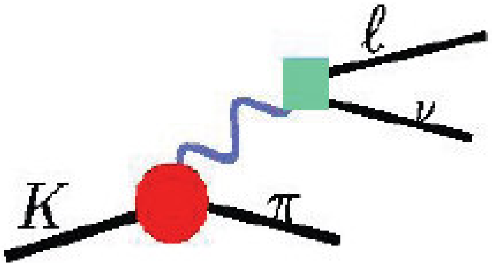
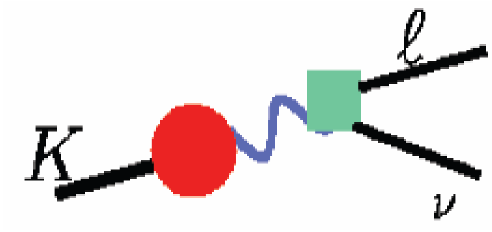
Kaon WG

Test of the Standard Model with Kaon decays





Barbara Sciascia
LNF-INFN
for the Kaon WG, FlaviaNet
PhiPsi08 - LNF, 7 April 2008

Introduction

Analysis of leptonic and semileptonic kaon decays data

- provide precise determination of **fundamental SM couplings**;
- set stringent SM tests, almost **free from hadronic uncertainties**;
- discriminate between **different NP scenarios**.

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{G_F^2 m_K^5}{192\pi^3} C_K S_{\rm ew} \, |V_{us}|^2 f_+(0)^2 \, I_K^\ell(\lambda_{+,0}) \, \left(1 + \delta^K_{SU(2)} + \delta^{K\ell}_{\rm em}\right)^2$$

$$\frac{\Gamma(K^\pm_{\ell 2(\gamma)})}{\Gamma(\pi^\pm_{\ell 2(\gamma)})} = \left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{f_K^2 m_K}{f_\pi^2 m_\pi} \left(\frac{1 - m_\ell^2/m_K^2}{1 - m_\ell^2/m_\pi^2}\right)^2 \times (1 + \delta_{\rm em})$$

- Test **unitarity of the quark mixing matrix** (V_{CKM}):

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \epsilon_{\rm NP} \qquad\qquad \epsilon_{\rm NP} \sim M_W^2/\Lambda_{\rm NP}^2$$

→ present precision on V_{us} (dominant source of error) and V_{ub} negligible ($|V_{ub}{}^2| \sim 10^{-5}$) set bounds on NP well above 1 TeV.

- Comparison of Ke3 and Kμ3 modes, tests the **lepton universality**.

The FlaviaNet Kaon working group

- **The FlaviaNet Kaon WG (www.lnf.infn.it/wg/vus/)**. Recent kaon physics results come from many experimental (ISTRA+, KLOE, KTeV, NA48) and theoretical (Lattice, χ_{PT},). The main purpose of this working group is to perform precision tests of the Standard Model and to determine with high accuracy fundamental couplings (such as V_{us}) using all existing (published and/or preliminary) data on kaon decays, taking correlations into account.

- WG note: ***Precision tests of the Standard Model with leptonic and semileptonic kaon decays***, arXiv:0801.1817 [hep-ph] 11 Jan 2008.

Summary of the talk

Global fits and averages:

- K_L, K_S, and $K^{\pm}$, dominant BRs and lifetime.
- Parameterization of the $K \rightarrow \pi$ interaction (form factor)

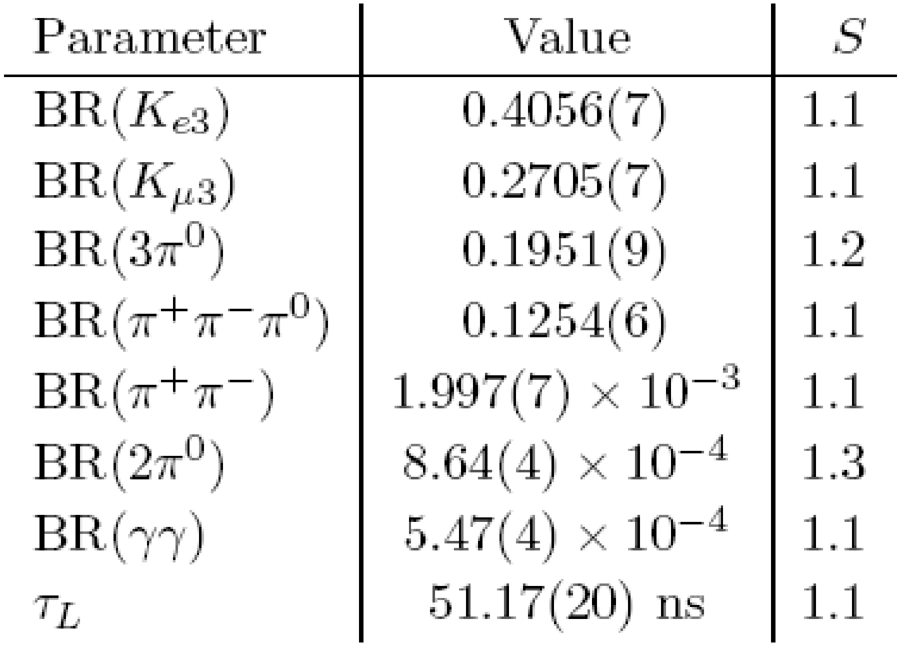
Physics results:

- $|V_{us}| \times f_+(0)$
- Test of lepton universality with $K_{\ell 3}$.
- $|V_{us}|/|V_{ud}| \times f_K/f_\pi$.
- Theoretical estimations of $f_+(0)$ and f_K/f_π.
- V_{us} and V_{ud} determinations.
- Bounds on helicity suppressed amplitudes.
- The special role of $BR(K^{\pm}e2)/BR(K^{\pm}\mu 2)$

K_L leading branching ratios and τ_L

18 input measurements:

- **5 KTeV** ratios
- **NA48** K_{e3} /2t and $\Gamma(3\pi^0)$
- **4 KLOE** BRs
- **KLOE**, **NA48** $\pi^+\pi^-/K_{l3}$
- **KLOE**, **NA48** $\gamma\gamma/3\pi^0$
- **PDG** ETAFIT for $\pi^+\pi^-/\pi^0\pi^0$
- **KLOE** τ_L from $3\pi^0$
- **Vosburgh** '72 τ_L



Parameter	Value	S
$\mathrm{BR}(K_{e3})$	0.4056(7)	1.1
$\mathrm{BR}(K_{\mu3})$	0.2705(7)	1.1
$\mathrm{BR}(3\pi^0)$	0.1951(9)	1.2
$\mathrm{BR}(\pi^+\pi^-\pi^0)$	0.1254(6)	1.1
$\mathrm{BR}(\pi^+\pi^-)$	$1.997(7) \times 10^{-3}$	1.1
$\mathrm{BR}(2\pi^0)$	$8.64(4) \times 10^{-4}$	1.3
$\mathrm{BR}(\gamma\gamma)$	$5.47(4) \times 10^{-4}$	1.1
τ_L	51.17(20) ns	1.1

8 free parameters, 1 constraint: ΣBR=1

Main differences wrt PDG06:

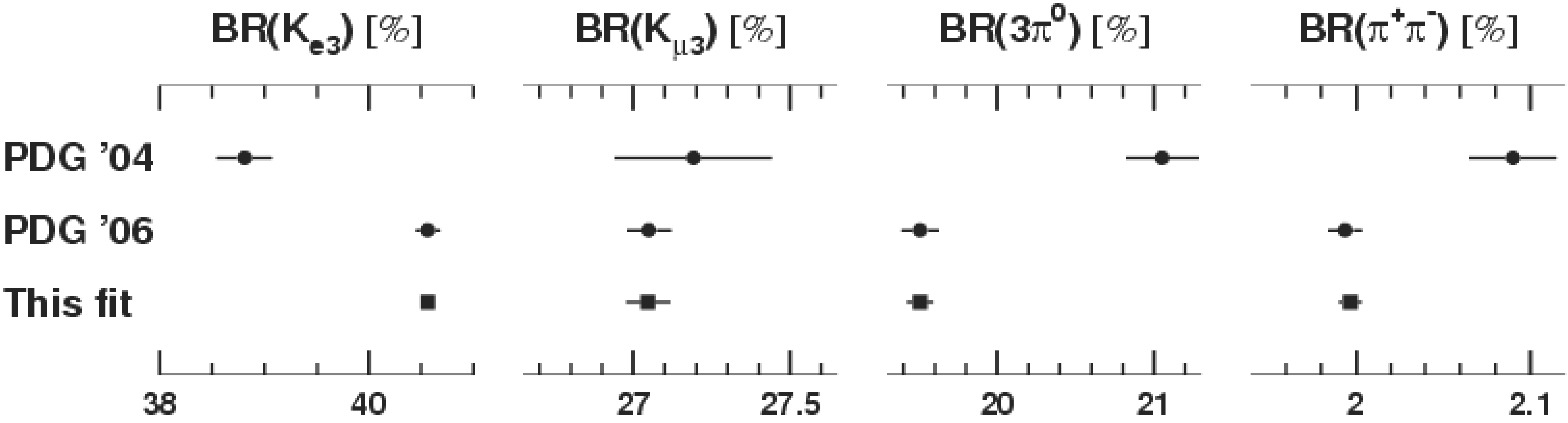
- For KLOE and KTeV, use values obtained before applying constraints.
- Make use of preliminary $\mathrm{BR}(3\pi^0)$ and new $\mathrm{BR}(\pi^+\pi^-)/\mathrm{BR}(\mathrm{Ke3})$ from NA48
- Fit parameter $\mathrm{BR}(\pi^+\pi^-)$ is understood to be inclusive of the DE component.

FlaviA net Kaon WG

Evolution of the average BR values

This fit χ^2/ndf = 20.2/11 (4.3%); PDG06 fit: χ^2/ndf = 14.9/9 (14.0%)
Minor differences wrt PDG06:

- contrast between KLOE BR($3\pi^0$) and other inputs involving BR($2\pi^0$) and BR($3\pi^0$)
- **treatment of the correlated KLOE and KTeV inputs**: more uniform scale factor in this fit and significantly smaller uncertainty for BR(Ke3).



K_S leading branching ratios and τ_S

4 input measurements:

KLOE BR(Ke3)/BR($\pi\pi$)
KLOE BR($\pi\pi$)/BR($\pi^0\pi^0$)
Universal lepton coupling
NA48 BR(Ke3)
τ_S: non CPT-constrained fit value, dominated by 2002 NA48 and 2003 KTeV measurements

4 free parameters: $K_S\pi\pi$, $K_S\pi^0\pi^0$,K_Se3,$K_S\mu$3 , 1 constraint: ΣBR=1

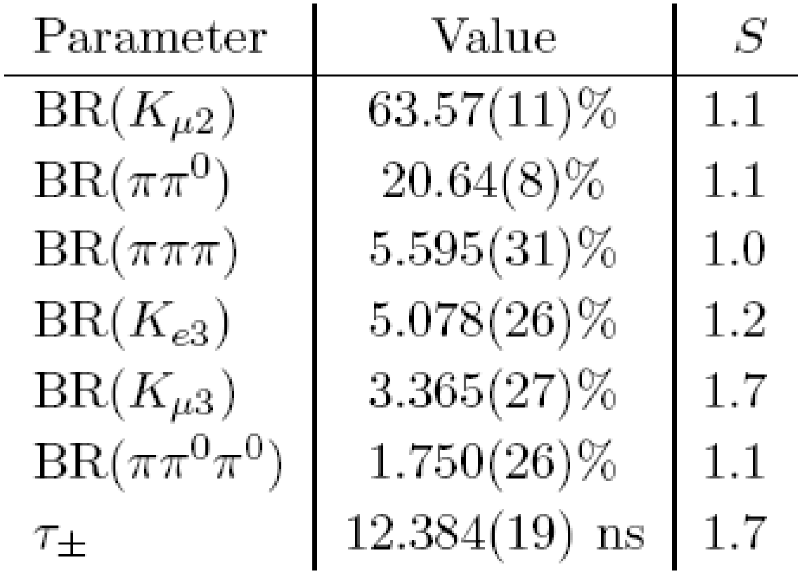
- **KLOE meas. completely determine the leading BR values**.
- NA48 Ke3 input improve the BR(Ke3) accuracy of about 10%.
- BR(K_Se3)/BR(K_Le3) from NA48 not included (need of a K_L and K_S combined fit)
- Combined fit would be useful in properly account for preliminary NA48 $\Gamma(K_L \to 3\pi^0)$ and PDG ETAFIT, used in the K_L fit.

$K^{\pm}$ leading branching ratios and $\tau_{\pm}$

26 input measurements:

- **5 older τ values in PDG**
- **2 KLOE τ**
- **KLOE BR(μν)**
- **KLOE *Ke3*, *Kμ3*, and *Kπ2* BRs**
- **ISTRA+ $K_{e3}/\pi\pi^0$**
- **NA48/2 $K_{e3}/\pi\pi^0$, $K_{\mu3}/\pi\pi^0$**
- **E865 K_{e3}/Kdal**
- **3 old $\pi\pi^0/\mu\nu$**
- **2 old *Ke3*/2 body**
- **3 *Kμ3/Ke3* (2 old)**
- **2 old + 1 KLOE results on 3π**

7 free parameters,
1 constraint: ΣBR=1



Parameter	Value	S
$\mathrm{BR}(K_{\mu2})$	63.57(11)%	1.1
$\mathrm{BR}(\pi\pi^0)$	20.64(8)%	1.1
$\mathrm{BR}(\pi\pi\pi)$	5.595(31)%	1.0
$\mathrm{BR}(K_{e3})$	5.078(26)%	1.2
$\mathrm{BR}(K_{\mu3})$	3.365(27)%	1.7
$\mathrm{BR}(\pi\pi^0\pi^0)$	1.750(26)%	1.1
$\tau_{\pm}$	12.384(19) ns	1.7

Don’t use the 6 BR meas. from Chiang;

- no implementation of radiative corrections
- 6 BR constrained to sum to unit.
- the correlation matrix not available.

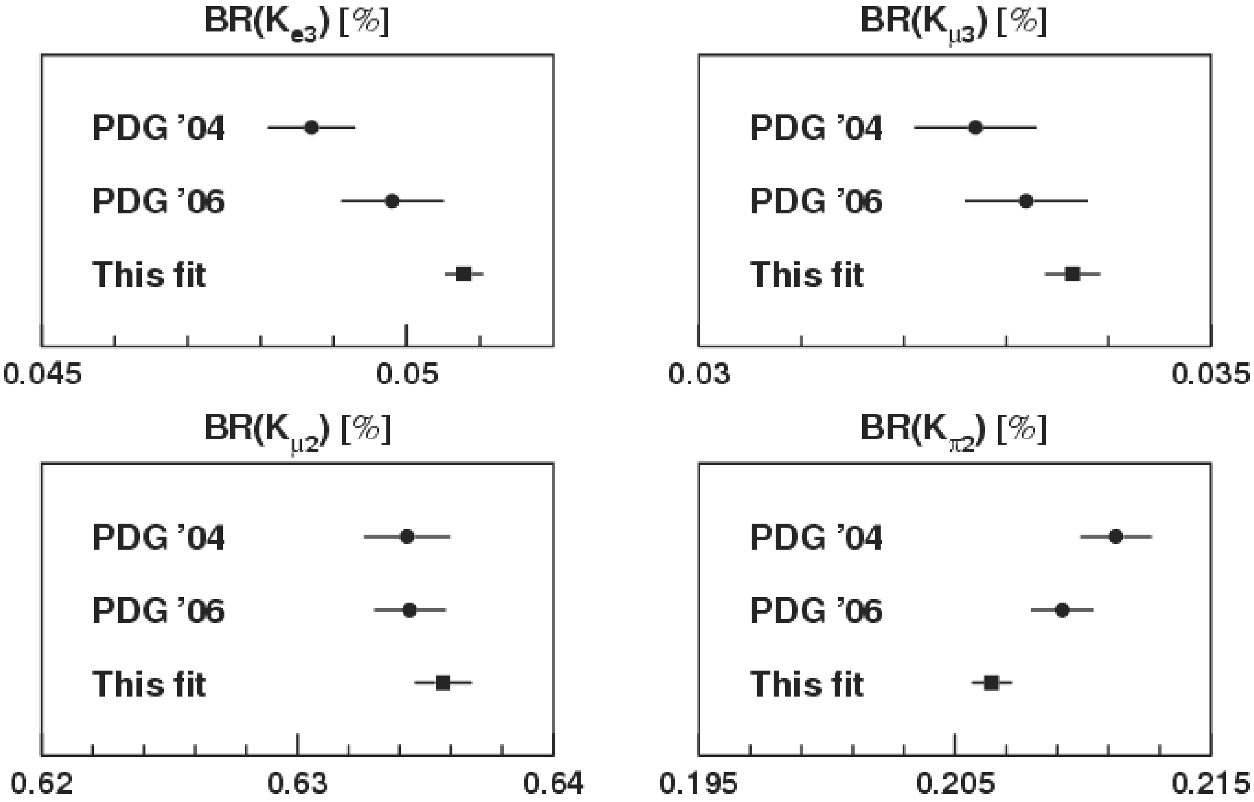
Try to discard many other old meas.:

- no recent meas. involving BR(πππ)
- fit instable if only recent are used.

Evolution of the average BR values

- This fit **χ^2/ndf = 42/20 (0.31%)**; PDG06 fit: χ^2/ndf = 30/19 (5.2%)
- If 5 older $\tau_\pm$ measurements replaced by PDG avg (with S=2.1), **χ^2/ ndf = 24/16 (8.4%)** with **no significant changes to central values or errors**.
- include many new results
- **some conflict among newer meas. involving BR(Ke3)**: the pulls are +1.04, -0.26, -0.73, and -2.13, for NA48, BNL-E865, ISTRA+, and KLOE respectively.

- Evolution of the BR($K_{\ell 3}$) and for the important normalization channels.



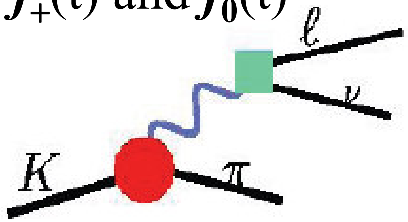
FlaviAnet Kaon WG

Parameterization of $K_{\ell 3}$ form factors

- Hadronic K→π matrix element is described by two form factors ***f***$_+$(t) and ***f***$_0$(t) defined by:

$$\langle \pi^- (k) | \bar{s} \gamma^\mu u | K^0 (p) \rangle = (p+k)^\mu f_+(t) + (p-k)^\mu f_-(t)$$

$$f_-(t) = \frac{m_K^2 - m_\pi^2}{t} \left(f_0(t) - f_+(t) \right)$$



- Experimental or theoretical inputs to define ***t***-dependence of ***f***$_{+,0}$(t).
- ***f***$_-$(t) term negligible for K$_{e3}$.
- Taylor expansion:

$$\tilde{f}_{+,0}(t) \equiv \frac{f_{+,0}(t)}{f_+(0)} = 1 + \lambda'_{+,0} \frac{t}{m_\pi^2} + \frac{1}{2} \lambda''_{+,0} \left(\frac{t}{m_\pi^2} \right)^2 + \; \ldots .$$

- Obtain λ′ ,λ″, from fit to data distributions (more accurate than theor. predictions).
- λ′ and λ″ are **strongly correlated: –95% for *f*$_+$(t), and –99.96% for *f*$_0$(t)**.

One parameter parameterizations:

- Pole parameterization
  (what vector/scalar state should be used?)

$$\tilde{f}_{+,0}(t) = \frac{M_{V,S}^2}{M_{V,S}^2 - t}$$

- Dispersive approach plus Kπ scattering data for both ***f***$_+$(***t***) and ***f***$_0$(***t***)

Vector form factor

Quadratic expansion:

- Measurements from ISTRA+, KLOE, KTeV, NA48 with K_Le3 and K^-e3 decays.
- **Good fit quality: χ^2/ndf=5.3/6(51%) for all data**; χ^2/ndf=4.7/4(32%) for K_L only
- The significance of the quadratic term is 4.2σ from all data and 3.5σ from K_L only.
- Using all data or K_L only changes the space phase integrals I^0_{e3} and $I^\pm_{e3}$ by 0.07% .
- Errors on I_{e3} are significantly smaller when K^- data are included.

A **pole parameterization** is in good agreement with present data:

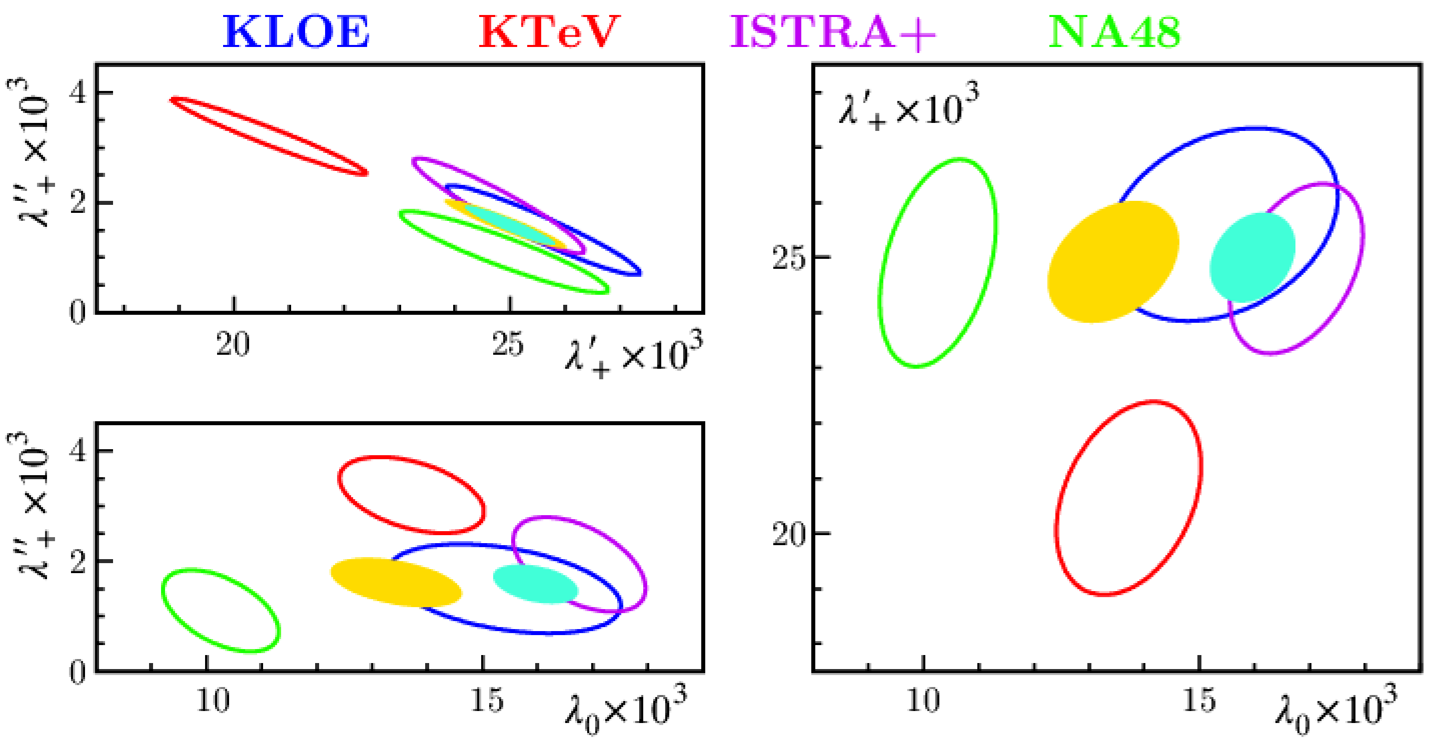
$$\tilde{f}_+(t) = M_V^2/(M_V^2 - t), \text{ with } M_V \sim 892 \text{ MeV} \qquad \lambda' = (m_{\pi+}/M)^2;\ \lambda'' = 2\lambda'^2$$

- KLOE, KTeV, NA48 quote value for M_V for pole fit to K_Le3 data (χ^2/ndf=1.8/2)
- The values for λ_+' and λ_+'' from pole expansion are in agreement with quadratic fit results.
- Using quadratic averages or pole fit results changes I^0_{e3} by 0.03% .

Improvements: **dispersive parameterization** for $f_+(t)$, with analytical and unitarity properties and a correct threshold behavior, (e.g.Passemar arXiv:0709.1235[hep-ph]) Dispersive results for λ_+ and λ_0 are in agreement with pole parameterization.

Vector and scalar form factor from $K_{\mu3}$

- λ_+', λ_+'' and λ_0 measured for $K\mu3$ from ISTRA+, KLOE, KTeV, and NA48.
- **NA48 result lies out of correlation directions in the [λ_+', λ_+'', λ_0] space**.
- Fit probability varies from 1×10^{-6} (with NA48) to 22.3% (without NA48).



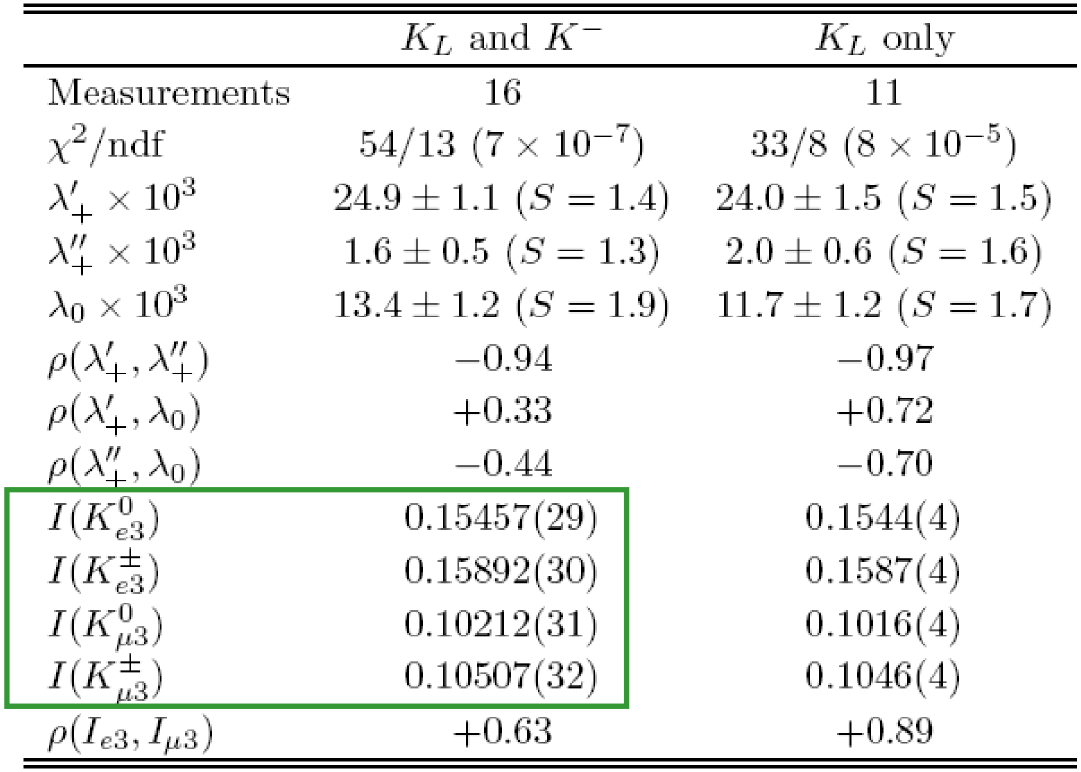
1σ contour for all the experimental results.

- Neglecting a quadratic term in the param. of scalar FF implies: $\boldsymbol{\lambda_0'\to\lambda_0'+3.5\lambda_0''}$
- Because of correlation, is not possible measure $\boldsymbol{\lambda_0''}$ at any plausible level of stat.

FlaviA net Kaon WG

Vector and scalar form factor from $K_{\ell 3}$

- Slope parameters λ_+', λ_+'' and λ_0 from ISTRA+, KLOE, KTeV, and NA48.



	K_L and K^-	K_L only
Measurements	16	11
χ^2/ndf	$54/13$ (7×10^{-7})	$33/8$ (8×10^{-5})
$\lambda_+' \times 10^3$	24.9 ± 1.1 $(S = 1.4)$	24.0 ± 1.5 $(S = 1.5)$
$\lambda_+'' \times 10^3$	1.6 ± 0.5 $(S = 1.3)$	2.0 ± 0.6 $(S = 1.6)$
$\lambda_0 \times 10^3$	13.4 ± 1.2 $(S = 1.9)$	11.7 ± 1.2 $(S = 1.7)$
$\rho(\lambda_+', \lambda_+'')$	-0.94	-0.97
$\rho(\lambda_+', \lambda_0)$	$+0.33$	$+0.72$
$\rho(\lambda_+'', \lambda_0)$	-0.44	-0.70
$I(K^0_{e3})$	0.15457(29)	0.1544(4)
$I(K^\pm_{e3})$	0.15892(30)	0.1587(4)
$I(K^0_{\mu 3})$	0.10212(31)	0.1016(4)
$I(K^\pm_{\mu 3})$	0.10507(32)	0.1046(4)
$\rho(I_{e3}, I_{\mu 3})$	$+0.63$	$+0.89$

Space integral used for the $|V_{us}|f_+(0)$ determination

Averages of quadratic fit results for Ke3 and Kμ3 slopes.

- Adding Kμ3 data to the fit doesn’t cause significant changes to I^0_{e3} and $I^\pm_{e3}$.
- The significance of the quadratic term λ_+'' is strong (3.6σ from to fit to all data).
- **NA48: Δ[I(Kμ3)] = 0.6%, but Ke3+Kμ3 average gives $\Delta[V_{us}f_+(0)] = -0.08\%$.**

Determination of $|V_{us}|\times f_+(0)$

$$\Gamma(K_{l3(\gamma)}) = \frac{C_K^2\, G_F^2 M_K^5}{192\pi^3}\, S_{EW}\, |V_{us}|^2\, |f_+^{K^0\pi^-}(0)|^2\, I_{K\ell}(\lambda_{+,0})\, (1+\delta^K_{SU(2)}+\delta^{K\ell}_{em})^2$$

with $K = K^+, K^0;\ \ell = e, \mu$ and $C_K^2 = 1/2$ for K^+, 1 for K^0

Inputs from theory:

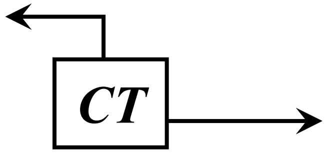
S_{EW}	Universal short distance EW correction (1.0232)
$\delta^K_{SU(2)}$	Form factor correction for strong SU(2) breaking
$\delta^{K\ell}_{em}$	Long distance EM effects
$f_+^{K^0\pi^-}(0)$	Hadronic matrix element at zero momentum transfer (t=0)

Inputs from experiment:

$\Gamma(K_{l3(\gamma)})$	**Branching ratios** with well determined treatment of radiative decays; **lifetimes**
$I_{K\ell}(\lambda)$	Phase space integral: λ's parameterize form factor dependence on t :

K_{e3}: only λ_+

$K_{\mu 3}$: need λ_+ and λ_0



SU(2) and *em* corrections

	$\delta^{K}_{SU(2)}$(%)	$\delta^{K\ell}_{em}$(%)
K^0e3	0	+0.57(15)
$K^0\mu3$	0	+0.80(15)
K^+e3	+2.36(22)%	+0.08(15)
$K^+\mu3$	+2.36(22)%	+0.05(15)

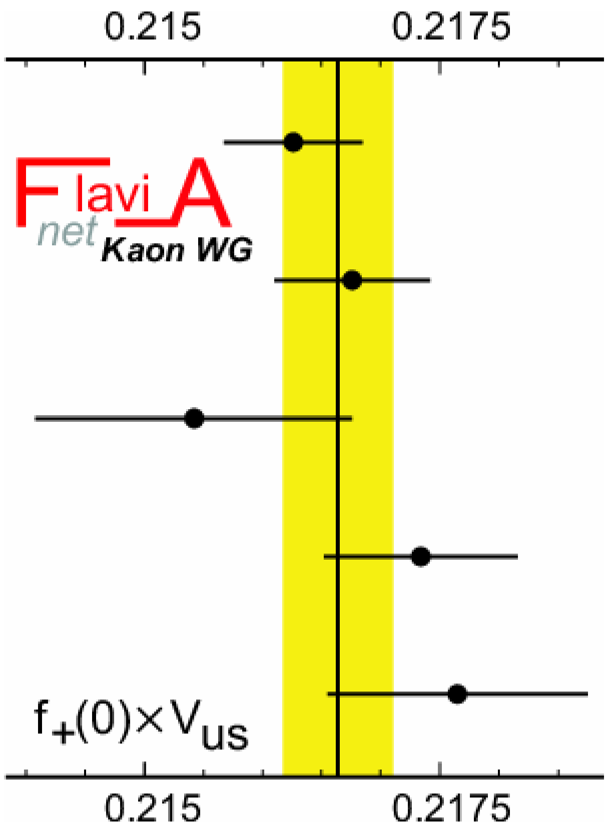
$$\begin{pmatrix} 1.0 & 0.1 & 0.8 & -0.1 \\ & 1.0 & -0.1 & 0.8 \\ & & 1.0 & 0.1 \\ & & & 1.0 \end{pmatrix}$$

(values used to extract $|V_{us}|f_+(0)$)

- δ_{em} for full phase space: all measurements assumed fully inclusive.
- **Different estimates of δ_{em} agree within the quoted errors**.
- Available correlation matrix between different corrections.

Determination of $|V_{us}|\times f_+(0)$

$$\Gamma(K_{l3(\gamma)}) = \frac{C_K^2 G_F^2 M_K^5}{192\pi^3} S_{EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}(\lambda_{+,0}) (1+\delta^K_{SU(2)}+\delta^{K\ell}_{em})^2$$



		% err	Approx. contr. to % err from:			
			BR	τ	δ	$I_{K\ell}$
K_Le3	0.2163(6)	0.28	0.09	0.19	0.15	0.09
$K_L\mu3$	0.2168(7)	0.31	0.10	0.18	0.15	0.15
K_Se3	0.2154(13)	0.67	0.65	0.03	0.15	0.09
$K^\pm e3$	0.2173(8)	0.39	0.26	0.09	0.26	0.09
$K^\pm\mu3$	0.2176(11)	0.51	0.40	0.09	0.26	0.15

Average: $|V_{us}|\,f_+(0) = 0.2166(5)$ $\chi^2/\text{ndf} = 2.74/4\ (60\%)$

Test of Lepton Universality from Kℓ3

- Test of Lepton Flavor Universality: comparing Ke3 and Kμ3 modes constraints possible anomalous LF dependence in the leading weak vector current. Evaluate $R_{K\mu3/Ke3}$:

$$\frac{\Gamma(K_{\mu3})}{\Gamma(K_{e3})} = \left(\frac{G_F^\mu}{G_F^e}\right)^2 \frac{I_K^\mu}{I_K^e} \frac{(1+\delta_K^\mu)^2}{(1+\delta_K^e)^2}$$

Compare experimental results with SM prevision:

$$\mathbf{r}_{\mu\mathbf{e}} = \frac{(R_{K\mu3/Ke3})_{\text{obs}}}{(R_{K\mu3/Ke3})_{\text{SM}}} = \frac{\Gamma(K_{\mu3})}{\Gamma(K_{e3})} \frac{I_K^e}{I_K^\mu} \frac{(1+\delta_K^e)^2}{(1+\delta_K^\mu)^2} = \frac{\mathbf{G}_\mathbf{F}^\mu}{\mathbf{G}_\mathbf{F}^\mathbf{e}}$$

Using FlaviaNet results get accuracy ~0.5%,

K_L $r_{\mu e}$ = 1.0049(61)
$K^\pm$ $r_{\mu e}$ = 1.0029(86)
Average $r_{\mu e}$ = 1.0043(52)

Comparable with other determinations:

- τ decays: $(r_{\mu e})_\tau$ = 1.0005(41) (PDG06)
- π decays: $(r_{\mu e})_\pi$ = 1.0042(33)

- Comparing values obtained for K_L and $K^\pm$ (**without $\delta^K_{SU(2)}$ correction**) allows the empirical evaluation of SU(2) breaking correction : **2.81(38)%**.

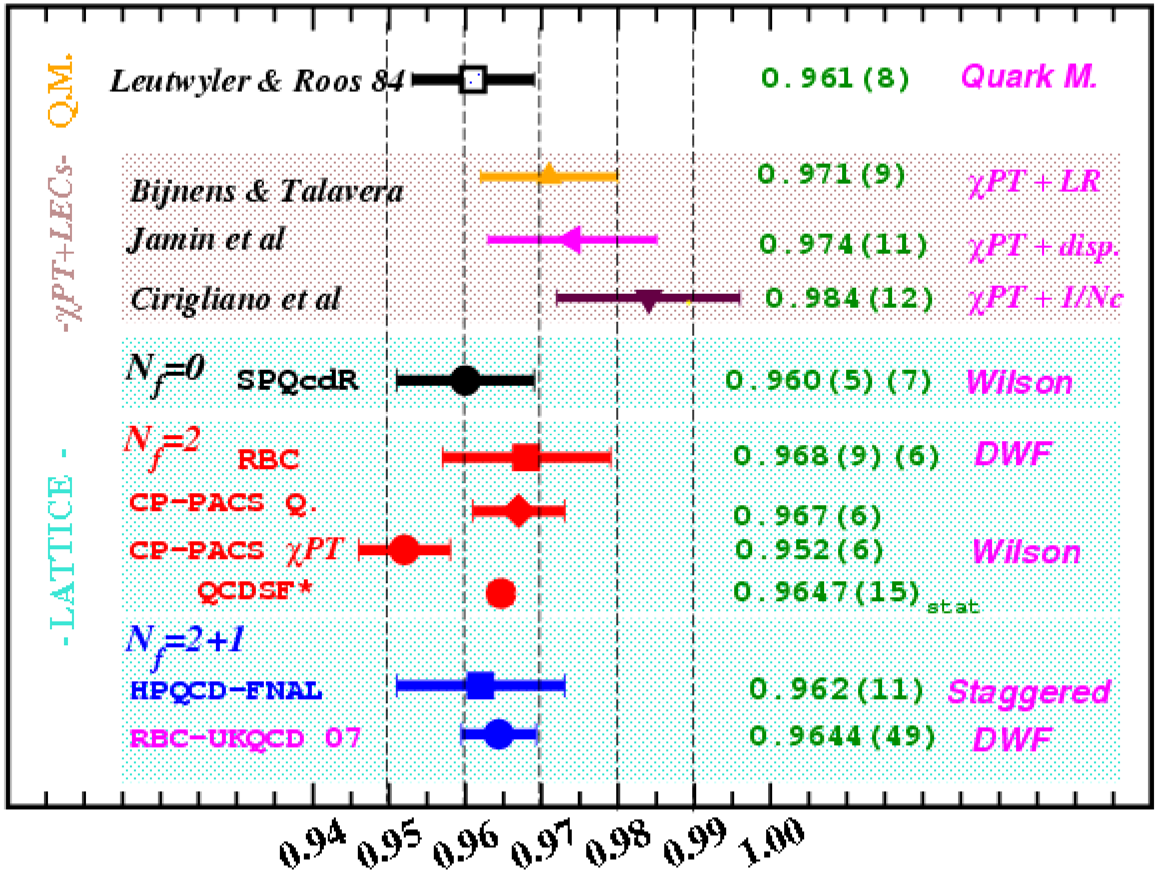
To be compared with χ_{PT} prediction **2.36(22)%.** Recent analyses point to **~3%**.

Theoretical estimate of $f_+(0)$

$$\Gamma(K_{l3(\gamma)}) = \frac{C_K^2\, G_F^2 M_K^5}{192\pi^3}\, S_{EW}\, |V_{us}|^2\, |f_+^{K^0\pi^-}(0)|^2\, I_{K\ell}(\lambda_{+,0})\, (1+\delta^K_{SU(2)}+\delta^{K\ell}_{em})^2$$

Leutwyler & Roos estimate still widely used:
$f_+(0) = 0.961(8)$.

Lattice evaluations generally agree well with this value; use RBC-UKQCD07 value:
$f_+(0) = 0.9644(49)$ (0.5% accuracy, also syst. err.).



FlaviAnet Kaon WG

V_{us}/V_{ud} determination from BR($K_{\mu2}$)

$$\frac{\Gamma(K_{\mu2(\gamma)})}{\Gamma(\pi_{\mu2(\gamma)})} = \frac{|V_{us}|^2}{|V_{ud}|^2} \times \frac{f_K}{f_\pi} \times \frac{M_K(1-m_\mu^2/M_K^2)^2}{m_\pi(1-m_\mu^2/m_\pi^2)^2} \times (1+\alpha(C_K-C_\pi))$$

Inputs from experiment:

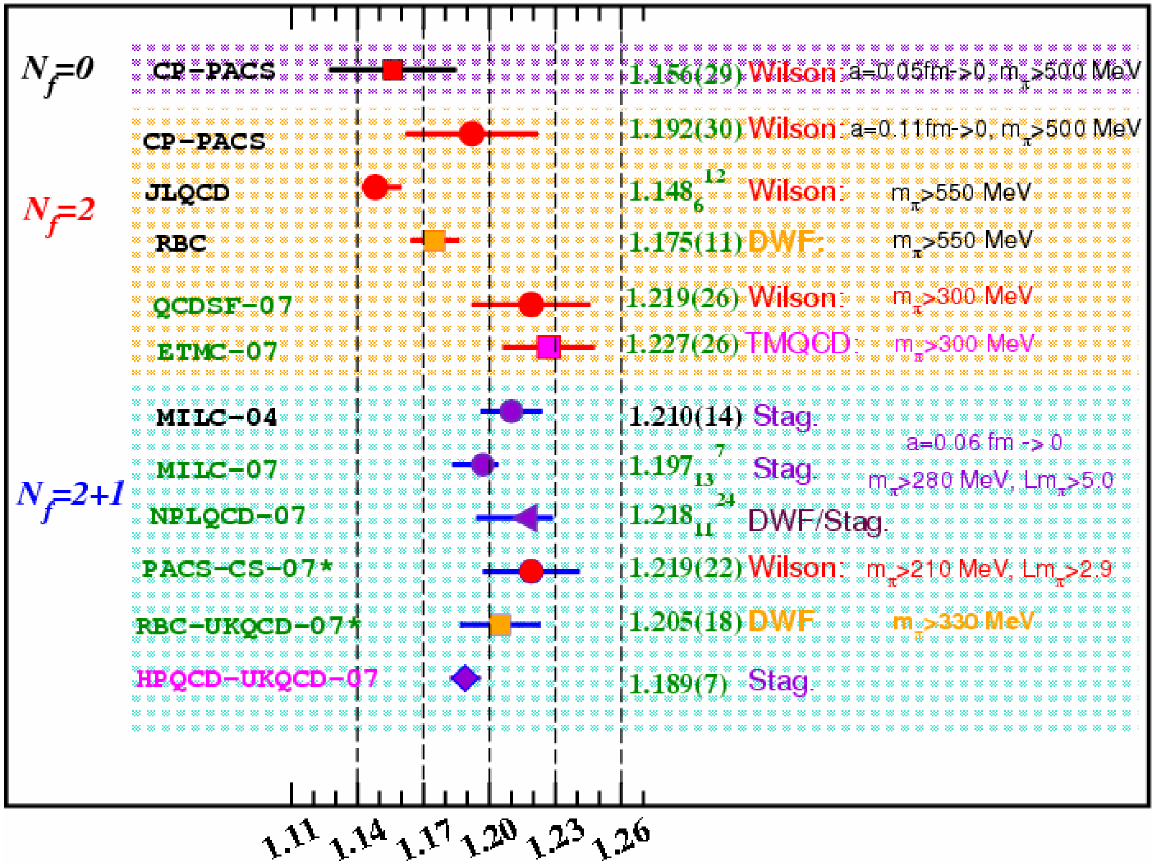
$\Gamma(\pi,K_{l2(\gamma)})$ **BR** with well determined treatment of radiative decays; **lifetimes**

Inputs from theory:

$C_{K,\pi}$ Rad. inclusive EW corr.

f_K/f_π Not protected by the Ademollo-Gatto theorem: only lattice.

- Lattice calculation of f_K/f_π and radiative corrections benefit of cancellations.
- Use HPQCD-UKQCD07 value: $f_K/f_\pi = 1.189(7)$.



A test of lattice calculations

Scalar form factor $f_0(t) = \tilde{f}_0(t)\, f_+(0)$ extrapolation at **Callan-Treiman** point:

$$\tilde{f}_0(\Delta_{K\pi}) = \frac{f_K}{f_\pi}\frac{1}{f(0)} + \Delta_{CT}, \quad \Delta_{CT} \simeq -3.4 \times 10^{-3}$$

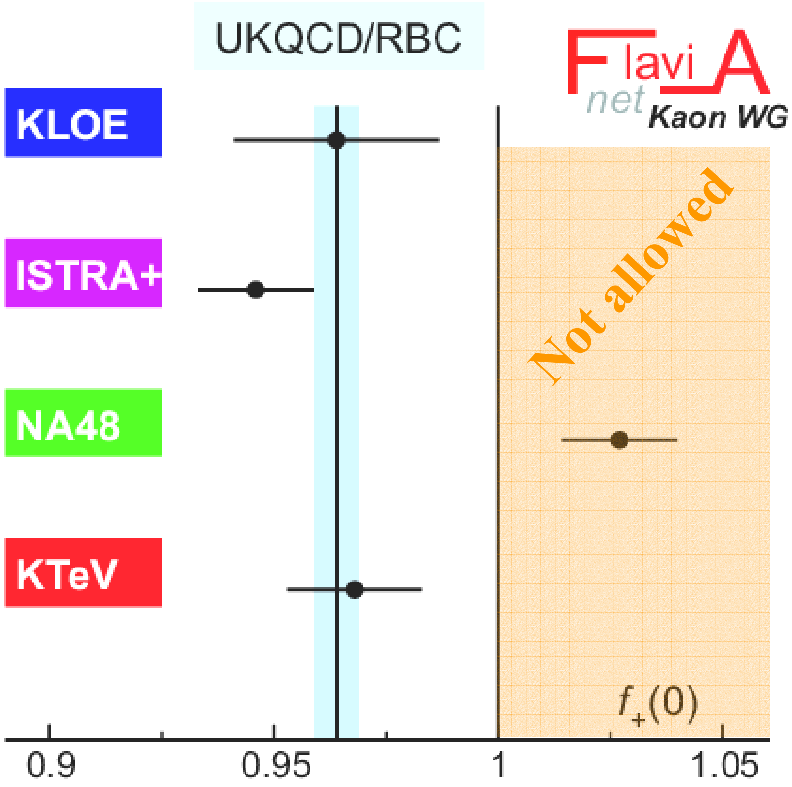
- links $f_+(0)$ and f_K/f_π with λ_0 measured in Kμ3 decays.

$\tilde{f}_0(\Delta_{K\pi})$ is evaluated fitting $K_L\mu3$ with a dispersive parameterization

$$\tilde{f}_0(t) = \exp\left(\frac{t}{\Delta_{K\pi}}\log(C - G(t))\right)$$

G(t) from Kπ scattering data.
To fit we use a 3rd order expansion

From CT, using f_K/f_π=1.189(7) [HPQCD-UKQCD07] obtain: $f_+(0)$=0.964(23) in agreement with RBC/UKQCD07 value: $f_+(0) = 0.9644(49)$.

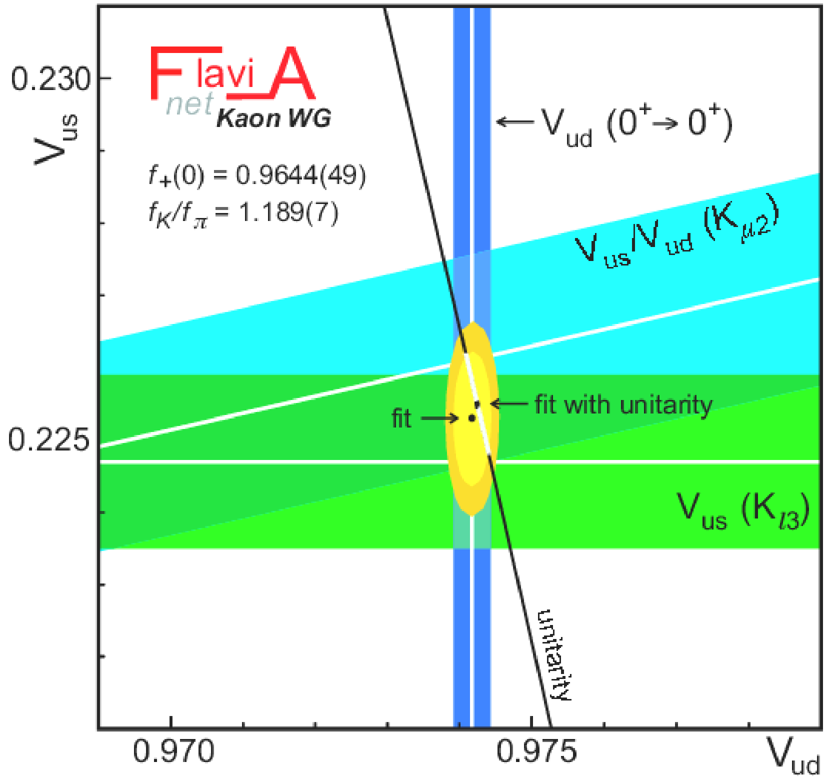


V_{ud}, V_{us} and V_{us}/V_{ud}

K13: $|V_{us}| f_+(0) = 0.2166(5)$ and $f_+(0) = 0.964(5)$, obtain $|\mathbf{V_{us}}| = \mathbf{0.2246(12)}$

K12: $|V_{us}|/|V_{ud}|\, f_K/f_\pi = 0.2760(6)$ and $f_K/f_\pi = 1.189(7)$, obtain $|\mathbf{V_{us}}|/|\mathbf{V_{ud}}| = \mathbf{0.2321(15)}$

V_{ud} from nuclear β decay: $\mathbf{V_{ud} = 0.97418(26)}$ [Hardy-Towner, nucl-th 0710.3181]



Fit (no CKM unitarity constraint):

$\mathbf{V_{ud} = 0.97417(26);\ V_{us} = 0.2253(9)}$

$\chi^2/\mathrm{ndf} = 0.65/1\ (41\%)$

- Unitarity: $\mathbf{1 - V_{ud}^2 - V_{us}^2 = 0.0002(6)}$

- The test on the unitarity of CKM can be also interpreted as a **test of the universality of lepton and quark gauge coupling**:

$$G_{\mathrm{CKM}} \equiv G_\mu \left[|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2\right]^{1/2}$$
$$= (1.1662 \pm 0.0004) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

$$G_\mu = (1.166371 \pm 0.000007) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

Fit (with CKM unitarity constraint): $\mathbf{V_{us} = 0.2255(7)}$ $\chi^2/\mathrm{ndf} = 0.8/2\ (67\%)$

$K_{\mu 2}$: sensitivity to NP

Comparison of V_{us} from $K_{\ell 2}$ (helicity suppressed) and from $K_{\ell 3}$ (helicity allowed)
To reduce theoretical uncertainties study the quantity:

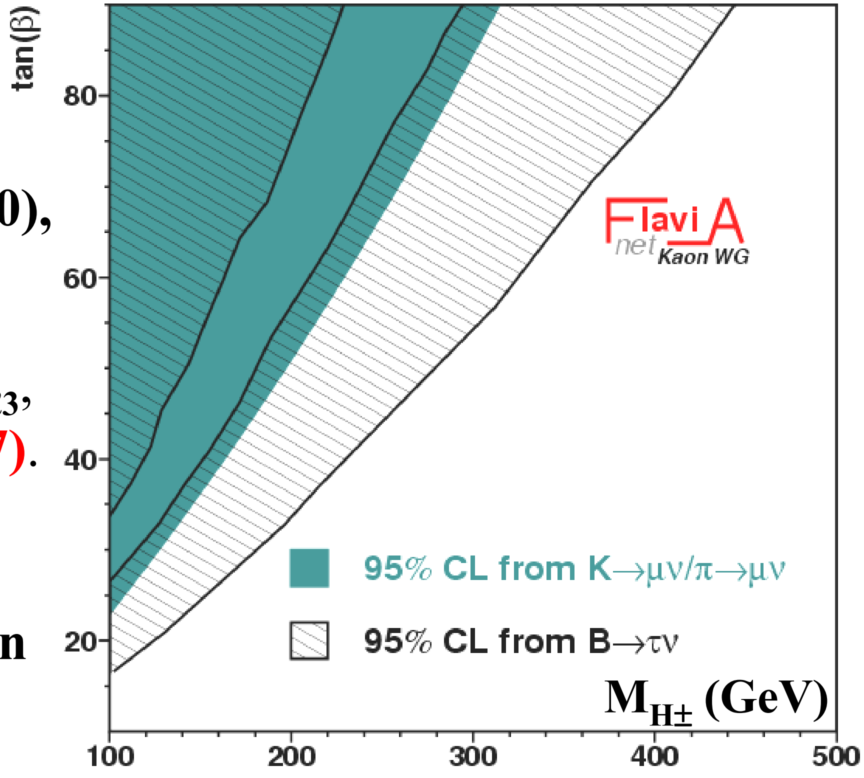
$$R_{l23} = \left| \frac{V_{us}(K_{\ell 2})}{V_{us}(K_{\ell 3})} \times \frac{V_{ud}(0^+ \to 0^+)}{V_{ud}(\pi_{\ell 2})} \right|$$

Within SM $R_{\ell 23} = 1$; NP effects can show as scalar currents due to a charged Higgs:

$$R_{l23} = \left| 1 - \frac{m_{K+}^2}{M_{H+}^2} \left(1 - \frac{m_d}{m_s}\right) \frac{\tan^2\beta}{1+\epsilon_0 \tan\beta} \right|$$

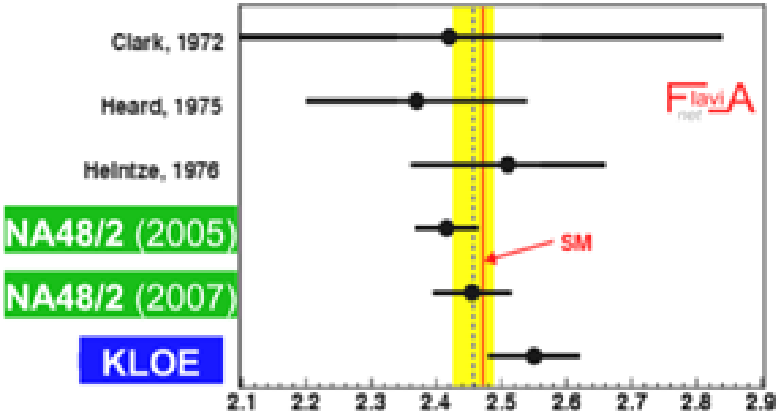
$R_{\ell 23}$ is accessible via $BR(K_{\mu 2})/BR(\pi_{\mu 2})$, $V_{us}f_+(0)$, and V_{ud}, and $f_K/f_\pi/f_+(0)$ determinations.

- Using $K^\pm$ fit results, assuming unitarity for $K_{\ell 3}$, and using $f_K/f_\pi/f_+(0)$ from lattice: **$R_{\ell 23} = 1.004(7)$**.
- Uncertainty dominated by $f_K/f_\pi/f_+(0)$.
- 95% CL excluded region (with $\varepsilon_0 \sim 0.01$).
- In $\tan\beta$-$M_{H\pm}$ plane, $R_{\ell 23}$ **fully cover the region uncovered by $BR(B \to \tau\nu)$**.

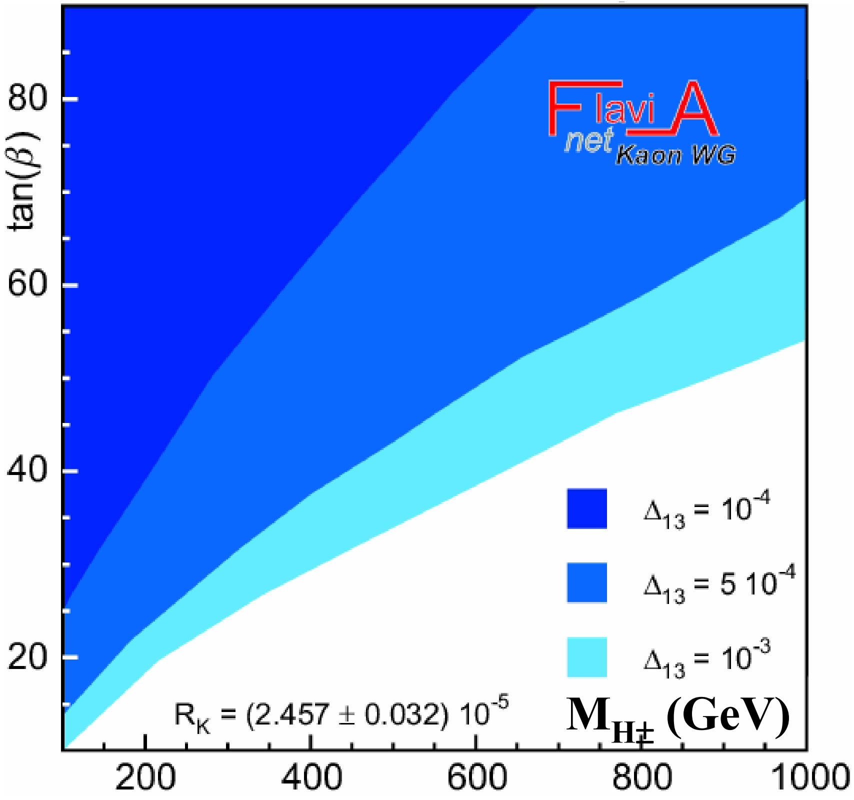


Measurement of $R_K = \Gamma(K_{e2})/\Gamma(K_{\mu2})$

- PDG06: 5% precision from 3 old mnts
- 2 preliminary meas. from NA48 (see M.Raggi talk); waiting for new data result.
- 1 preliminary from KLOE see (A.Passeri talk); waiting for final.



- **New average**: $R_K = 2.457(32) \times 10^{-5}$.
- Perfect agreement with SM expectations: $R_K^{SM} = 2.477(1) \times 10^{-5}$.
- In SUSY(MSSM) LFV appear at 1-loop level (effective $H^+ \ell \nu_\tau$ Yukawa interaction). For moderately large tanβ values, enhance R_K up to few %.
- The world average gives strong constrains for tanβ and $M_{H\pm}$.
- **95%-CL excluded regions in the tanβ - M_H plane, for $\Delta_{13} = 10^{-4}$, 0.5×10^{-4}, 10^{-3}**.



Conclusions

- Dominant K_S, K_L, and $K^{\pm}$ BRs, and lifetime known with very good accuracy.
- Dispersive approach for form factors.
- Constant improvements from lattice calculations of $f_+(0)$ and f_K/f_π:
  Callan-Treiman relation allows checks from measurements;
  lack of sys. errors; need to average different evaluations.

- $|V_{us}| f_+(0)$ at 0.2% level.
- Test of LU with Kl3 decays with 0.5% accuracy.
- $|V_{us}|$ measured with 0.4 accuracy (with $f_+(0)= 0.9644(49)$)
  Dominant contribution to uncertainty on $|V_{us}|$ still from $f_+(0)$.
  CKM unitarity test satisfied at 0.3σ level
  test of lepton-quark universality

- Comparing $|V_{us}|$ values from $K\mu2$ and Kl3, exclude large region in the (m_H,$\tan\beta$) plane, complementary to results from $B\to\tau\nu$ decays.

Additional information

$K^{\pm}$ leading branching ratios and $\tau_{\pm}$

FlaviA net Kaon WG

No significant differences in the fit if the final KLOE measurement of $K^{\pm}$ lifetime is used instead of the preliminary one (FlaviaNet note):

FlaviaNet note

$\tau^{\pm}$(KLOE) = 12.347(30) ns

Parameter	Value	S
BR($K_{\mu 2}$)	63.57(11)%	1.1
BR($\pi\pi^0$)	20.64(8)%	1.1
BR($\pi\pi\pi$)	5.595(31)%	1.0
BR(K_{e3})	5.078(26)%	1.2
BR($K_{\mu 3}$)	3.365(27)%	1.7
BR($\pi\pi^0\pi^0$)	1.750(26)%	1.1
$\tau_{\pm}$	12.384(19) ns	1.7

← 5.593(30)%

← 1.749(26)%

← 12.379(19) ns

Dispersive parameterization

$$\tilde{f}_+(t) = \exp\left[\frac{t}{m_\pi^2}\left(\Lambda_+ + H(t)\right)\right]$$

$$\tilde{f}_+(t) = 1 + \lambda_+\frac{t}{m^2} + \frac{{\lambda_+}^2 + p_2}{2}\left(\frac{t}{m^2}\right)^2 + \frac{{\lambda_+}^3 + 3p_2\lambda_+ + p_3}{6}\left(\frac{t}{m^2}\right)^3$$

p_n	$\tilde{f}_+(t)$	$\tilde{f}_0(t)$
$p_2 \times 10^4$	5.84 ± 0.93	4.16 ± 0.50
$p_3 \times 10^4$	0.30 ± 0.02	0.27 ± 0.01

Table 1: Constants appearing in the dispersive form of vector and scalar form factors.

$$\tilde{f}_0(t) = \exp\left[\frac{t}{\Delta_{K\pi}}(\ln C - G(t))\right]$$

$$\tilde{f}_0(t) = 1 + \lambda_0\frac{t}{m^2} + \frac{{\lambda_0}^2 + p_2}{2}\left(\frac{t}{m^2}\right)^2 + \frac{{\lambda_0}^3 + 3p_2\lambda_0 + p_3}{6}\left(\frac{t}{m^2}\right)^3$$

With or without NA48 Kμ3 data

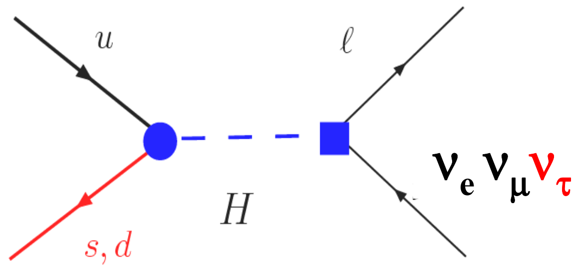
	K_L and K^-	K_L only	K_L and K^-	K_L only
Measurements	16	11	13	8
χ^2/ndf	$54/13$ (7×10^{-7})	$33/8$ (8×10^{-5})	$13/9$ (24.9%)	$9/5$ (12.3%)
$\lambda'_+\times10^3$	24.9 ± 1.1 $(S=1.4)$	24.0 ± 1.5 $(S=1.5)$	25.0 ± 0.8	24.5 ± 1.1
$\lambda''_+\times10^3$	1.6 ± 0.5 $(S=1.3)$	2.0 ± 0.6 $(S=1.6)$	1.6 ± 0.4	1.8 ± 0.4
$\lambda_0\times10^3$	13.4 ± 1.2 $(S=1.9)$	11.7 ± 1.2 $(S=1.7)$	16.0 ± 0.8	14.8 ± 1.1
$\rho(\lambda'_+,\lambda''_+)$	-0.94	-0.97	-0.94	-0.95
$\rho(\lambda'_+,\lambda_0)$	$+0.33$	$+0.72$	$+0.26$	$+0.28$
$\rho(\lambda''_+,\lambda_0)$	-0.44	-0.70	-0.37	-0.38
$I(K^0_{e3})$	0.15457(29)	0.1544(4)	0.15459(20)	0.15446(27)
$I(K^\pm_{e3})$	0.15892(30)	0.1587(4)	0.15894(21)	0.15881(28)
$I(K^0_{\mu3})$	0.10212(31)	0.1016(4)	0.10268(20)	0.10236(28)
$I(K^\pm_{\mu3})$	0.10507(32)	0.1046(4)	0.10559(20)	0.10532(29)
$\rho(I_{e3},I_{\mu3})$	$+0.63$	$+0.89$	$+0.59$	$+0.62$

wNA48-w/oNA48: -0.00002 (0.01%) -0.00006 (0.04%) $\Delta I(K^0e3)$
-0.00002 (0.01%) -0.00011 (0.07%) $\Delta I(K^\pm e3)$
-0.00056 (0.55%) -0.00076 (0.75%) $\Delta I(K^0\mu3)$
-0.00052 (0.49%) -0.00072 (0.69%) $\Delta I(K^\pm\mu3)$

Lepton universality from $K_{e2}/K_{\mu2}$

SM: no hadronic uncertainties (no f_K) → 0.4×10^{-3}

In MSSM, LFV can give up to % deviations [Masiero, Paradisi, Petronzio]

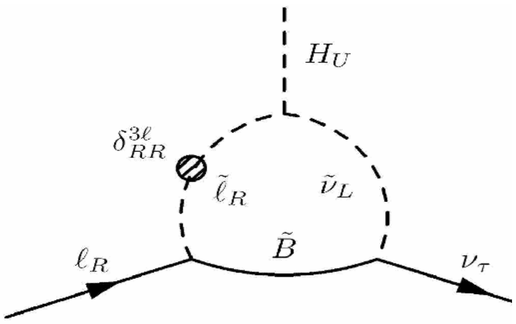


NP dominated by contribution of $e\nu_\tau$

$$R_K \approx \frac{\Gamma(K\to e\nu_e)+\Gamma(K\to e\nu_\tau)}{\Gamma(K\to\mu\nu_\mu)}$$

with effective coupling:

$$eH^{\pm}\nu_\tau \to \frac{g_2}{\sqrt{2}}\frac{m_\tau}{M_W}\Delta_R^{31}\tan^2\beta$$



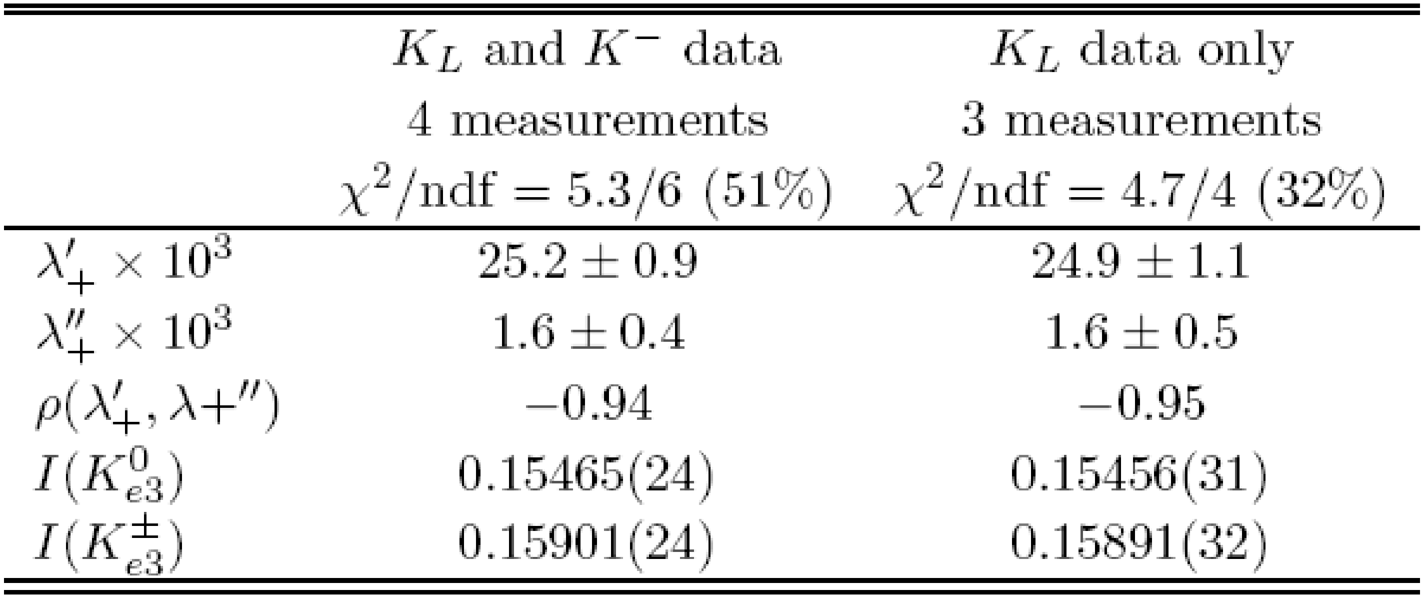
$$R_K \approx R_K^{SM}[\,1+\frac{m_K^{\,4}}{m_H^{\,4}}\,\frac{m_\tau^{\,2}}{m_e^{\,2}}\,|\Delta^R_{31}|^2\tan^6\beta]$$

1% effect (Δ^R_{31} ~5x10^{-4}, tanβ~40, m_H~500GeV) not unnatural

Present accuracy on R_K @ 6% ; need for precise (<1%) measurements

Vector form factor from K_{e3}

- Quadratic from ISTRA+, KLOE, KTeV, NA48 with K_L and K^- decays.

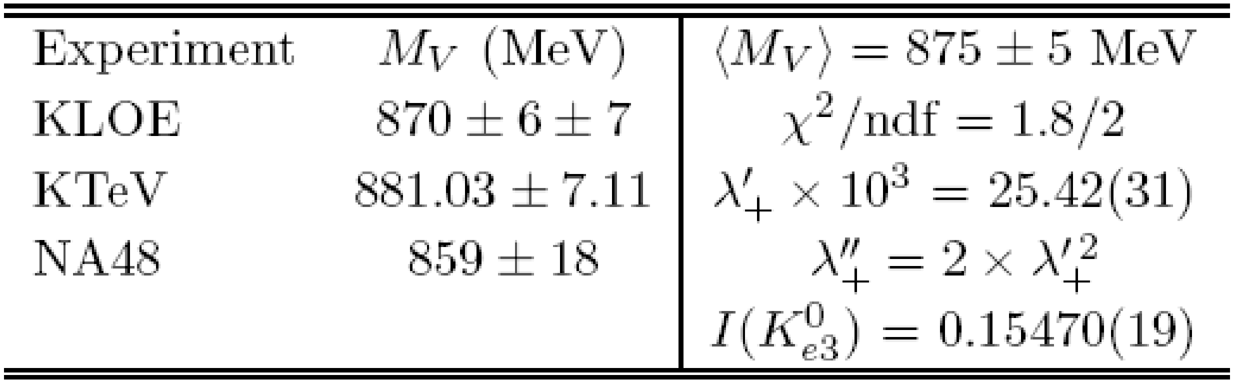


	K_L and K^- data	K_L data only
	4 measurements	3 measurements
	$\chi^2/\text{ndf} = 5.3/6$ (51%)	$\chi^2/\text{ndf} = 4.7/4$ (32%)
$\lambda'_+ \times 10^3$	25.2 ± 0.9	24.9 ± 1.1
$\lambda''_+ \times 10^3$	1.6 ± 0.4	1.6 ± 0.5
$\rho(\lambda'_+, \lambda+'')$	-0.94	-0.95
$I(K^0_{e3})$	0.15465(24)	0.15456(31)
$I(K^\pm_{e3})$	0.15901(24)	0.15891(32)

- The significance of the quadratic term is 4.2σ from all data and 3.5σ from K_L only.
- Using all data or K_L only changes the space phase integrals I^0_{e3} and $I^\pm_{e3}$ by 0.07% .
- Errors on I_{e3} are significantly smaller when K^- data are included.

Vector form factor from K_{e3}

- KLOE, KTeV, NA48 quote value for M_V for pole fit to K_Le3 data.



Experiment	M_V (MeV)	$\langle M_V \rangle = 875 \pm 5$ MeV
KLOE	$870 \pm 6 \pm 7$	$\chi^2/\mathrm{ndf} = 1.8/2$
KTeV	881.03 ± 7.11	$\lambda'_+ \times 10^3 = 25.42(31)$
NA48	859 ± 18	$\lambda''_+ = 2 \times \lambda'^2_+$
		$I(K^0_{e3}) = 0.15470(19)$

- The values for $\lambda_+'$and λ_+'' from pole expansion are in agreement with quadratic fit results.

- Using quadratic averages or pole fit results changes I^0_{e3} by 0.03% .