Heavy quark masses from lattice QCD

ALPHA Collaboration

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PHIPSI 08

International Workshop on e^+e^- collisions from Phi to Psi

I.N.F.N., Laboratori Nazionali di Frascati, Italy

7. – 10. April 2008

Based on a lot of past, recent and ongoing work of the ALPHA Collaboration, in particular

- *Precision computation of the strange quark's mass in quenched QCD*
  J. Garden, J. H., R. Sommer and H. Wittig, Nucl. Phys. B 571 (2000) 237
- *A precise determination of the charm quark's mass in quenched QCD*
  J. Rolf and S. Sint, J. High Energy Phys. 0212 (2002) 007
- *The* $\mathrm{D_s}$*-meson decay constant in the continuum limit of quenched QCD*
  J. H. and A. Jüttner, in preparation

- *Computation of the strong coupling in QCD with two dynamical flavours*
  M. Della Morte, R. Frezzotti, J. H., J. Rolf, R. Sommer and U. Wolff, Nucl. Phys. B 713 (2005) 378
- *Non-perturbative quark mass renormalization in two-flavor QCD*
  M. Della Morte, R. Hoffmann, F. Knechtli, J. Rolf, R. Sommer, I. Wetzorke and U. Wolff,
  Nucl. Phys. B 729 (2005) 117

- *Non-perturbative heavy quark effective theory*
  J. H. and R. Sommer, J. High Energy Phys. 0402 (2004) 022
- *Effective heavy-light meson energies in small-volume quenched QCD*
  J. H. and J. Wennekers, J. High Energy Phys. 0402 (2004) 064
- *Heavy Quark Effective Theory computation of the mass of the bottom quark*
  M. Della Morte, N. Garron, M. Papinutto and R. Sommer, J. High Energy Phys. 0701 (2007) 007
- *Towards a non-perturbative matching of HQET and QCD with dynamical light quarks*
  M. Della Morte, P. Fritzsch, J. H., H. Meyer, H. Simma and R. Sommer, PoS LAT2007 (2007) 246
- ALPHA Collaboration, in progress

Outline

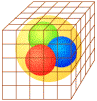
1. Computing quark masses with lattice QCD
   - Generic strategy & Illustration
   - The charm quark's mass
2. The bottom quark's mass
   - Non-perturbative Heavy Quark Effective Theory
   - Application: Computation of M_{b}
   - Status in two-flavour QCD
3. Challenges & Outlook

Lattice QCD

‘Ab initio’ approach to determine phenomenologically relevant key parameters

$$\mathcal{L}_{\mathrm{QCD}}[g_0, m_f] = -\frac{1}{2g_0^2}\,\mathrm{Tr}\{F_{\mu\nu}F_{\mu\nu}\} + \sum_{f=u,d,s,\ldots} \overline{\psi}_f\{\gamma_\mu(\partial_\mu + g_0 A_\mu) + m_f\}\psi_f$$

$$\underbrace{\begin{bmatrix} F_\pi \\ m_\pi \\ m_K \\ m_D \\ m_B \end{bmatrix}}_{\text{Experiment}} \quad \overset{\mathcal{L}_{\mathrm{QCD}}[g_0,m_f]}{\Longrightarrow} \quad \underbrace{\begin{bmatrix} \Lambda_{\mathrm{QCD}} \\ M_u, M_d \\ M_s \\ M_c \\ M_b \end{bmatrix}}_{\text{QCD parameters (RGIs)}} \quad + \quad \underbrace{\begin{bmatrix} F_D \\ F_B \\ B_K, B_B \\ \xi \\ \ldots \end{bmatrix}}_{\text{Predictions}}$$



$\overset{\mathcal{L}_{\mathrm{QCD}}[g_0,m_f]}{\Longrightarrow}$ means formulation of QCD on a Euclidean lattice with:

- Gauge invariance
- Locality
- Unitarity
- Applicable for all scales
- Technical issues/obstacles
  - Continuum limit & Renormalization
  - Computing resources of > 1 TFlop/s

Lattice QCD

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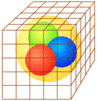
$$\underbrace{\begin{bmatrix} F_\pi \\ m_\pi \\ m_K \\ m_D \\ m_B \end{bmatrix}}_{\text{Experiment}} \xrightarrow{\mathcal{L}_{\rm QCD}\,[g_0, m_f]} \underbrace{\begin{bmatrix} \Lambda_{\rm QCD} \\ M_u, M_d \\ M_s \\ M_c \\ M_b \end{bmatrix}}_{\text{QCD parameters (RGIs)}} + \underbrace{\begin{bmatrix} F_D \\ F_B \\ B_K, B_B \\ \xi \\ \ldots \end{bmatrix}}_{\text{Predictions}}$$



$\xrightarrow{\mathcal{L}_{\rm QCD}\,[g_0, m_f]}$ means formulation of QCD on a Euclidean lattice with:

- Gauge invariance
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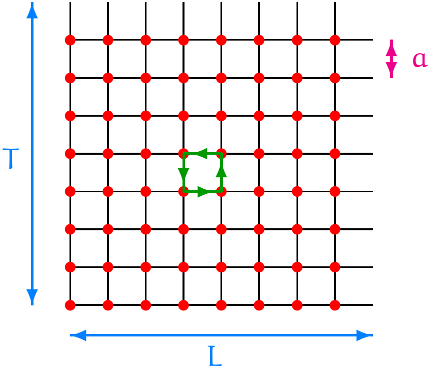
- Applicable for all scales
- Technical issues/obstacles
  - Continuum limit & Renormalization
  - $\mathrm{O}(1/\sqrt{t_{\rm CPU}})$ statistical errors

$U_\mu(x) = e^{iag_0 A_\mu(x)}$ $\psi(x)$



- Lattice cutoff $a^{-1} \sim \Lambda_{UV}$
- Finite volume $L^3 \times T$
- Lattice action

$$S[U,\overline{\psi},\psi] = S_G[U] + S_F[U,\overline{\psi},\psi]$$

$$S_G = \frac{1}{g_0^2}\sum_p \mathrm{Tr}\{1 - U(p)\}$$

$$S_F = a^4 \sum_x \overline{\psi}(x) D[U] \psi(x)$$

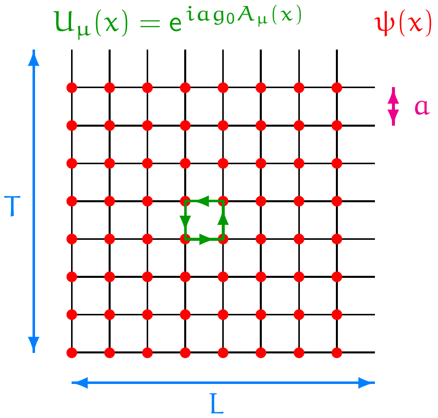
EVs: represented as path integrals

$$\mathcal{Z} = \int \mathcal{D}[U]\mathcal{D}[\overline{\psi},\psi]\, e^{-S[U,\overline{\psi},\psi]} = \int \mathcal{D}[U] \prod_f \det\left(\not{D} + m_f\right) e^{-S_G[U]}$$

$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int \prod_{x,\mu} dU_\mu(x)\, O \prod_f \det\left(\not{D} + m_f\right) e^{-S_G[U]} \;\hat{=}\; \text{thermal average}$$

Stochastic evaluation with *Monte Carlo methods*

$\rightarrow$ Observables $\langle O \rangle = \frac{1}{N}\sum_{n=1}^{N} O_n \pm \Delta_O$ from numerical simulations



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EVs: represented as path integrals

Towards *realistic* QCD simulations — Systematics

(1) *dynamical* fermion effects $\leftrightarrow$ quenched approximation: $\det(\not{D} + m_f) = 1$

(2) lattice artefacts cause discretization errors $\rightarrow$ $\langle O\rangle^{\mathrm{lat}} = \langle O\rangle^{\mathrm{cont}} + O(a^p)$

(3) quark mass restrictions $\rightarrow$ $a \ll m_q^{-1} \ll L$: extrapolate by ChPT & HQET

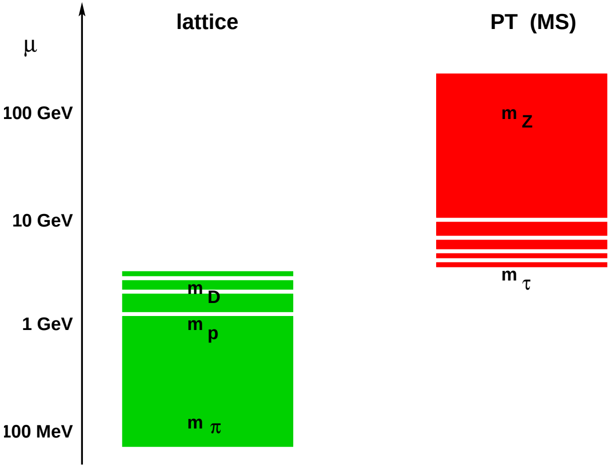
(4) energy range restrictions $\rightarrow$ $\mu < a^{-1} \lesssim 4\,\mathrm{GeV}$ if (typically) $a \gtrsim 0.05\,\mathrm{fm}$

Computing quark masses with lattice QCD

- Generic strategy & Illustration
- The charm quark's mass

Scale problem and generic strategy

Multiple scale problems always difficult for a numerical/lattice treatment



$1/L \ll \mu, E, |\mathbf{p}| \ll 1/a$ $\mu_{\rm pert} \gg \Lambda_{\rm QCD}$

Scale problem and generic strategy

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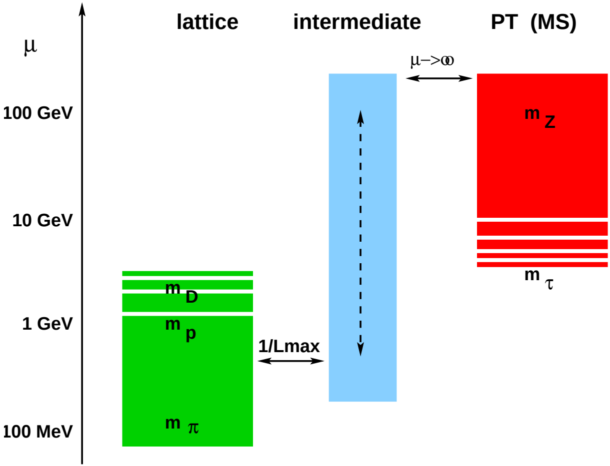


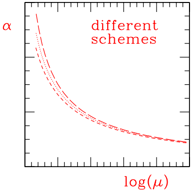
intermediate = Schrödinger functional, finite-volume scheme

Scale dependence of QCD parameters

- Renormalization group (RG) equations, $\bar{g} \equiv \bar{g}(\mu)$

$$\mu \frac{\partial \bar{g}}{\partial \mu} = \beta(\bar{g}) \quad \overset{\bar{g}\to 0}{\sim} \quad -\bar{g}^3 \left\{ b_0 + b_1 \bar{g}^2 + b_2 \bar{g}^4 + \ldots \right\}$$

$$\mu \frac{\partial \overline{m}}{\partial \mu} = \tau(\bar{g})\, \overline{m} \qquad \tau(\bar{g}) \quad \overset{\bar{g}\to 0}{\sim} \quad -\bar{g}^2 \left\{ d_0 + d_1 \bar{g}^2 + \ldots \right\}$$

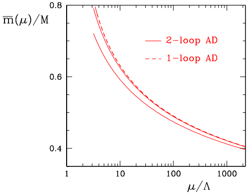


- Solution leads to *exact* equations in a mass-independent scheme

$$\Lambda \equiv \mu \left(b_0 \bar{g}^2\right)^{-b_1/(2b_0^2)} e^{-1/(2b_0\bar{g}^2)} \exp\left\{ -\int_0^{\bar{g}} dg \left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g} \right] \right\}$$

$$M \equiv \overline{m}(\mu) \left(2b_0 \bar{g}^2\right)^{-d_0/(2b_0)} \exp\left\{ -\int_0^{\bar{g}} dg \left[\frac{\tau(g)}{\beta(g)} - \frac{d_0}{b_0 g} \right] \right\}$$

RG invariant quark mass



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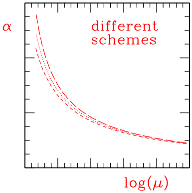

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RG invariant quark mass

- *Simple* relations between different renormalization schemes:

$$S \rightarrow S' \qquad \alpha \rightarrow \alpha' = \alpha + c\,\alpha^2 + O(\alpha^3)$$

$\Rightarrow$ M is scale & scheme independent

Scale dependence of QCD parameters

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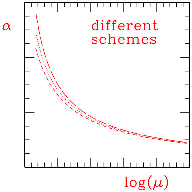

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RG invariant quark mass

- *Simple* relations between different renormalization schemes:

$$\frac{\Lambda_{S'}}{\Lambda_S} = e^{\frac{c}{4\pi b_0}}, \quad \frac{\overline{m}_{S'}(\mu)}{\overline{m}_S(\mu)} = 1 + O\left(\alpha(\mu)\right) \xrightarrow{\mu\to\infty} 1 \;\Rightarrow\; M_{S'} = M_S \equiv M$$

$\Rightarrow$ Choose *convenient* scheme (i.e. physical coupling) to compute M

The basic equation for the RGI quark mass

Definition of the running mass through the PCAC relation:

$$\partial_\mu A_\mu^{su} = (m_u + m_s)\, P^{su} \qquad \begin{cases} A_\mu^{su} = \overline{u}\gamma_\mu\gamma_5\, s : & \text{axial vector current} \\ P^{su} = \overline{u}\gamma_5\, s : & \text{pseudoscalar density} \end{cases}$$

Upon renormalization in the lattice regularized theory $(g_0 \leftrightarrow a)$:

$$\underbrace{\overline{m}_u(\mu) + \overline{m}_s(\mu)}_{\text{renormalized \& running}} = \frac{Z_A(g_0)}{Z_P(g_0,\mu)} \times \underbrace{\frac{\langle 0 \,|\, \partial_\mu A_\mu^{su} \,|\, K^-(\mathbf{p}=0) \rangle}{\langle 0 \,|\, P^{su} \,|\, K^-(\mathbf{p}=0) \rangle}}_{m_u + m_s = \frac{F_K m_K^2}{G_K} :\ \text{bare PCAC}}$$

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- Scale & scheme dependence via Z_P, which is poorly convergent in PT
  $\rightarrow$ NP determination needed [ALPHA Collaboration 1999 $(N_f = 0)$ & 2005 $(N_f = 2)$]
  (Z_A fixed by requiring a chiral Ward identity from Euclidean current algebra)
- Analogously for the charm sector:

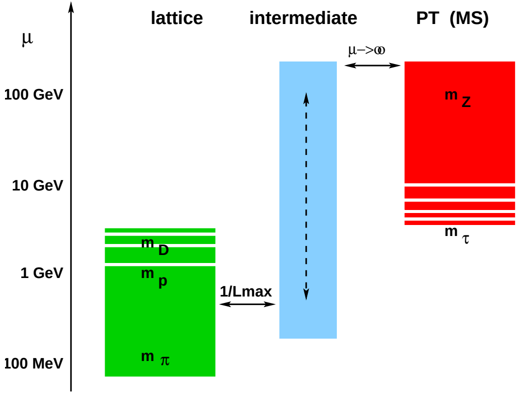
$$\overline{m}_s(\mu) + \overline{m}_c(\mu) = \frac{Z_A(g_0)}{Z_P(g_0,\mu)} \underbrace{\frac{\langle 0 \,|\, \partial_\mu A_\mu^{cs} \,|\, D_s^+(\mathbf{p}=0) \rangle}{\langle 0 \,|\, P^{cs} \,|\, D_s^+(\mathbf{p}=0) \rangle}}_{m_s + m_c:\ \text{bare PCAC}} \qquad \begin{cases} A_\mu^{cs} = \bar{s}\gamma_\mu\gamma_5\, c \\ P^{cs} = \bar{s}\gamma_5\, c \end{cases}$$

with *same* Z_P in a mass/flavour independent renormalization scheme

Now split up the problem according to the generic strategy:

$$
\begin{aligned}
M_s &= \frac{M}{\overline{m}(\mu)}\,\overline{m}_s(\mu) = \frac{M}{\overline{m}(\mu)}\,\frac{Z_A(g_0)}{Z_P(g_0,\mu)}\,\underbrace{m_s(g_0)}_{\text{bare PCAC}} \\
&\equiv Z_M(g_0)\times m_s(g_0) \qquad\qquad (\text{recall: } g_0 \leftrightarrow a)
\end{aligned}
$$

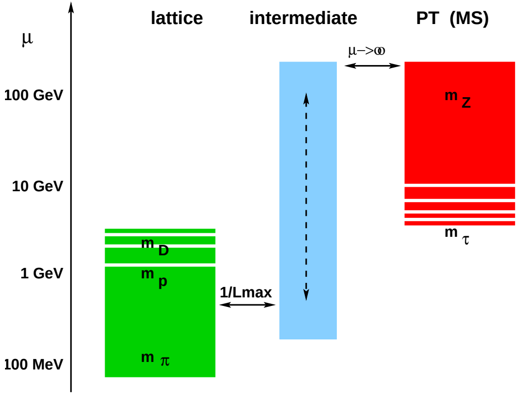
$$
Z_M(g_0) = \frac{M}{\overline{m}(\mu)}\,\frac{Z_A(g_0)}{Z_P(g_0,\mu)} = \underbrace{\frac{M}{\overline{m}(\mu_{\mathrm{pert}})}}_{\text{PT}}\,\underbrace{\frac{\overline{m}(\mu_{\mathrm{pert}})}{\overline{m}(\mu_{\mathrm{had}})}}_{\text{NP}}\,\underbrace{\frac{Z_A(g_0)}{Z_P(g_0,\mu_{\mathrm{had}})}}_{\text{“easy”}}
$$



Basic equation for the RGI quark mass

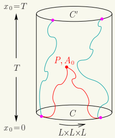
$$Z_M(g_0) = \underbrace{\frac{Z_A(g_0)}{Z_P(g_0,\mu_{had})}}_{\text{lattice}} \quad \underbrace{\frac{\overline{m}(\mu_{pert})}{\overline{m}(\mu_{had})}}_{\textbf{NP}\text{: interm. \& CL}} \quad \underbrace{\frac{M}{\overline{m}(\mu_{pert})}}_{\text{PT}}$$

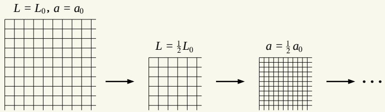
$$M_i = Z_M(g_0)\, m_i(g_0) \qquad (\text{here: } i = s, c)$$



NP calculation of $\overline{m}(\mu_{\rm pert})/\overline{m}(\mu_{\rm had})$ in the intermediate *SF* scheme

Schrödinger Functional $=$ QCD partition function in a cylinder (Dirichlet BCs in x_0)





$\Rightarrow$ NP running from (finite-volume) correlation functions & a recursive technique

Illustration: M_s $(N_f = 0)$ [Garden, H., Sommer & Wittig, NPB571(2000)237]

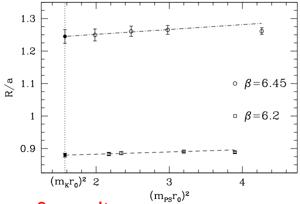
Physics inputs & Lattice QCD computation

- Set bare strange mass in the Lagrangian s.th. m_K/F_K = experiment
  $\Rightarrow$ amounts to slightly extrapolate to physical quark mass: $m_{PS} \rightarrow m_K$
- MeV-scale by fixing the static quark potential to phenomenology: $r_0 = 0.5\,\mathrm{fm}$
- Evaluate ratio $R \equiv F_K/G_K$ of K-meson matrix elements in numerical lattice simulations and combine this with Z_M:

$$M_s + M_u = Z_M \frac{F_K}{G_K} m_K^2$$

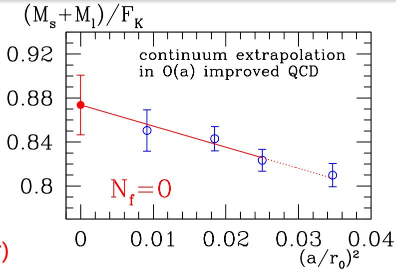
[NP Z_M: ALPHA Collaboration 1999 & 2005 $(N_f = 0, 2)$]

(ChPT "cheat": We actually compute $M_s + M_\ell$, $M_\ell = \frac{1}{2}(M_u + M_d)$, $\frac{M_s}{M_\ell} = 24.4$)



$N_f = 0$ result:

$\overline{m}_s^{\overline{MS}}(2\,\mathrm{GeV}) = 97(4)$ MeV ($\lesssim$ 15% quenching error)



The charm quark's mass

Rolf & Sint, JHEP0212(2002)007
ALPHA Collaboration & INFN/TOV, in progress

Calculation in same spirit as for strange

- Physics input: bare charm mass in $\mathcal{L}_{\mathrm{QCD}}$ s.th. m_{D_s}/F_K = experiment
- Additional complication in the charm sector:
  - ▶ $O(am_{q,c})$ cutoff effects become relevant

$$M_c = Z_M \left[1 + (b_A - b_P)\, am_{q,c}\right] m_c = Z_M \frac{Z_m Z_P}{Z_A}\, m_{q,c}\left(1 + b_m\, am_{q,c}\right)$$

$\rightarrow$ remove $O(am_{q,c})$ NP'ly [ALPHA Collaboration 2000 ($N_f = 0$) & in progress ($N_f = 2$)]

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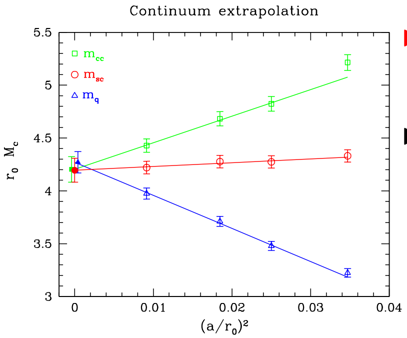

- ▶ $N_f = 0$, $r_0 = 0.5$ fm: $M_c = 1654(45)$ MeV

$$\Rightarrow \quad \overline{m}_c^{\overline{\mathrm{MS}}}(\overline{m}_c) = 1301(34)\ \mathrm{MeV}$$

- ▶ Similar analysis for $N_f > 0$ in progress

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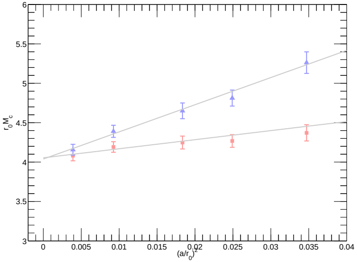


$a \simeq 0.03\,$fm added
(byproduct of work with A. Jüttner)

- ▶ $N_f=0,\ r_0=0.5\,\text{fm}:\ M_c=1654(45)\,\text{MeV}$

  $\Rightarrow\quad \overline{m}_c^{\overline{\rm MS}}(\overline{m}_c)\;=\;1301(34)\,\text{MeV}$
- ▶ Similar analysis for $N_f>0$ in progress
- ▶ Charm *just* doable, but $aM_b \simeq 4aM_c$!

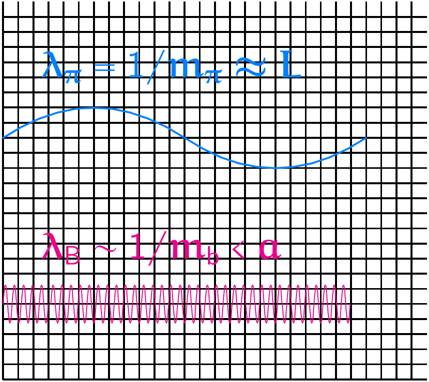
  $\rightarrow$ for b-quarks, continuum limit $a \rightarrow 0$ can't be controlled in this way

  $\Rightarrow$ need for an *effective* theory

The bottom quark's mass

- Non-perturbative Heavy Quark Effective Theory
- Application: Computation of M_{b}
- Status in two-flavour QCD

Lattice HQET — Why ?



- Light quarks: too light
  - Widely spread objects
  - Finite-volume errors due to light pions
- b-quark: too heavy
  - Extremely localized object
  - B-mesons with a propagating b-quark on the lattice require *very* small $a < \frac{1}{m_b}$, beyond today's computing resources

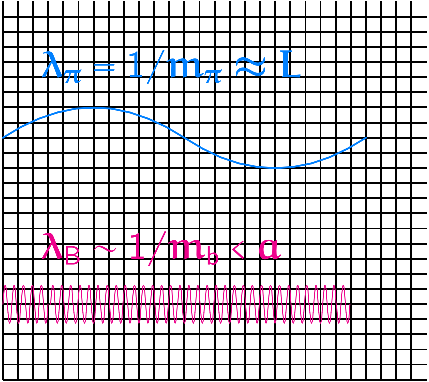
$\Rightarrow$ Recourse to an *effective* theory for the b-quark:

Heavy **Q**uark **E**ffective **T**heory

[Eichten, 1988; Eichten & Hill, 1990]

$$\overline{\psi}_b \{\gamma_\mu D_\mu + m_b\} \psi_b \quad \longrightarrow$$

Lattice HQET — Why ?



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$\boxed{\textbf{Heavy Quark Effective Theory}}$

[Eichten, 1988; Eichten & Hill, 1990]

$$\mathcal{L}_{\rm HQET}(x) = \overline{\psi}_h(x)\Big[\underbrace{D_0 + m_b}_{\text{static limit}} - \frac{\omega_{\rm kin}}{2m_b}\mathbf{D}^2 - \frac{\omega_{\rm spin}}{2m_b}\boldsymbol{\sigma}\cdot\mathbf{B}\Big]\psi_h(x) + \ldots$$

Non-perturbative formulation of HQET

Beyond the static approximation

$$\overline{\psi}_h \left[\nabla_0^* + \delta m \right] \psi_h \quad \longrightarrow \quad \text{Eichten-Hill action}$$

$$\overline{\psi}_h \left(-\tfrac{1}{2}\,\mathbf{D}^2 \right) \psi_h \;\equiv\; \mathcal{O}_{\mathrm{kin}} \quad \longrightarrow \quad \left\{ \begin{array}{l} \text{kinetic energy from heavy} \\ \text{quark's residual motion} \end{array} \right.$$

$$\overline{\psi}_h \left(-\tfrac{1}{2}\,\boldsymbol{\sigma}\cdot\mathbf{B} \right) \psi_h \;\equiv\; \mathcal{O}_{\mathrm{spin}} \quad \longrightarrow \quad \left\{ \begin{array}{l} \text{chromomagnetic interaction} \\ \text{with the gluon field} \end{array} \right.$$

- $\delta m, \omega_i(g_0, m)$ must be determined such that HQET matches QCD
- <u>Analogously:</u> Composite fields in the effective theory, e.g.

$$A_0^{\mathrm{HQET}}(x) = \underbrace{Z_A^{\mathrm{HQET}}}_{1+\mathrm{O}(g_0^2)} \underbrace{\overline{\psi}_l(x)\gamma_0\gamma_5\psi_h(x)}_{A_0^{\mathrm{stat}}(x)} + \underbrace{c_A^{\mathrm{HQET}}}_{\propto 1/m} \overline{\psi}_l(x)\gamma_j\gamma_5\overleftarrow{D}_j\psi_h(x) + \ldots$$

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- EVs $=$ Functional integral representation at the quantum level:

$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\varphi]\, O[\varphi]\, e^{-(S_{\rm rel} + S_{\rm HQET})} \qquad \mathcal{Z} = \int \mathcal{D}[\varphi]\, e^{-(S_{\rm rel} + S_{\rm HQET})}$$

Now the *integrand* is expanded in a *power series* in $1/m$

$$\exp\{-S_{\rm HQET}\} =$$

$$\exp\left\{-a^4 \textstyle\sum_x \mathcal{L}_{\rm stat}(x)\right\}$$
$$\times \left\{1 - a^4 \textstyle\sum_x \mathcal{L}^{(1)}(x) + \frac{1}{2}\left[a^4 \sum_x \mathcal{L}^{(1)}(x)\right]^2 - a^4 \sum_x \mathcal{L}^{(2)}(x) + \ldots\right\}$$

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$$\Rightarrow \quad \langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\varphi] \, e^{-S_{\rm rel} - a^4 \sum_x \mathcal{L}_{\rm stat}(x)} \, O \, \left\{ 1 - a^4 \textstyle\sum_x \mathcal{L}^{(1)}(x) + \ldots \right\}$$

Important (but not automatic) implications of this definition of HQET

- $1/m$ – terms appear only as *insertions* of local operators
  $\Rightarrow$ Power counting: Renormalizability at any given order in $1/m$
- $\Leftrightarrow$ Existence of the continuum limit with universality
- Effective theory $=$ Continuum asymptotic expansion in $1/m$

Mass renormalization pattern in HQET

Already at the level of $\mathcal{L}_{\rm stat}(x) = \overline{\psi}_{\rm h}(x)\,[\,\nabla_0^* + \delta m\,]\,\psi_{\rm h}(x)$:

Linear divergence $\delta m \propto a^{-1}$ originates from mixing of $\overline{\psi}_{\rm h} D_0 \psi_{\rm h}$ with $\overline{\psi}_{\rm h}\psi_{\rm h}$

$$m_{\rm b}^{\overline{\rm MS}} \;=\; Z_{\rm pole}^{\overline{\rm MS}} \times m_{\rm pole} \qquad m_{\rm pole} \;=\; m_{\rm B} - E_{\rm stat} - \delta m$$

$$\left[\begin{array}{rl} m_{\rm B}: & \text{(exp.) B-meson mass} \\ E_{\rm stat}: & \text{static binding energy} \end{array}\right]$$

$$\delta m \;=\; \frac{c(g_0)}{a} \;\sim\; e^{1/(2b_0 g_0^2)} \times \left\{ c_1 g_0^2 + c_2 g_0^4 + \ldots \right\}$$

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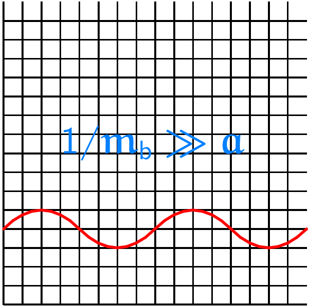
- In PT:
  uncertainty $=$ truncation error $\sim e^{1/(2b_0 g_0^2)}\, c_{n+1}\, g_0^{2n+2} \xrightarrow{g_0 \to 0} \infty$!
  - $\Rightarrow$ Non-perturbative $c(g_0)$ needed
  - $\Rightarrow$ *NP renormalization (resp. matching to QCD) of HQET required for the continuum limit to exist*
- Power-law divergences even worse at the level of $1/m$ – corrections:
  $a^{-1} \rightarrow a^{-2}$

NP matching in *finite* volume [H. & Sommer, JHEP0402(2004)022]

Need to treat the b as particle with finite mass (while $am_b \ll 1$ to keep a– effects small) and $Lm_b \gg 1$ to apply HQET

$\Rightarrow$ Trick: start with QCD in a *small* volume, $V = L^4 \approx (0.4\,\mathrm{fm})^4$

QCD



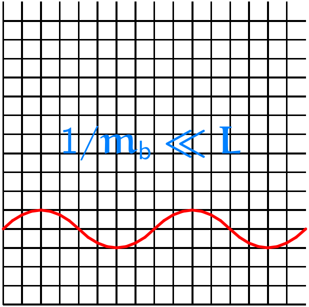
Matching conditions

$$\boxed{\Phi_k^{\mathrm{QCD}} = \Phi_k^{\mathrm{HQET}}}$$

for observables Φ_k

(renormalized quantities, computable for $a \to 0$)

HQET



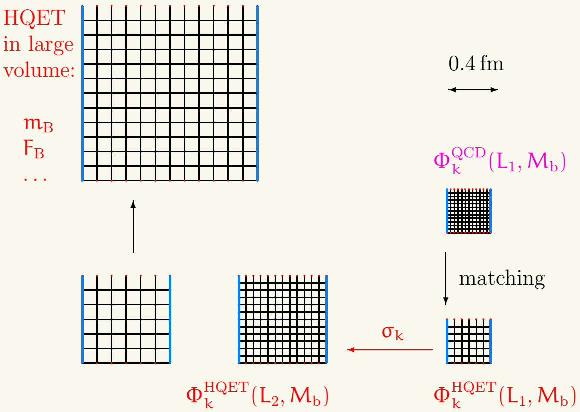
- HQET parameters fixed by relating them to QCD observables in small V
- Rather than simulating a propagating ‘real’ relativistic b-quark, one aims at determining the *NP heavy quark mass dependence* of Φ_k^{QCD} in finite V

Connecting small and large volumes

- Matching volume: $L = L_1 \simeq 0.4\,\mathrm{fm}$, very small lattice spacings
- Gap to large volumes and practicable lattice spacings, where physical quantities (e.g. m_B, F_{B_s}) may be extracted, bridged by a . . .

Finite-size scaling step

[Lüscher, Weisz & Wolff, 1991; ALPHA Collaboration, 1993-2006]



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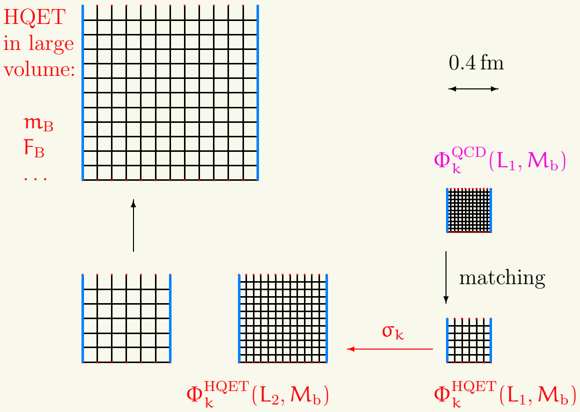

- Fully non-perturbative, continuum limit can be taken everywhere
- Use the *QCD Schrödinger Functional*, $L \to 2L$ via Step Scaling Functions

$$\Phi_k^{\rm HQET}(2L) = \sigma_k\Big(\{\Phi_j^{\rm HQET}(L), j = 1, \ldots, N\}\Big) \qquad 2L = 2L_1 \simeq 0.8\,\text{fm}$$

$\rightarrow$ Large V ($L \simeq 2$ fm) at same resolution, where a B-meson fits comfortably

Computation of M_b

H. & Sommer, JHEP0402(2004)022
Della Morte, Garron, Sommer & Papinutto, JHEP0701(2007)007

Non-trivial matching problem:

$$\{m_{\mathrm{bare}} + \delta m, \omega_{\mathrm{kin}}, \omega_{\mathrm{spin}}\} \text{ from QCD} \quad \text{s.th.} \quad m_{\mathrm{pole}} + \delta m \;\leftrightarrow\; M_b$$

$\Rightarrow$ $N = 3$ matching conditions:

$$\Phi_k^{\mathrm{QCD}}(L, M) = \Phi_k^{\mathrm{HQET}}(L, M) \qquad k = 1, 2, 3$$

[M : RGI heavy quark mass]

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Basic equation in leading order of HQET (static approximation)

$$m_B = E_{\rm stat} - E_{\rm stat}^{\rm sub} + E_{\rm stat}^{\rm sub} \quad \text{with} \quad E_{\rm stat}^{\rm sub} = E(L_1, M_b) \quad \text{[matching to QCD]}$$

$$= \underbrace{E_{\rm stat} - E_{\rm stat}(L_2)}_{a \to 0 \text{ in HQET}} + \underbrace{E_{\rm stat}(L_2) - E_{\rm stat}(L_1)}_{a \to 0 \text{ in HQET } (\sigma_m)} + \underbrace{E(L_1, M_b)}_{\overset{a\to 0}{\equiv} \Phi_2/L_1 \text{ in QCD}} \quad (\star)$$

- Divergent static quark's self-energy δm cancels in *differences* !
- $\Phi_2(L_1, M)$ carries entire (relativistic) heavy quark mass dependence

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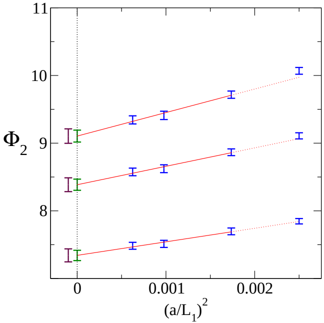

Quenched result

Della Morte, Garron, Sommer & Papinutto
JHEP01(2007)007

- $L_1 \simeq 0.4\,\text{fm}\,,\; L_2 = 2L_1$
- Use $r_0 m_B^{(exp)}, r_0 = 0.5\,\text{fm}$ & solve $(\star)$

$\Rightarrow \quad M_b^{stat} = (6771 \pm 99)\,\text{MeV}$

- ▶ NP renormalization & Continuum limit
- ▶ Error dominated by that on Z_M ($\simeq 1\%$) in $L_1 M = Z_M\, Z\,(1 + b_m a m_q) \times L_1 m_q$

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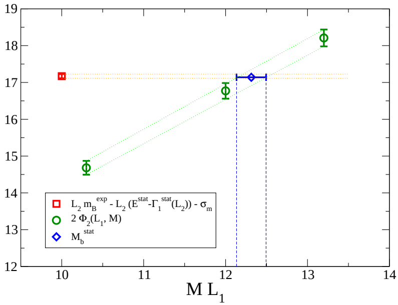

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Inclusion of $1/m$–terms

Della Morte, Garron, Sommer & Papinutto, JHEP0701(2007)007

m_B at next-to-leading order of HQET

$$m_B = m_{\text{bare}} + E_{\text{stat}} + \omega_{\text{kin}} E_{\text{kin}} + \omega_{\text{spin}} E_{\text{spin}}$$

- $E_{\text{kin}}, E_{\text{spin}}$ associated with $\overline{\psi}_h(-\frac{1}{2}\mathbf{D}^2)\psi_h$ and $\overline{\psi}_h(-\frac{1}{2}\boldsymbol{\sigma}\cdot\mathbf{B})\psi_h$ in $\mathcal{L}^{(1)}$
  $\rightarrow$ Three observables Φ_1, Φ_2, Φ_3 required in the matching step
- Considering the *spin-averaged* B-meson instead, ω_{spin} cancels:

$$m_B^{(av)} = \frac{1}{4} m_B + \frac{3}{4} m_B^* = m_{\text{bare}} + E_{\text{stat}} + \omega_{\text{kin}} E_{\text{kin}}$$

$\rightarrow$ Only *two* obervables Φ_1, Φ_2 necessary

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Strategy

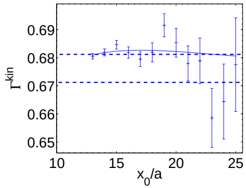
$$\begin{array}{ccc}
\boxed{\text{Experiment}} & & \boxed{\text{Lattice with } am_b \ll 1} \\
m_B = 5.4\,\text{GeV} & L_1 \simeq 0.4\text{fm},\ L_2 = 2L_1 & \Phi_1(L_1, M),\ \Phi_2(L_1, M) \\
\downarrow & u_1 = \bar{g}^2(L_1) & \downarrow \\
\Phi_1^{HQET}(L_2),\ \Phi_2^{HQET}(L_2) & \xleftarrow[\sigma_1^{kin}(u_1),\ \sigma_2^{kin}(u_1)]{\sigma_m(u_1)} & \Phi_1^{HQET}(L_1),\ \Phi_2^{HQET}(L_1)
\end{array}$$

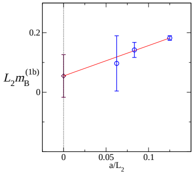
Matching formula $=$ Static part $+$ $1/m$ – correction :

$$
\begin{aligned}
L_2 m_B^{(av)} \;=\; & \left. L_2 m_B^{\rm stat} \right\} \;=\; L_2 \left[E_{\rm stat} - \Gamma_1^{\rm stat}(L_2) \right] + \sigma_m(u_1) + 2\Phi_2(L_1, M_b) \\
+ & \left. L_2 m_B^{(1)} \right\} \;=\; \sigma_2^{\rm kin}(u_1)\Phi_1(L_1, M_b) + L_2 \left[E_{\rm kin} - \Gamma_1^{\rm kin}(L_2) \right] \omega_{\rm kin}
\end{aligned}
$$

Most difficult piece encountered in the calculation

Large-volume HQET matrix element $\left[E_{\rm kin} - \Gamma_1^{\rm kin}(L_2) \right]$ in the $1/m_b$ – contribution





Result in the quenched approximation

$$
\overline{m}_b^{\overline{\rm MS}}(\overline{m}_b) \;=\; 4.374(64)\ {\rm GeV} \underbrace{-\ 0.027(22)\ {\rm GeV}}_{O(\Lambda^2/m_b)} + \underbrace{O(\Lambda^3/m_b^2)}_{\text{negligible}}
$$

Further ingredients:

- Finite- and large-volume B-meson energies extracted from static-light SF correlators
  $\leftarrow$ HYP-smeared static actions [Hasenfratz & Knechtli, 2001; ALPHA Collaboration 2004/05]
- Consistency with results from different matching observables/conditions checked

Status in two-flavour QCD

Della Morte, Fritzsch, H., Meyer, Simma & Sommer, PoS LAT2007(2007)246

ALPHA Collaboration, in progress

Matching to QCD in finite V with a relativistic b-quark

$N_f = 2$ degenerate massless sea quarks

- Evaluation of QCD heavy-light valence quark correlation functions ($m_{\rm light}^{\rm val} = m^{\rm sea} = 0$ in the SF)
- NP quark mass renormalization in $V = L_1^4 \approx (0.5\,{\rm fm})^4$ finished:

  $z \equiv L_1 M \;=\; Z_M Z\,(1 + b_m a m_q) \times L_1 m_q$ known in terms of bare parameters

  $\rightarrow$ enables to calculate the heavy quark mass dependence of observables

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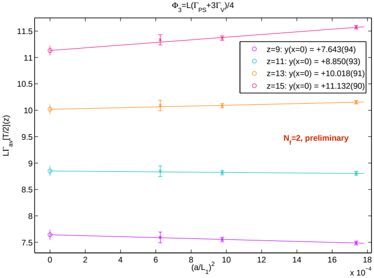
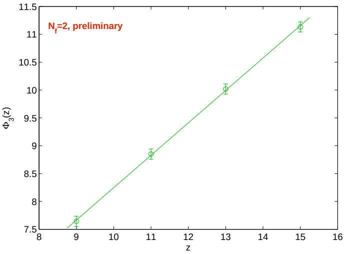
Example for a CL extrapolation: spin-averaged B-energy $\equiv L_1 \times \left[\, \frac{1}{4}\,\Gamma_{PS} + \frac{3}{4}\,\Gamma_V \,\right]$





Challenges & Outlook

- Challenges:
  - (M_s), M_c for $N_f = 2$:
    - ◇ smaller lattice spacings and smaller quark masses are under way by various groups/collaborations
    - ◇ appear feasible owing to algorithmic/technical improvements: domain decomposition, low-mode deflation & all-to-all quark propagators
      [Lüscher, '03 – '05; Del Debbio et al., '07; Lüscher, '07; Foley et al., '05]

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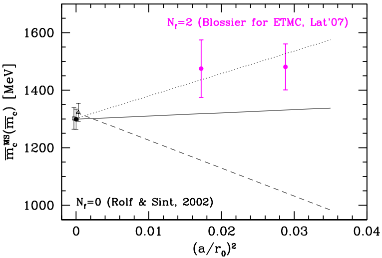
    - ◇ with current results (i.e. status reported @ Lattice 2007), it is still quite a bit early for a continuum limit (e.g. B. Blossier for ETMC: $\overline{m}_c^{\overline{MS}}(\overline{m}_c) = 1478(75)\,\text{MeV}$)



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  - M_b for $N_f = 2$ through non-perturbative HQET is in progress:
    - error for M_b is dominated by $Z_M(g_0)$ $\longrightarrow$ reduce by a factor 2 ?!
    - alternative/complementary approach: combination of static results and (TOV) step scaling method to turn QCD *extra*polations into *inter*polations
      [de Divitiis et al., '03; Guazzini, Sommer & Tantalo, '07]

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    - smaller lattice spacings and smaller quark masses are under way by various groups/collaborations
    - appear feasible owing to algorithmic/technical improvements: domain decomposition, low-mode deflation & all-to-all quark propagators [Lüscher, '03 – '05; Del Debbio et al., '07; Lüscher, '07; Foley et al., '05]
  - M_b for $N_f = 2$ through non-perturbative HQET is in progress:
    - error for M_b is dominated by $Z_M(g_0)$ $\rightarrow$ reduce by a factor 2 ?!
    - alternative/complementary approach: combination of static results and (TOV) step scaling method to turn QCD *extra*polations into *inter*polations [de Divitiis et al., '03; Guazzini, Sommer & Tantalo, '07]
  - **Light up & down quarks, large volumes and chiral extrapolation**
- Why should we face the challenges ?
  - LQCD: "No" assumptions to determine fundamental SM parameters
  - Superb experimental input available: m_p, m_K, m_D, m_B, . . .

Challenges & Outlook

- Bottomline conclusions:
  - Non-perturbative (NP) tools for renormalization are fully developed
  - Quenched computations are under good control, particularly thanks to
    - NP $\mathrm{O}(a)$ improvement
    - NP renormalization
  - LQCD with dynamical quarks, $N_f \geqslant 2$:
    Significant progress & even more expected in the next 2 – 3 years