

# $\tau$ Physics: Theory Overview

A. Pich

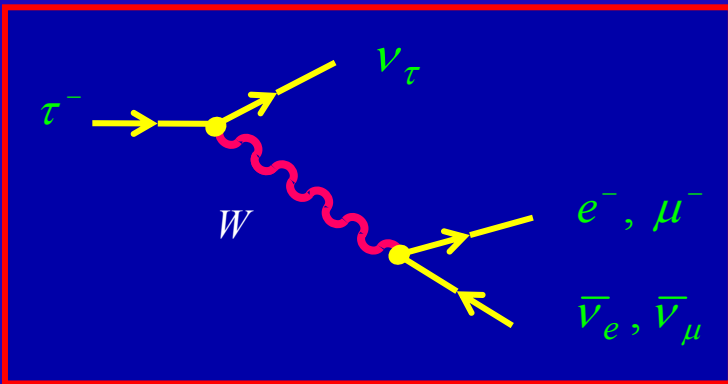
IFIC, Valencia



PhiPsi08

Frascati

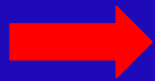
7-10 April 2008



$$\Gamma(\tau \rightarrow \nu_\tau l \bar{\nu}_l) = \frac{G_F^2 m_\tau^5}{192 \pi^3} f(m_l^2/m_\tau^2) r_{EW}$$

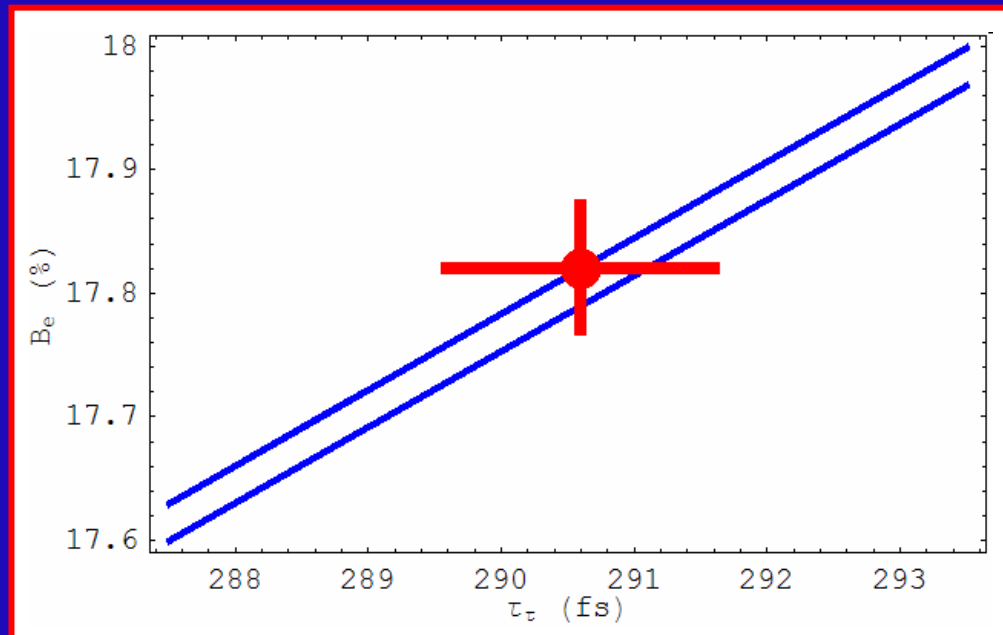
$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$$

$$r_{EW} = 0.9960 \quad (\text{Marciano-Sirlin})$$



$$B_e = \frac{B_\mu}{0.972564 \pm 0.000010} = \frac{\tau_\tau}{(1632.1 \pm 1.4) \times 10^{-15} \text{ s}}$$

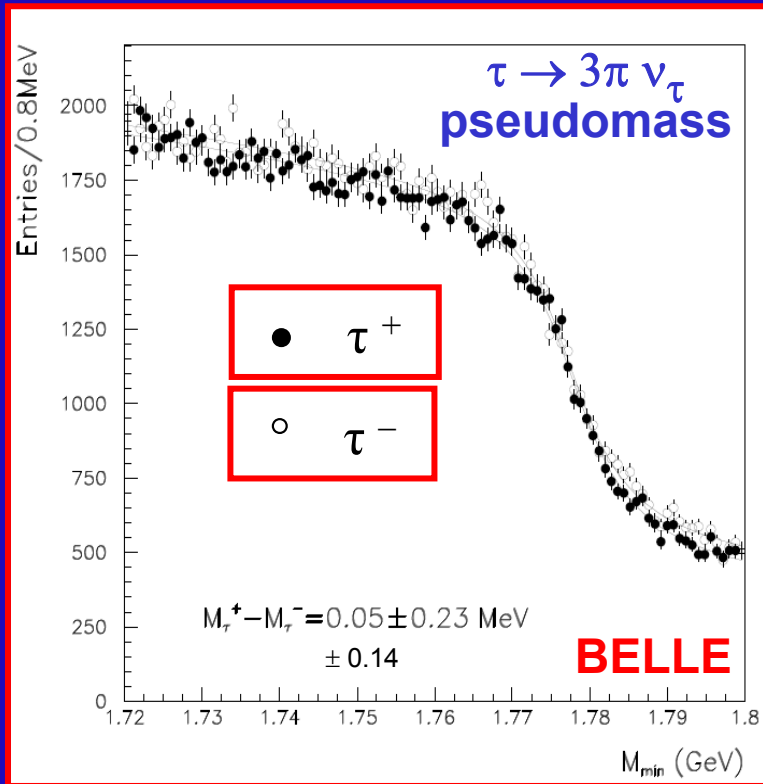
$$\left( \frac{B_\mu}{B_e} \right)_{\text{exp}} = 0.9725 \pm 0.0039$$



$\tau_\tau, \text{Br}(\tau \rightarrow \mu), \text{Br}(\tau \rightarrow e) \sim 0.3\%$  precision

**Future Improvements:** BABAR, BELLE, KEDR, BESIII, SuperB, ...

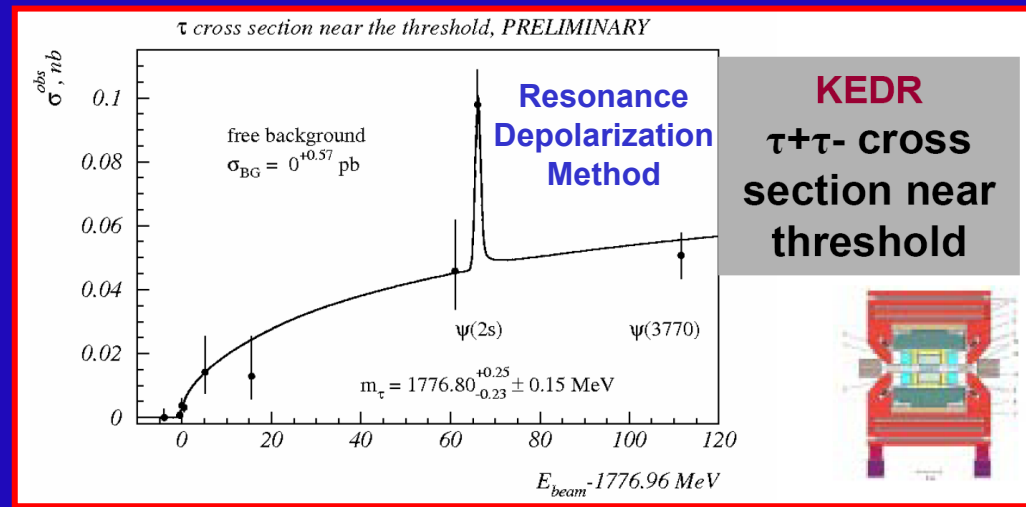
$\delta m_\tau \sim 0.023 \text{ MeV}$  (12.7 ppm) BESIII



$$m_\tau = 1776.99^{+0.29}_{-0.26} \text{ MeV} \quad (\text{PDG06})$$

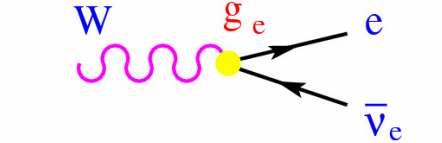
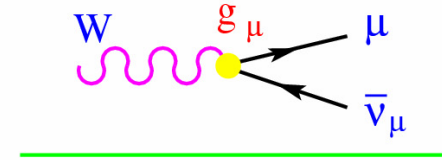
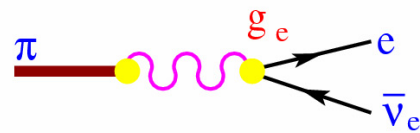
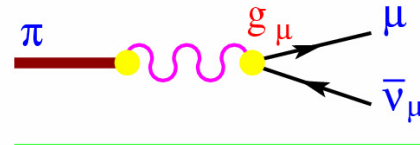
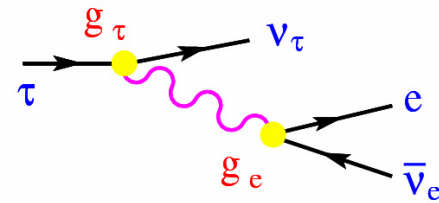
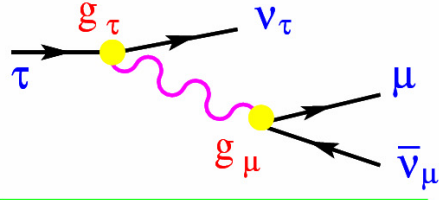
$$1776.61 \pm 0.13 \pm 0.35 \text{ MeV} \quad (\text{BELLE})$$

$$1776.81 \pm 0.25^{+0.25}_{-0.23} \pm 0.15 \text{ MeV} \quad (\text{KEDR})$$

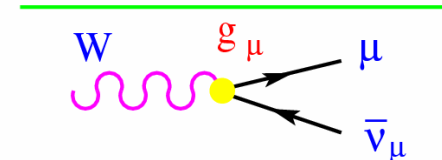
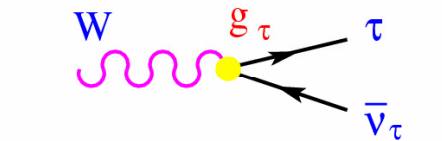
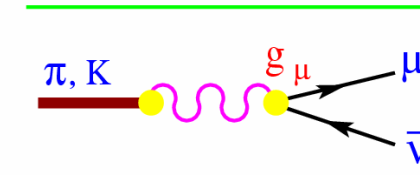
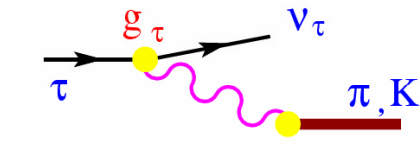
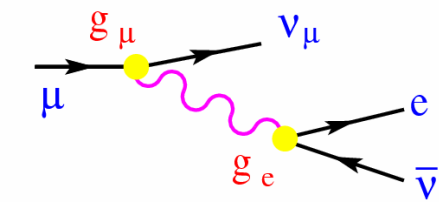
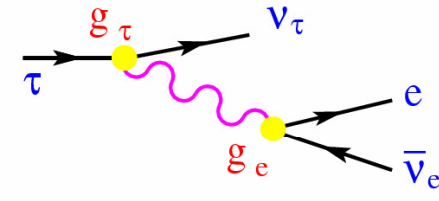


# LEPTON UNIVERSALITY

$\frac{g_\mu}{g_e}$



$\frac{g_\tau}{g_\mu}$



# CHARGED CURRENT UNIVERSALITY

$$|g_\mu / g_e|$$

$B_{\tau \rightarrow \mu} / B_{\tau \rightarrow e}$	$1.0000 \pm 0.0020$
$B_{\pi \rightarrow \mu} / B_{\pi \rightarrow e}$	$1.0021 \pm 0.0016$
$B_{K \rightarrow \mu} / B_{K \rightarrow e}$	$1.004 \pm 0.007$
$B_{K \rightarrow \pi\mu} / B_{K \rightarrow \pi e}$	$1.002 \pm 0.002$
$B_{W \rightarrow \mu} / B_{W \rightarrow e}$	$0.997 \pm 0.010$

$$|g_\tau / g_\mu|$$

$B_{\tau \rightarrow e} \tau_\mu / \tau_\tau$	$1.0006 \pm 0.0022$
$\Gamma_{\tau \rightarrow \pi} / \Gamma_{\pi \rightarrow \mu}$	$0.996 \pm 0.005$
$\Gamma_{\tau \rightarrow K} / \Gamma_{K \rightarrow \mu}$	$0.979 \pm 0.017$
$B_{W \rightarrow \tau} / B_{W \rightarrow \mu}$	$1.039 \pm 0.013$

$$|g_\tau / g_e|$$

$B_{\tau \rightarrow \mu} \tau_\mu / \tau_\tau$	$1.0005 \pm 0.0023$
$B_{W \rightarrow \tau} / B_{W \rightarrow e}$	$1.036 \pm 0.014$

# LEPTON FLAVOUR VIOLATION

90% CL Upper Limits on  $\text{Br}(l^- \rightarrow X^-)$

[BABAR / BELLE]

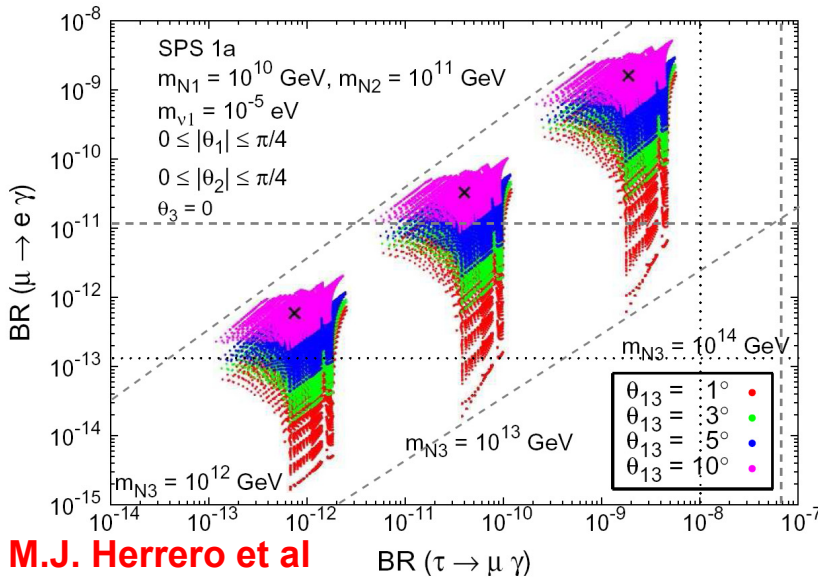
Decay	U.L.	Decay	U.L.	Decay	U.L.
$\mu^- \rightarrow e^- \gamma$	$1.2 \cdot 10^{-11}$	$\mu^- \rightarrow e^- e^+ e^-$	$1.0 \cdot 10^{-12}$	$\mu^- \rightarrow e^- \gamma \gamma$	$7.2 \cdot 10^{-11}$
$\tau^- \rightarrow e^- \gamma$	$1.1 \cdot 10^{-7}$	$\tau^- \rightarrow e^- e^+ e^-$	$3.6 \cdot 10^{-8}$	$\tau^- \rightarrow e^- e^+ \mu^-$	$2.7 \cdot 10^{-8}$
$\tau^- \rightarrow \mu^- \gamma$	$4.5 \cdot 10^{-8}$	$\tau^- \rightarrow e^- \mu^+ \mu^-$	$3.7 \cdot 10^{-8}$	$\tau^- \rightarrow \mu^- e^+ \mu^-$	$2.3 \cdot 10^{-8}$
$\tau^- \rightarrow e^- e^- \mu^+$	$2.0 \cdot 10^{-8}$	$\tau^- \rightarrow \mu^- \mu^+ \mu^-$	$3.2 \cdot 10^{-8}$	$\tau^- \rightarrow e^- \pi^0$	$8.0 \cdot 10^{-8}$
$\tau^- \rightarrow \mu^- \pi^0$	$1.1 \cdot 10^{-7}$	$\tau^- \rightarrow e^- \eta'$	$1.6 \cdot 10^{-7}$	$\tau^- \rightarrow \mu^- \eta'$	$1.3 \cdot 10^{-7}$
$\tau^- \rightarrow e^- \eta$	$9.2 \cdot 10^{-8}$	$\tau^- \rightarrow \mu^- \eta$	$6.5 \cdot 10^{-8}$	$\tau^- \rightarrow e^- K^{*0}$	$7.8 \cdot 10^{-8}$
$\tau^- \rightarrow e^- K_S$	$5.6 \cdot 10^{-8}$	$\tau^- \rightarrow \mu^- K_S$	$4.9 \cdot 10^{-8}$	$\tau^- \rightarrow \mu^- \rho^0$	$6.8 \cdot 10^{-8}$
$\tau^- \rightarrow e^- K^+ K^-$	$1.4 \cdot 10^{-7}$	$\tau^- \rightarrow e^- K^+ \pi^-$	$1.6 \cdot 10^{-7}$	$\tau^- \rightarrow e^- \pi^+ K^-$	$3.2 \cdot 10^{-7}$
$\tau^- \rightarrow \mu^- K^+ K^-$	$2.5 \cdot 10^{-7}$	$\tau^- \rightarrow \mu^- K^+ \pi^-$	$3.2 \cdot 10^{-7}$	$\tau^- \rightarrow \mu^- \pi^+ K^-$	$2.6 \cdot 10^{-7}$
$\tau^- \rightarrow e^- \pi^+ \pi^-$	$1.2 \cdot 10^{-7}$	$\tau^- \rightarrow \mu^- \pi^+ \pi^-$	$2.9 \cdot 10^{-7}$	$\tau^- \rightarrow \Lambda \pi^-$	$7.2 \cdot 10^{-8}$
$\tau^- \rightarrow e^+ K^- K^-$	$1.5 \cdot 10^{-7}$	$\tau^- \rightarrow e^+ K^- \pi^-$	$1.8 \cdot 10^{-7}$	$\tau^- \rightarrow e^+ \pi^- \pi^-$	$2.0 \cdot 10^{-7}$
$\tau^- \rightarrow \mu^- K^{*0}$	$5.9 \cdot 10^{-8}$	$\tau^- \rightarrow e^- \phi$	$7.3 \cdot 10^{-8}$	$\tau^- \rightarrow \mu^- \omega$	$8.9 \cdot 10^{-8}$
$\tau^- \rightarrow \mu^+ K^- K^-$	$4.4 \cdot 10^{-7}$	$\tau^- \rightarrow \mu^+ K^- \pi^-$	$2.2 \cdot 10^{-7}$	$\tau^- \rightarrow \mu^+ \pi^- \pi^-$	$0.7 \cdot 10^{-7}$



# Impact of $\theta_{13}$ on LFV processes

(All plotted points lead to 'viable BAU' and respect EDM bounds)

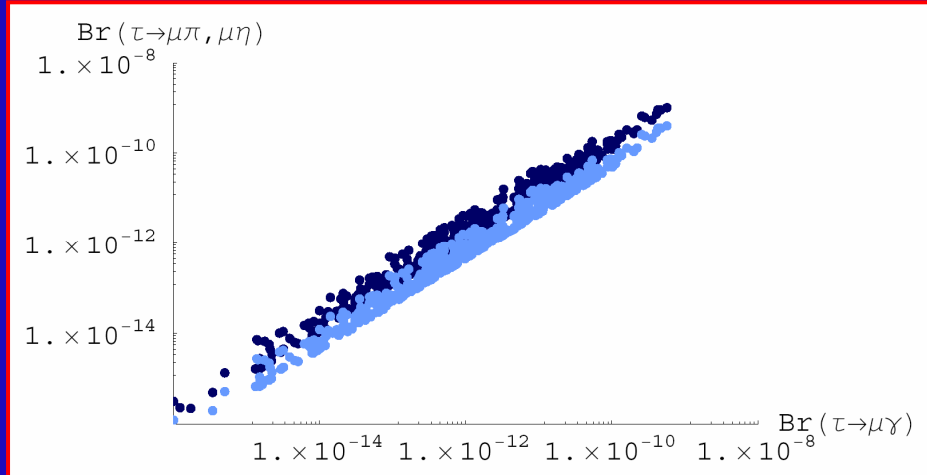
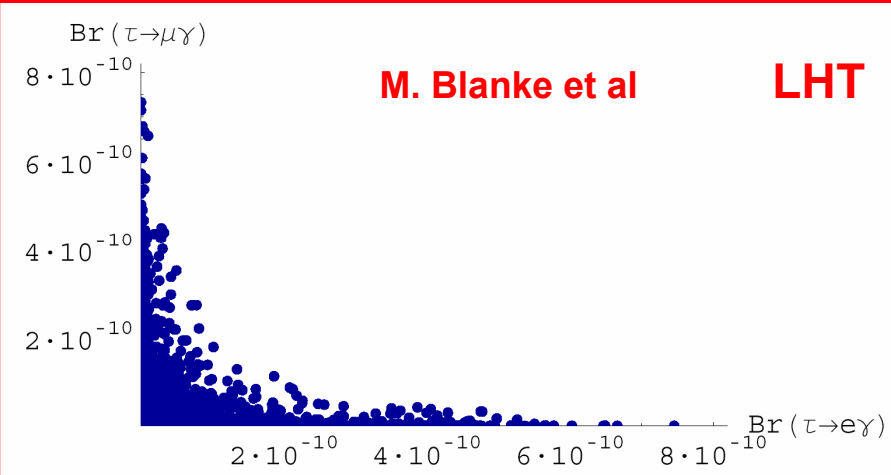
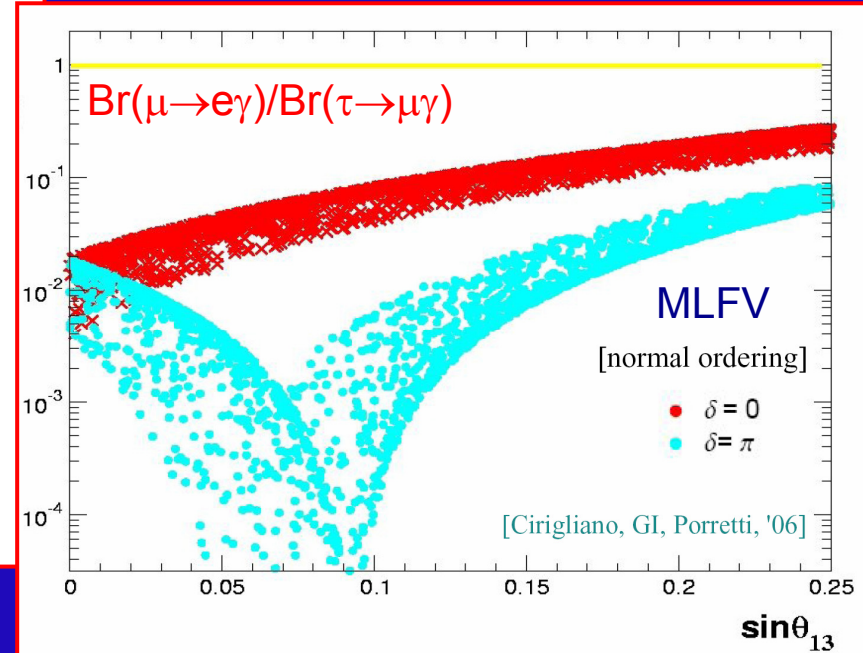
$$(-\pi/4 \lesssim \arg\theta_1 \lesssim \pi/4, 0 \lesssim \arg\theta_2 \lesssim \pi/4)$$



**MEG:**  $\text{Br}(\mu \rightarrow e\gamma) \sim 10^{-13}$

**Prism:**  $\text{Pr}(\mu \rightarrow e) \sim 10^{-18}$

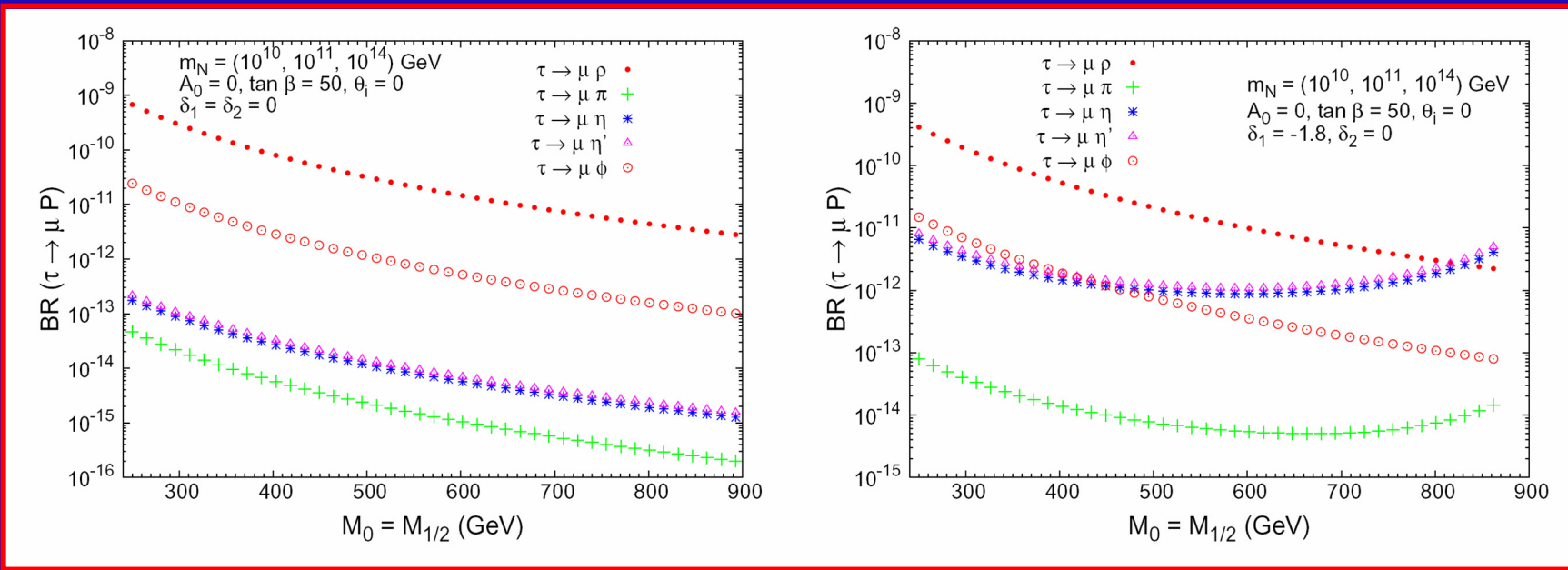
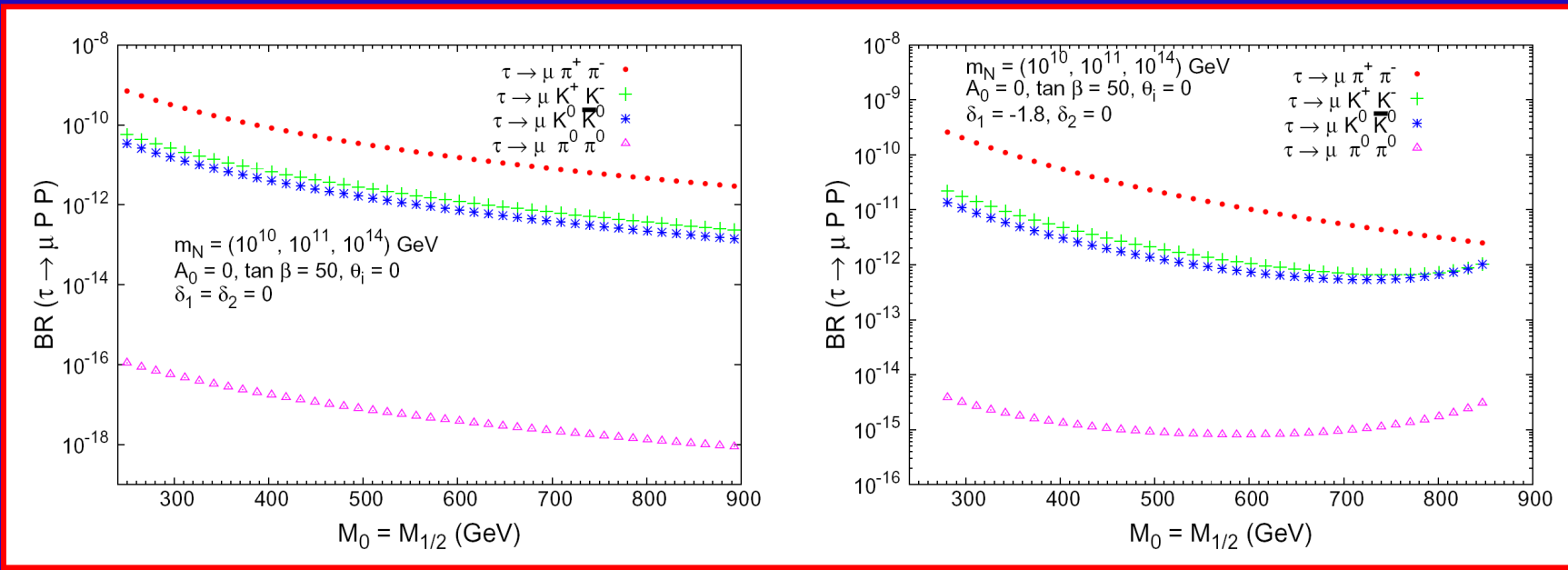
**SuperB:**  $\text{Br}(\tau \rightarrow \mu\gamma) \sim 10^{-9}$



CMSSM

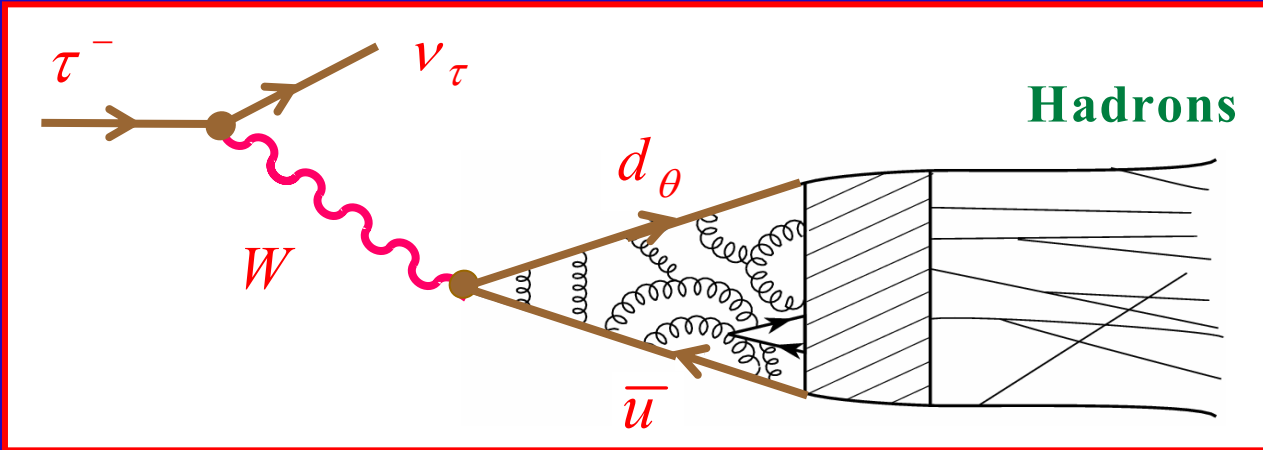
Constrained MSSM–Seesaw Scenarios

NUHM





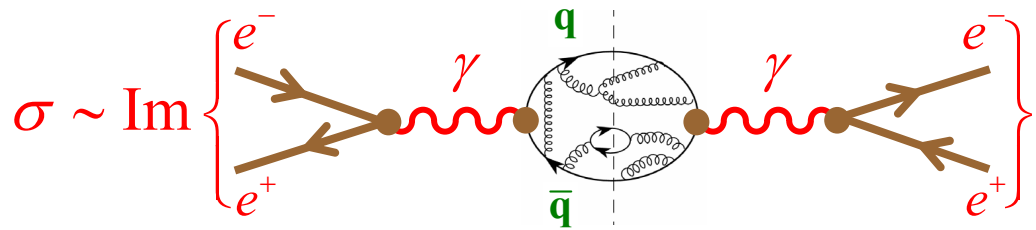
# HADRONIC TAU DECAY



$$d_\theta = V_{ud} d + V_{us} s$$

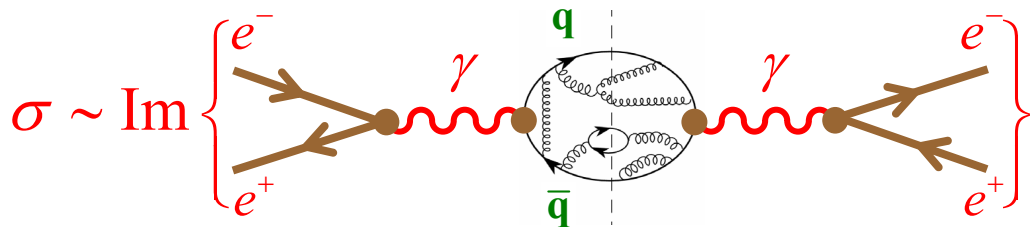
Only lepton massive enough to decay into hadrons

$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{Hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} \approx N_c \quad ; \quad R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.639 \pm 0.011$$



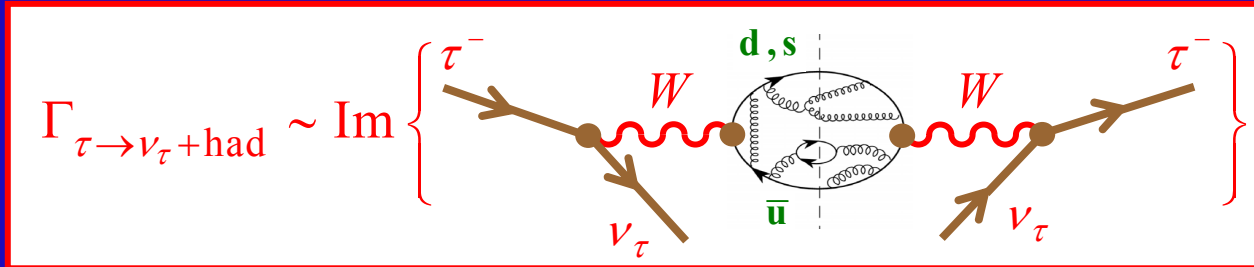
$$\frac{\sigma(e^+e^- \rightarrow \text{had})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 12\pi \text{Im} \Pi_{\text{em}}(s)$$

$$\Pi_{\text{em}}^{\mu\nu}(q) \equiv i \int d^4x e^{iqx} \langle 0 | T [J_{\text{em}}^\mu(x) J_{\text{em}}^\nu(0)] | 0 \rangle = (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{\text{em}}(q^2)$$



$$\frac{\sigma(e^+e^- \rightarrow \text{had})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 12\pi \text{Im} \Pi_{\text{em}}(s)$$

$$\Pi_{\text{em}}^{\mu\nu}(q) \equiv i \int d^4x e^{iqx} \langle 0 | T [J_{\text{em}}^\mu(x) J_{\text{em}}^\nu(0)] | 0 \rangle = (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{\text{em}}(q^2)$$



$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{had})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^{m_\tau^2} dx \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[ \left(1 + 2 \frac{s}{m_\tau^2}\right) \text{Im} \Pi^{(1)}(s) + \text{Im} \Pi^{(0)}(s) \right]$$

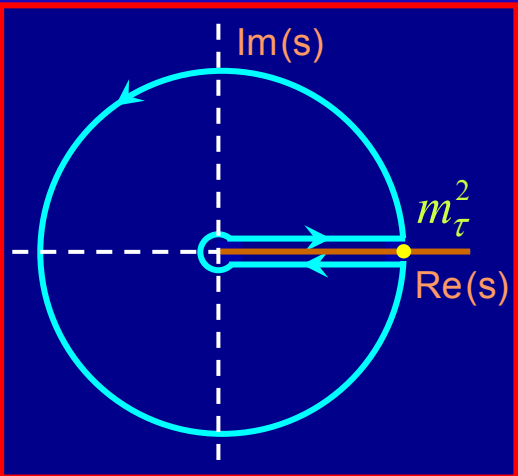
$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 \left[ \Pi_{ud,V}^{(J)}(s) + \Pi_{ud,A}^{(J)}(s) \right] + |V_{us}|^2 \left[ \Pi_{us,V}^{(J)}(s) + \Pi_{us,A}^{(J)}(s) \right]$$

$$\Pi_{ij,J}^{\mu\nu}(q) \equiv i \int d^4x e^{iqx} \langle 0 | T [J_{ij}^\mu(x) J_{ij}^\nu(0)^\dagger] | 0 \rangle = (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{ij,J}^{(1)}(q^2) + q^\mu q^\nu \Pi_{ij,J}^{(0)}(q^2)$$

$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{had})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^1 dx (1-x)^2 \left[ (1+2x) \text{Im} \Pi^{(1)}(x m_\tau^2) + \text{Im} \Pi^{(0)}(x m_\tau^2) \right]$$



$$R_\tau = 6\pi i \oint_{|x|=1} dx (1-x)^2 \left[ (1+2x) \Pi^{(0+1)}(x m_\tau^2) - 2x \Pi^{(0)}(x m_\tau^2) \right]$$



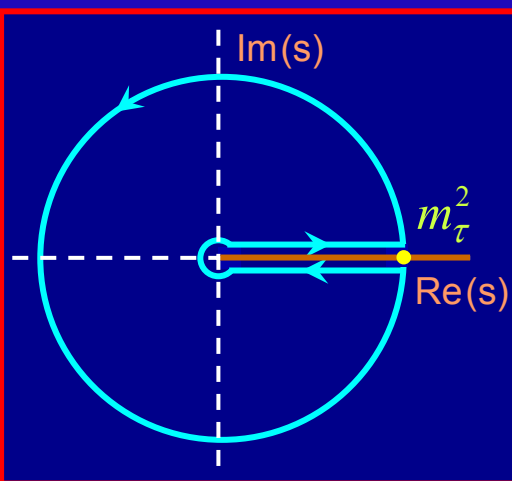
$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{had})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^1 dx (1-x)^2 \left[ (1+2x) \text{Im} \Pi^{(1)}(x m_\tau^2) + \text{Im} \Pi^{(0)}(x m_\tau^2) \right]$$



$$R_\tau = 6\pi i \oint_{|x|=1} dx (1-x)^2 \left[ (1+2x) \Pi^{(0+1)}(x m_\tau^2) - 2x \Pi^{(0)}(x m_\tau^2) \right]$$

$$\Pi^{(J)}(s) = \sum_{D=2n} \frac{C_D^{(J)}(s, \mu) \langle O_D(\mu) \rangle}{(-s)^{D/2}}$$

OPE



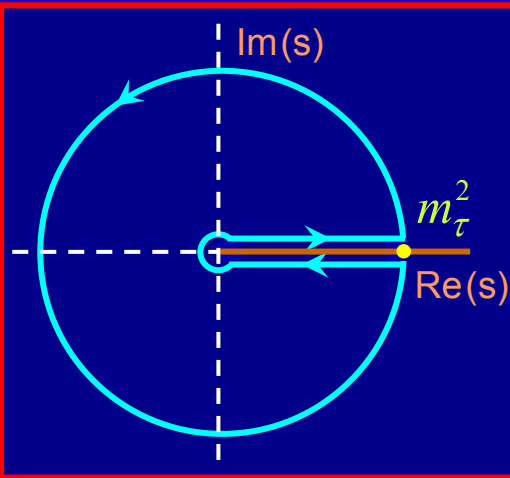
$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{had})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^1 dx (1-x)^2 \left[ (1+2x) \text{Im} \Pi^{(1)}(x m_\tau^2) + \text{Im} \Pi^{(0)}(x m_\tau^2) \right]$$



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$$\Pi^{(J)}(s) = \sum_{D=2n} \frac{C_D^{(J)}(s, \mu) \langle O_D(\mu) \rangle}{(-s)^{D/2}}$$

OPE



$$R_\tau = N_C S_{EW} (1 + \delta_P + \delta_{NP}) = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

$$S_{EW} = 1.0201(3)$$

Marciano-Sirlin, Braaten-Li, Erler

;

$$\delta_{NP} = 0.0001 \pm 0.0017$$

Fitted from data

$$\delta_P = a_\tau + 5.20 a_\tau^2 + 26 a_\tau^3 + \dots \approx 20\%$$

;

$$a_\tau \equiv \alpha_s(m_\tau) / \pi$$



Perturbative: ( $m_q=0$ )

$$K_4 = 49.076$$

Baikov-Chetyrkin-Kühn

$$-s \frac{d}{ds} \Pi^{(0+1)}(s) = \frac{1}{4\pi^2} \sum_{n=0} K_n \left( \frac{\alpha_s(-s)}{\pi} \right)^n ; \quad K_0 = K_1 = 1 \quad , \quad K_2 = 1.63982 \quad , \quad K_3 = 6.37101$$

➔ 
$$\delta_P = \sum_{n=1} K_n A^{(n)}(\alpha_s) = a_\tau + 5.20 a_\tau^2 + 26 a_\tau^3 + 127 a_\tau^4 + \dots$$

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} (1 - 2x + 2x^3 - x^4) \left( \frac{\alpha_s(-s)}{\pi} \right)^n = a_\tau^n + \dots ; \quad a_\tau \equiv \alpha_s(m_\tau) / \pi$$

## Perturbative: ( $m_q=0$ )

$$K_4 = 49.076$$

Baikov-Chetyrkin-Kühn

$$-s \frac{d}{ds} \Pi^{(0+1)}(s) = \frac{1}{4\pi^2} \sum_{n=0} K_n \left( \frac{\alpha_s(-s)}{\pi} \right)^n ; \quad K_0 = K_1 = 1 \quad , \quad K_2 = 1.63982 \quad , \quad K_3 = 6.37101$$

$$\longrightarrow \delta_P = \sum_{n=1} K_n A^{(n)}(\alpha_s) = a_\tau + 5.20 a_\tau^2 + 26 a_\tau^3 + 127 a_\tau^4 + \dots$$

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} (1 - 2x + 2x^3 - x^4) \left( \frac{\alpha_s(-s)}{\pi} \right)^n = a_\tau^n + \dots ; \quad a_\tau \equiv \alpha_s(m_\tau) / \pi$$

## Power Corrections:

$$\Pi_{\text{OPE}}^{(0+1)}(s) \approx \frac{1}{4\pi^2} \sum_{n \geq 2} \frac{C_{2n} \langle O_{2n} \rangle}{(-s)^n}$$

$$C_4 \langle O_4 \rangle \approx \frac{2\pi}{3} \langle 0 | \alpha_s G^{\mu\nu} G_{\mu\nu} | 0 \rangle$$

$$\delta_{\text{NP}} \approx \frac{-1}{2\pi i} \oint_{|x|=1} dx (1 - 3x^2 + 2x^3) \sum_{n \geq 2} \frac{C_{2n} \langle O_{2n} \rangle}{(-xm_\tau^2)^n} = -3 \frac{C_6 \langle O_6 \rangle}{m_\tau^6} - 2 \frac{C_8 \langle O_8 \rangle}{m_\tau^8}$$

Suppressed by  $m_\tau^6$  [additional chiral suppression in  $C_6 \langle O_6 \rangle^{V+A}$ ]

Similar predictions for  $R_{\tau,V}$ ,  $R_{\tau,A}$ ,  $R_{\tau,S}$  and the moments

$$R_{\tau}^{kl}(s_0) \equiv \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{m_{\tau}^2}\right)^l \frac{dR_{\tau}}{ds}$$

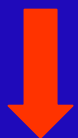
Different sensitivity to power corrections through  $k, l$

The non-perturbative contribution to  $R_{\tau}$  can be obtained from the invariant-mass distribution of the final hadrons:

$$\delta_{\text{NP}} = 0.0001 \pm 0.0017$$

Davier et al. (ALEPH data)

**ALEPH**



$$\alpha_s(m_\tau^2) = 0.344 \pm 0.009$$

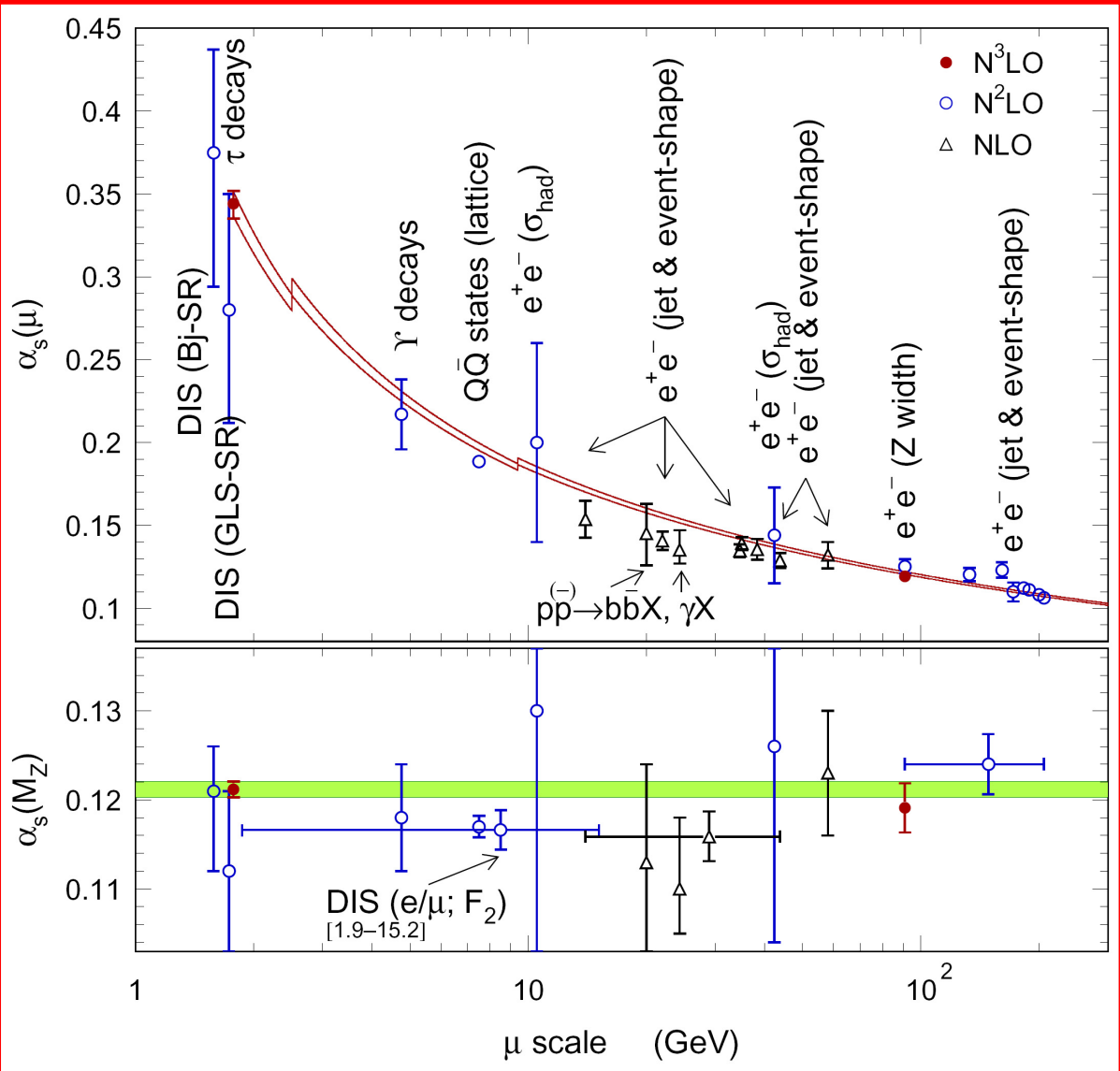
$$\alpha_s(M_Z^2) = 0.1212 \pm 0.0011$$

$$\alpha_s(M_Z^2)_{Z\text{width}} = 0.1191 \pm 0.0027$$

**The most precise test of Asymptotic Freedom**

$$\alpha_s^\tau(M_Z^2) - \alpha_s^Z(M_Z^2) = 0.0021 \pm 0.0011_\tau \pm 0.0027_Z$$

$$R_{\tau,V} = 1.783 \pm 0.011 \quad ; \quad R_{\tau,A} = 1.695 \pm 0.011 \quad ; \quad R_{\tau,V+A} = 3.478 \pm 0.010$$



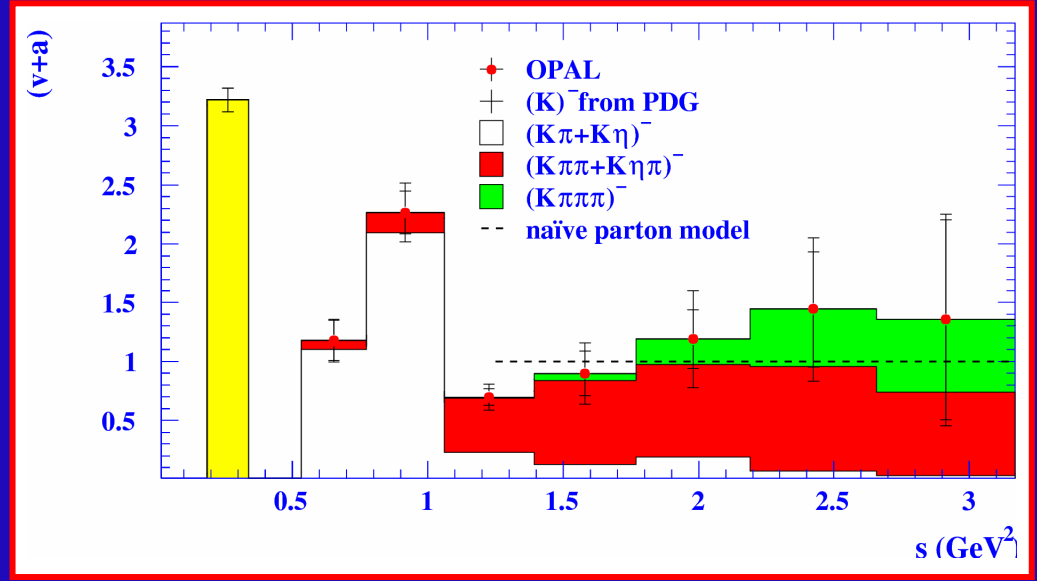
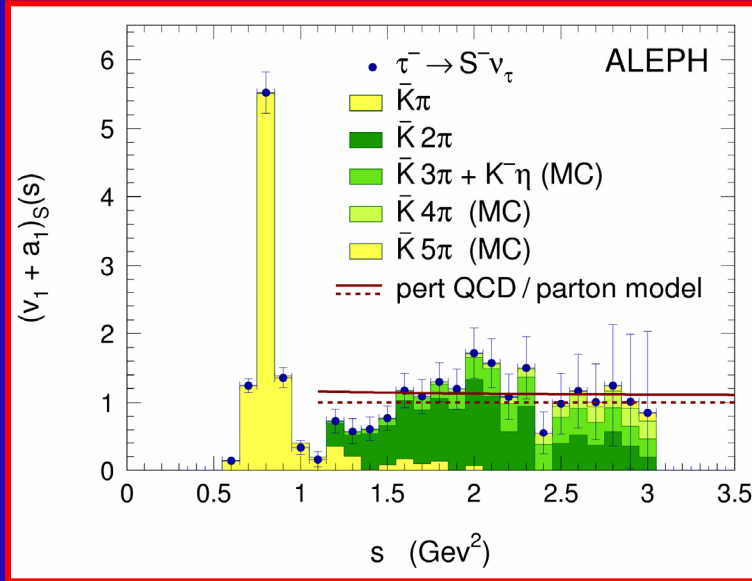
# SU(3) Breaking

$$R_{\tau}^{kl} = N_C S_{\text{EW}} \left\{ \left( |V_{ud}|^2 + |V_{us}|^2 \right) \left[ 1 + \delta^{kl(0)} \right] + \sum_{D \geq 2} \left[ |V_{ud}|^2 \delta_{ud}^{kl(D)} + |V_{us}|^2 \delta_{us}^{kl(D)} \right] \right\}$$



$$\delta R_{\tau}^{kl} \equiv \frac{R_{\tau, V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau, S}^{kl}}{|V_{us}|^2} \approx N_C S_{\text{EW}} \sum_{D \geq 2} \left[ \delta_{ud}^{kl(D)} - \delta_{us}^{kl(D)} \right]$$

# Strange Spectral Function: SU(3) Breaking



(k,l)	ALEPH	OPAL
(0,0)	$0.39 \pm 0.14$	$0.26 \pm 0.12$
(1,0)	$0.38 \pm 0.08$	$0.28 \pm 0.09$
(2,0)	$0.37 \pm 0.05$	$0.30 \pm 0.07$
(3,0)	$0.40 \pm 0.04$	$0.33 \pm 0.05$
(4,0)	$0.40 \pm 0.04$	$0.34 \pm 0.04$

$$\delta R_{\tau}^{kl} \equiv \frac{R_{\tau, V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau, S}^{kl}}{|V_{us}|^2} \approx 24 \frac{m_s^2(m_{\tau}^2)}{m_{\tau}^2} \Delta_{kl}(\alpha_s)$$

➔  **$m_s(m_{\tau})$  determination**

**$V_{us}$  and QCD uncertainties**



$$\delta R_{\tau}^{kl} \equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} \approx 24 \frac{m_s^2(m_{\tau}^2)}{m_{\tau}^2} \Delta_{kl}(\alpha_s)$$

Known to  $\mathcal{O}(\alpha_s^3)$

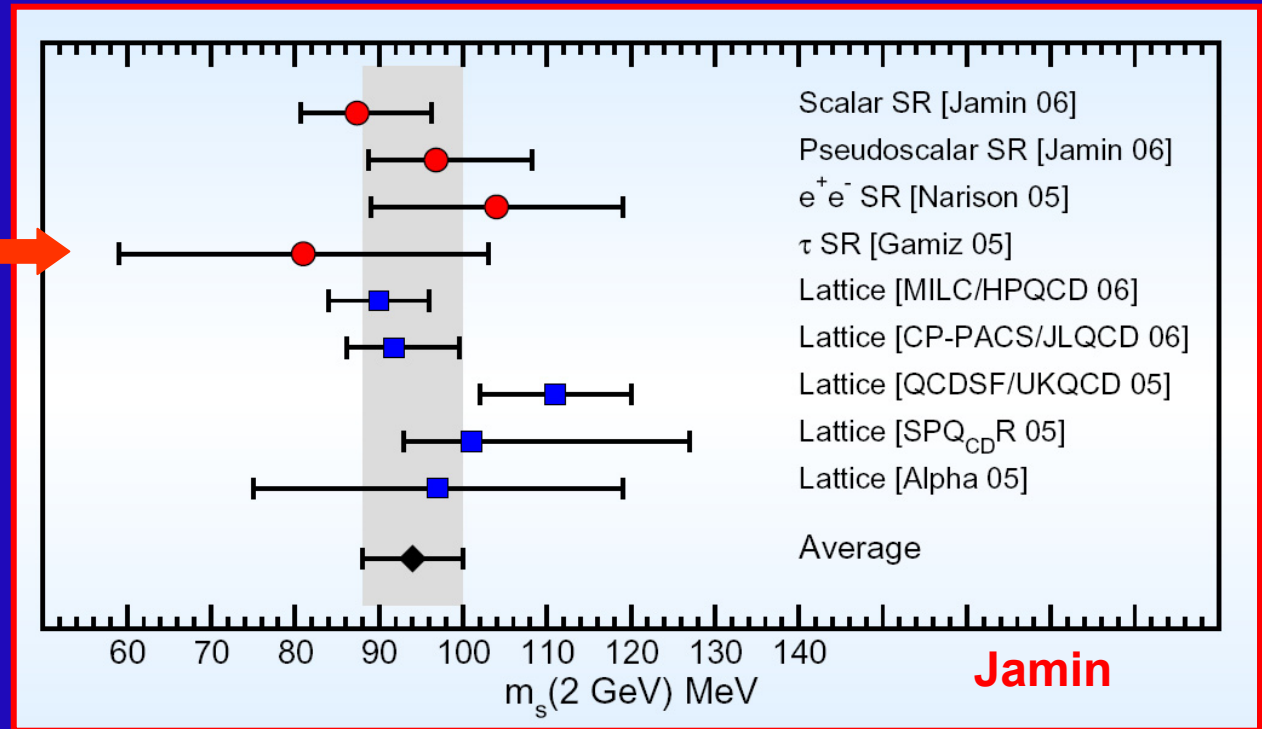
- $\Delta_{kl}(\alpha_s)$  gets **longitudinal (J=0)** and **transverse (J=0+1)** contributions
- Divergent QCD series for J=0
- **Longitudinal contribution determined through data:**
  - Kaon pole ( $K \rightarrow \mu\nu$ ) (dominant J=0 contribution)
  - Pion pole ( $\pi \rightarrow \mu\nu$ )
  - $(K\pi)_{J=0}$  (S-wave  $K\pi$  scattering)
  - ...

- Smaller uncertainties

	$R_{us,A}^{00,L}$	$R_{us,V}^{00,L}$	$R_{ud,A}^{00,L}$
Theory:	$-0.144 \pm 0.024$	$-0.028 \pm 0.021$	$-(7.79 \pm 0.14) \cdot 10^{-3}$
Phenom:	$-0.135 \pm 0.003$	$-0.028 \pm 0.004$	$-(7.77 \pm 0.08) \cdot 10^{-3}$

$$\delta R_{\tau,th}^{00} \equiv \underbrace{0.1544 (37)}_{J=0} + \underbrace{0.062 (15)}_{m_s(m_{\tau}) = 0.100 (10)} = 0.216 (16)$$

OPAL  $\tau$  data



Large uncertainty from  $V_{us}$



Strong sensitivity to  $V_{us}$

$$|V_{us}|^2 = \frac{R_{\tau,S}^{00}}{\frac{R_{\tau,V+A}^{00}}{|V_{ud}|^2} - \delta R_{\tau,\text{th}}^{00}}$$

Gámiz-Jamin-Pich-Prades-Schwab

**$\tau$  data:**  $R_{\tau,S}^{00} = 0.1686$  (47)

$$R_{\tau,V+A}^{00} = 3.471$$
 (11)

**PDG 06:**  $|V_{ud}| = 0.97377$  (27)

$$|V_{us}|^2 = \frac{R_{\tau,S}^{00}}{\frac{R_{\tau,V+A}^{00}}{|V_{ud}|^2} - \delta R_{\tau,\text{th}}^{00}}$$

Gámiz-Jamin-Pich-Prades-Schwab

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PDG 06:  $|V_{ud}| = 0.97377$  (27)

$$\delta R_{\tau,\text{th}}^{00} = 0 \quad \longrightarrow \quad |V_{us}| = 0.215$$
 (3)

$$|V_{us}|^2 = \frac{R_{\tau,S}^{00}}{\frac{R_{\tau,V+A}^{00}}{|V_{ud}|^2} - \delta R_{\tau,\text{th}}^{00}}$$

Gámiz-Jamin-Pich-Prades-Schwab

**$\tau$  data:**  $R_{\tau,S}^{00} = 0.1686$  (47)

$R_{\tau,V+A}^{00} = 3.471$  (11)

**PDG 06:**  $|V_{ud}| = 0.97377$  (27)

$\delta R_{\tau,\text{th}}^{00} = 0 \quad \longrightarrow \quad |V_{us}| = 0.215$  (3)

Taking as input (from non  $\tau$  sources)  $m_s(m_\tau) = 100 \pm 10$  MeV :

$\delta R_{\tau,\text{th}}^{00} = 0.216$  (16)



$|V_{us}| = 0.2212 \pm 0.0031_{\text{exp}} \pm 0.0005_{\text{th}}$

$$|V_{us}|^2 = \frac{R_{\tau,S}^{00}}{\frac{R_{\tau,V+A}^{00}}{|V_{ud}|^2} - \delta R_{\tau,\text{th}}^{00}}$$

Gámiz-Jamin-Pich-Prades-Schwab

$\tau$  data:  $R_{\tau,S}^{00} = 0.1686$  (47)

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Taking as input (from non  $\tau$  sources)  $m_s(m_\tau) = 100 \pm 10$  MeV :

$\delta R_{\tau,\text{th}}^{00} = 0.216$  (16)  $\longrightarrow$

$|V_{us}| = 0.2212 \pm 0.0031_{\text{exp}} \pm 0.0005_{\text{th}}$

$\mathbf{K}_{13}$ :  $|V_{us}| = 0.2233 \pm 0.0024$

$[f_+(0) = 0.97 \pm 0.01]$

The  $\tau$  could give the most precise  $V_{us}$  determination



# First Measurements from Babar and Belle

Mode	$\mathcal{B}(10^{-3})$ [15]	Updated $\mathcal{B}(10^{-3})$ with results from [20–22]
$K^-$	$6.81 \pm 0.23$	[Replace with $7.15 \pm 0.03$ ]
$K^- \pi^0$	$4.54 \pm 0.30$	Average with $4.16 \pm 0.18 \Rightarrow 4.26 \pm 0.16$ ( $S = 1.0$ )
$\bar{K}^0 \pi^-$	$8.78 \pm 0.38$	Average with $8.08 \pm 0.26 \Rightarrow 8.31 \pm 0.28$ ( $S = 1.3$ )
$K^- \pi^0 \pi^0$	$0.58 \pm 0.24$	
$\bar{K}^0 \pi^- \pi^0$	$3.60 \pm 0.40$	
$K^- \pi^+ \pi^-$	$3.30 \pm 0.28$	Average with $2.73 \pm 0.09 \Rightarrow 2.80 \pm 0.16$ ( $S = 1.9$ )
$K^- \eta$	$0.27 \pm 0.06$	
$(\bar{K}3\pi)^-$ (estimated)	$0.74 \pm 0.30$	
$K_1(1270)^- \rightarrow K^- \omega$	$0.67 \pm 0.21$	
$(\bar{K}4\pi)^-$ (estimated) and $K^{*-} \eta$	$0.40 \pm 0.12$	
Sum	$29.69 \pm 0.86$	Updated Estimate: $28.44 \pm 0.74$ [ $28.78 \pm 0.71$ ]

S. Banerjee  
KAON'07

Smaller  $\tau \rightarrow K$  branching ratios  $\rightarrow$  smaller  $R_{\tau,S}$   $\rightarrow$  smaller  $V_{us}$

$$R_{\tau,S}^{00} \Big|_{\text{OLD}} = 0.1686 \quad (47) \quad \rightarrow \quad R_{\tau,S}^{00} \Big|_{\text{NEW}} = 0.1617 \quad (40)$$

$$|V_{us}|_{\text{OLD}} = 0.2212 \pm 0.0031_{\text{exp}} \pm 0.0005_{\text{th}}$$

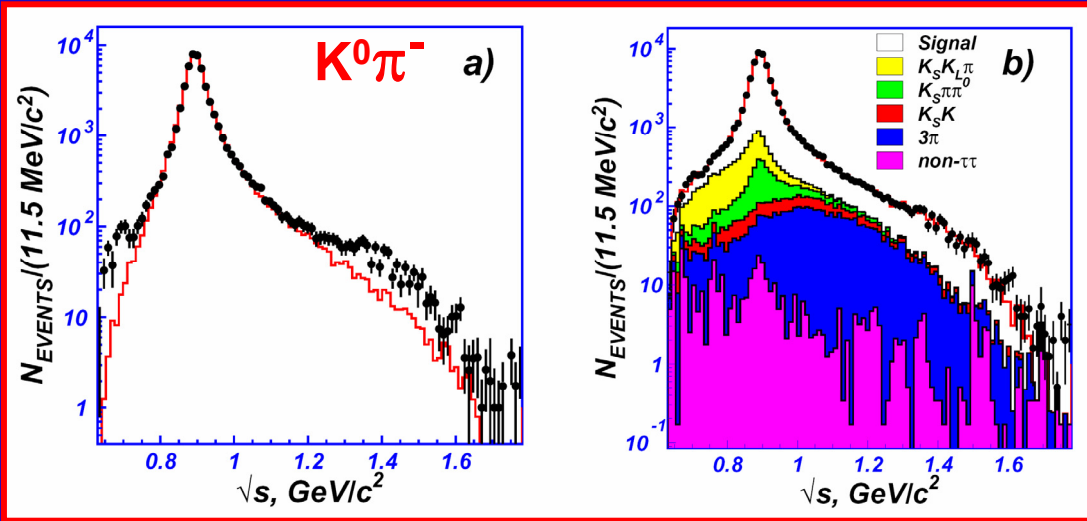
$$|V_{us}|_{\text{NEW}} = 0.2165 \pm 0.0026_{\text{exp}} \pm 0.0005_{\text{th}}$$

**Much more data coming. Precise measurement expected soon**

# Huge number of $\tau^+\tau^-$ events at the B Factories

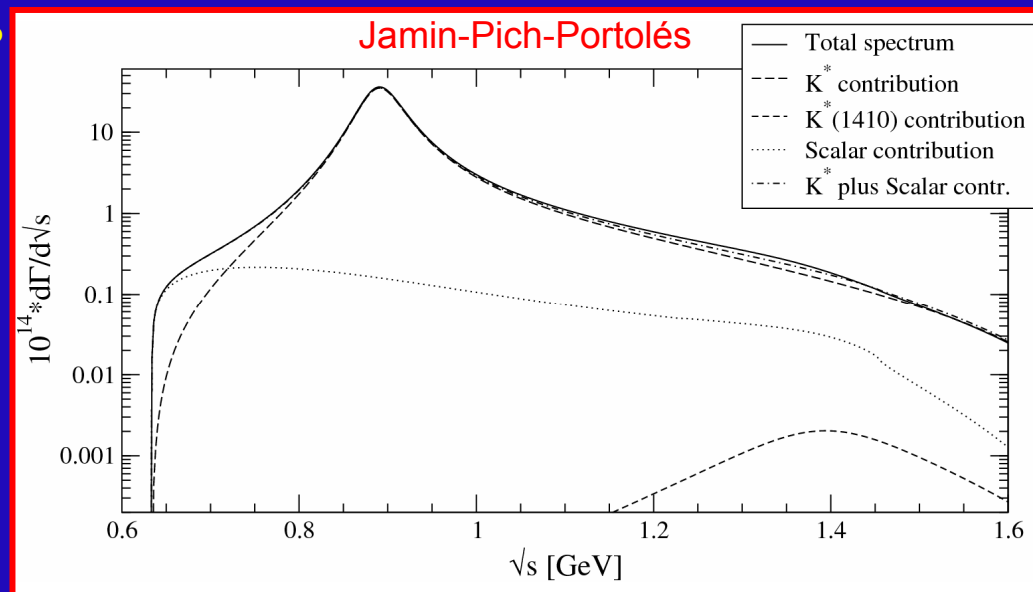
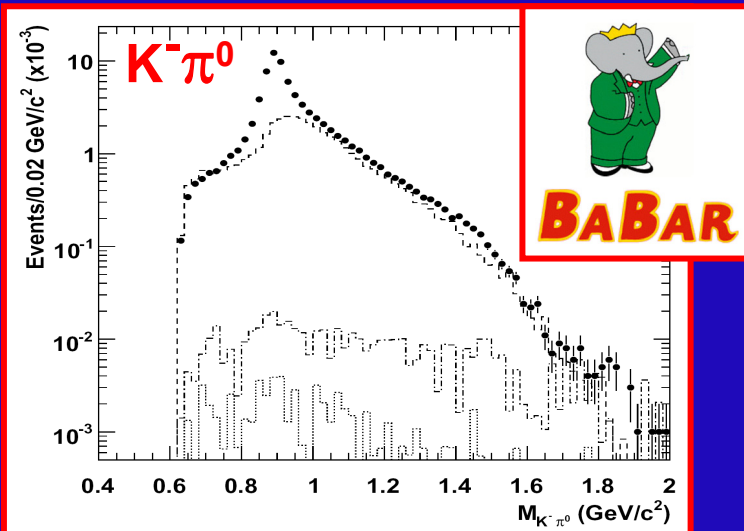


$$\text{Br}(\tau^- \rightarrow K_S \pi^- \nu_\tau) = (0.404 \pm 0.002_{\text{stat}} \pm 0.013_{\text{syst}}) \%$$



Ongoing  
data  
analysis

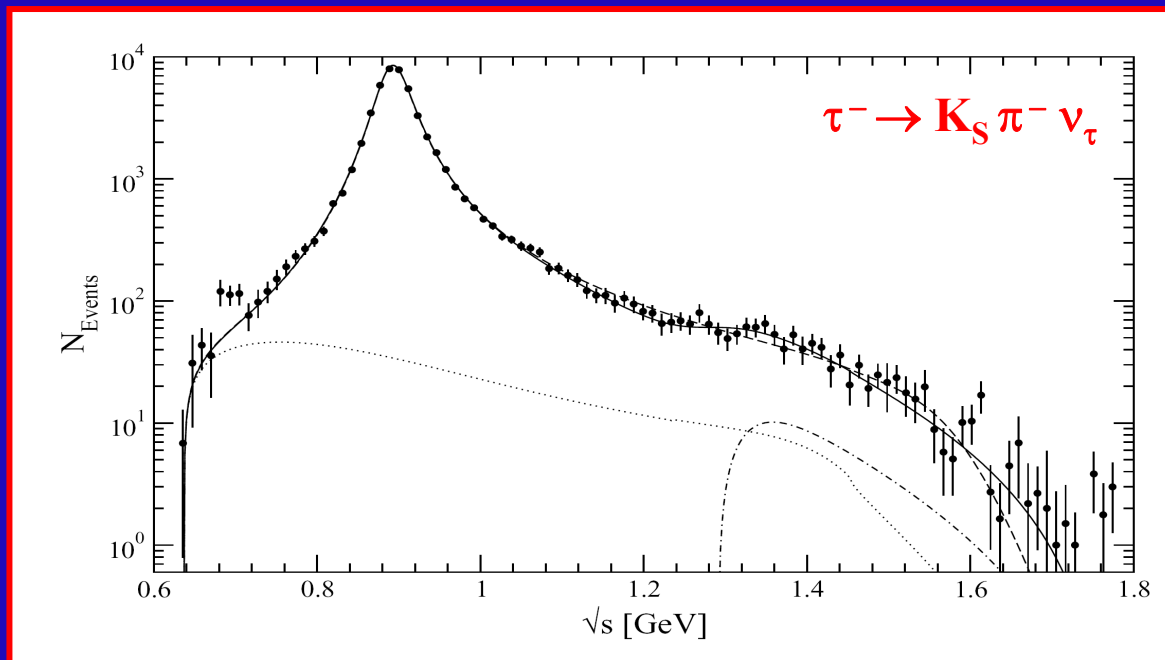
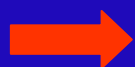
$$\text{Br}(\tau^- \rightarrow K^- \pi^0 \nu_\tau) = (0.416 \pm 0.003_{\text{stat}} \pm 0.018_{\text{syst}}) \%$$



Shape fit:

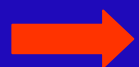
$$M_{K^*} = 895.3 \pm 0.2 \text{ MeV}$$

$$\Gamma_{K^*} = 47.5 \pm 0.4 \text{ MeV}$$

 $R_{\chi T}$  normalization fixed

$$\text{Br}(\tau^- \rightarrow K_S \pi^- \nu_\tau)_{\text{th}} = (0.427 \pm 0.024) \%$$

$$\text{Br}(\tau^- \rightarrow K_S \pi^- \nu_\tau)_{\text{Belle}} = (0.404 \pm 0.002_{\text{stat}} \pm 0.013_{\text{syst}}) \%$$

Prediction for  $K_{13}$  Form Factor Slopes:

$$\lambda'_+ = (25.20 \pm 0.33) 10^{-3} \quad ; \quad \lambda''_+ = (12.85 \pm 0.31) 10^{-4} \quad ; \quad \lambda'''_+ = (9.56 \pm 0.28) 10^{-5}$$

$$\text{EXP: (Flavianet Kaon WG)} \quad \lambda'_+ = (25.2 \pm 0.9) 10^{-3} \quad ; \quad \lambda''_+ = (16 \pm 4) 10^{-4}$$

# SUMMARY

The  $\tau$  is a clean laboratory to test the Standard Model

- Precision physics
- QCD tests
- Open questions:  $(g-2)_\mu$
- Hints of new physics:  $\nu$ 's
- Future facilities:  $\tau$ cF, SuperB
- Tool for NP searches: LHC

**There is an exciting future ahead**

# The $\tau$ could give the most precise $V_{us}$ determination

- From present  $\tau$  data one gets:

$$|V_{us}| = 0.2165 \pm 0.0026_{\text{exp}} \pm 0.0005_{\text{th}}$$

- Accuracy similar already to  $K_{l3}$ :

$$|V_{us}| = 0.2234 \pm 0.0023 \quad [f_+(0) = 0.97 \pm 0.01]$$

**Interesting challenge for the B Factories & BESIII**

# A simultaneous $m_s$ & $V_{us}$ fit could be possible

However:

- Perturbative QCD corrections need to be better understood (CIPT)

$$\Delta_{00}(\alpha_s)^{L+T} = 0.753 + 0.214 + 0.065 - 0.063 + \dots$$

$$\Delta_{10}(\alpha_s)^{L+T} = 0.912 + 0.334 + 0.192 + 0.069 + \dots$$

$$\Delta_{20}(\alpha_s)^{L+T} = 1.055 + 0.451 + 0.330 + 0.232 + \dots$$

$$\Delta_{30}(\alpha_s)^{L+T} = 1.190 + 0.571 + 0.484 + 0.432 + \dots$$

$$\Delta_{40}(\alpha_s)^{L+T} = 1.324 + 0.697 + 0.657 + 0.676 + \dots$$

**Sizeable theoretical uncertainties**

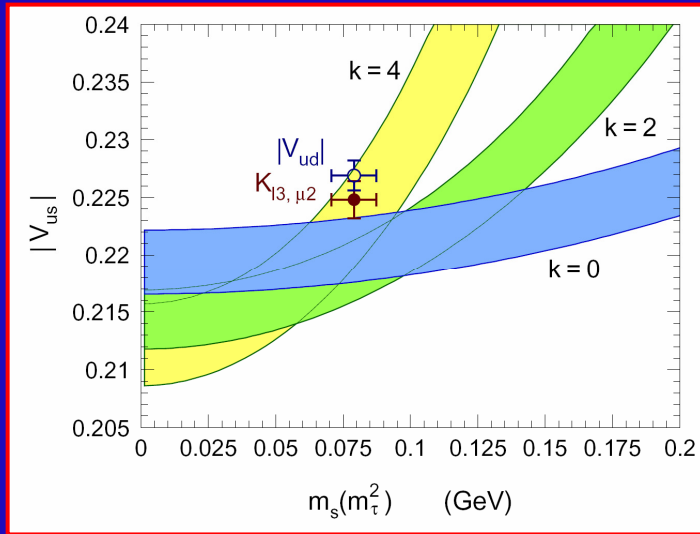
Resummations, pinched weights (Maltman & Wolfe), ...

- Not enough sensitivity with present data

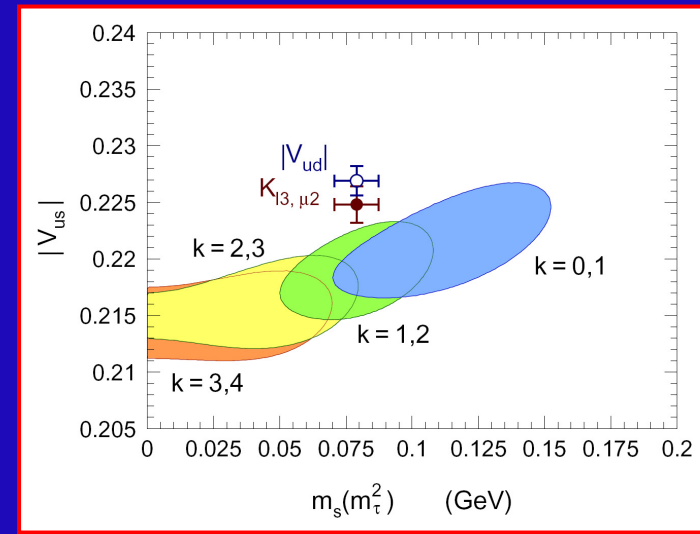
**Large correlations. Low statistics. Missing decay modes ...**



# ALEPH



# Davier-Höcker-Zhang '05



Taking  $V_{us} = 0.2225 (21)$  :

Chen et al '01 , J=0 included

$(k, l)$	$m_s$ (MeV)	$\sigma_{m_s}$ (MeV)						
		exp.	$ V_{us} $	$\alpha_s$	$\langle m_s \bar{s}s \rangle$	trunc.	R-scale	th.
(0,0)	132	26	13	2	4	9	9	14
(1,0)	120	13	9	3	4	10	11	16
(2,0)	117	10	7	3	6	14	14	21
(3,0)	117	9	8	2	8	19	16	27
(4,0)	103	7	5	3	9	20	19	29

$$m_s(m_\tau) = (120^{+21}_{-26}) \text{ MeV}$$

$$m_s(2 \text{ GeV}) = (116^{+20}_{-25}) \text{ MeV}$$

Gámiz et al '03 , J=0 excluded

Moment	$m_s(m_\tau)$ [MeV]
(0,0)	$192 \pm 72$
(1,0)	$164 \pm 31$
(2,0)	$137 \pm 20$
(3,0)	$115 \pm 17$
(4,0)	$100 \pm 17$

$$m_s(m_\tau) = (122 \pm 17) \text{ MeV}$$

$$m_s(2 \text{ GeV}) = (117 \pm 17) \text{ MeV}$$

- Strong  $k$  dependence with ALEPH data ( $m_s$  decreases with increasing  $k$ )

Spectral function underestimated at large invariant masses

➔ Missing events / modes ( $K\pi\pi, K\pi\pi\pi, \dots$ )

- Much better behaviour with OPAL data:

Gámiz et al '05,  $J=0$  excluded

Moment	$m_s(m_\tau)$ [MeV]
(2,0)	$89 \pm 39$
(3,0)	$84 \pm 27$
(4,0)	$78 \pm 22$

(0,0) ➔  $V_{us} = 0.2208 (34)$  ➔

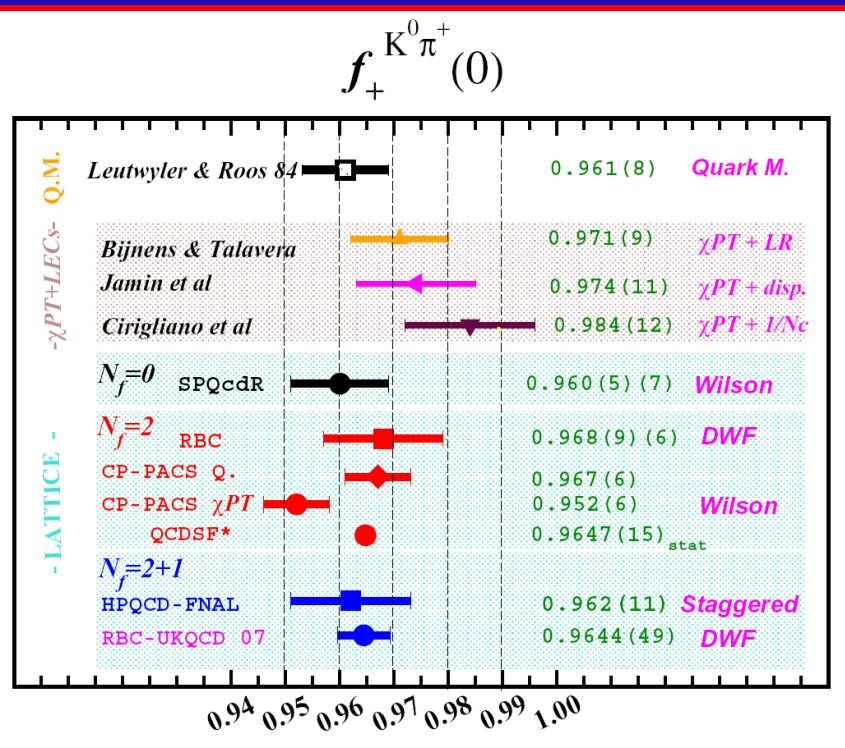
$m_s(m_\tau) = (84 \pm 23) \text{ MeV}$  ,  $m_s(2 \text{ GeV}) = (81 \pm 22) \text{ MeV}$

- $\tau \rightarrow K\nu$  from  $K \rightarrow \mu\nu$  + OPAL:

$V_{us} = 0.2220 (33)$

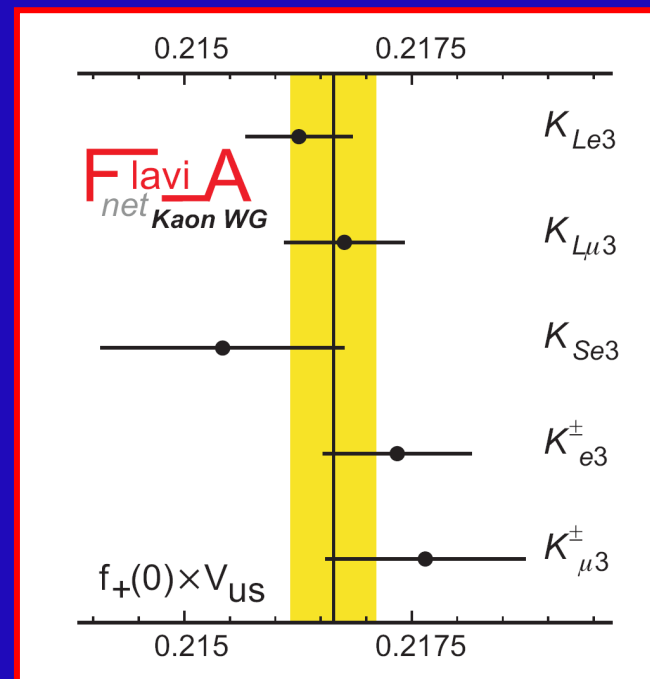
# $K_{l3}$ Decays

Large  $O(p^6)$  ChPT correction (Bijnens-Talavera)



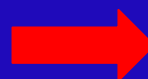
$O(p^4)$

$O(p^6)$



$$|f_+(0) V_{us}| = 0.2166 \pm 0.0005$$

$$f_+(0) = 0.97 \pm 0.01$$



$$|V_{us}| = 0.2233 \pm 0.0024$$