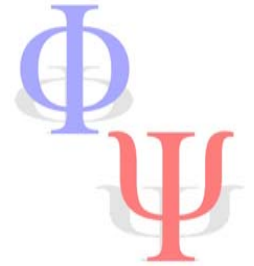


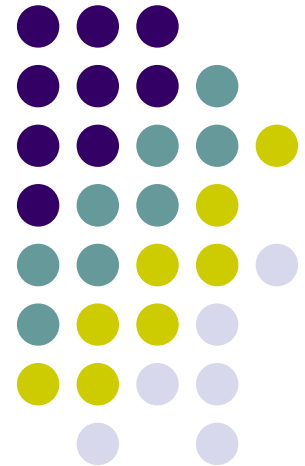
Results on Tau Spectral Functions from Belle

$$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$$



International Workshop on e^+e^- collisions
from Phi to Psi, Frascati, 7-10 April 2008

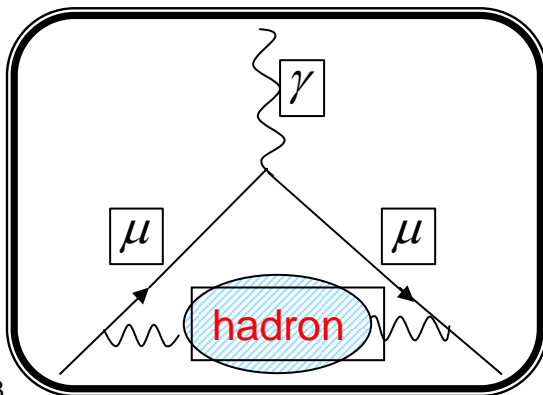
Hisaki Hayashii
Nara Women's University



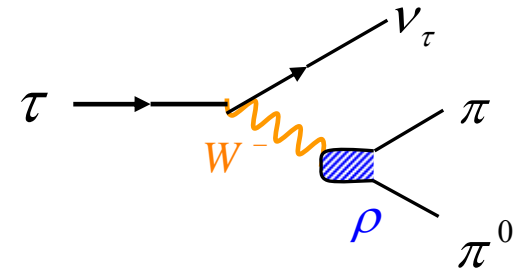
Introduction



- Hadronic decays of tau lepton provide clean environment for studying the dynamics of hadronic states. $\alpha_s(s)$, a_μ , V_{us} , m_s
- $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$
 - has the largest Br
 - CVC relation with $e^+e^- \rightarrow \pi^+ \pi^-$
 - Plays an important role for the h. v. p. term in the muon anomalous magnetic moment. $a_\mu = (g_\mu - 2)/2$



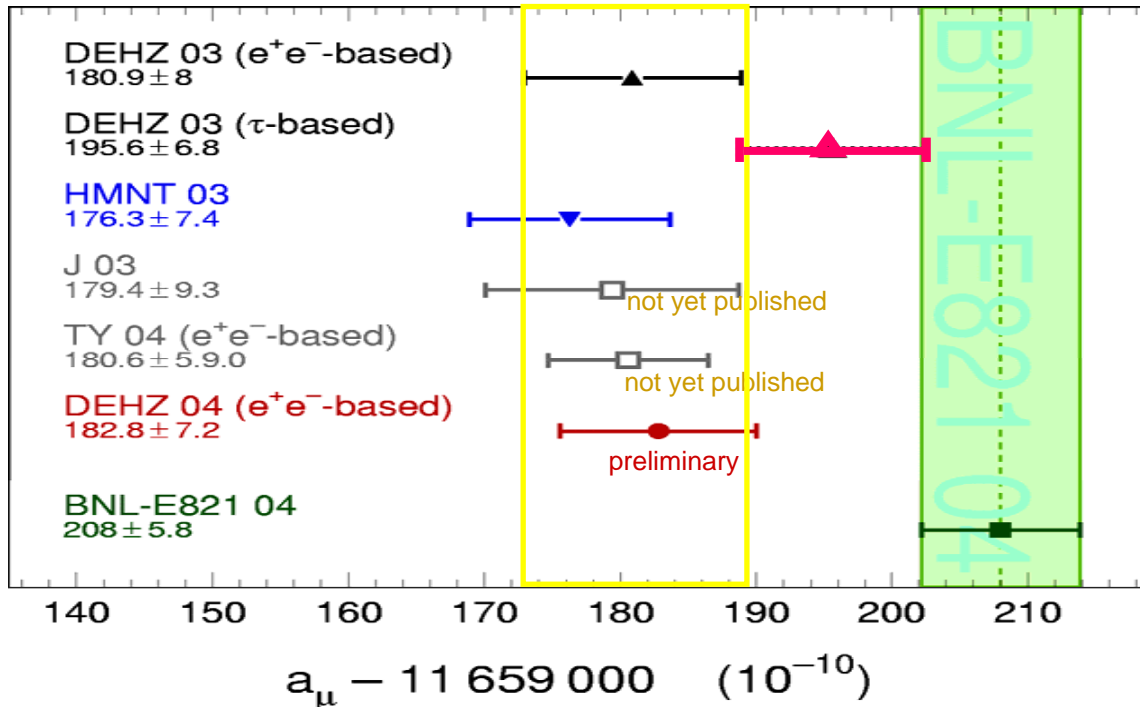
← +CVC



Introduction (cont.)



- Recent data indicate that there is a systematic difference between the 2π system in e^+e^- reaction and τ -decays, even after applying known iso-spin violation correction. Main difference btw a_μ^τ and a_μ^{ee} is from 2π mode.



ICHEP-2006(M.Davier et al.):

$a_\mu(\text{exp}) - a_\mu(\text{th})$ is

3.3σ

A hint to New Physics beyond the SM!

What should be measured



- In this talk, We present update results from Belle experiment based on $5.6 \times 10^6 \tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decays (72.2/fb).
 - Final for 72.2/fb data.

$$a_\mu^{\pi\pi} = \frac{\alpha(0)^2}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} v^{\pi\pi}(s) \quad \longleftarrow \text{spectral function}$$

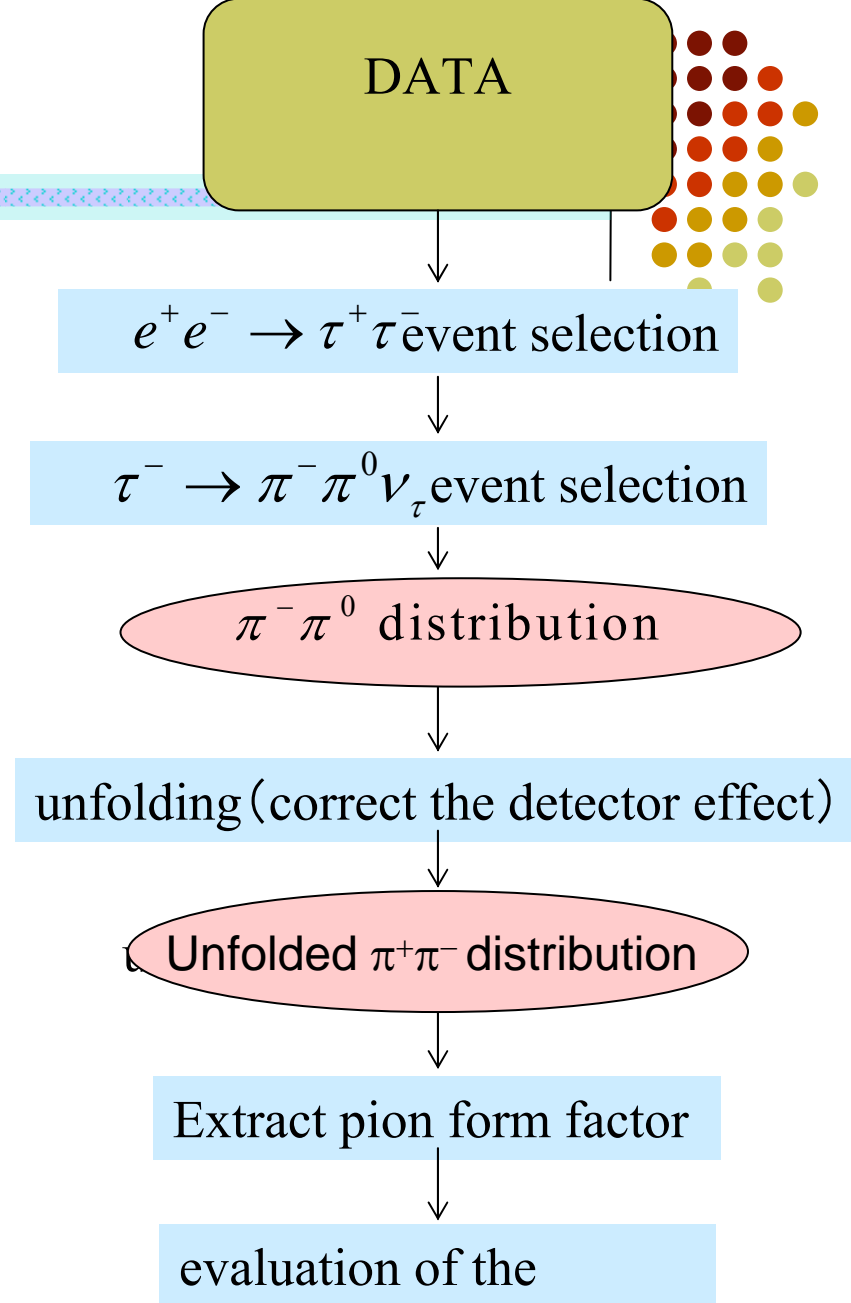
$$v^{\pi\pi}(s) = \frac{m_\tau^2}{6\pi |V_{ud}|^2 S_{EW}} \cdot \frac{B_{\pi\pi^0}}{B_e} \cdot \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \right]^{-1} \cdot \frac{1}{N_{\pi\pi^0}} \frac{dN_{\pi\pi^0}}{ds}$$

■ Branching Fraction

■ Mass Spectrum

Analysis Procedure

- $e^+e^- \rightarrow \tau^+\tau^-$ selection
- $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$ selection
- Background subtraction
- Unfolding
- Br measurement
- Extract pion form factor
 $|F_\pi(s)|^2$
- Fit with Breit-Wigner Form
- Evaluation of a_μ



Event Selection



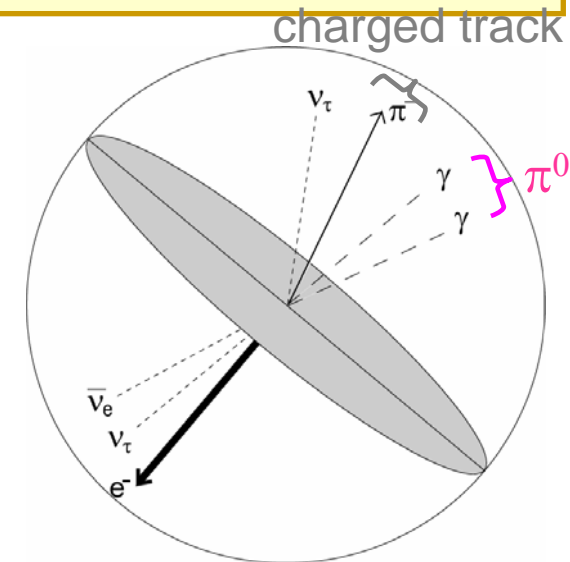
$e^+e^- \rightarrow \tau^+\tau^-$ Selection

- Low multiplicity:
Number of charged tracks: 2 or 4, net charge=0
- Beam background rejection: Event Vertex Position
- Physics background rejection:
 - Use Missing Mass and Missing Angle information. (Bhabha, 2photon)
 - Low track and gamma multiplicity. (qq continuum)

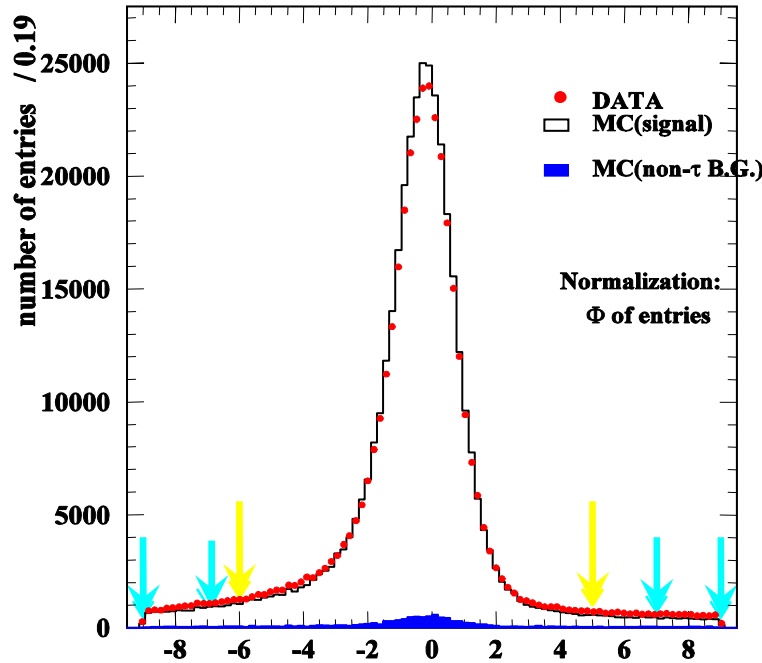
$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ Selection

- one charged track in the event hemisphere.
- one π^0 in the event hemisphere.
- No additional γ with $E_\gamma \geq 200 \text{ MeV}$
- Tag-side condition:
2,4 prong + any γ /1 prong + no γ

8 April 2008



π^0 Signal



Signal region

$$-6 < S_{\gamma\gamma} < 5$$

Sideband region

right: $-9 < S_{\gamma\gamma} < -7$
left: $7 < S_{\gamma\gamma} < 9$

- Sideband region is used to estimate the non- π^0 background

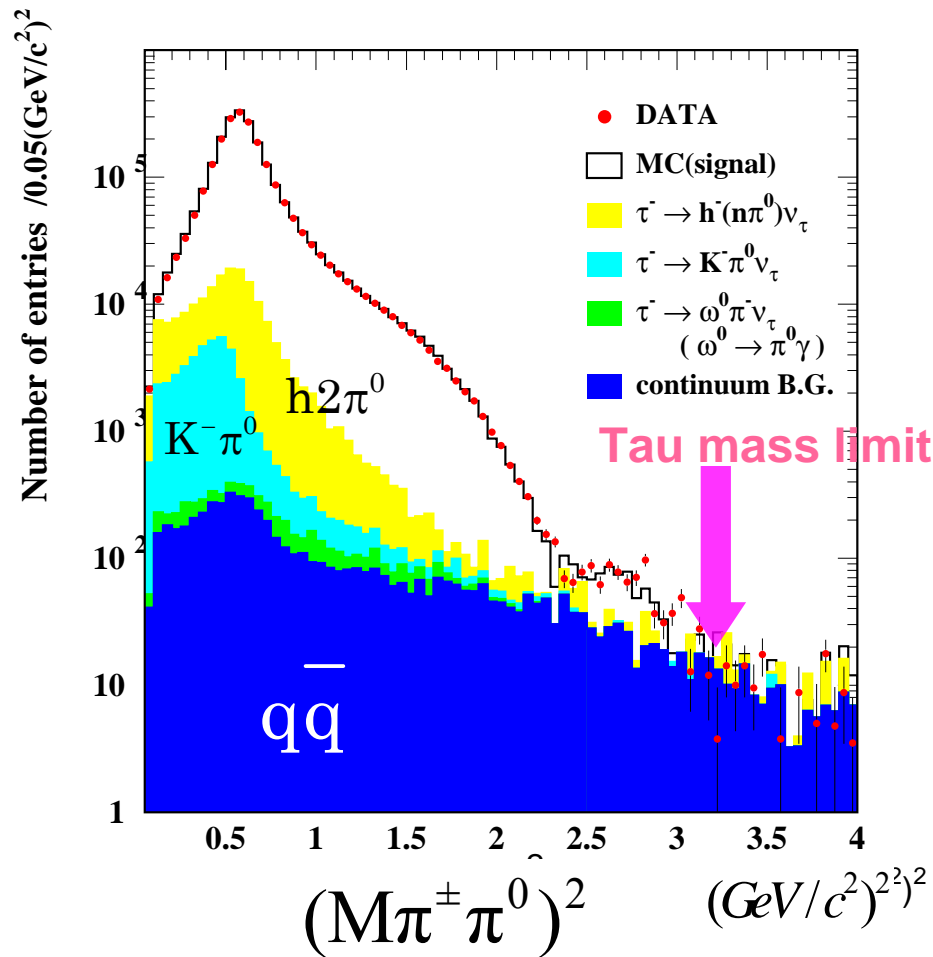
Generally background free, but non-negligible BK in the low $M_{\pi\pi^0}$ region.

$$S_{\gamma\gamma} \equiv \frac{(m_{\gamma\gamma} - m_{\pi^0})}{\sigma_{\gamma\gamma}}$$

$$\sigma_{\gamma\gamma} : 5 - 8 \text{ MeV}$$

$$5.6 \text{ M } \tau^- \rightarrow \pi^- \pi^0 \nu_\tau \text{ events}$$

$m^2_{\pi\pi}$ distribution



The signal level is different more than 4th order of magnitude between $\rho(770)$ and $\rho''(1700)$.

Background

- non- τ B.G.

$$\frac{qq}{\tau^- \rightarrow h^-(n\pi^0)\nu_\tau} = 2.22 \pm 0.05\%$$

- feed down B.G.

$$\frac{h \geq 2\pi^0\nu_\tau}{\tau^- \rightarrow h^-(n\pi^0)\nu_\tau} = 6.0\%$$

$$\frac{K^-\pi^0\nu_\tau}{\tau^- \rightarrow h^-(n\pi^0)\nu_\tau} = 1.6\%$$

$$\frac{\tau^- \rightarrow \pi^-\omega\nu_\tau (\omega \rightarrow \pi^0\gamma)}{\tau^- \rightarrow h^-(n\pi^0)\nu_\tau} = 0.5\%$$

BG is important at threshold and ρ'' region.

Some important features



New MC set

- New τ MC (include ρ'' to TAUOLA MC)

More reliable estimate of continuum BG

- Estimate continuum using BG enriched sample.
- Require stronger cut for tag side
 - 1 charged track + no γ

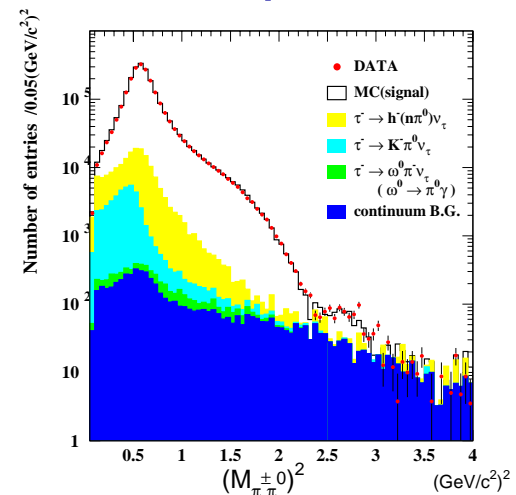
More reliable estimate of feed-down BG.

some modes have small Br but dominate in the special region. i.e. threshold region

- $\tau^- \rightarrow \pi^- \omega \nu_\tau (\omega \rightarrow \pi^0 \gamma)$
- $\tau^- \rightarrow \pi^- \pi^0 K_L \nu_\tau$ etc.

Extensive systematic study

- Non- π^0 background, efficiency

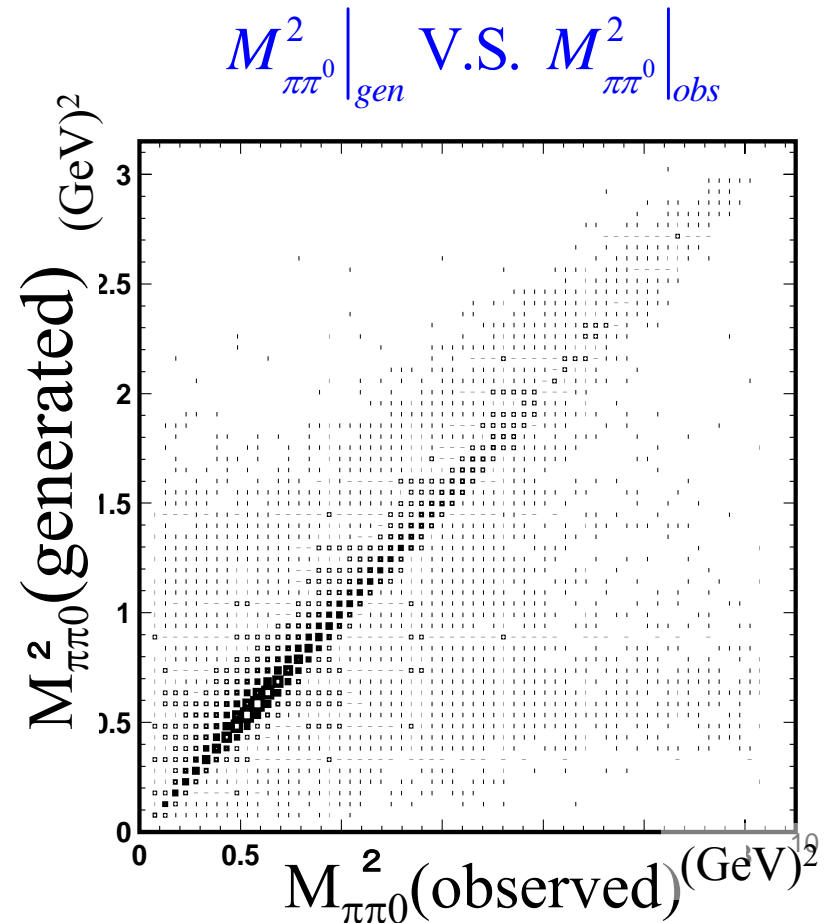
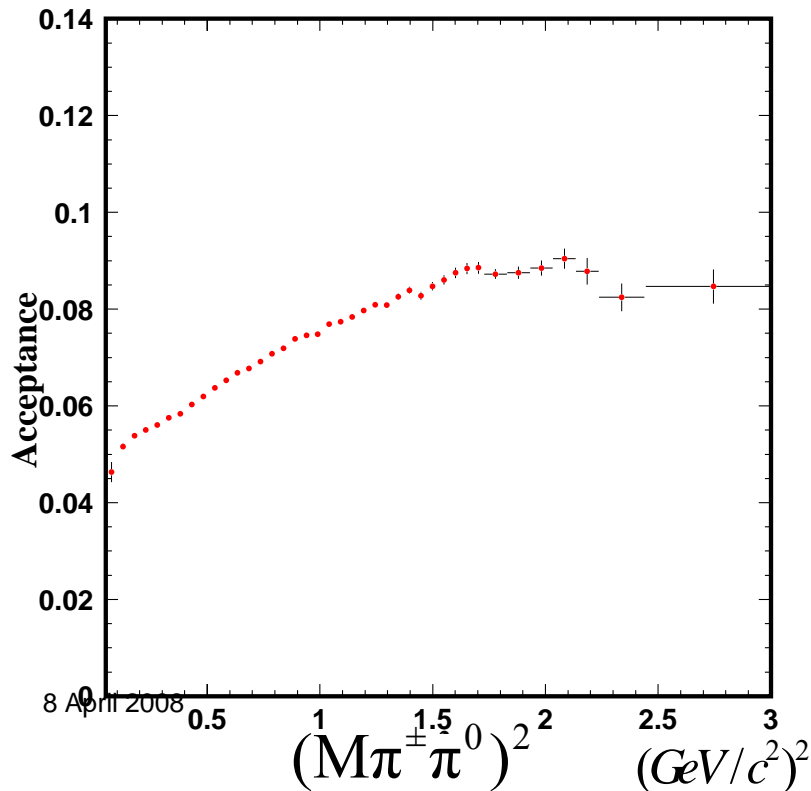


Acceptance



Data are Unfolded with the Singular Value Decomposition (SVD) method.

Acceptance



Results



- Br
- Mass Spectrum
- Pion form factor
- Extract resonance Parameters
- Comparison with previous exp.
- Evaluation of a_{μ} and iso-spin violation correction.

Result (1) Branching Fraction



- Normalized to the number of τ -pairs

$$Br_{2\pi} = \frac{N_{2\pi}^{obs}}{2N_{\tau\tau}^{obs}} \cdot \frac{(1 - b^{feed-down} - b^{non-\tau}) \cdot \epsilon^{\tau\tau}}{(1 - b_{bg}^{\tau\tau})} \cdot \frac{1}{\epsilon_{2\pi}^{ID} \epsilon_{2\pi}^{\tau}}$$

- Tau-pair selection

acceptance : $32.6 \pm 0.05\%$

$f_b = \frac{\epsilon_{2\pi}^{\tau\tau}}{\epsilon^{\tau\tau}}$: 1.112 ± 0.003

background : $7.4 \pm 0.03\%$

- $\pi\pi^0$ selection

Acceptance: $41.0 \pm 0.1\%$

Background

feed down: $7.02 \pm 0.08\%$

qq-conti. : $2.22 \pm 0.05\%$

Belle

$25.17 \pm 0.04 \pm 0.40$

CLEO

$25.42 \pm 0.12 \pm 0.42$

L3

$24.60 \pm 0.35 \pm 0.50$

ALEPH

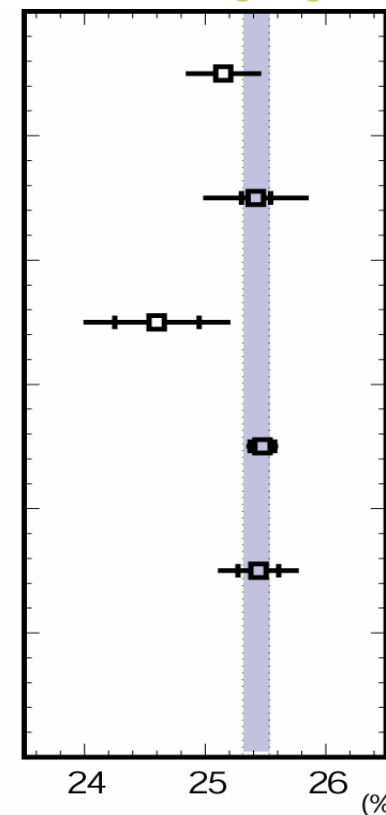
$25.474 \pm 0.101 \pm 0.085$

OPAL

$25.44 \pm 0.17 \pm 0.29$

Average

25.42 ± 0.11



Br = $(25.17 \pm 0.04(\text{stat}) \pm 0.40(\text{sys}))\%$

Systematic on Br Measurement



	$\Delta B/B$ (%)
■ Source	
Tracking efficiency	0.47
π^0 efficiency	1.4
Background in τ -pair	0.36
Feed down background	0.16
Continuum background	0.20
γ veto	0.20
Trigger	0.32
MC statistics	0.08
Total	1.6

■ Systematic is dominated by the uncertainty of the π^0 efficiency

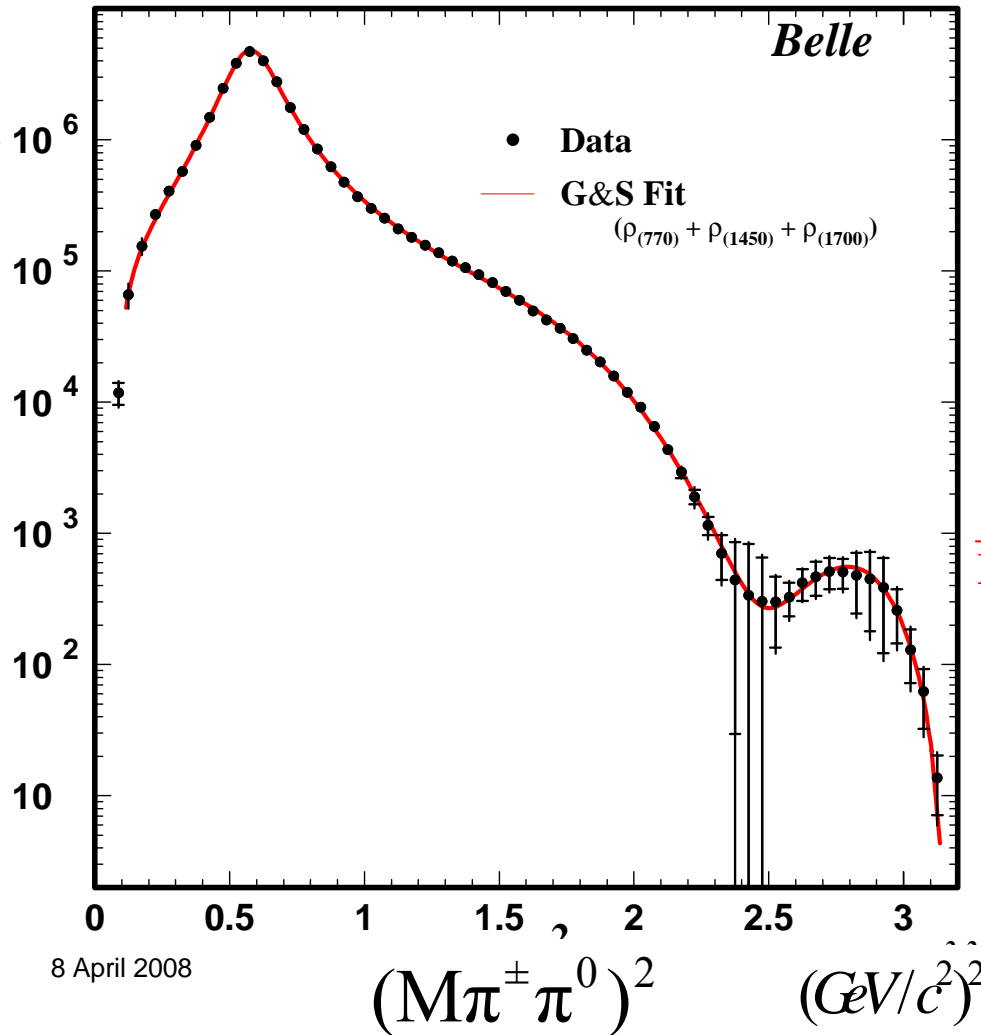
- Calibrated by η signals ($\eta \rightarrow \pi^0 \pi^0 \pi^0 / \eta \rightarrow \gamma \gamma$).
- Checked by using electron tracks.
- ($\gamma \gamma \rightarrow \pi^0 \pi^0$ can use to improve π^0 efficiency in near future.)

Result (2): Mass spectrum



Unfolded Results

Number of entries / 0.05 (GeV/c²)²



Mass spectra

= Phase space

× Form Factor

$$\frac{1}{N_{\pi^0}} \frac{dN_{\pi^0}}{ds} = \frac{6\pi |V_{ud}|^2 S_{EW}}{m_t^2} \times \frac{B_e}{B_{\pi\pi}} \left[\left(1 - \frac{s}{m_t^2}\right)^2 \left(1 + \frac{2s}{m_t^2}\right) \right] v^{\pi\pi^0}(s)$$

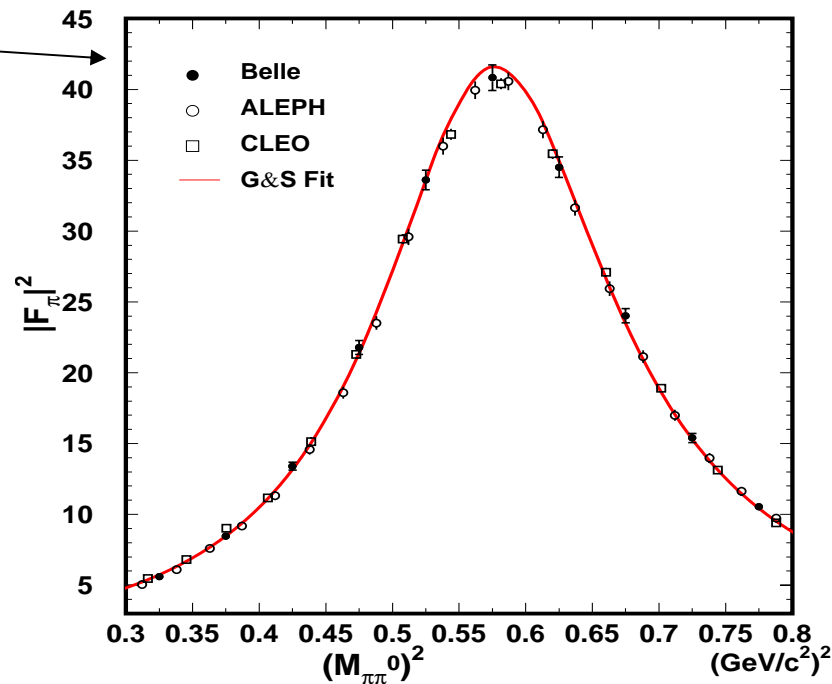
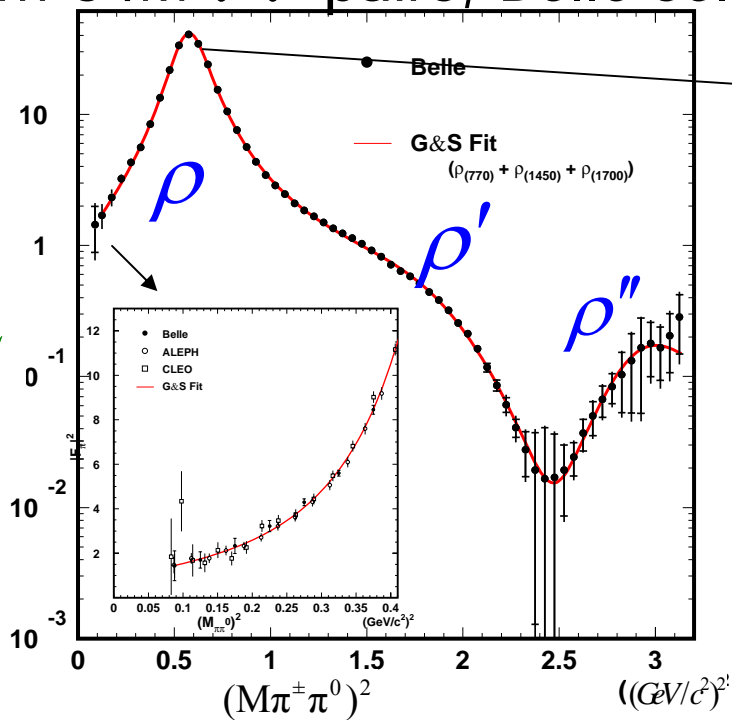
$$v^{\pi\pi^0}(s) = \frac{\beta^3(s)}{12\pi} |F_{\pi}(s)|^2$$

Result (3) Pion Form Factor $|F_\pi|^2$



From 64M $\tau^+\tau^-$ pairs, Belle selects 5.5M $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$ events!

$|F_\pi|^2$
 π
 2



■ Error bars include both statistical and systematic

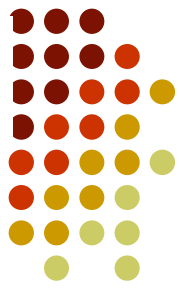
■ Interference between ρ' and ρ''

■ — Fit with BW

$$B_{\text{Belle}} = (25.17 \pm 0.04 \pm 0.40)\%$$

$$B_{\text{ALEPH}} = (25.471 \pm 0.097 \pm 0.085)\%^{15}$$

Systematic on the Mass Spectrum/Form Factor



$M_{\pi\pi}^2$	threshold	ρ region	ρ' region	ρ'' region
Unfolding (MC)	0.79	0.31	1.5	1.50
Unfolding (cond)	0.53	0.09	0.58	9.19
BKG (continuum)	0.09	0.01	0.52	5.76
BKG (feed-down)	0.65	0.10	---	0.50
BKG(non- π^0)	4.80	---	---	---
Acceptance	1.44	0.03	0.15	0.40
Energy scale	1.08	0.59	0.05	0.50
Total	5.3	0.7	1.8	11.4

(%)

← Check by known MC distribution

Systematic	Region	Dominant Factor
0.7-1.8%	ρ, ρ'	Energy scale/Unfolding
5%	threshold	Background (π^0 , feed down)
10%	ρ''	Continuum BG

Result(4) Resonance parameters



- Fit with BW form $\rho(770), \rho'(1400), \rho''(1700)$

$$F_{\pi}(s) = \frac{1}{1 + \beta + \gamma} (\underbrace{BW_{\rho}} + \beta \underbrace{BW_{\rho'}} + \gamma \underbrace{BW_{\rho''}})$$

- Gounaris-Sakurai (GS) parameterization

$$BW_{\rho}^{G\&S} = \frac{M_{\rho}^2 + d(s)M_{\rho}\Gamma_{\rho}(s)}{(M_{\rho}^2 - s) + f(s) - i\sqrt{s}\Gamma_{\rho}(s)}$$

- 10 fit parameters

The normalization of the GS form is given by $|F_{\pi}(0)|^2 = 1$

M_{ρ}, Γ_{ρ} : ρ mass and width

$M_{\rho'}, \Gamma_{\rho'}$: ρ' mass and width

$M_{\rho''}, \Gamma_{\rho''}$: ρ'' mass and width

β, ϕ_{β} : ρ' amplitude

γ, ϕ_{γ} : ρ'' amplitude

Fit Results



Fit parameter	Norm fixed
Norm $ F_\pi(0) ^2$	[1.0]
M_ρ (MeV)	774.5 ± 0.2 ± 0.5
Γ_ρ (MeV)	148.1 ± 0.4 ± 1.7
$M_{\rho'}$ (MeV)	1446 ± 7 ± 28
$\Gamma_{\rho'}$ (MeV)	434 ± 16 ± 60
$ \beta $	$0.15 \pm 0.05 \pm_{0.04}^{0.15}$
ϕ_β (degree)	$202 \pm 4 \pm_8^{41}$
$M_{\rho''}$ (MeV)	1728 ± 17 ± 89
$\Gamma_{\rho''}$ (MeV)	$164 \pm 21 \pm_{26}^{89}$
$ \gamma $	$0.028 \pm 0.020 \pm_{0.009}^{0.059}$
ϕ_γ (degree)	$24 \pm 9 \pm_{28}^{118}$
$\chi^2/\text{d.o.f}$	80/52

PDG2006

$\tau, e+e^-$	hadronic
775.4 ± 0.4	766.5 ± 1.1
146.4 ± 1.1	150.2 ± 2.4
	1459 ± 11

←→ 147 ± 40

1720 ± 20

250 ± 100

■ All ρ, ρ', ρ'' resonance parameters are determined at the same time!

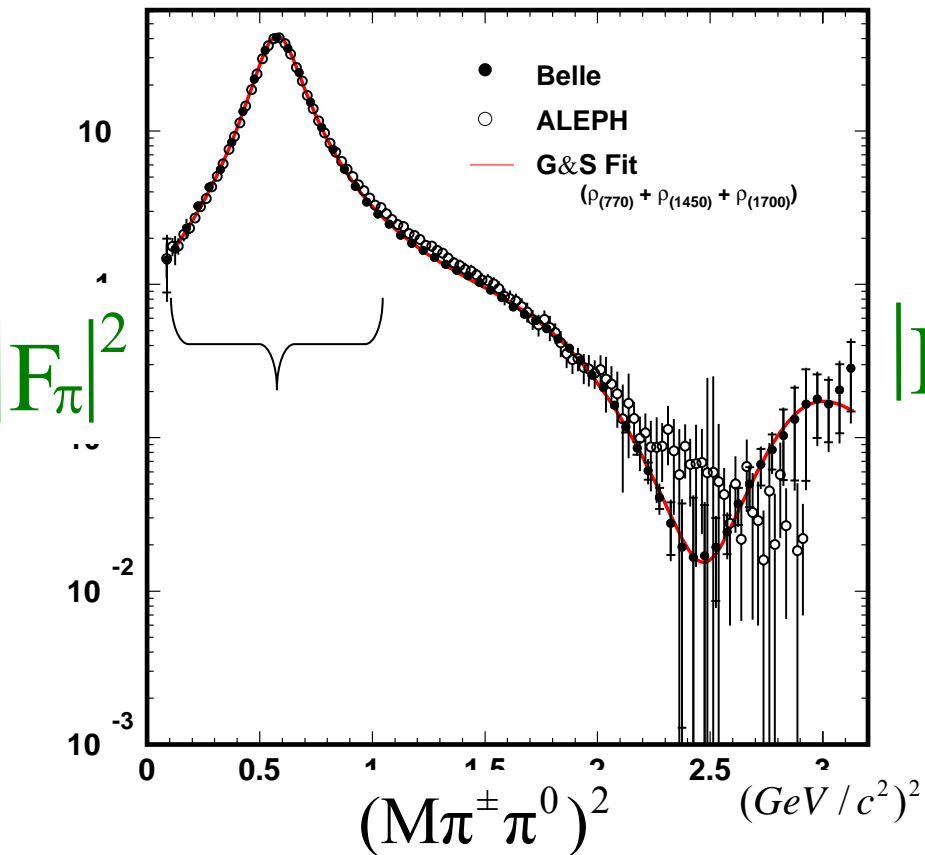


Most precise !

Comparison with previous exp.

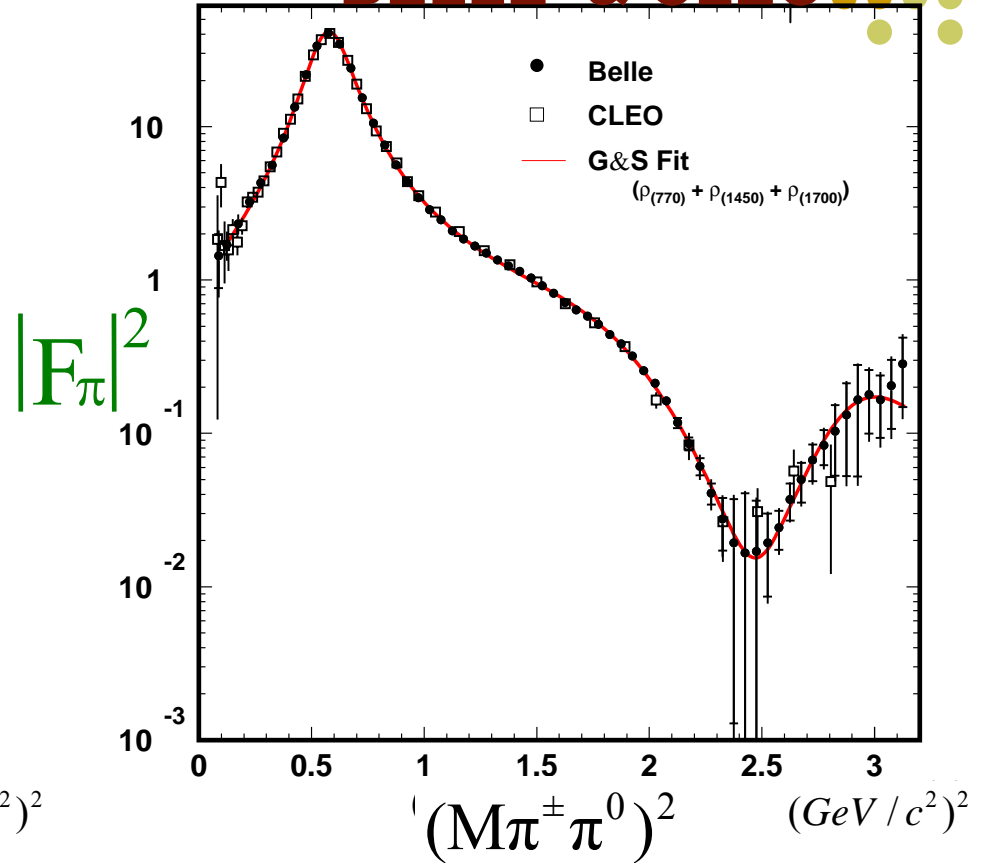


BELLE & ALEPH



Ref: Phys. Rep.421 (2005) 191

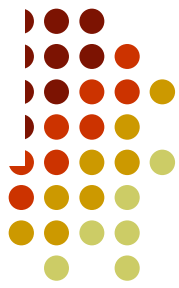
BELLE & CLEO



Ref: Phys. Rev. D61, 112002(2000) 1

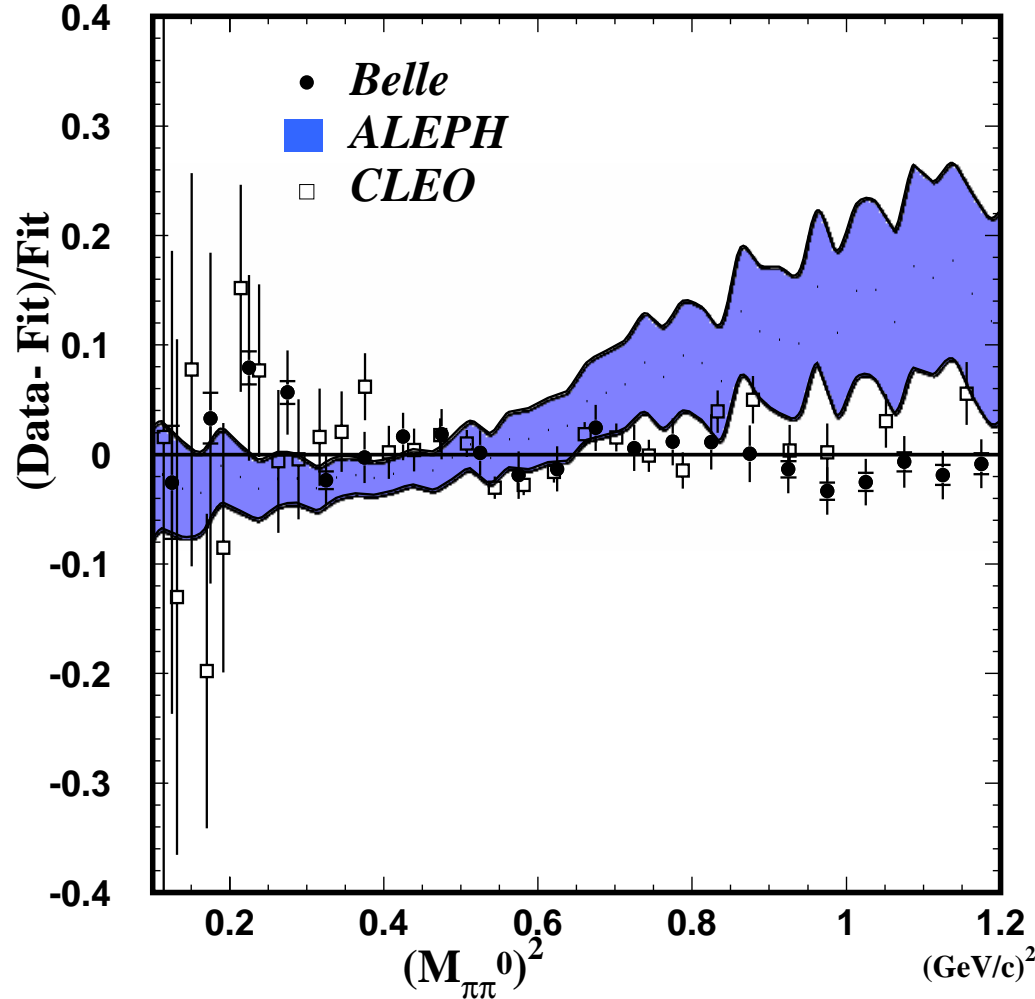
■ Agree with previous exp. of data.

■ Our result is more precise especially in high mass region.



Detailed Comparison: ρ, ρ' region

Fit: Fit to Belle Data



- Belle-CLEO consistent,
 - ALEPH is higher
- at $(M_{\pi\pi})^2 > 0.8 \text{ GeV}^2$;



Effect on $a_{\mu}^{2\pi}$?

i

Results (5) $a_\mu(2\pi)$



$$a_\mu^{had, LO} = \frac{\alpha^2}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} v^{\pi\pi}(s) \quad v^{\pi\pi}(s) = \frac{m_\tau^2}{6\pi |V_{ud}|^2 S_{EW}} \cdot \frac{B_{\pi\pi^0}}{B_e} \cdot \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \right]^{-1} \cdot \frac{1}{N_{\pi\pi^0}} \frac{dN_{\pi\pi^0}}{ds}$$

range (GeV ²)	Belle	ALEPH
0.08-0.20	39.57	38.23
0.20-0.35	70.65	66.89
0.35-0.50	123.30	119.20
0.50-0.65	196.90	194.20
0.65-0.80	62.37	62.39
0.80-0.95	15.64	16.41
0.95-1.10	5.74	6.50
1.10-1.25	2.86	3.27
1.25-1.40	1.65	1.89
1.40-3.20	2.43	2.77
0.08-3.20	521.1	511.5

X 10⁻¹⁰

Belle > ALEPH

Belle < ALEPH

No iso-spin
violation correction
is applied

← Total

Iso-spin Violation Correction



■ Ref. V. Cirigliano et al., J. High Energy Phys. 08, 002(2002)
 A.Flores-Tlalpa et al., Phys. Rev. D 74, 071301 (2006)

$\times 10^{-10}$

Source	Correction	Uncertainty
Short distance rad. Cor (S_{EW})	-12.0	+/- 0.2
Long distance rad. Cor. (G_{EM})	-1.0	
$m_{\pi^-} \neq m_{\pi^0}$ (in phase space)	-7.0	
ρ - ω interference	+3.5	+/- 0.6
$m_{\pi^-} \neq m_{\pi^0}$ (in decay width)	+4.2	
Electromagnetic decays	-1.4	
$m_{\rho^0} \neq m_{\rho^-}$	-	+/- 1.4
Total	-13.7	+/- 2.0

ρ - ω interference



- ρ - ω interference effects are estimated using following form for the amplitude.

$$F_{\pi}^0(s)_{\rho-\omega} = -\frac{\theta_{\rho\omega}}{3m_{\rho}^2} \frac{s}{m_{\omega}^2 - s - im_{\omega}\Gamma_{\omega}}$$

$$\theta_{\rho\omega} = (-3.3 \pm 0.7) \times 10^{-3} \text{ GeV}^2$$

$$m_{\omega} = 0.783 \text{ GeV}$$

$$\Gamma_{\omega} = 0.00844 \text{ GeV}$$

Results (5) $a_{\mu}(2\pi)$



● Integrated region: $\sqrt{s} = m_{\pi} - 1.8 \text{ GeV}/c^2$

● After applying the known iso-spin violation correction.

■ Belle(τ)

$$a_{\mu}^{\pi\pi} = (519.4 \pm 1.5(\text{exp.}) \pm 2.7(\text{Br}) \pm 2.5(\text{isospin})) \times 10^{-10}$$

● ALEPH, CLEO, OPAL (τ) Ref. Eur. Phys. J. C27, 497 (2003)

$$a_{\mu}^{\pi\pi} = (520.1 \pm 2.4(\text{exp.}) \pm 2.7(\text{Br}) \pm 2.5(\text{isospin})) \times 10^{-10}$$

● CMD2, SND (e^+e^-) Ref. Nucl. Phys. Proc. Suppl. 169, 288 (2007)

$$a_{\mu}^{\pi\pi} = (504.6 \pm 3.1(\text{exp.}) \pm 0.9(\text{rad.})) \times 10^{-10}$$



τ results are higher than those from e^+e^- .

Summary



- We have studied $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ using high statistics Belle data
- Br measurement: $\text{Br}(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau) = (25.17 \pm 0.04 \pm 0.40)\%$
(1.6% accuracy)
Preliminary
- Precise 2π mass spectrum and the pion form factor are determined. We can provide them by a table.
- In addition to $\rho(700)$, $\rho'(1400)$, the production of $\rho''(1700)$ is unambiguously identified and its parameters are determined.
- Our results for $a_\mu^{\pi\pi}$ agree with the previous τ based results but are higher than those from $e^+ e^-$.



Backup Slide

External parameters



Item	Value	$\Delta a_{\mu}^{\pi\pi}$ (unit : $\times 10^{-10}$)
S_{EW} : electroweak radiative correction	1.0233 ± 0.0006	± 0.32
$ V_{ud} $: CKM matrix element	0.9734 ± 0.0008	± 0.42
B_e : $\tau^- \rightarrow e^- \bar{\nu}_e \nu_{\tau}$ Branching Fraction	$(17.84 \pm 0.06)\%$	± 1.82
$B_{\pi\pi^0}$: $\tau^- \rightarrow \pi^- \pi^0 \nu_{\tau}$ Branching Fraction	$(25.42 \pm 0.11)\%$	± 2.30
Total		± 3.0

$B_{\pi\pi^0}$: $\tau^- \rightarrow \pi^- \pi^0 \nu_{\tau}$ Branching Fraction

World average is calculated combining our new result and the preceding measurements of other experiments.

Belle
 $25.15 \pm 0.04 \pm 0.31$

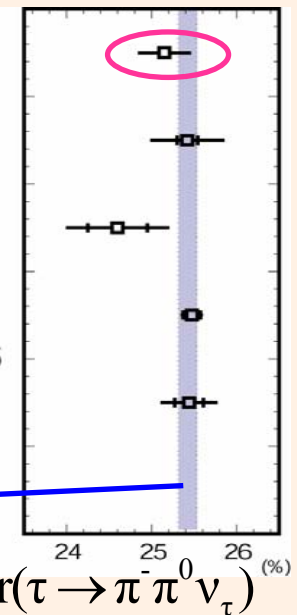
CLEO
 $25.42 \pm 0.12 \pm 0.42$

L3
 $24.60 \pm 0.35 \pm 0.50$

ALEPH
 $25.474 \pm 0.101 \pm 0.085$

OPAL
 $25.44 \pm 0.17 \pm 0.29$

Average
 25.42 ± 0.11



Iso-spin Violation Correction



■ Ref. V. Cirigliano et al., J. High Energy Phys. 08, 002(2002)
 A.Flores-Tlalpa et al., Phys. Rev. D 74, 071301 (2006)

$\times 10^{-10}$

Source	Correction	Uncertainty
Short distance rad. Cor (S_{EW})	-12.0	+/- 0.2
Long distance rad. Cor. (G_{EM})	-1.0	
$m_{\pi^-} \neq m_{\pi^0}$ (in phase space)	-7.0	
ρ - ω interference	+3.5	+/- 0.6
$m_{\pi^-} \neq m_{\pi^0}$ (in decay width)	+4.2	
Electromagnetic decays	-1.4	
$m_{\rho^0} \neq m_{\rho^-}$	-	+/- 1.4
Total	-13.7	+/- 2.0

Internal Systematic Error

$$m_{\pi\pi}^2 \geq 0.25 \text{ GeV}^2$$

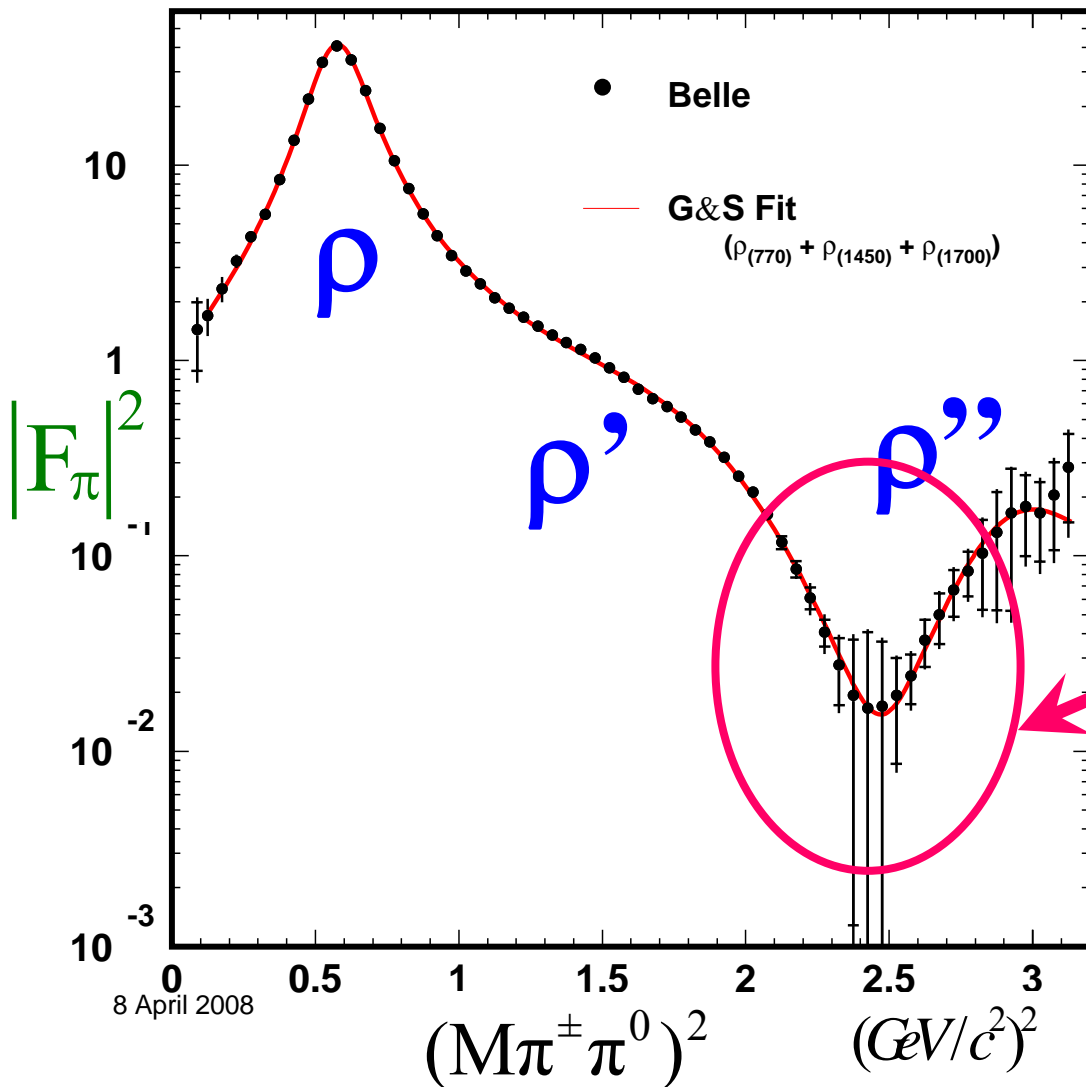


source	$\Delta a_{\mu}^{\pi\pi}$ (unit : $\times 10^{-10}$)
Background estimation	
▪ non- τ (ee \rightarrow hadron)	± 0.11
▪ feed-down $h \geq 2\pi^0\nu$	± 0.09
▪ feed-down $K^- \pi^0\nu$	± 0.15
π^0/γ selection efficiency/shape cuts	± 0.35
Energy scale	± 0.10
Gamma veto	± 0.93
γ /track overlap	0.24
Tagging Dependence	< 0.1
Smearing/Migration effect	
Total	± 1.04

Result(3) Pion Form

Factor $|F_\pi|^2$

From 64M $\tau^+\tau^-$ pairs, Belle selects 5.6M $\tau^- \pi^- \pi^0 \nu_\tau$ events!



◆ — Fit with BW Forms. → Next slide

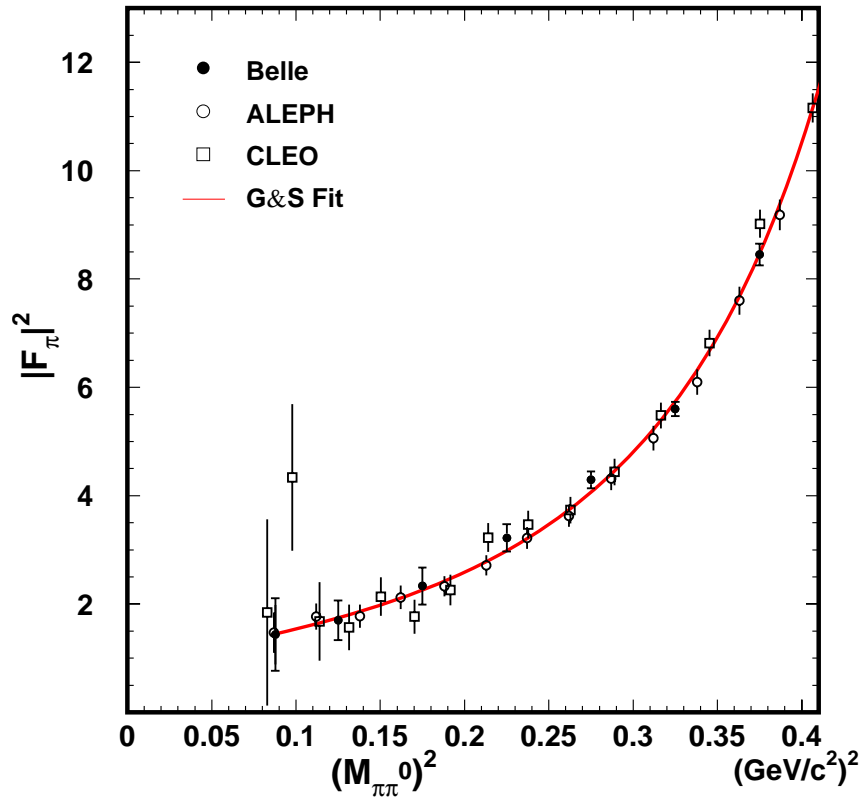
Dip at $s=2.5 \text{ GeV}^2$

◆ Interference between ρ' and ρ''

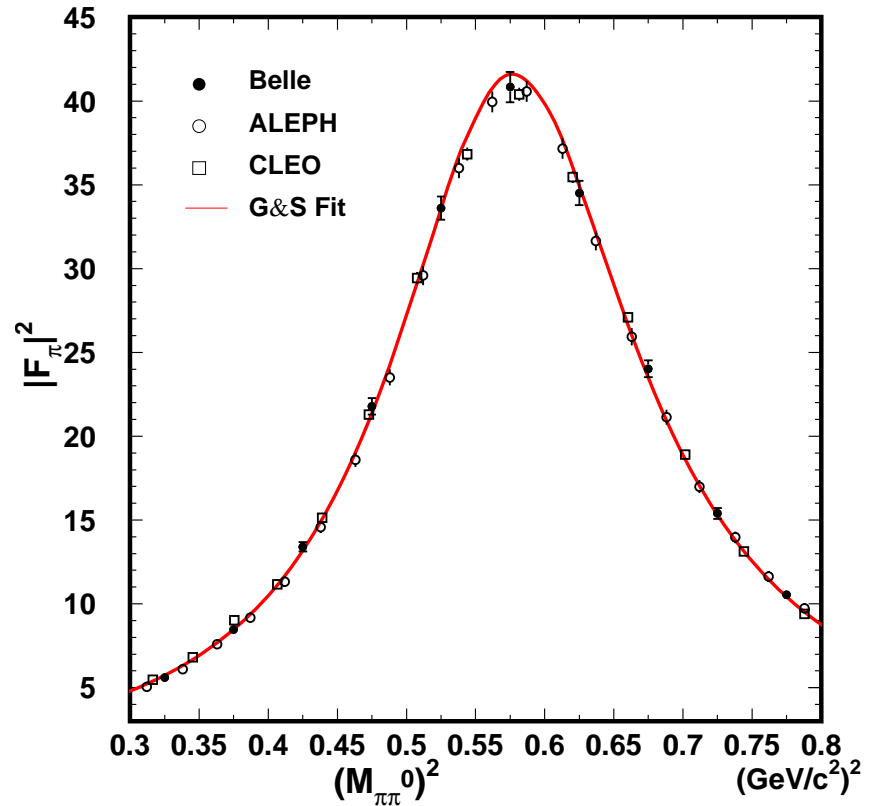
Pion Form Factor $|F_\pi|^2$ (linear scale)



Low mass region



ρ mass region



Error bars include both statistical and systematic

Systematic on the mass distribution (1)



■ Unfolding procedure

- Checked by Signal MC (UNF①)
- Unfolding condition : value ± 5 (UNF②)

■ Acceptance (Accept.)

$-\pi^0$ efficiency uncertainty

- Effect of γ -track isolation
 - Change a cut on the cluster-track distance (default and tighter one(30cm))

■ Momentum or energy scale (ENS)

- Change E_γ by it's uncertainty estimated from the π^0 mass peak position. ($\pm 0.2\%$)



Systematic on the mass distribution (2)

■ Background

● continuum BG (BKG①)

- estimate at the mass region higher than m_τ
- uncertainty is estimated to be 10%

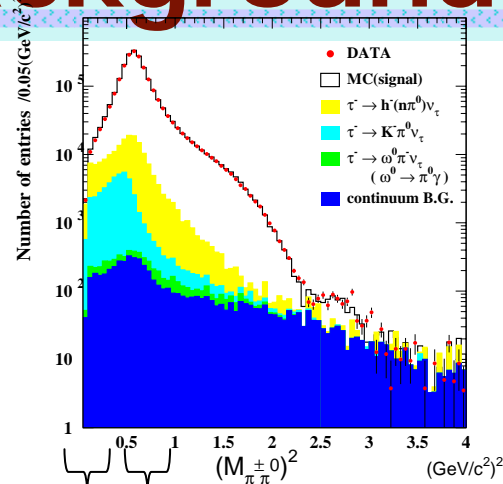
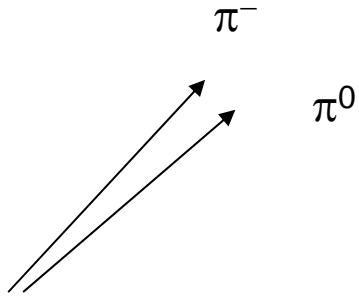
● Feed down BG (BKG②)

- dominated by $\tau^- \rightarrow \pi^- 2\pi^0 \nu_\tau$
- systematic is estimated by changing the Br in PDG by 1σ

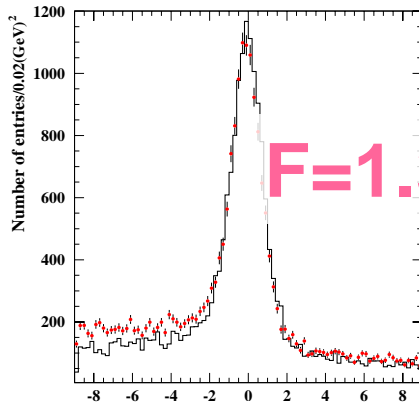
● non- π^0 BG (BKG③)

- dominated in the low $(M_{\pi\pi^0})^2$ region.
- In this region, the size of the non- π^0 background in the lower $M_{\gamma\gamma}$ side is different between data and MC. This difference is estimated as a systematic.

Non- π^0 background (details).



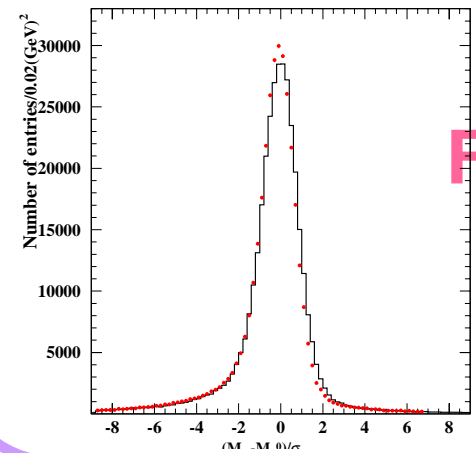
$$0.10 < (M_{\pi\pi^0})^2 < 0.15$$



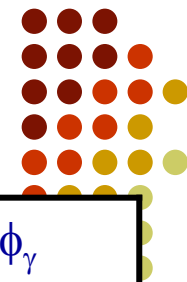
8 April 2008

$M_{\gamma\gamma}$

$$0.55 < (M_{\pi\pi^0})^2 < 0.60$$

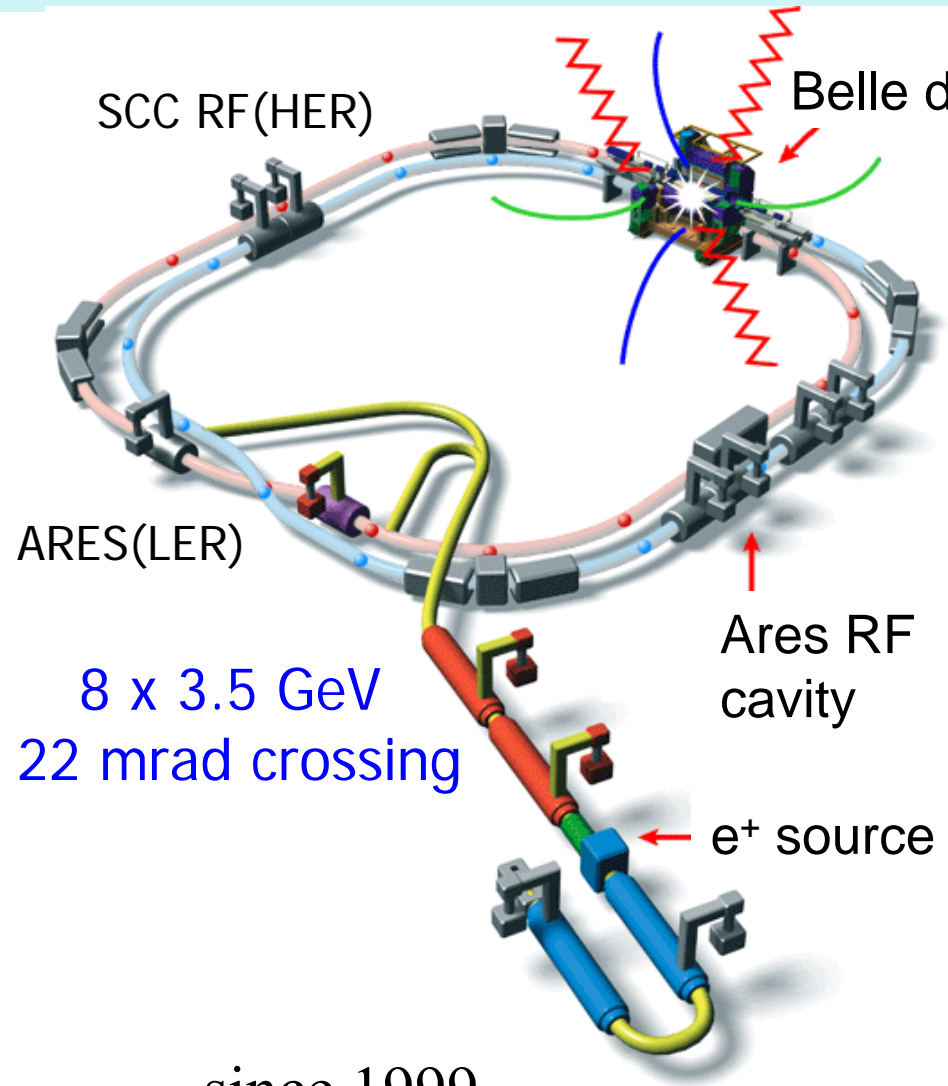


Systematic of resonance parameters



Source of systematics	$M\rho$ (MeV)	$\Gamma\rho$ (MeV)	$M\rho'$ (MeV)	$\Gamma\rho'$ (MeV)	β	ϕ_β (deg.)	$M\rho''$ (MeV)	$\Gamma\rho''$ (MeV)	γ	ϕ_γ (deg.)
Fit bias	0.3	1.6	25	49	0.028	4	75	10	0.038	13
Unfold	0.3	0.3	4	24	0.001	4	11	14	0.002	12
B.G.	0.3	--	11	25	+0.14 -0.03	+41 - 5	13	+86 -10	+0.053 -0.020	+117 - 22
Acceptance	--	0.1	1	4	---	0.6	0.1	7	---	1
Momentum scale	0.3	0.6	2	1	---	2	45	15	---	1
total	0.5	1.7	28	60	+0.15 -0.04	+41 -8	89	+89 -26	+0.065 -0.009	+118 - 28

The KEKB Collider

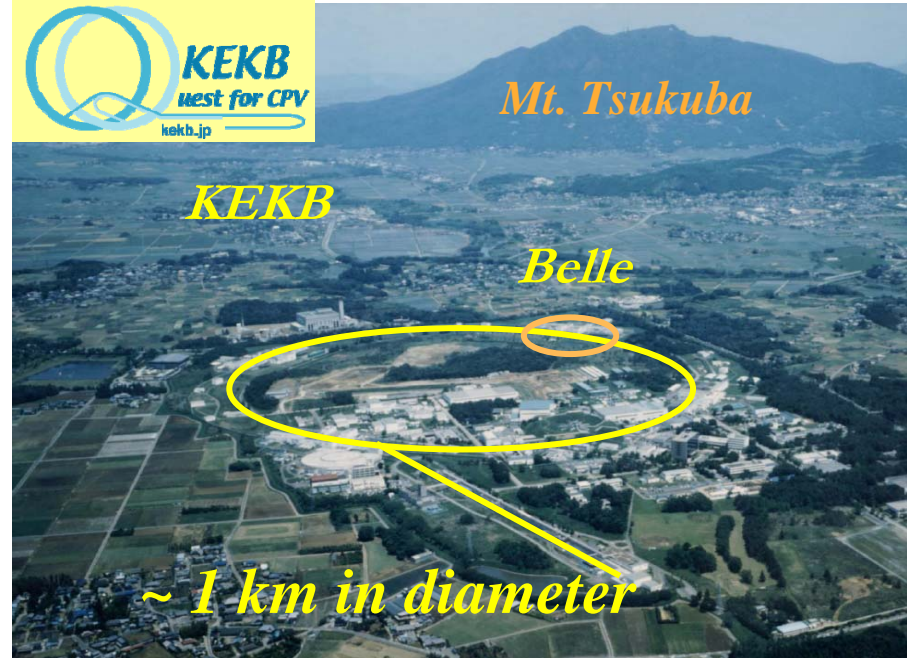


World record:

$$L = 1.7118 \times 10^{34} / \text{cm}^2 / \text{sec}$$



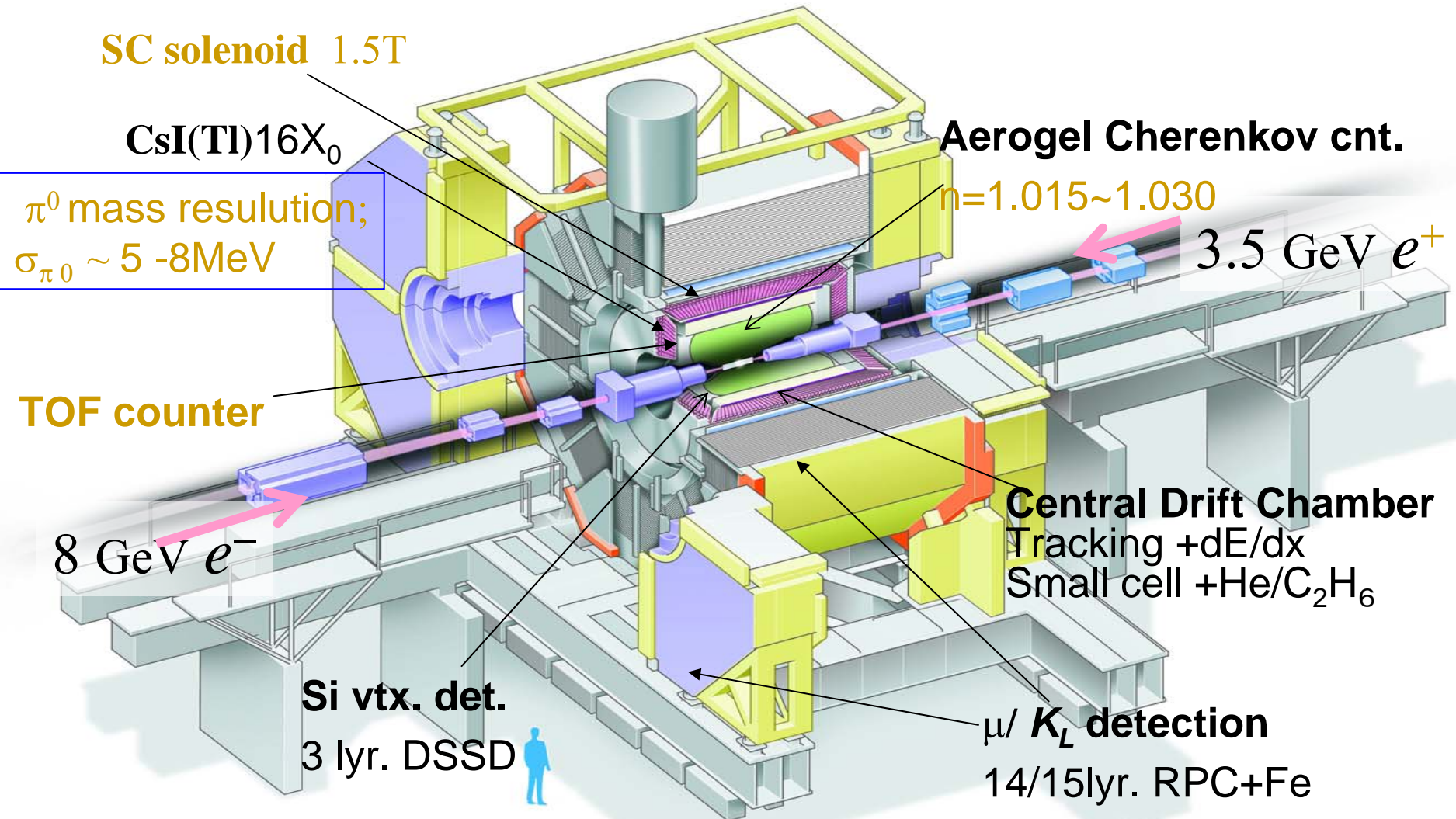
First successful op. of Crab cavities



since 1999

8 April 2008

Belle detector



Good tracking and particle identification