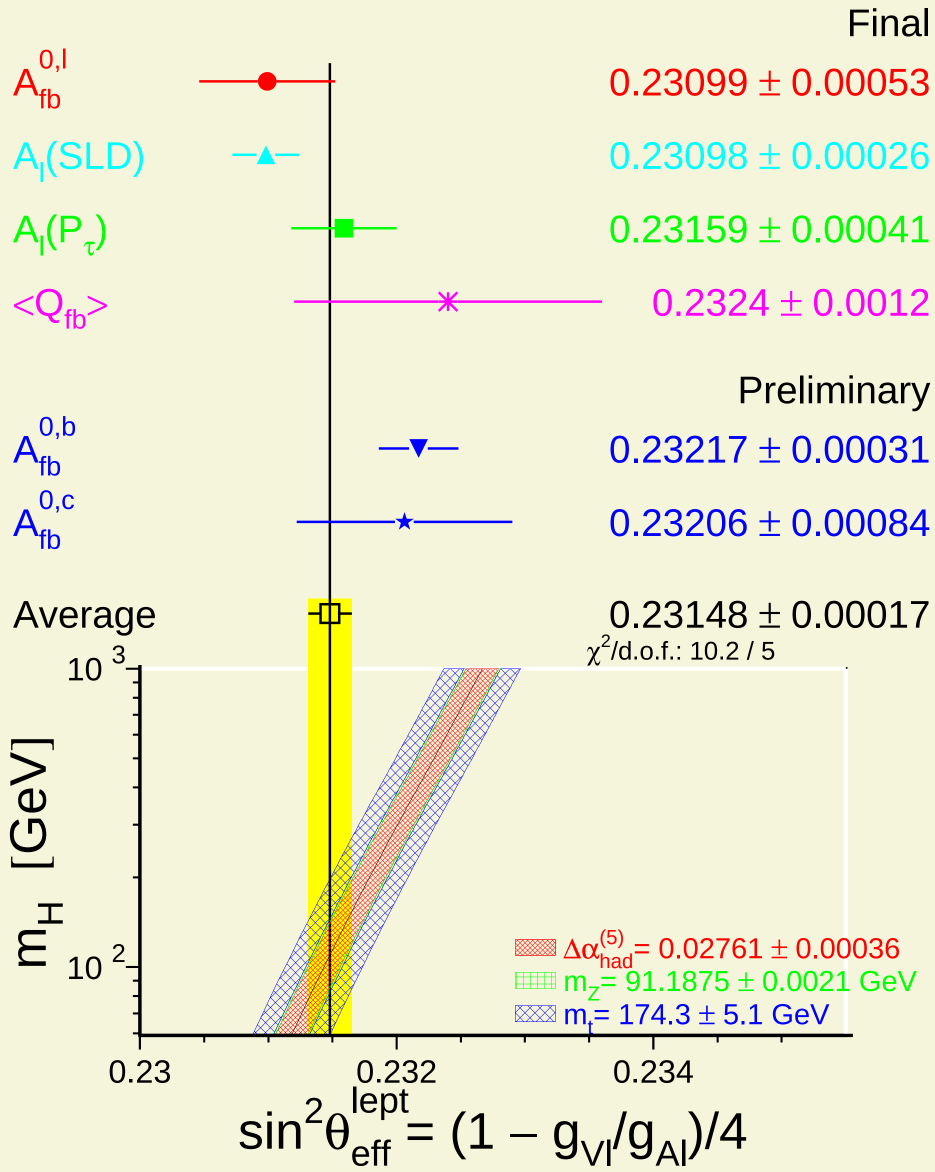
$e^+e^- \to \tau$: $\Rightarrow \delta m_H \sim -19$ **GeV**

Direct lower bound:

$m_H > 114$ **GeV at 95% CL**

Indirect upper bound:

$m_H < 193$ **GeV at 95% CL**



What is the point once m_H has been measured by the LHC? $\Rightarrow \sin^2\Theta_{\rm eff}$ **tuns into an excellent monitor for new physics!**

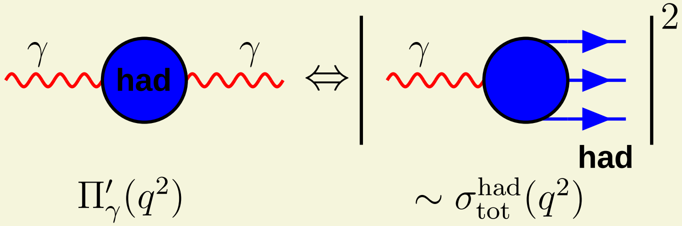
③ Evaluation of $\alpha(M_Z)$

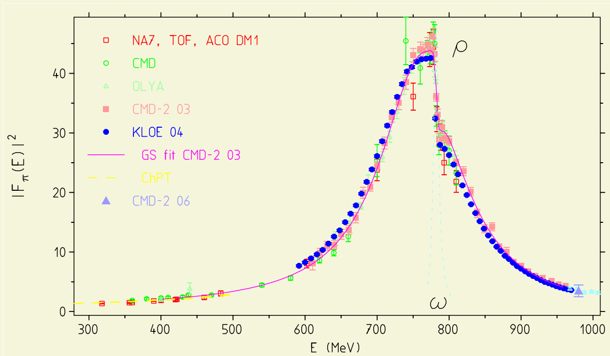
Non-perturbative hadronic contributions $\Delta\alpha^{(5)}_{\rm had}(s)$ can be evaluated in terms of $\sigma(e^+e^- \to \mathrm{hadrons})$ data via dispersion integral:

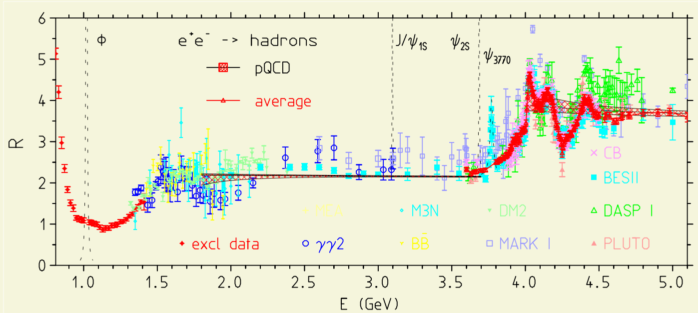
$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha s}{3\pi}\left(\;\; \mathrm{P}\!\!\int_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R^{\rm data}_\gamma(s')}{s'(s'-s)} + \mathrm{P}\!\!\int_{E^2_{\rm cut}}^{\infty} ds' \frac{R^{\rm pQCD}_\gamma(s')}{s'(s'-s)} \right)$$

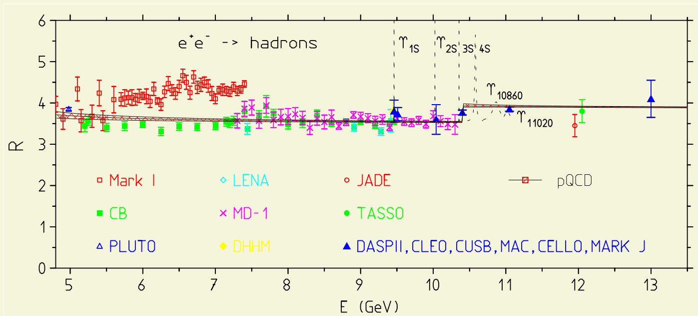
where

$$R_\gamma(s) \equiv \frac{\sigma^{(0)}(e^+e^-\to\gamma^*\to\mathrm{hadrons})}{\frac{4\pi\alpha^2}{3s}}$$









Compilation:
Theory = pQCD:
Jegerlehner 06
Groshny et al. 91, Chetyrkin et al. 97

Evaluation FJ 2006 update: at $M_Z = 91.19$ GeV

- $R(s)$ data up to $\sqrt{s} = E_{cut} = 5$ GeV
  and for Υ resonances region between 9.6 and 13 GeV
- perturbative QCD from 5.0 to 9.6 GeV
  and for the high energy tail above 13 GeV

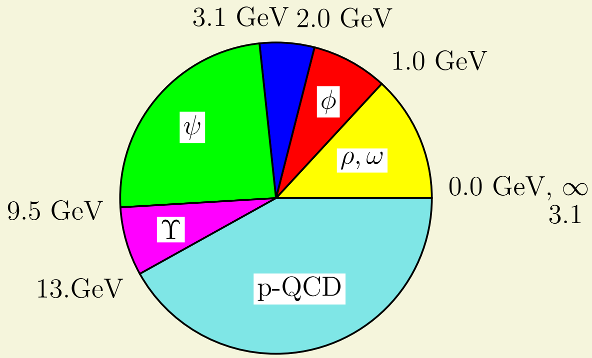
$\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2)$	$=$	0.027594 ± 0.000219	
		0.027469 ± 0.000149	Adler
$\alpha^{-1}(M_Z^2)$	$=$	128.944 ± 0.033	
		128.955 ± 0.014	Adler

Table 1: Contributions and uncertainties $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)^{\rm data}\cdot 10^4$. Direct integration method. In red the results relevant for VEPP-2000/DAFNE-II.

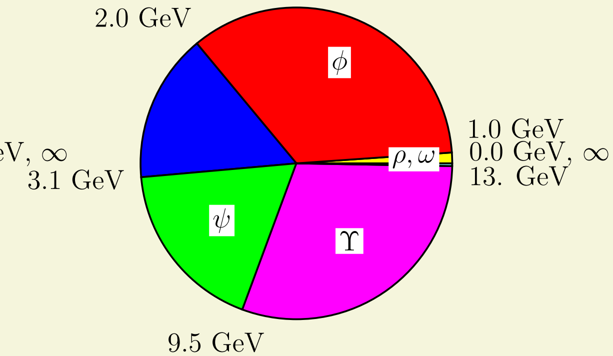
Energy range	$\Delta\alpha^{(5)}_{\rm had}[\%](\rm error)\times 10^4$	rel. err.	abs. err.
ρ,ω $(E<2M_K)$	36.36 [13.2](0.20)	0.6 %	0.9 %
$2M_K<E<2$ GeV	21.46 [7.8](1.31)	6.1 %	35.9 %
2 GeV $<E<M_{J/\psi}$	15.73 [5.7](0.88)	5.6 %	16.2 %
$M_{J/\psi}<E<M_\Upsilon$	66.98 [24.3](0.84)	1.3 %	14.9 %
$M_\Upsilon<E<E_{\rm cut}$	19.69 [7.1](1.24)	6.3 %	32.0 %
$E_{\rm cut}<E$ pQCD	115.71 [41.9](0.06)	0.0 %	0.1 %
$E<E_{\rm cut}$ data	160.23 [58.1](2.19)	1.4 %	99.9 %
total	275.94 [100.0](2.19)	0.8 %	100.0 %

Table 2: Contributions and uncertainties $\Delta\alpha^{(5)}_{\rm had}(-M_0^2)^{\rm data}\cdot 10^4$ ($M_0=2.5$ GeV). Adler function method. In red the results relevant for VEPP-2000/DAFNE.

Energy range	$\Delta\alpha^{(5)}_{\rm had}[\%](\rm error)\times 10^4$	rel. err.	abs. err.
ρ,ω ($E<2M_K$)	33.40 [45.4](0.19)	0.6 %	3.1 %
$2M_K<E<2$ GeV	16.23 [22.1](0.92)	5.7 %	73.4 %
2 GeV $<E<M_{J/\psi}$	7.91 [10.8](0.44)	5.6 %	16.9 %
$M_{J/\psi}<E<M_\Upsilon$	13.95 [19.0](0.27)	1.9 %	6.4 %
$M_\Upsilon<E<E_{\rm cut}$	0.96 [1.3](0.06)	6.2 %	0.3 %
$E_{\rm cut}<E$ pQCD	1.09 [1.5](0.00)	0.1 %	0.0 %
$E<E_{\rm cut}$ data	72.45 [98.5](1.07)	1.5 %	100.0 %
total	73.54 [100.0](1.07)	1.5 %	100.0 %



contributions



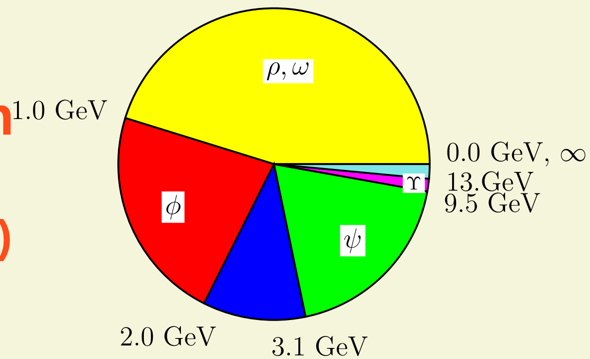
error^2

direct integration of data

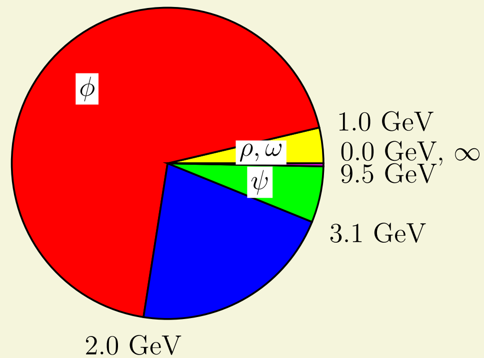
$\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2)$

integration via Adler function

$\Delta\alpha^{(5)}_{\rm had}(-M_0^2)^{\rm data}$ **($M_0 = 2.5$ GeV)**



contributions



error^2

present distribution of contributions and errors

④ Testing non–perturbative hadronic effects via the Adler function

Adler function in terms of experimental data:

$$D(Q^2) = Q^2 \left(\int_{4m_\pi^2}^{E^2_{\rm cut}} \frac{R^{\rm data}(s)}{(s+Q^2)^2} ds \; + \; \int_{E^2_{\rm cut}}^{\infty} \frac{R^{\rm pQCD}(s)}{(s+Q^2)^2} ds \right)$$

$Q^2 = -q^2$ is the squared Euclidean momentum transfer,

s is the center of mass energy squared for hadron production in e^+e^-–annihilation

Adler function from pQCD and OPE:

Basic object: photon vacuum polarization amplitude

$$\Pi^\gamma_{\mu\nu}(q) \;\; = \;\; i \int d^4x e^{iqx} < 0|T J^\gamma_\mu\,(x)\; J^\gamma_\nu\,(0)\;|0>= -\left(q^2\, g_{\mu\nu} - q_\mu q_\nu\right)\; \Pi'_\gamma\,(q^2)$$

Adler function:

$$D(-s) = -(12\pi^2)\, s\, \frac{d\Pi'_\gamma\,(s)}{ds} = \frac{3\pi}{\alpha} s \frac{d}{ds} \Delta\alpha_{\rm had}(s)$$

is the hadronic contribution to the shift of the fine structure constant at scale q^2.

Contributions:

$$D(Q^2)=D^{(0)}(Q^2)+D^{(1)}(Q^2)+D^{(2)}(Q^2)+\cdots+D^{\rm NP}(Q^2)$$

One–loop:

$$D^{(0)}(Q^2)=\sum_f Q_f^2 N_{cf} H^{(0)}$$

with $H\equiv(12\pi^2)\,\dot{\Pi}'_V\;,\quad \dot{\Pi}'_V\equiv -s\,d\Pi'_V/ds$, in terms of the vector current amplitude Π'_V.

Explicitly:

$$H^{(0)}=1+\frac{3y}{2}-\frac{3y^2}{4}\frac{1}{\sqrt{1-y}}\ln\xi\quad y=4m_f^2/s\quad,\quad \xi=\frac{\sqrt{1-y}-1}{\sqrt{1-y}+1}$$

where ξ is taking values $0\leq\xi\leq 1$ for $s\leq 0$.

Asymptotically:

$$H^{(0)}\to\begin{cases}\frac{1}{5}\frac{Q^2}{m_f^2}-\frac{3}{70}\left(\frac{Q^2}{m_f^2}\right)^2+\frac{1}{105}\left(\frac{Q^2}{m_f^2}\right)^3+\cdots & Q^2\ll m_f^2\\ 1-6\frac{m_f^2}{Q^2}-12\left(\frac{m_f^2}{Q^2}\right)^2\ln\frac{m_f^2}{Q^2}+24\left(\frac{m_f^2}{Q^2}\right)^3\left(\ln\frac{m_f^2}{Q^2}+1\right)+\cdots & Q^2\gg m_f^2\end{cases}\qquad (1)$$

and this behavior determines the quark parton model (QPM) (leading order QCD) property of the Adler function: heavy quarks ($m_f^2\gg Q^2$) decouple like Q^2/m_f^2 while light modes ($m_f^2\ll Q^2$) contribute $Q_f^2N_{cf}$ to $D^{(0)}$.

Two–loop: **known analytically** (Broadhurst 85 and others)

$$D^{(1)}(Q^2) = \frac{\alpha_s(Q^2)}{\pi} \sum_f Q_f^2 N_{cf}\, H^{(1)}$$

where $H^{(1)} = (12\pi^2)\, \dot{\Pi}_V^{\prime (2)}(-Q^2, m_f^2)$.

Three–loop: **known as low and large momentum expansion** (Chetyrkin, Harlander, Kühn, Steinhauser 96/97)

$$D^{(2)}(Q^2) = \left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 \sum_f Q_f^2 N_{cf}\, H^{(2)}$$

where $H^{(2)} = (12\pi^2)\, \dot{\Pi}_V^{\prime (3)}(-Q^2, m_f^2)$.

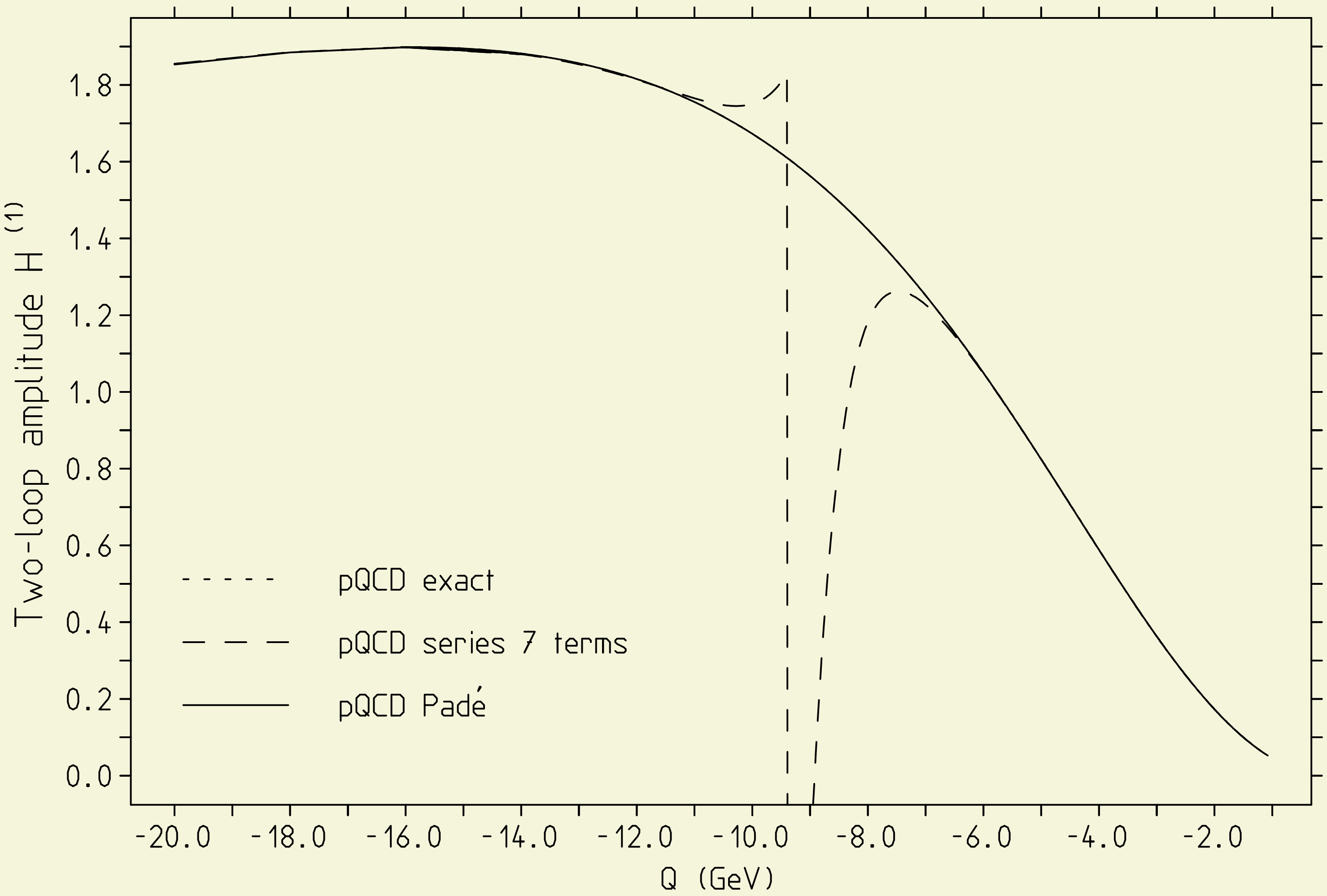
Both series expansions diverge at the boundary of the circle of convergence $Q^2 = 4m^2 \Rightarrow$ problem in the region where mass effects are of the order of unity in the Euclidean region. Apply a conformal mapping (Schwinger 48)

$$y^{-1} = \frac{-Q^2}{4m^2} \to \omega = \frac{1-\sqrt{1-1/y}}{1+\sqrt{1-1/y}}$$

from the complex negative q^2 half–plane to the interior of the unit circle $|\omega| < 1$ together with Padé

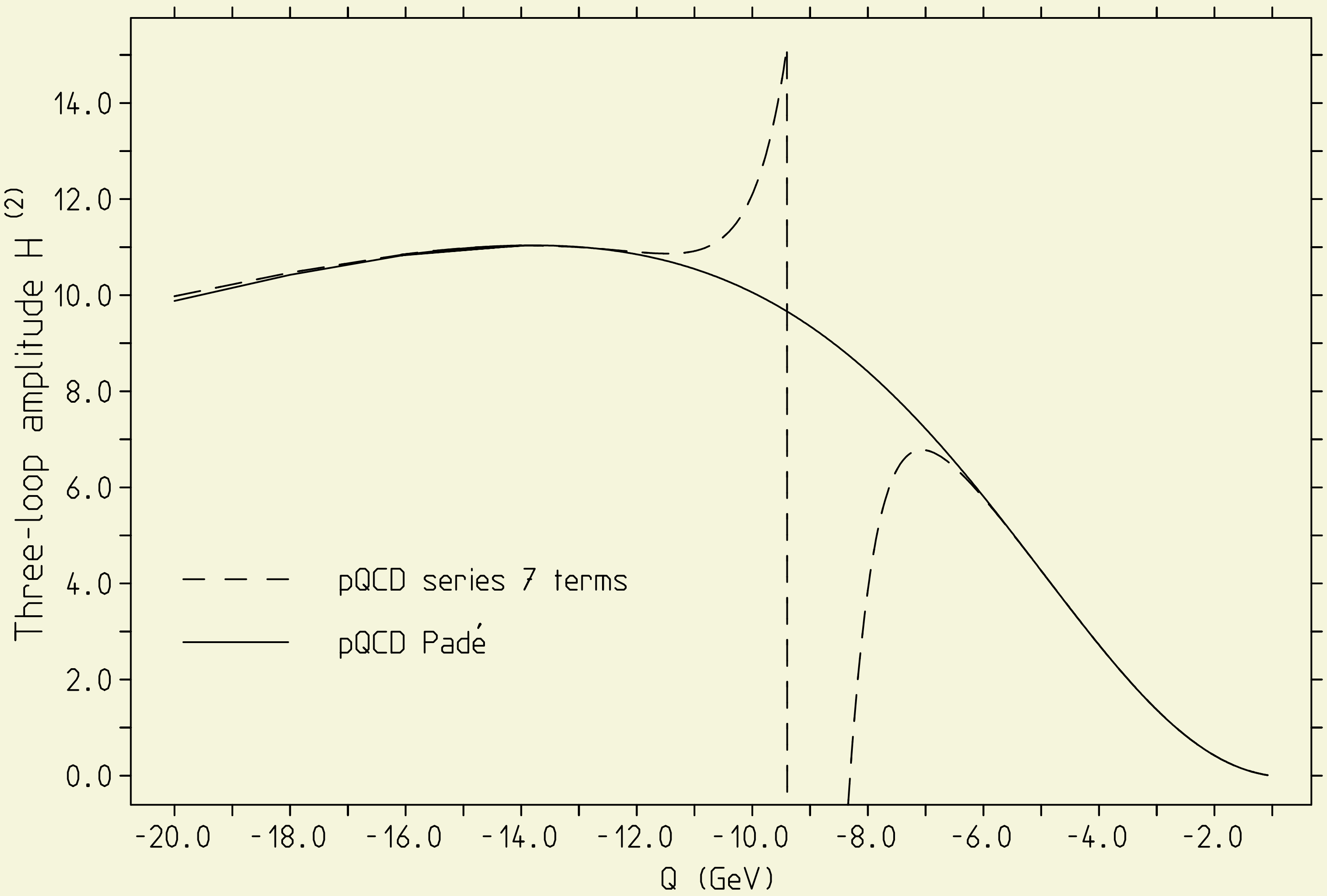
resummation (Fleischer and Tarasov 94). The Padé approximant provides a good estimation to much higher values of $1/y$ up to about $1/y \sim 4$. This is displayed in $(\Rightarrow)$ the Figures. Padé improvement allows us to obtain reliable results also in the relevant Euclidean "threshold region", around $y = 1$.

Adler–function: pQCD two-loop



(Eidelman, F.J., Kataev, Veretin 98)

Adler–function: pQCD three-loop



(Eidelman, F.J., Kataev, Veretin 98)

- **Four–loop in/and high energy limit:**

$$D(Q^2)=3\sum_f Q_f^2\times\left(1+a+c_1a^2+c_2a^3+\cdots\right)$$

with
$a=\alpha_s(Q^2)/\pi$,
$c_1=1.9857-0.1153n_f$,
$c_2=18.2428-4.2159n_f+0.0862n_f^2-1.2395\left(\sum Q_f\right)^2/(3\sum Q_f^2)$.

The corresponding formula for $R(s)$ only differs at the 4–loop level due to the effect from the analytic continuation from the Euclidean to the Minkowski region which yields $c_2^R=c_2^D-\frac{\pi^2}{3}\frac{\beta_0^2}{16}$ with $\beta_0=11-2/3\,n_f$.

Numerically the 4–loop term proportional to c_2 amounts to -0.0036% at 100 GeV and increases to about 0.32% at 2.5 GeV.

- **Non perturbative effects?**

Parametrize NP effects at sufficiently large energies and away from resonances as prescribed by the OPE. Non–vanishing gluon and light quark condensates (Shifman, Vainshtein, Zakharov 79) imply the leading power corrections

$$\begin{aligned} D^{\rm NP}(Q^2) &= \sum_{q=u,d,s} Q_q^2 N_{cq}\,(8\pi^2)\cdot\Bigg[\frac{1}{12}\left(1-\frac{11}{18}\frac{\alpha_s(\mu^2)}{\pi}\right)\frac{<\frac{\alpha_s}{\pi}GG>}{Q^4} \\ &+ 2\left(1+\frac{\alpha_s(\mu^2)}{3\pi}+\left(\frac{47}{8}-\frac{3}{4}l_{q\mu}\right)\left(\frac{\alpha_s(\mu^2)}{\pi}\right)^2\right)\frac{<m_q\bar{q}q>}{Q^4} \\ &+\left(\frac{4}{27}\frac{\alpha_s(\mu^2)}{\pi}+\left(\frac{4}{3}\zeta_3-\frac{88}{243}-\frac{1}{3}l_{q\mu}\right)\left(\frac{\alpha_s(\mu^2)}{\pi}\right)^2\right)\sum_{q'=u,d,s}\frac{<m_{q'}\bar{q}'q'>}{Q^4}+\cdots\Bigg] \end{aligned}$$

where $a \equiv \alpha_s(\mu^2)/\pi$ and $l_{q\mu} \equiv \ln(Q^2/\mu^2)$.
$< \frac{\alpha_s}{\pi}GG >$ and $< m_q\bar{q}q >$ are the scale-invariantly defined condensates. Sum rule estimates of the condensates yield typically (large uncertainties)

$$\begin{aligned} &< \tfrac{\alpha_s}{\pi}GG >\sim \quad (0.389\ \text{GeV})^4 \\ &< m_q\bar{q}q >\sim -(0.098\ \text{GeV})^4 \quad \text{for} \quad q=u,d \\ &< m_q\bar{q}q >\sim -(0.218\ \text{GeV})^4 \quad \text{for} \quad q=s \end{aligned}$$

Note that the above expansion is just a parameterization of the high energy tail of NP effects associated with the existence of non–vanishing condensates. There are other kind of NP phenomena like bound states, resonances, instantons and what else. The dilemma with is that it works only for Q^2 large enough

and it has been successfully applied in heavy quark physics. It fails do describe NP physics at lower Q^2, once it starts to be numerically relevant pQCD starts to fail because of the growth of the strong coupling constant.

The virtues of this analysis are obvious:

- **no problems with physical threshold and resonances**
- **pQCD is used only where we can check it to work (Euclidean, $Q^2 \gtrsim$ 2.5 GeV).**
- **no manipulation of data, no assumptions about global or local duality.**
- **non–perturbative "remainder" $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ is mainly sensitive to low energy data !!!**

⑤ $\Delta\alpha^{\rm had}$ via the Adler function

✗ use old idea: Adler function: Monitor for comparing theory and data

$$D(-s) \doteq \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\Delta\alpha_{\rm had}(s) = -\left(12\pi^2\right)\, s\, \frac{d\Pi'_\gamma(s)}{ds}$$

$$\Rightarrow \quad D(Q^2) = Q^2 \int\limits_{4m_\pi^2}^{\infty} ds\, \frac{R(s)}{(s+Q^2)^2}$$

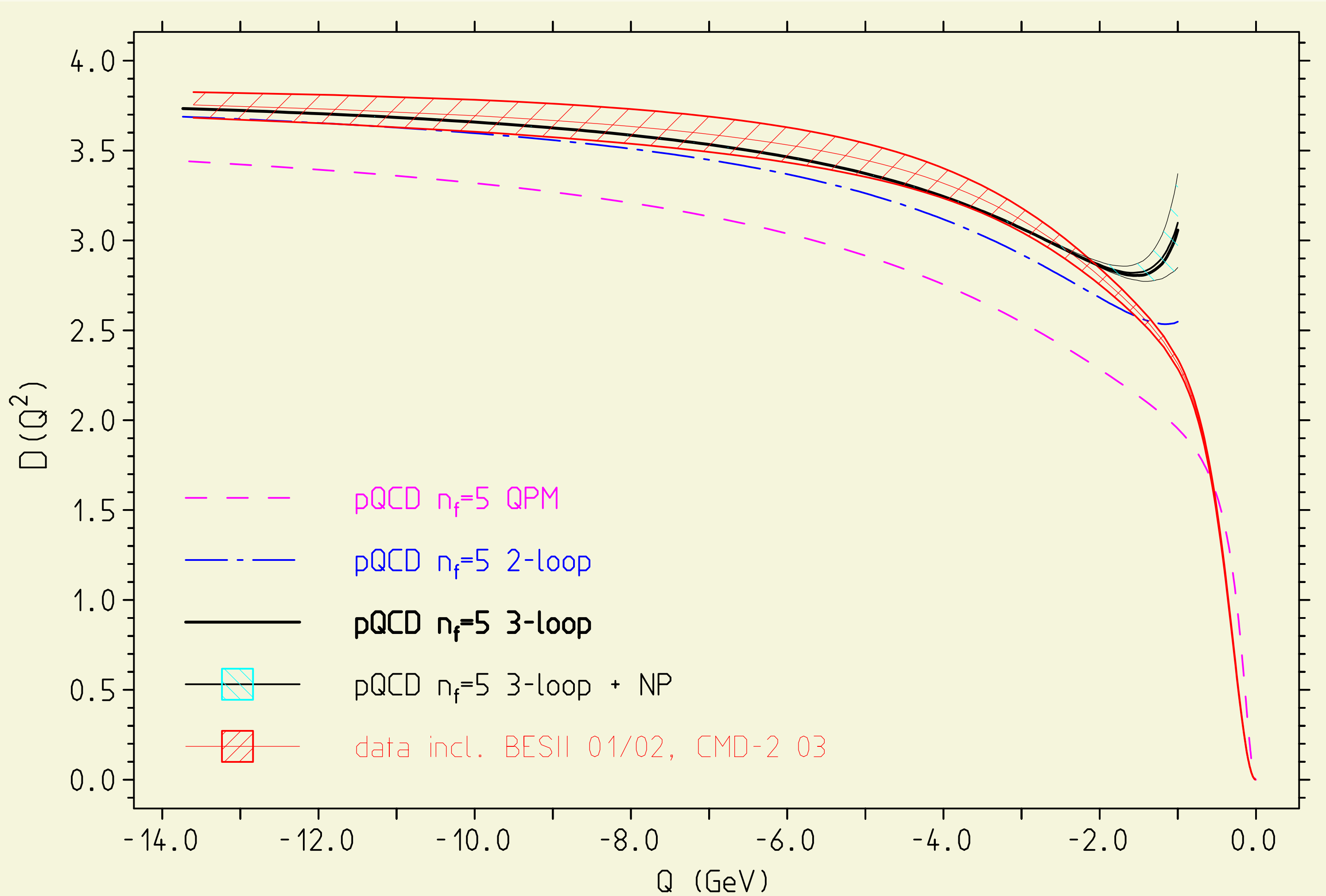
pQCD $\leftrightarrow R(s)$	pQCD $\leftrightarrow D(Q^2)$
very difficult to obtain in theory	smooth simple function in Euclidean region

Conservative conclusion:

- time-like approach: pQCD works well in "perturbative windows" 3.00 - 3.73 GeV, 5.00 - 10.52 GeV and 11.50 - ∞ (Kühn,Steinhauser)
- space-like approach: pQCD works well for $\sqrt{Q^2=-q^2} > 2.5$ GeV (see plot)

"Experimental" Adler–function versus theory (pQCD + NP)

Error includes statistical + systematic here (incontrast to most R-plots showing statistical errors only)!



(Eidelman, F.J., Kataev, Veretin 98, FJ 06 update (BES, CMD-2)) theory based on results by Chetyrkin, Kühn et al.

⇒ **pQCD works well to predict $D(Q^2)$ down to $s_0=(2.5\,\textbf{GeV})^2$; use this to calculate**

$$\Delta\alpha_{\rm had}(-Q^2)\sim\frac{\alpha}{3\pi}\int dQ'^2\,\frac{D(Q'^2)}{Q'^2}$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2)\;=\;\left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2)-\Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD}+\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}$$

and obtain, for $s_0=(2.5\,\textbf{GeV})^2$: **(FJ 98/05)**

$$\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}\;=\;0.007354\pm0.000107$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2)\;=\;0.027469\pm0.000107\pm0.000103$$

Status of QCD parameters: (see talks by Kühn, Sturm and Heitger[lattice])

$$\alpha_s(M_Z) =$$

$$m_c(m_c) = 1.286(13) \text{ GeV} \quad \Leftrightarrow \quad M_c^{3-\rm loop} = 1.666(17) \text{ GeV}$$

$$m_b(m_c) = 4.164(25) \text{ GeV} \quad \Leftrightarrow \quad M_b^{3-\rm loop} = 4.800(29) \text{ GeV}$$

(Kühn, Steinhauser, Sturm)

For lattice QCD status see talk by Heitger!

Needed improvements (integral part of this strategy):

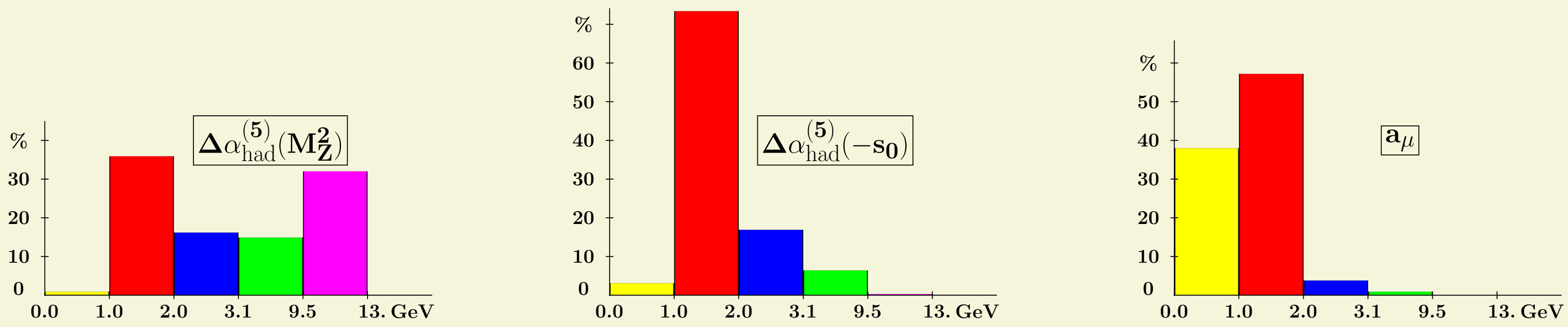
- 4–loop massive pQCD calculation of Adler function $\Leftrightarrow$ series expansion plus Padé improvement; essentially equivalent to 4–loop calculation of $R(s)$
- m_c improvement; sum rule and lattice QCD evaluations
- α_s in low Q^2 region

Testable models: – “analytized” α_s

– instanton liquid model current vs. constituent quark masses

(Shirkhov et al, Dorokhov)

Comparison of error profiles between $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$, $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ and a_μ:



⑥ Possible improvement by DAFNE-2

Future: ILC requirement: improve by factor 10 in accuracy

- **direct integration of data: 58% from data 42% p-QCD**

  $\Delta\alpha_{\rm had}^{(5)\,{\rm data}} \times 10^4 = 162.72 \pm 4.13$ **(2.5%)**

  1% overall accuracy ± 1.63

  1% accuracy for each region (divided up as in table)

  added in quadrature: ± 0.85

  Data: [4.13] vs. [0.85] $\Rightarrow$ **improvement factor 4.8**

  $\Delta\alpha_{\rm had}^{(5)\,{\rm pQCD}} \times 10^4 = 115.57 \pm 0.12$ **(0.1%)**

  Theory: no improvement needed !

- **integration via Adler function: 26% from data 74% p-QCD**

  $\Delta\alpha_{\rm had}^{(5)\,{\rm data}} \times 10^4 = 073.61 \pm 1.68$ **(2.3%)**

  1% overall accuracy ± 0.74

1% accuracy for each region (divided up as in table)

added in quadrature: ± 0.41

Data: [2.25] vs. [0.46] $\Rightarrow$ **improvement factor 4.9 (Adler vs Adler)**

[4.13] vs. [0.46] $\Rightarrow$ **improvement factor 9.0 (Standard vs Adler)**

$$\Delta\alpha^{(5)\,\rm pQCD}_{\rm had} \times 10^4 = 204.68 \pm 1.49$$

Theory: (QCD parameters) has to improve by factor 10 ! $\longrightarrow \pm 0.20$

Requirement may be realistic:

- **pin down experimental errors to 1% level in all non-perturbative regions up to 10 GeV**
- **switch to Adler function method**
- **improve on QCD parameters, mainly on** m_c **and** m_b

⑦ Conclusion

- Recent and future high precision experiments on $a_\mu = (g-2)/2$ (BNL/KEK project may gain factor 10?) and $\sin^2\Theta_{\rm eff}$, etc. (LEP/SLD→TESLA/ILC) imposed and further impose a lot of pressure to theory and experiment to improve, in particular, in reducing the hadronic uncertainties which mainly are due to the experimental errors of $R(s)^{\rm exp}_{\rm had}$.
- In electroweak precision physics at non-zero energies (note $E \sim m_\mu$ in $(g-2)_\mu$) there is now way around determining $\alpha_{\rm eff}(E)$ via precision measurements of $\sigma_{\rm hadronic}$ or lattice QCD simulations via Adler function approach (which is a very difficult long term project).
- Needs for linear collider (like ILC): requires $\sigma_{\rm had}$ at 1% level up to the $\Upsilon \Rightarrow \delta\alpha(M_Z)/\alpha(M_Z) \sim 5 \times 10^{-5}$. New cross section measurements at VEPP-2000, DAFNE-II, radiative return measurements at DAFFNE-I, BABAR and Belle, and at CLEOc and BESS-III are able to reduce the hadronic uncertainties to ???? for a_μ and to ???? for $\alpha(M_Z)$. Together with improved mesurements of the top mass m_t from LHC, at present this would allow to get much better Higgs boson mass limits but much more than that.

- Future precision physics requires dedicated effort on $\sigma_{\rm had}$ experimentally as well as theoretically (radiative corrections, final state radiation from hadrons etc.)
- Improving hadronic cross section measurements must be seen as a global effort in particular in the context of ILC project, which primarily makes sense as a high precision physics project. The $\sigma_{\rm hadronic}$ efforts have to be pushed at any machine able to perform such a measurement up tp 10 GeV! One has to see this activity as an integral part of the international linear collider (ILC) project and to ask for support by the international community. However, a more $\alpha_{\rm eff}$ will be needed at many other places: $\alpha_{\rm eff}(m_p)$, Bhabha,...
- Projects VEPP-2000 and DAFNE–II can play a major role in this respect. What is required is a scan measurement with a good energy calibration (preferable using resonance depolarization). In radiative return at higher energies and multiplicities one has to precisely reconstruct the invariant mass event by event which I think is a difficulty. Dedicated Monte Carlo simulations has to be done to study what precision in which scenario can be achieved.
- Don't believe people claiming very small errors and that everything has been solved already or that some other lab is already doing the same; in high precision physics

any experiment becomes a real challenge and I think at least two experiments should be performed for cross check.

- Note complementary approach important: direct $R(s)$ integration vs. Adler $D(Q^2)$; in particular for the latter as well as for $(g-2)_\mu$ projects like DAFNE–II are a real need!