Hadronic Contributions to the theoretical value of $(g-2)_\mu$ and $\alpha(q^2)$

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LIVERPOOL THEORETICAL PHYSICS

I. Input for dispersion integrals: $\sigma^0_{\rm had}$

II. Recent developments in $g-2$

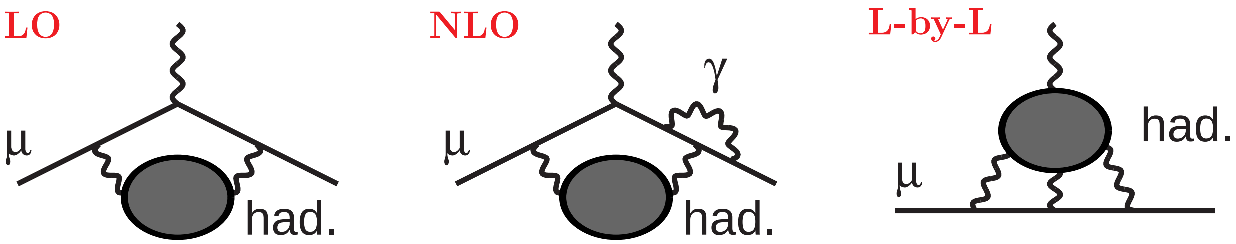
III. $\alpha_{\rm QED}$ at low and high energies

IV. The next round

For details and Refs: Hagiwara+Martin+Nomura+T: PRD 69(2004)093003; PLB 649(2007)173.

I. Input for dispersion integrals: $\sigma^0_{\rm had}$

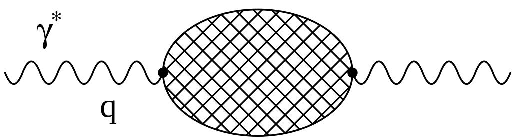
- $a_\mu^{\rm had} \quad = \quad a_\mu^{\rm had,LO} \quad + \quad a_\mu^{\rm had,NLO} \quad + \quad a_\mu^{\rm had,Light-by-Light}$



▶ Hadronic contributions from low γ virtualities not calculable with perturbative QCD

▶ rely instead on *dispersion relations*, using experimental data for $\sigma^0_{\rm had}(s)$:

$$a_\mu^{\rm had,LO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^\infty {\rm d}s\, \sigma^0_{\rm had}(s) K(s)\,, \quad \text{with } K(s) = \frac{m_\mu^2}{3s}\cdot(0.63\ldots1)$$

- 

$$\Delta\alpha^{(5)}_{\rm had}(q^2) \;=\; -\frac{q^2}{4\pi^2\alpha}\, P\int_{m_\pi^2}^\infty \frac{\sigma^0_{\rm had}(s)\,{\rm d}s}{s-q^2}$$

▶ Weighting of K extremely towards small s, less so for $\alpha(q^2) \;=\; \alpha\,/\,\big(1-\Delta\alpha_{\rm lep}(q^2)-\Delta\alpha_{\rm had}(q^2)\big)$.

▶ σ^0 means without running $\boldsymbol{\alpha}$ $\rightsquigarrow$ iteration needed.

→ Data input for $\sigma^0_{\rm had}(s)$ from the experiment CMD-2 at Novosibirsk:

(Still the most precise $e^+e^- \to \pi^+\pi^-$ data with only 0.6% sys. error)

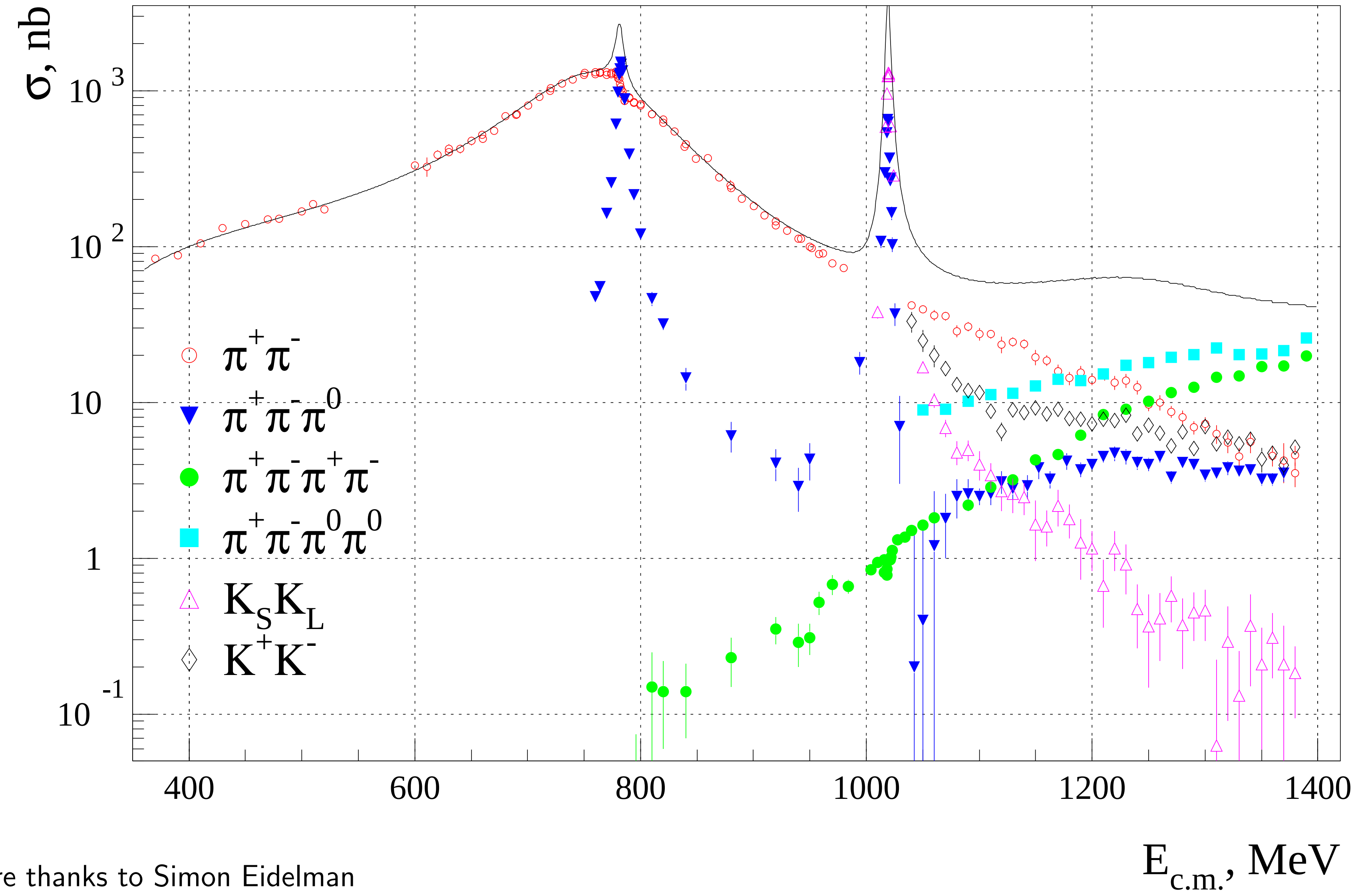


Figure thanks to Simon Eidelman

How to get the hadronic vac.-pol. contributions with precision:

- Lowest energies most important, i.e. the hadronic channels 2π, 3π, KK, 4π, 5π, etc. Have to sum ~ 24 exclusive channels and inclusive data for $\sqrt{s}$ above $1.43 - 2$ GeV to get total $\sigma_{\rm had}$ with high precision.

  $\longrightarrow$ Use of *state-of-the-art* perturbative QCD only above ~ 11.09 GeV.

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How to get the hadronic vac.-pol. contributions with precision: HMNT

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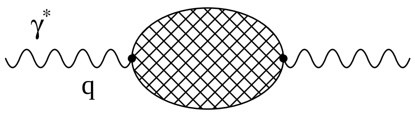
- Before averaging and $\sum$: Check Radiative Corrections of each data set:

$\longrightarrow$ Additional final state photons must be fully *included/estimated*

$\longrightarrow$ For σ^0, running $\alpha(q^2)$ effects must be *subtracted* (otherwise double-counting with $a_\mu^{\rm had, NLO}$)



$\longrightarrow$ but effects can cancel in $\sigma_{\rm had}/\sigma_{\rm norm}$, and corrections often done already partly... MANY COMPLICATIONS

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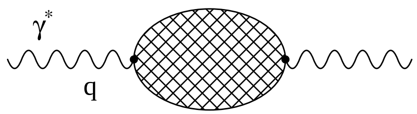


$\rightarrow$ but effects can cancel in $\sigma_{\rm had}/\sigma_{\rm norm}$, and corrections often done already partly.

$\longrightarrow$ PRECISION ONLY FROM TH + MC + EXP

→ Important detail: Use of time-like running of $\alpha(s)$:

$$\alpha(s) = \alpha / \left(1 - \Delta\alpha_{\rm lep}(s) - \Delta\alpha^{(5)}_{\rm had}(s) - \Delta\alpha^{\rm top}(s)\right)$$

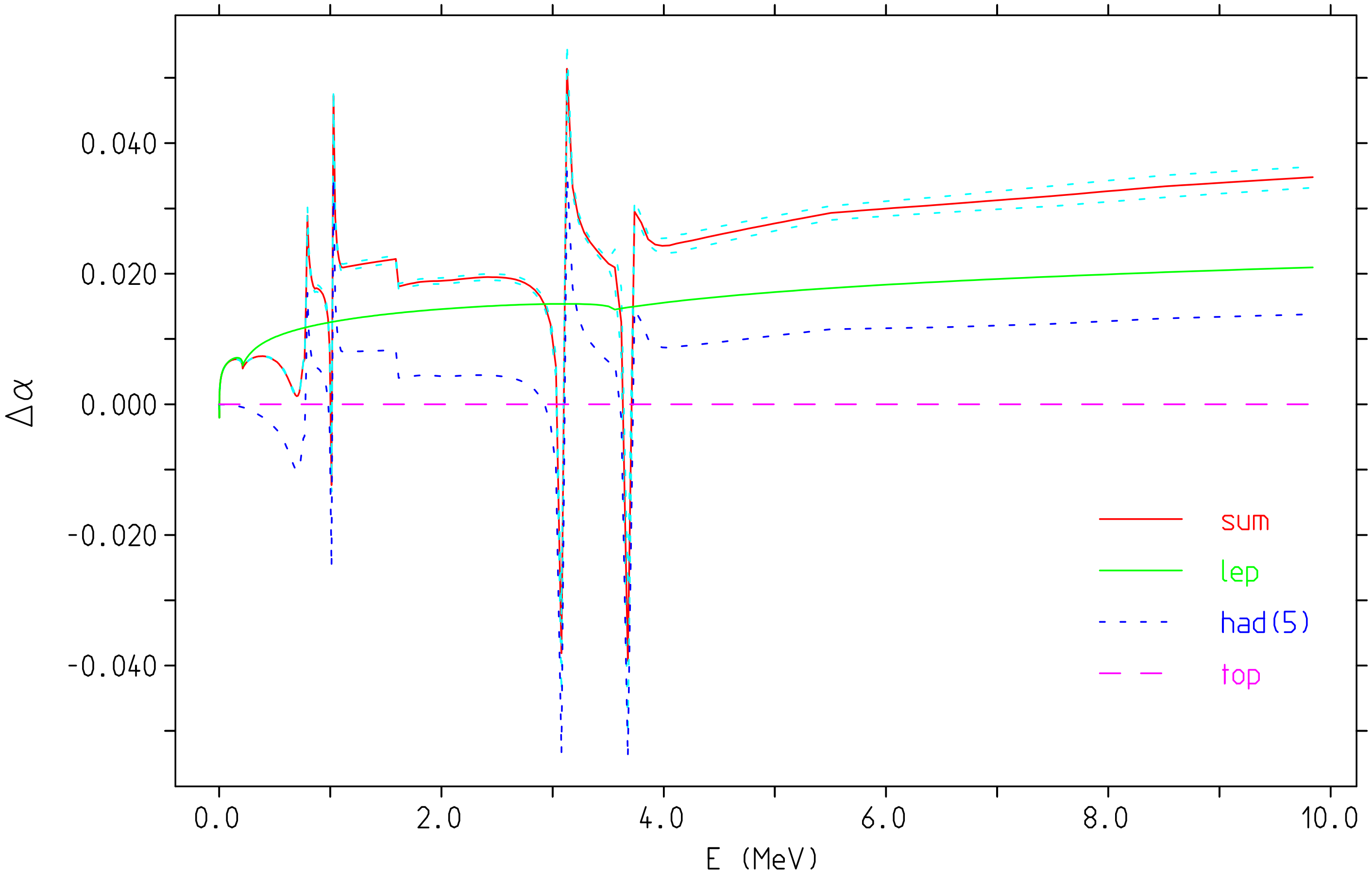
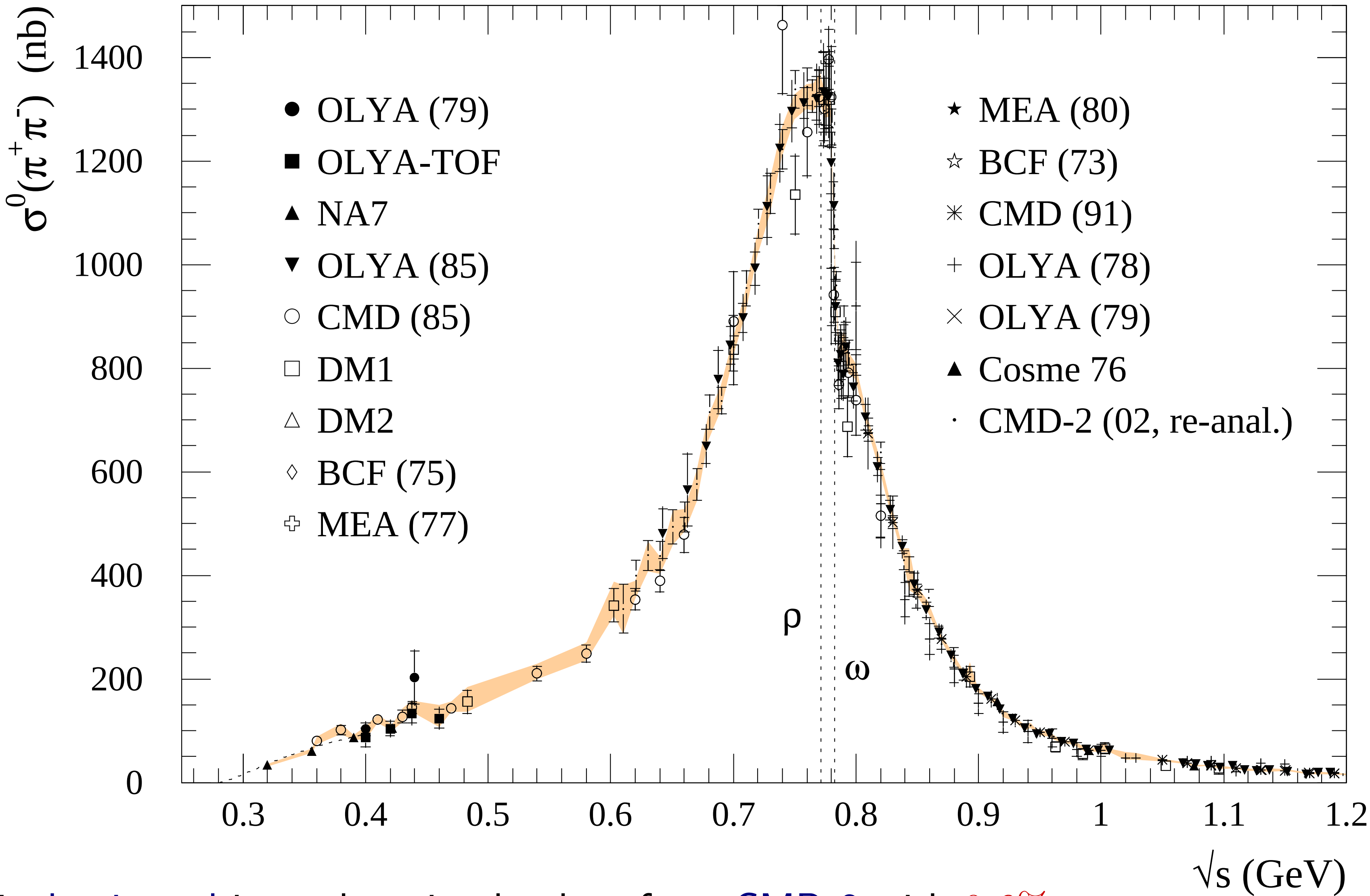


Figure from Fred Jegerlehner

→ In total these radiative corrections lead to an additional uncertainty of $\delta a_\mu^{\rm had,VP+FSR} \simeq 1.8 \times 10^{-10}$ [$\sim 10 \cdot \Delta a_\mu^{\rm EW}$] (HMNT analysis)

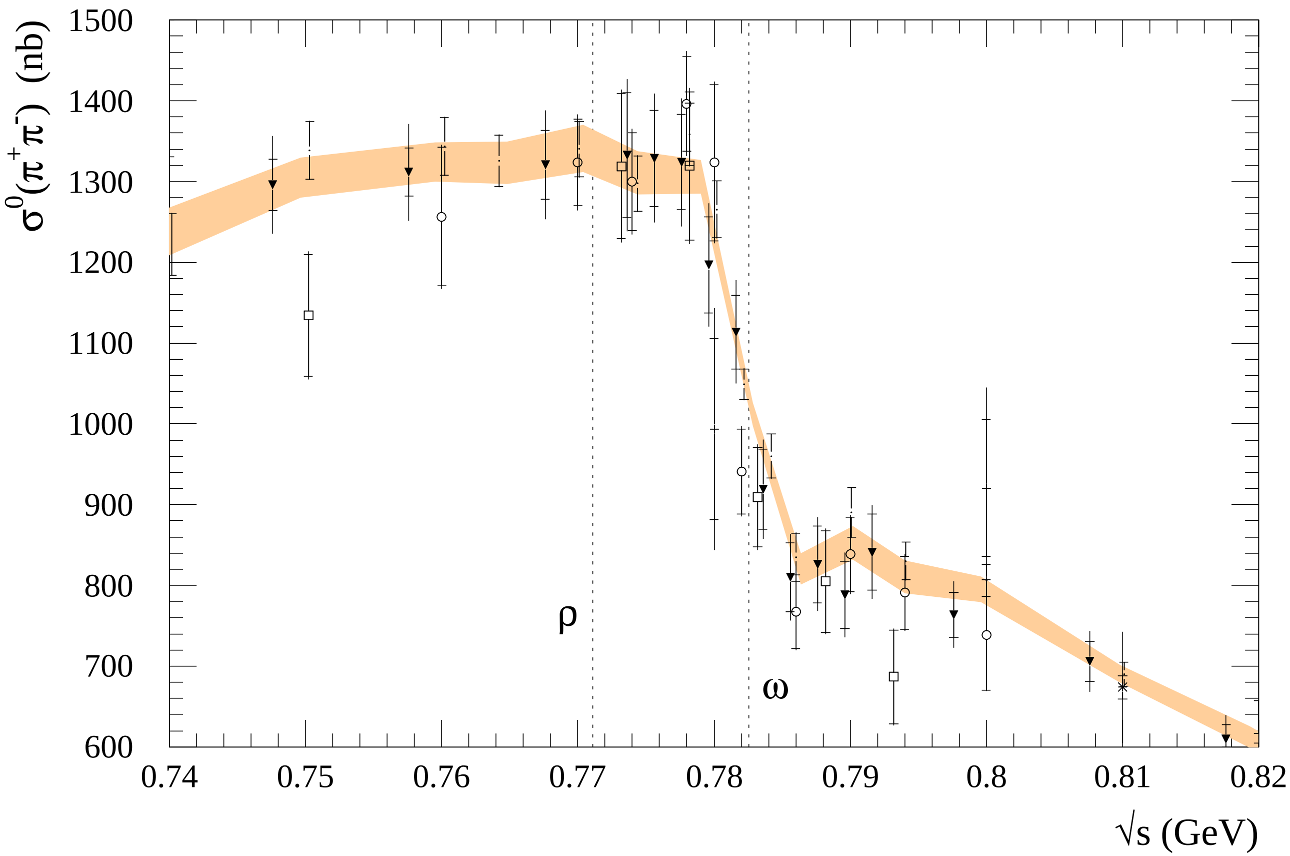
The most important channel: $\pi^+\pi^-$ ($\sim 72\%$ of total LO-hadronic!)

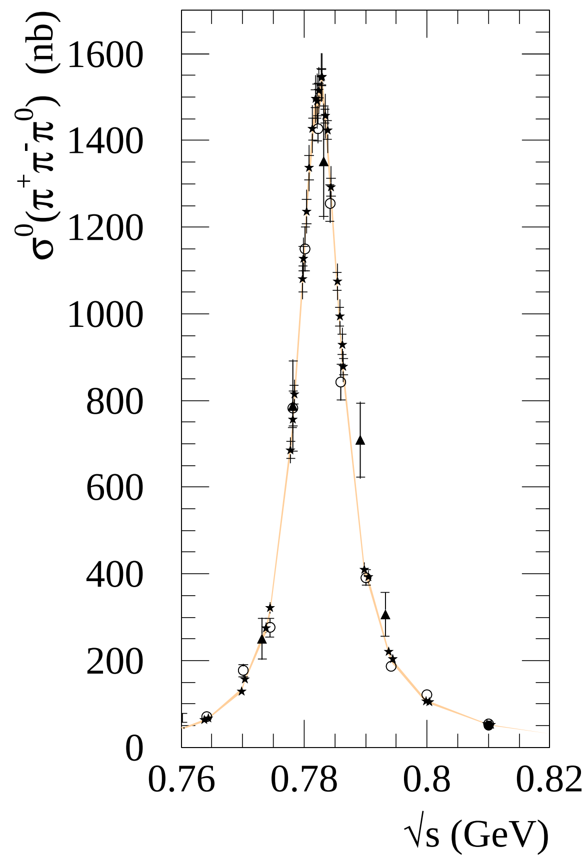


Fit dominated in peak region by data from CMD-2 with 0.6% sys. error.

Recently new, precise CMD-2, SND and KLOE data for ρ peak and tail regions.

Zoom in the region of $\rho-\omega$ interference in $\pi^+\pi^-$ $e^+e^- \to \omega \to 3\pi$

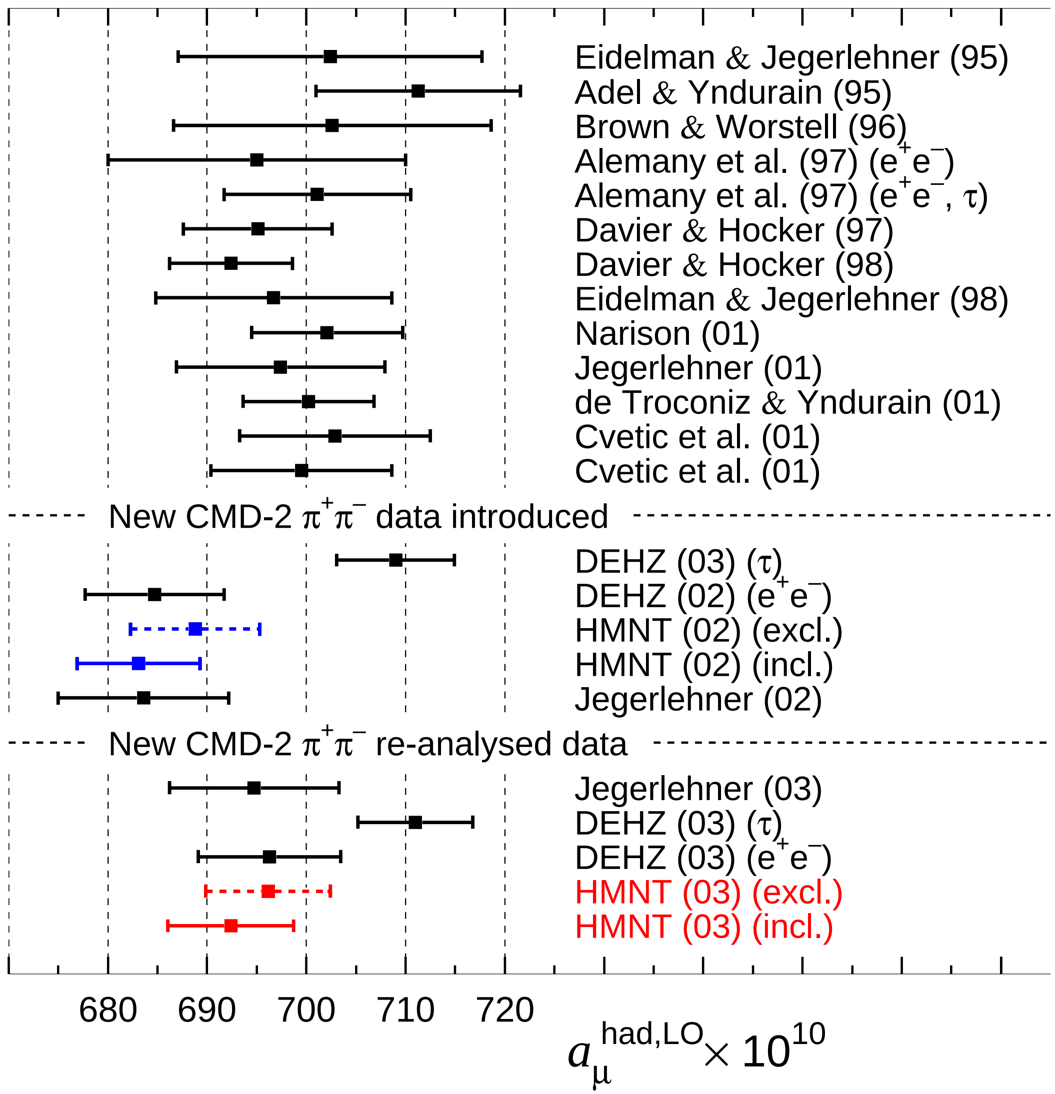




$\longrightarrow$ Dense data allow *direct data integration*; no bias from parametrizations.

$\longrightarrow$ NO problem with interference effects, resonance tails, missing or double-counting of non-resonant backgrounds.

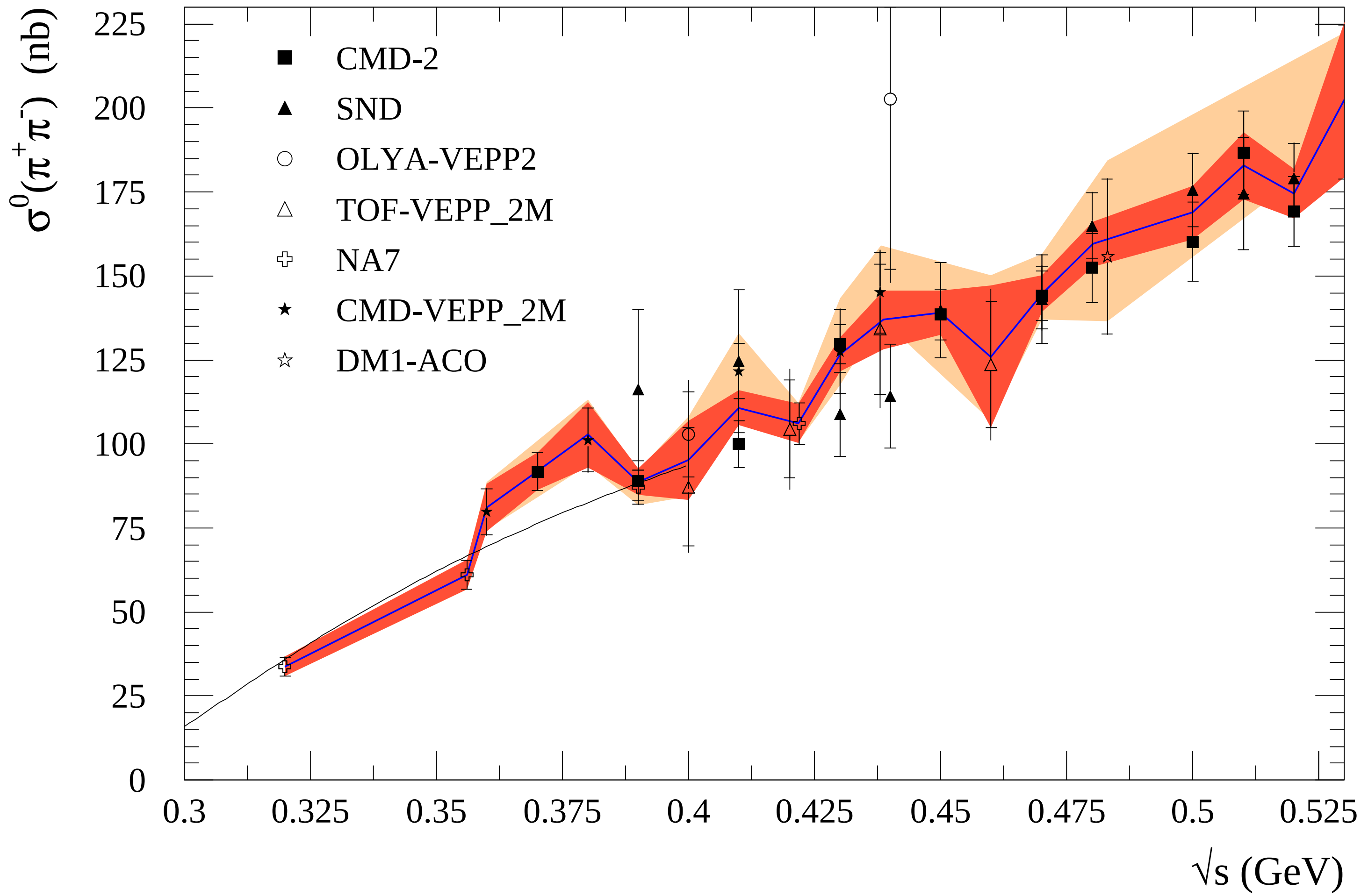
Brief history of different evaluations of $a_\mu^{\text{had, LO}}$:



$\longrightarrow$ Need for independent *checks* of CMD-2!

II. Recent developments in $g-2$

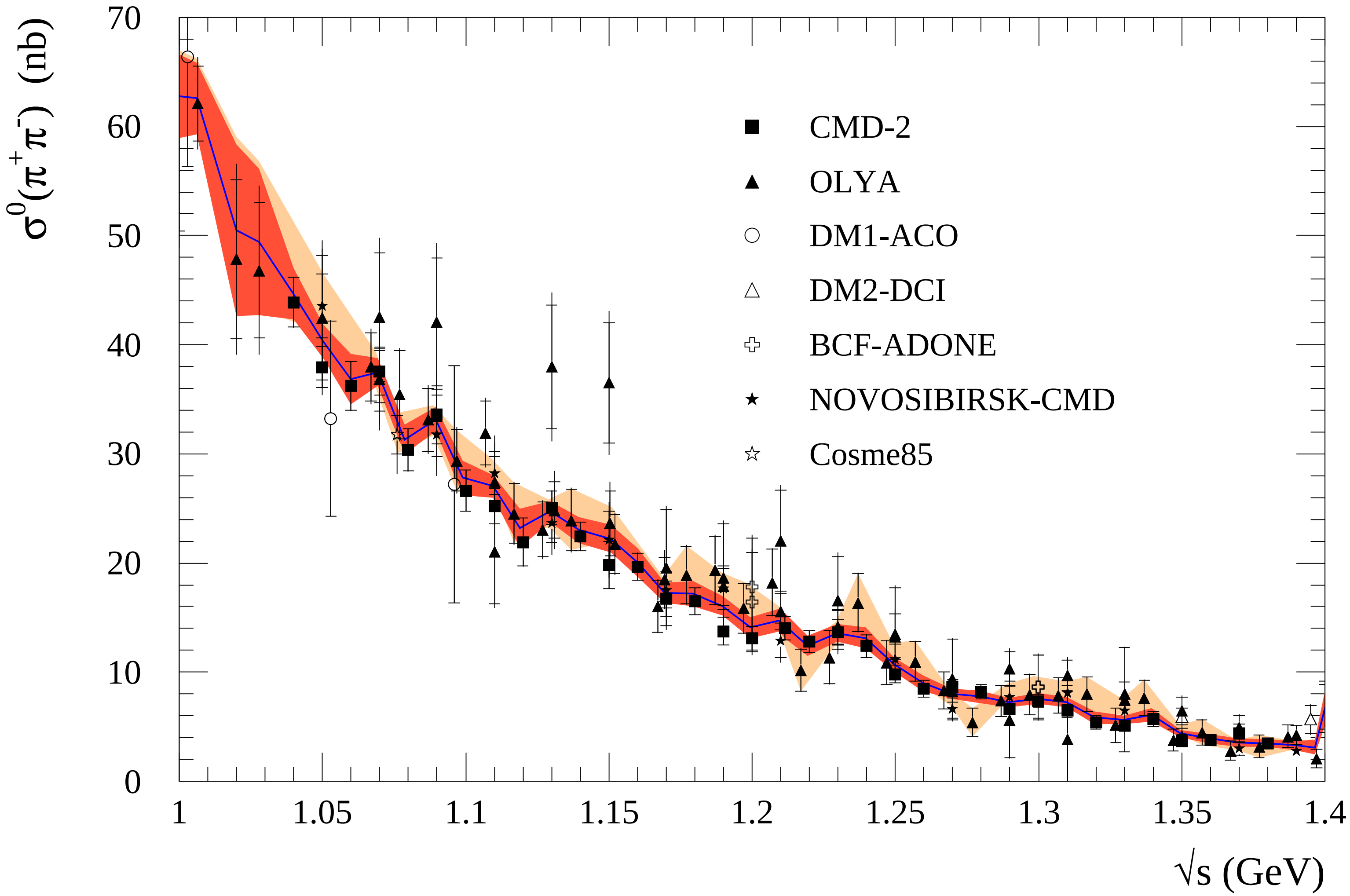
The troubled low-energy ρ tail: (orange: HMNT03, red: HMNT06)



– Data at lowest energy poor, use of chiral PT very close to threshold.

– Important role of few points from NA7 (one of the differences DEHZ – HMNT).

The 'high'-energy ρ tail:



$\longrightarrow$ Recent CMD-2, SND and KLOE analyses $\rightsquigarrow$ significant improvement of $\Delta a_\mu^{\mathrm{had,LO}}$!

The challengers of CMD-2: a) Direct scan, b) $\boldsymbol{\tau}$ spectral function, c) Radiative Return

a) SND:

– 2nd, independent experiment at VEPP-2M in Novosibirsk

– published $\sigma(e^+e^- \to \pi^+\pi^-)$ data, $0.4 < \sqrt{s} < 1.0$ GeV, 1.3% sys. error

– first not quite in agreement with CMD-2...

– But: Problems with Monte Carlos for $e^+e^- \to \mu^+\mu^-\gamma,\ \pi^+\pi^-\gamma$

– Re-analysis now in very good agreement with CMD-2:

'SND – CMD-2': $-0.3 \pm 1.6\%$

– Bad agreement with τ based spectral function data

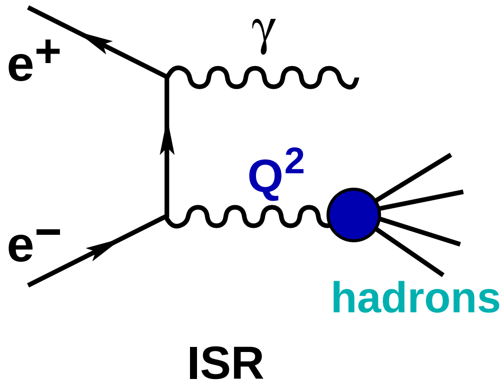
b) $\boldsymbol{\tau}$ spectral function:

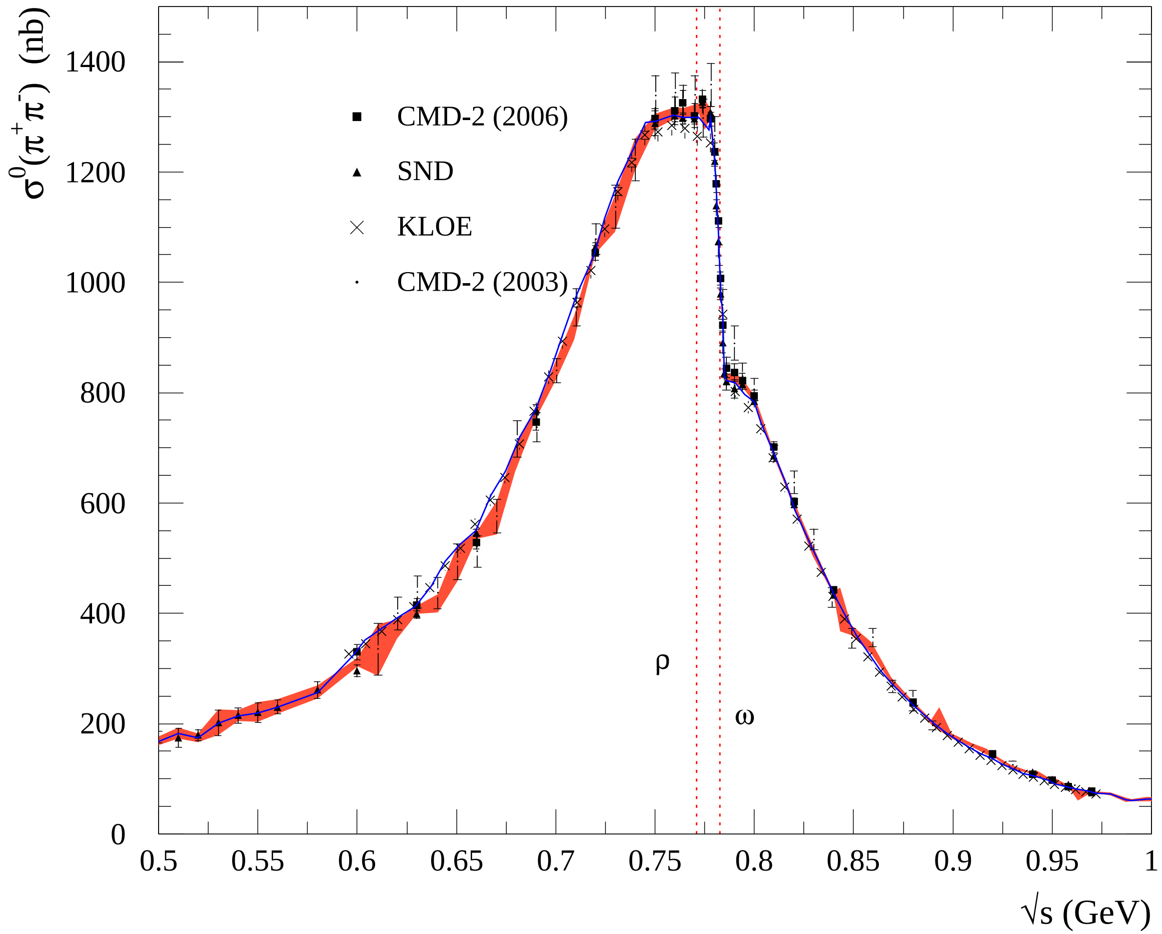
$\longrightarrow$ See talk of M. Benayoun and talks in τ session.

– Current 'consensus': Better NOT use τ data for $g-2$ predictions.

c) KLOE Radiative Return data (PLB606(2005)12) compared to CMD-2:

Radiative Return has recently developped (TH + EXP) into a powerful method with great potential, complementary to direct energy scan!

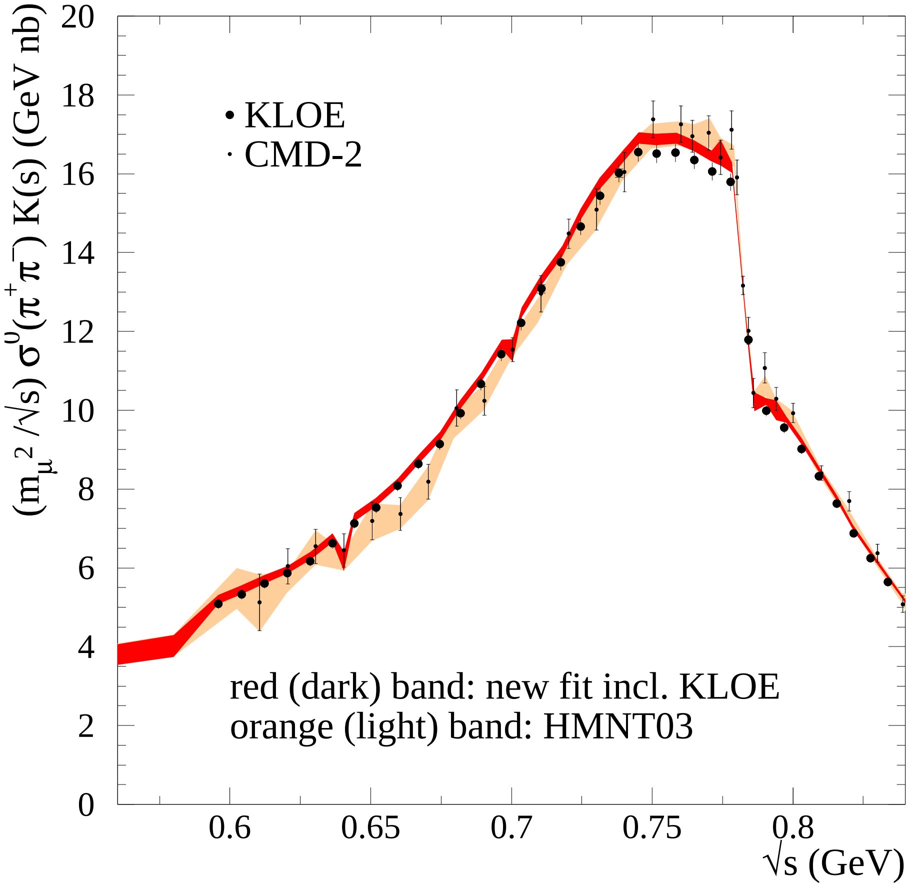




Phokara: $0.5 \rightarrow 0.2\%$ [Czyz et al.]

Good overall agreement with CMD-2, especially at larger $\sqrt{s}$ (where discrepancy with τ data is most significant)

But, locally, not such a good agreement between KLOE and CMD-2:



— Two different KLOE Rad. Ret. analyses (tagged/untagged) w. more data in progress.

  * Preliminary results very promising! Even smaller errors and better agreement.
  * Similar analysis from BaBar under way. ⟶ Exciting news at Glasgow, more tomorrow?!

— So far: HMNT integrate KLOE separately, then combine consistent results:

$a_\mu^{\pi\pi,LO}(\mathrm{fit\ w/o\ KLOE}, 0.6-0.97\ \mathrm{GeV}) = (384.3 \pm 2.5)\times 10^{-10}$, KLOE only: $(385.7 \pm 4.9)\times 10^{-10}$.

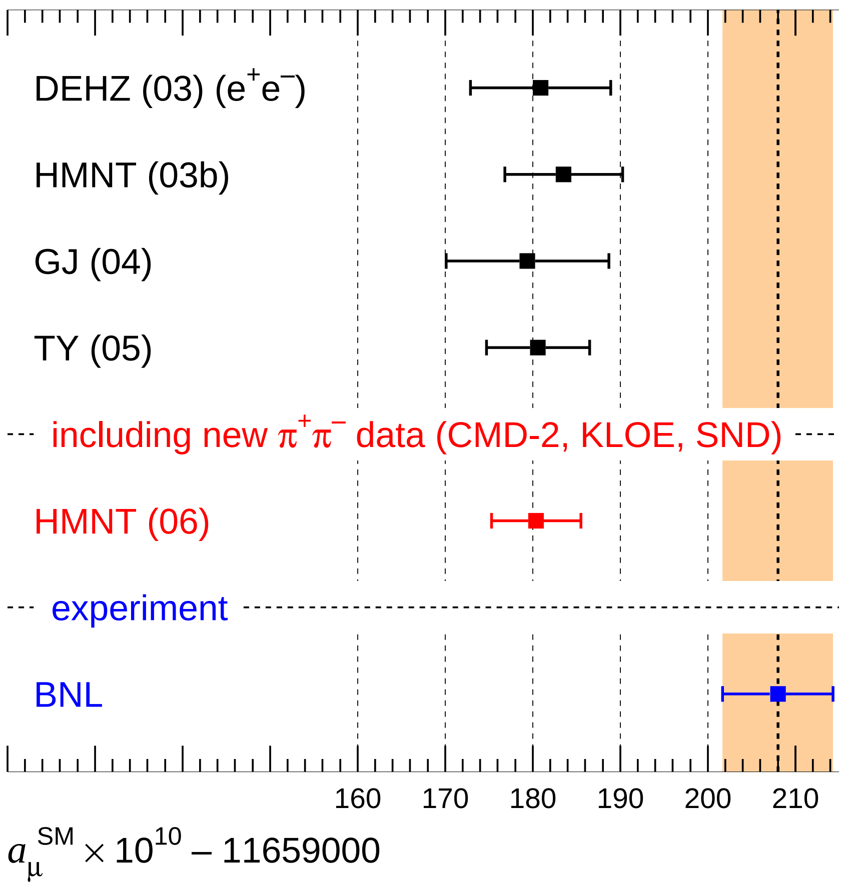
The different SM contributions numerically:

HMNT06

Source	contr. to $a_\mu \times 10^{11}$	remarks
QED	$116\,584\,718.09 \pm 0.16$	up to 5-loop! (Kinoshita+Nio, Passera)
	(was $116\,584\,719.35 \pm 1.43$)	▶ incl. recent updates of α
EW	154 ± 2	2-loop, Czarnecki+Marciano+Vainshtein
		(agrees very well with Knecht+Peris+Perrottet+deRafael)
LO hadr.	$7110 \pm 50 \pm 8 \pm 28$	Davier+Eidelman+Hoecker+Zhang '03b (τ)
	$6963 \pm 62 \pm 36$	Davier+Eidelman+Hoecker+Zhang '03b (e^+e^-)
	$\mathbf{6924 \pm 59 \pm 24}$	Hagiwara+Martin+Nomura+T 03
new data:	$\mathbf{6894 \pm 42 \pm 18}$	HMNT06, incl. recent CMD-2, SND, KLOE data
NLO hadr.	$\mathbf{-97.9 \pm 0.9}$	HMNT, in agreem. with Krause '97, Alemany+D+H '98
L-by-L	136 ± 25	▶ Melnikov+Vainshtein
< Dec. 2003:	80 ± 40	compilation from Nyffeler, `hep-ph/0203243`
		$\sim$ agrees (num.) w. Bijnens+Pallante+Prades
		and Hayakawa+Kinoshita *after*
< Nov. 2001:	(-85 ± 25)	the 'famous' sign error, $2.6\,\sigma \to 1.6\,\sigma$
$\sum$	$\mathbf{116591804 \pm 51}$	with HMNT06 (e^+e^-)

For the first time TH is (slightly) more precise than EXP:

a_μ^{SM} compared to BNL world av.



DEHZ 06: 180.5 ± 5.6 $[3.3\,\sigma]$

Jegerlehner 06: 179.3 ± 6.8 $[3.2\,\sigma]$

Recent changes

TH: Update of QED, up to 5–loop, new α:

was: $(116\ 584\ 719.35 \pm 1.43) \cdot 10^{-11}$

$\rightarrow$ is now: $(116\ 584\ 718.09 \pm 0.16) \cdot 10^{-11}$

TH: Improved LO hadr. (from e^+e^-):

Now, with new CMD-2, SND, KLOE:

$(6924 \pm 64) \cdot 10^{-11} \longrightarrow (6894 \pm 46) \cdot 10^{-11}$

EXP: BNL's '01 μ^- data [PRL92(2004)161802]:

$a_{\mu^-} = 11\ 659\ 214(8)(3) \times 10^{-10}\ (0.7 ppm)$

$\rightarrow a_\mu = 116\ 592\ 080(63) \times 10^{-11}\ (0.5 ppm)$

▶ With this input HMNT get:

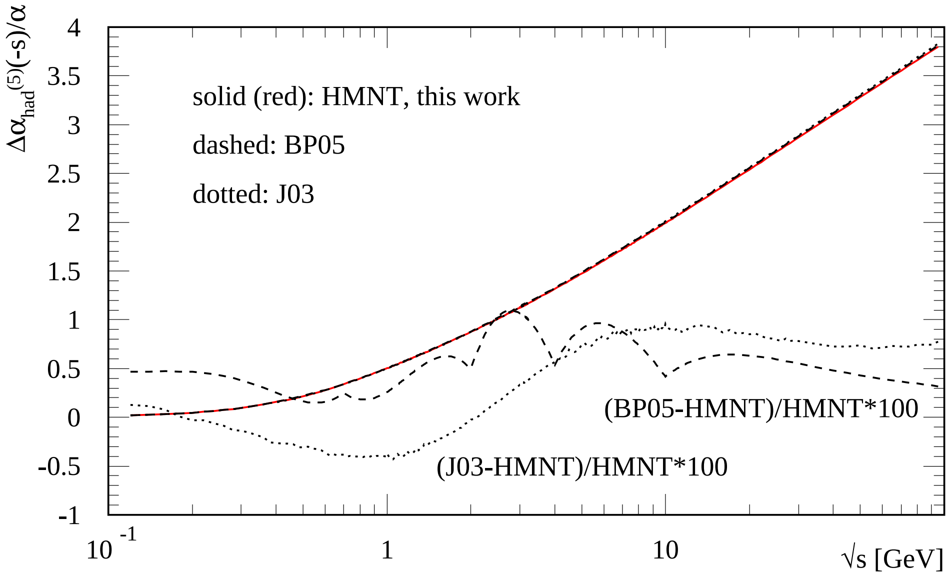
$a_\mu^{EXP} - a_\mu^{TH} = (27.6 \pm 8.1) \cdot 10^{-10}$, $\sim \mathbf{3.4\sigma}$

.. Discrepancy increased .. still not fully conclusive .. constrain SUSY ..

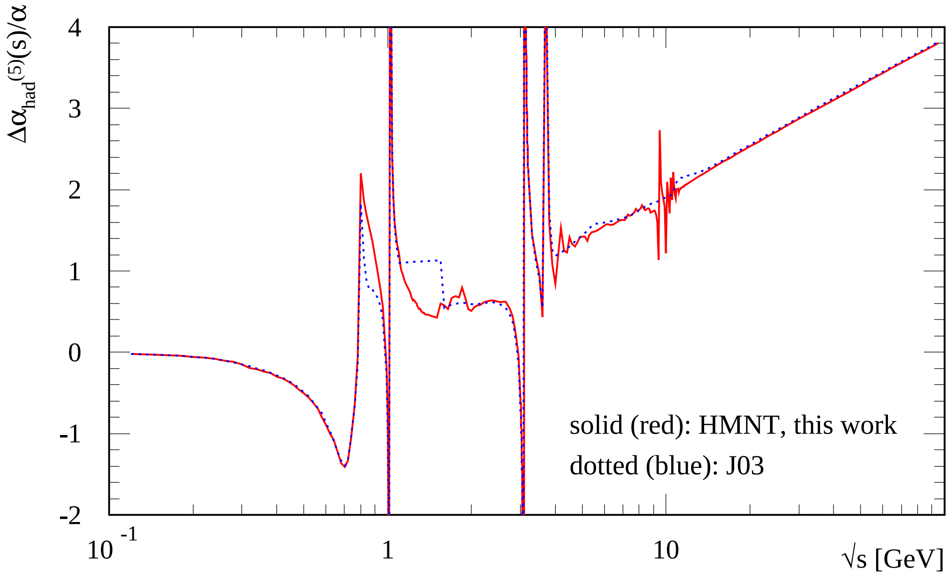
III. $\alpha_{\text{QED}}(q^2)$ at low and high energies

HMNT's evaluation of $\alpha_{\text{QED}}(q^2)$ compared to other parametrizations:

Spacelike (smooth $\alpha(q^2 < 0)$)



Timelike ($\alpha(q^2 > 0)$ follows resonance structure)



- Differences between parametrizations significant
- Slight shift in $\alpha(M_Z^2)$, see below
- What is in the MCs used by the experiments?
- $g - 2$ and EW precision fits ($\boldsymbol{m_H}$!) need best control/error

What about $\Delta\alpha(M_Z^2)$?

- With the same compilation of $\sigma_{\rm had}$ as for $g-2$ we find:

$$\boldsymbol{\Delta\alpha^{(5)}_{\rm had}(M_Z^2) \;=\; 0.02768 \pm 0.00022}$$

i.e. $\alpha(M_Z^2)^{-1} \;=\; 128.937 \pm 0.030$ (HMNT '06)

Other compilations:

Group	$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$	remarks
Burkhardt+Pietrzyk '05	0.02758 ± 0.00035	data driven
Troconiz+Yndurain '05	0.02749 ± 0.00012	pQCD
Kühn+Steinhauser '98	0.02775 ± 0.00017	pQCD
Jegerlehner '06	0.02761 ± 0.00023	data driven/pQCD
$(s_0^2 = (10\mathrm{GeV})^2)$	0.02759 ± 0.00017	Adler fct, pQCD
HMNT '06	0.02768 ± 0.00022	data driven

Adler function: $D(-s) \;=\; \dfrac{3\pi}{\alpha} s \dfrac{\mathrm{d}}{\mathrm{d}s}\Delta\alpha(s) \;=\; -(12\pi^2) s \dfrac{\mathrm{d}\Pi(s)}{\mathrm{d}s}$

allows use of pQCD and minimizes dependence on data.

IV. The next round

Where is improvement needed most urgently?

Pie diagrams of contributions to a_μ and $\alpha(M_Z)$ and their errors^2:

Critical regions:

$\rightarrow a_\mu$:

2π improved a lot, but

still ρ central regime,

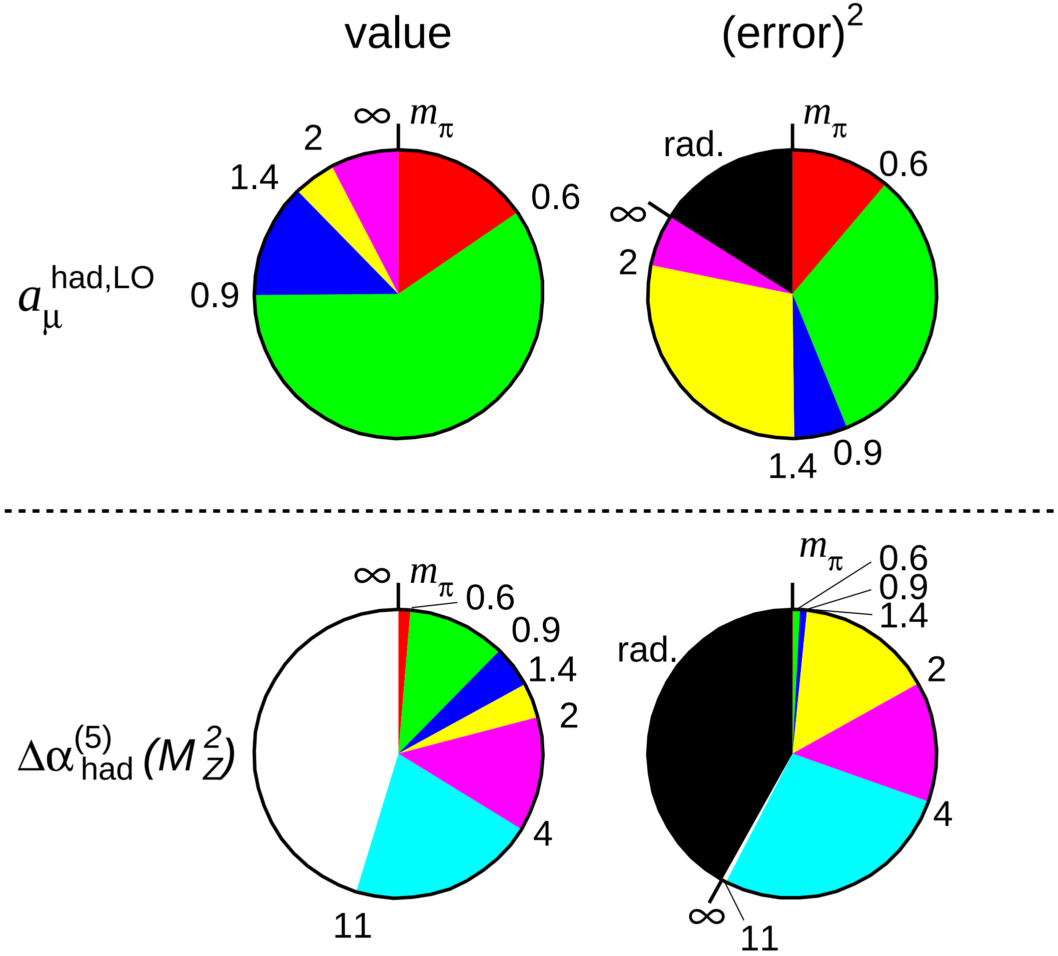
region below 2 GeV !

$\rightarrow \alpha(M_Z)$:

again below 2 GeV,

$+$ intermed./large energies,

w. *better* control of radcors!



Summary/Outlook:

- ▶ Impressive improvements of low energy hadronic data from many experiments
  $\rightsquigarrow$ determination of α_s, m_Q; input for $\alpha(q^2)$ and hadr. VP contr. to $g-2$.
- ▶ $\Delta\alpha(M_Z^2)$ and m_t: further tension in the EW precision fit.
- ▶ $\boldsymbol{g-2}$: deviation widened to $\mathbf{3.4}\,\boldsymbol{\sigma}$, possibly solved by (and constraining) SUSY;

— accuracy of/agreement between input data better than ever

— important checks positive; agreement between different groups

— VP still the leading uncertainty, but l-by-l could finally become the limiting factor

— TH for the first time slightly more accurate than EXP,
and more data to come/already there!

♣ Strong case and renewed interest/chances for a new $g-2$ experiment at BNL, Fermilab or possibly J-PARC, see D. Hertzog's talk.

Could TH match a drastically improved exp. accuracy in $g-2$?

The prospects are good:

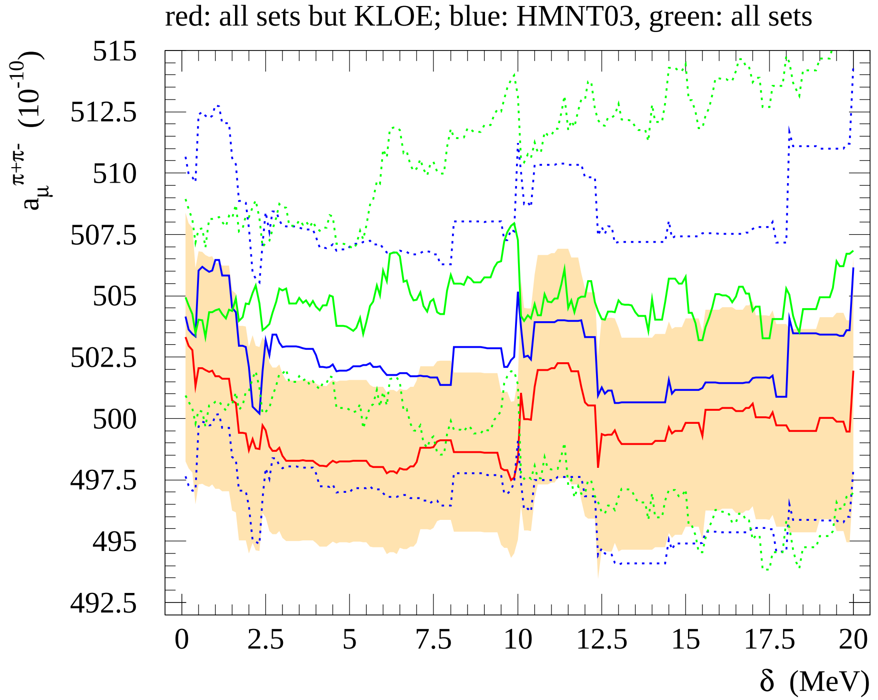
- Further Radiative Return analyses from KLOE eagerly awaited...
  $\longrightarrow$ check 2π down to threshold and hopefully combine to squeeze error.
- BaBar already very successful with RadRet for higher multiplicity channels
  $\hookrightarrow$ critical region $1.4\ldots 2$ GeV should improve soon.
  $\longrightarrow$ $\pi\pi\gamma$ analysis on the way.
- Opportunities for BELLE?
- With upcoming VEPP-2000 in Novosibirsk (and hopefully DAFNE-2 here in Frascati) improvement of factor 2–3 in reach!
- At higher energies, relevant for $\Delta\alpha$, more analyses from CLEO at Cornell and BES-II at BEPC in Beijing; soon BEPCII with BES-III.

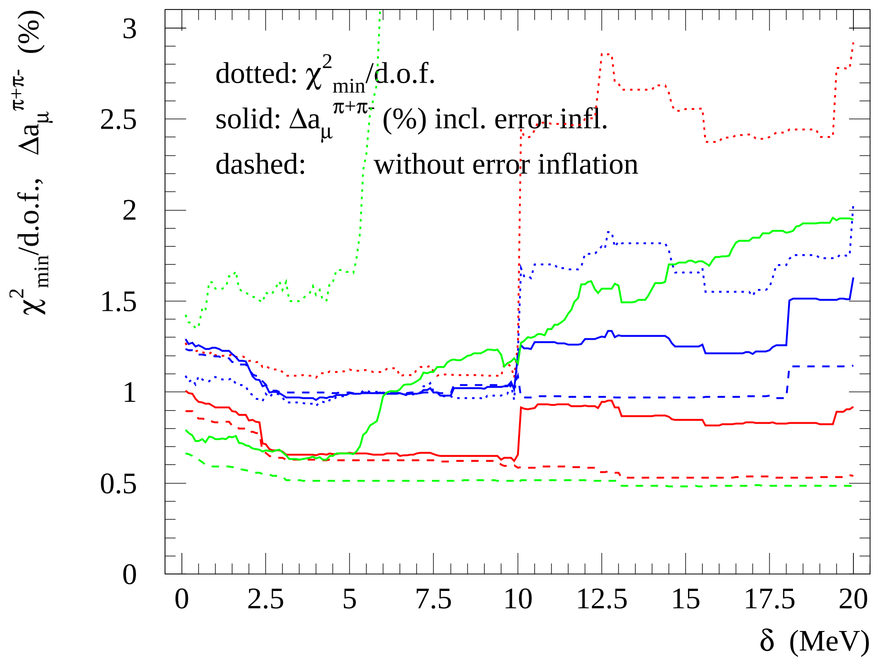
Extras:

Behaviour of $\pi^+\pi^-$ data fit

including new CMD-2, SND and KLOE data, as a function of the binning-parameter δ

contribution to a_μ with error band $\longrightarrow$





Without KLOE:

– a_μ and its error stable w.r.t. variation of the 'cluster' size

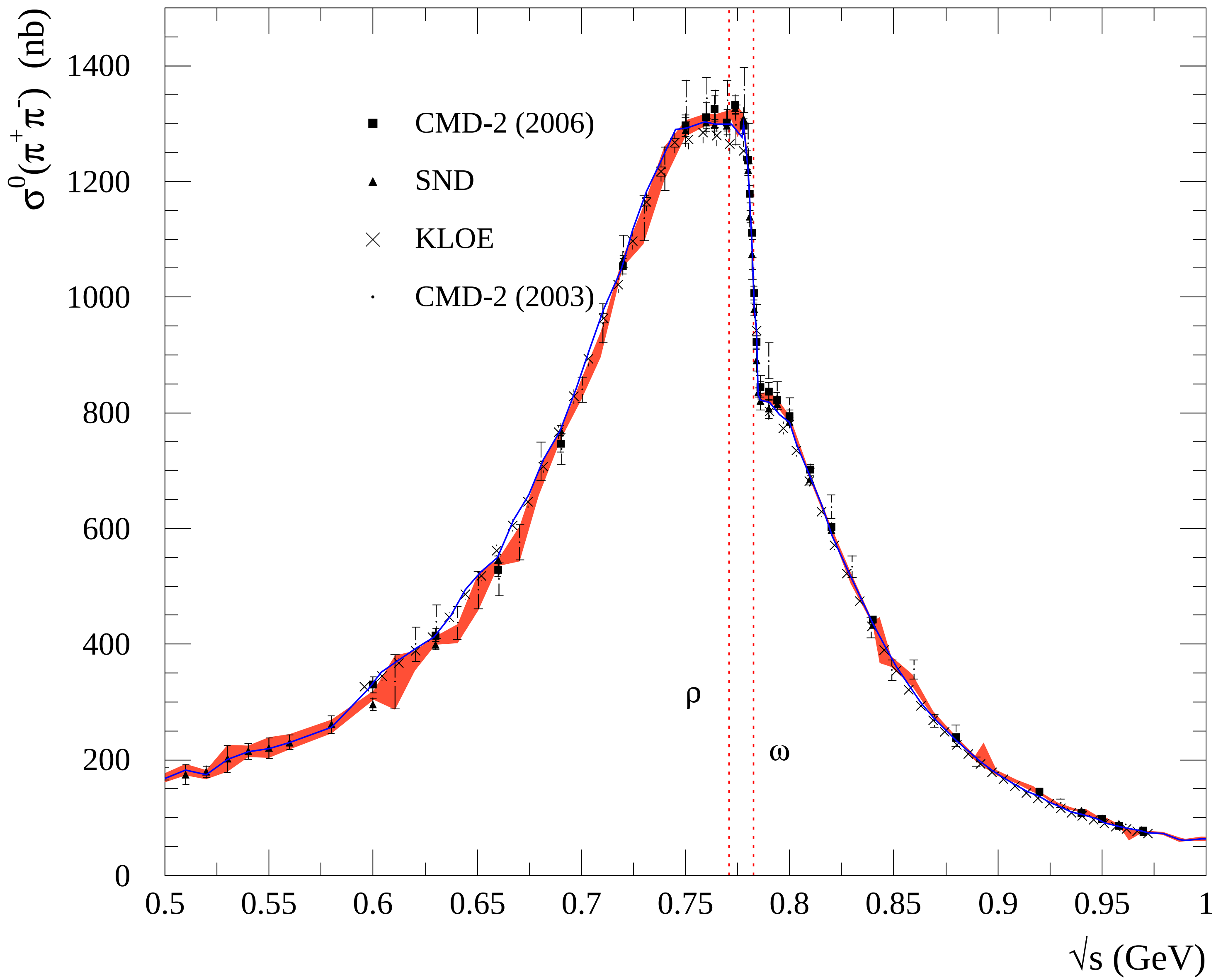
– significant improvement of error through clustering with $\delta \approx 4$ MeV

– too fine or wide clustering would worsen error *and* fit, which is otherwise good: $\chi^2_{\rm min}/d.o.f. \sim 1$

With KLOE:

– 'tension' in the unstable fit, bad χ^2_{min}

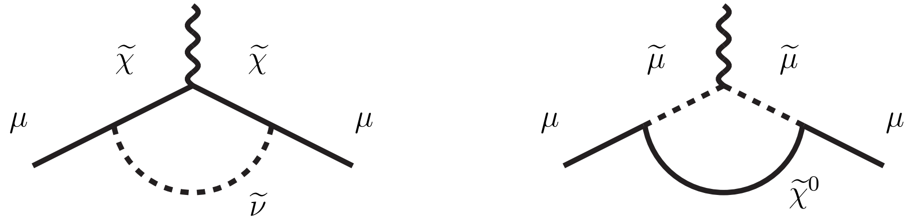
The most important channel: $\pi^+\pi^-$ (recent data only)

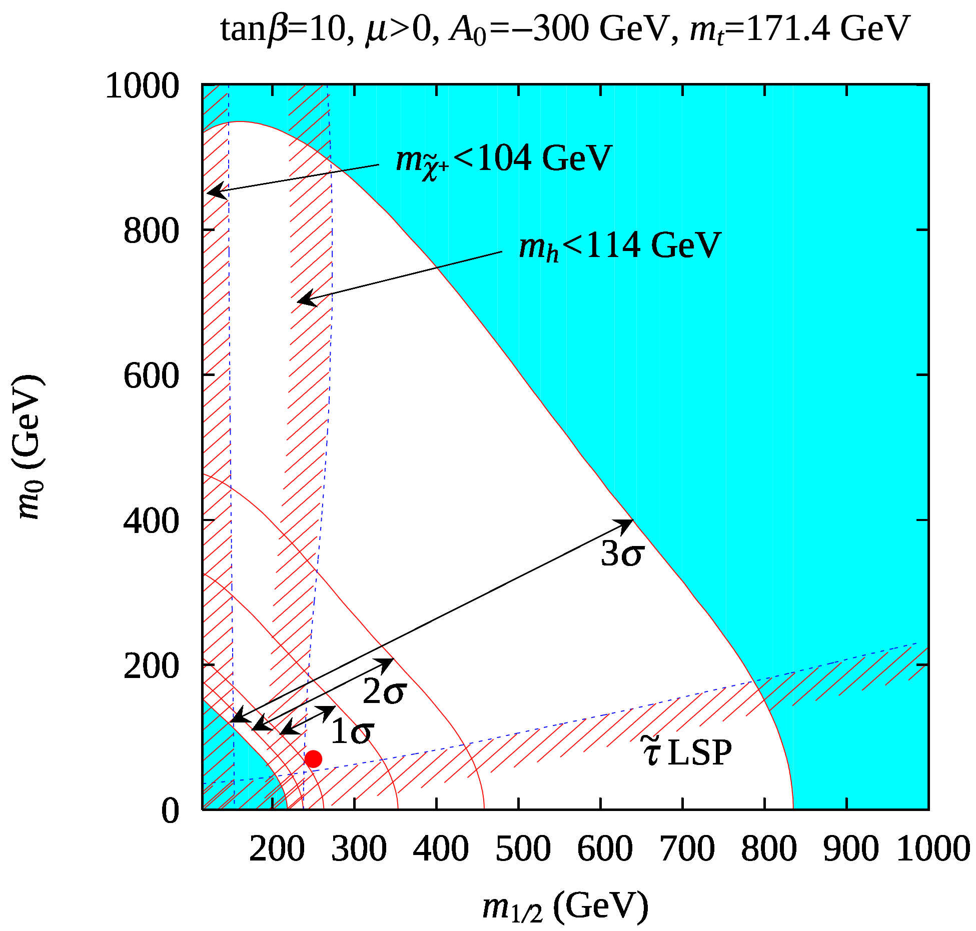


SUSY contributions in a_μ ?

$$a_\mu^{\mathrm{SUSY,1-loop}} \simeq \frac{\alpha}{8\pi \sin^2\theta_W} \tan\beta \; \mathrm{sign}(\mu) \frac{m_\mu^2}{M_{\mathrm{SUSY}}^2}$$

They mainly come from:





SUSY is a good candidate to explain $\Delta a_\mu = a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}}$, but

- no chargino at LEP
- so far no light Higgs
- $\tilde\tau$ prob. not LSP
- $+$ limits from direct searches
- SPS 1a′ in 1σ band from $g-2$

EW Precision Fits

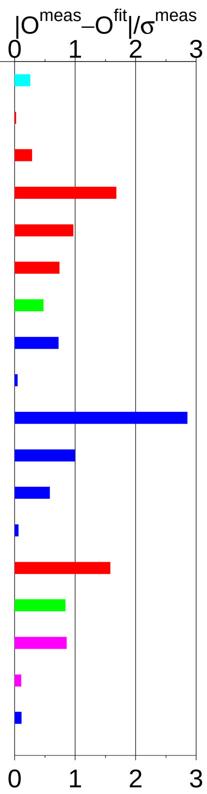
- With the same compil. of $\sigma_{\rm had}$ as for $g-2$ we find:

$$\boldsymbol{\Delta\alpha^{(5)}_{\rm had}(M_Z^2) \;=\; 0.02768 \pm 0.00022}$$

i.e. $\alpha(M_Z^2)^{-1} \;=\; 128.937 \pm 0.030$ (HMNT '06)

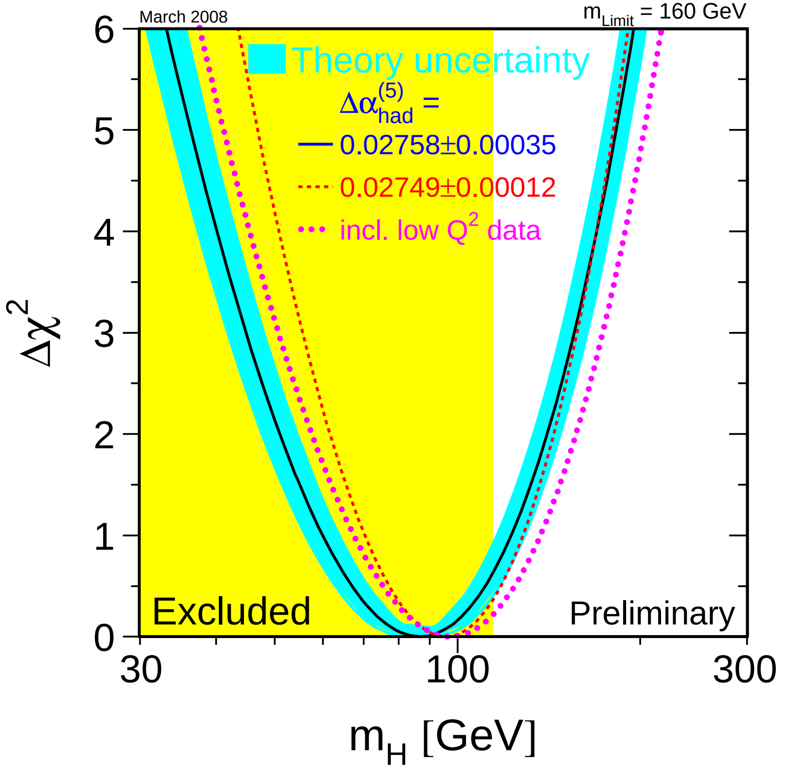
LEP EWWG 08:

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.743
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01643
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21581
R_c	0.1721 ± 0.0030	0.1722
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.398 ± 0.025	80.377
Γ_W [GeV]	2.097 ± 0.048	2.092
m_t [GeV]	172.6 ± 1.4	172.8



March 2008

Fit of the SM Higgs mass: EWWG 08



$\longrightarrow$ preferred m_H moves down w. higher $\Delta\alpha$

$\longrightarrow$ lower error lowers excl. limit

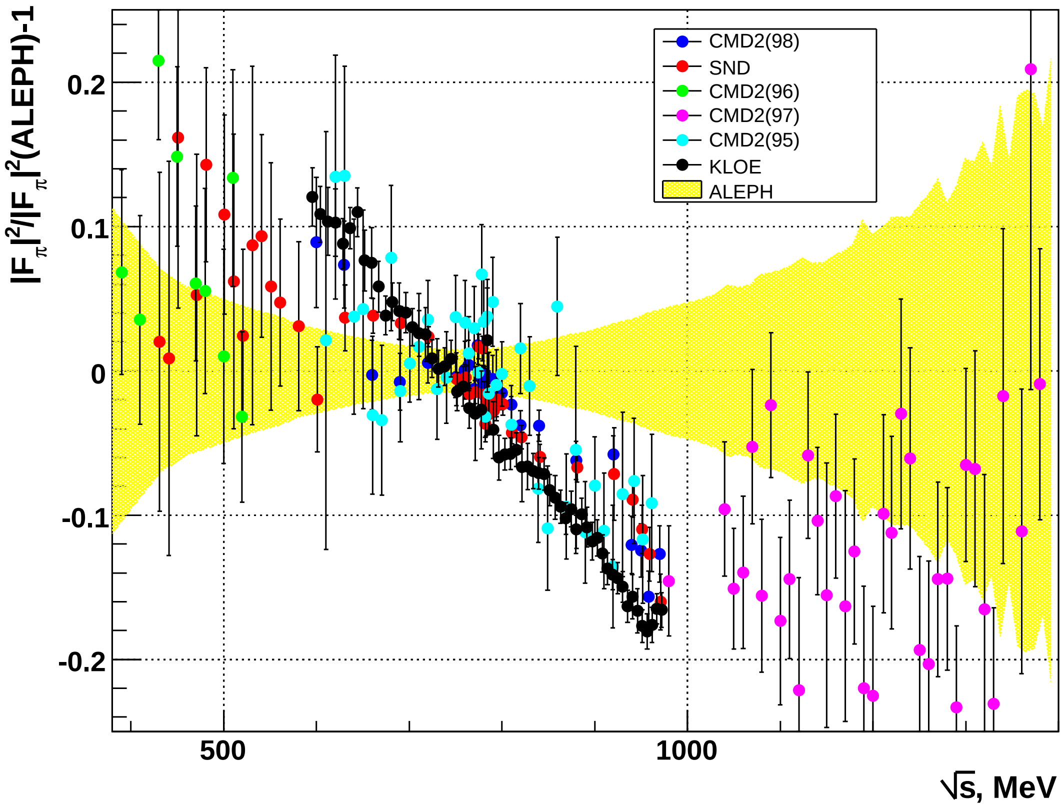
Difficult to 'cure' $g-2$ and m_H by changing $\sigma_{\rm had}$

τ spectral functions:

- CVC hypothesis (Isospin-symm.) connects $\tau^- \to hadrons$ to $e^+e^- \to hadrons$
- Sizeable Isospin-symmetry violations [from radiative corrections, mass differences ($m_{\pi^-} \neq m_{\pi^0}$), $\rho - \omega$ interf.] ($\longrightarrow$ Cirigliano+Ecker+Neufeld)
- Role of possible $\rho^0 - \rho^\pm$ mass difference?
- Width difference $\Gamma_{\rho^0} \neq \Gamma_{\rho^\pm}$?

  Large effects possible.

S. Eidelman (ICHEP06): τ compared to e^+e^- data

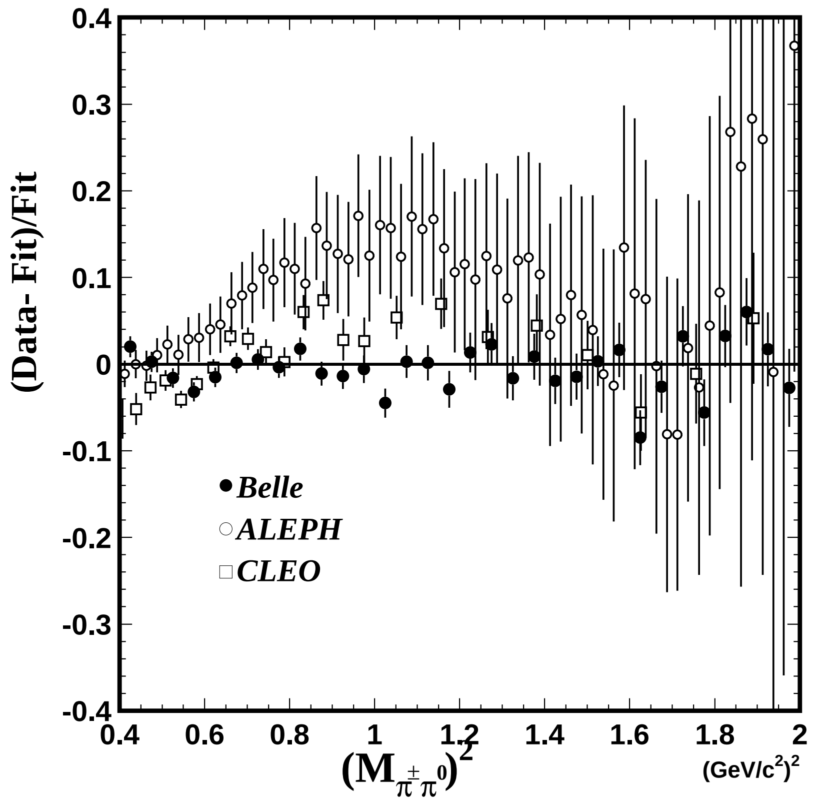


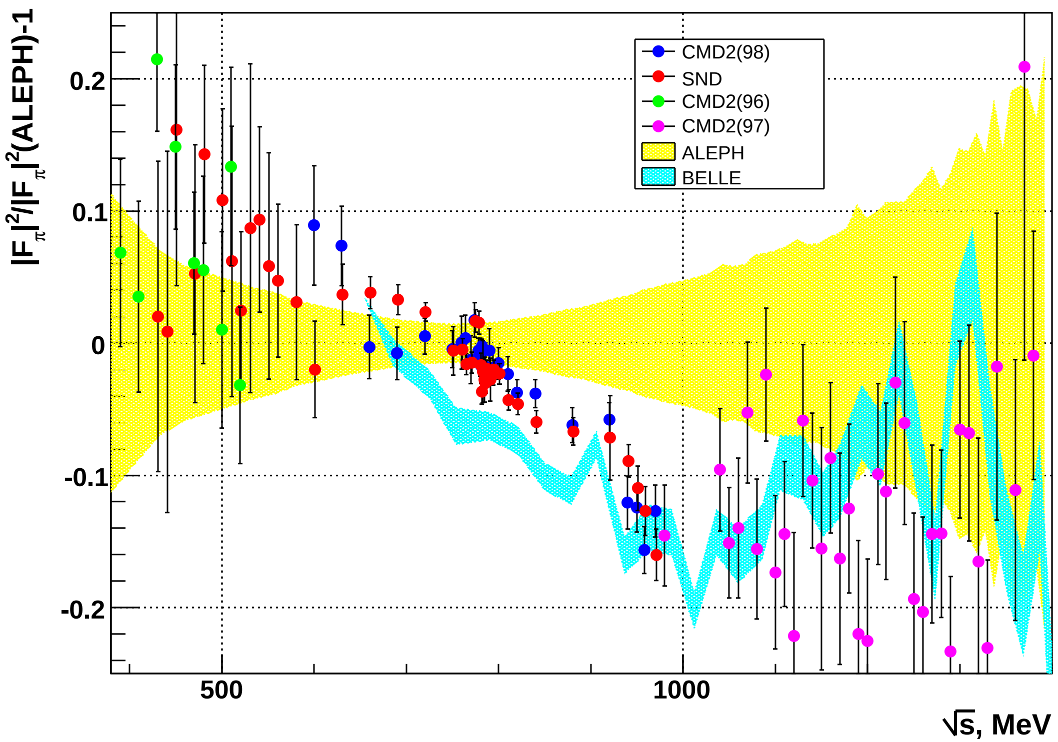
$\longrightarrow$ Bad disagreement between τ and e^+e^- data already for $[B_\tau - B_{CVC}]_{\pi\pi^0} : 4.5\,\sigma$ (not only for more complicated spectral functions)

$\hookrightarrow$ Is everything under control *at the % level*? Is something wrong with data? H^-?

- KLOE Radiative Return agrees with e^+e^- scan experiments.

- Prel. $\tau^- \to \pi^-\pi^0\nu_\tau$ anal. from *BELLE* at KEK, `hep-ex/0512071`; S. Eidelman



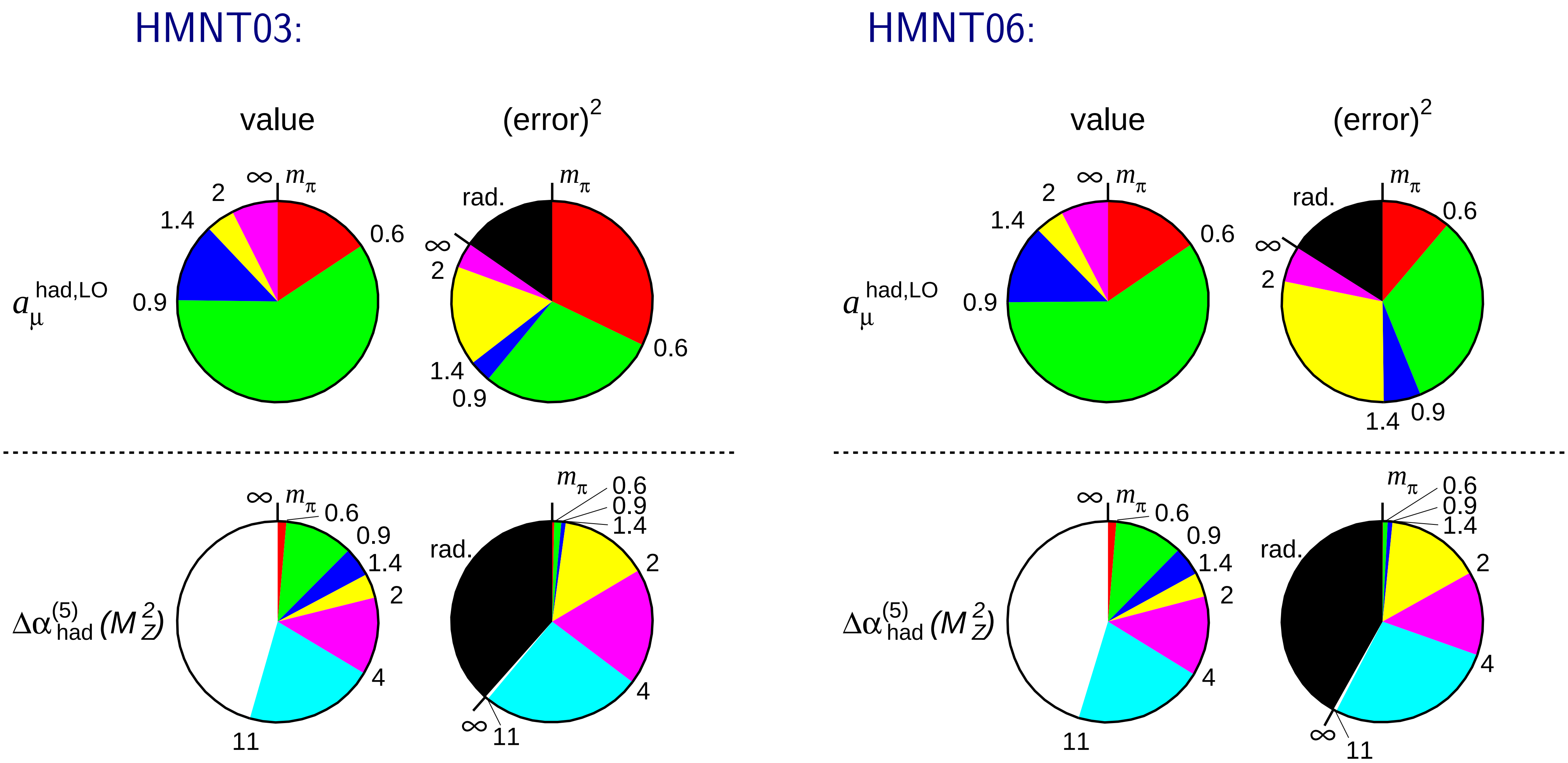


- *BELLE* τ spectral function is well below ALEPH and CLEO. Final result?
- Glasgow Workshop: Davier proposed solution of τ puzzle, but highly controversial..

$\longrightarrow$ sizeable $\Gamma_{\rho^0} \neq \Gamma_{\rho^\pm}$ corrections + smaller changes: $[B_\tau - B_{CVC}]_{\pi\pi^0} : 4.5 \;\to\; 2.6\,\sigma$

- New work from M. Benayoun et al., `arXiv:0711.4482`, EPJC xxx: consistency?

Pie diagrams of contributions to a_μ and $\alpha(M_Z)$ and their errors^2:



By far the biggest change was in $\pi\pi$: $502.78 \pm 5.02 \longrightarrow 498.46 \pm 2.87$