Theoretical descriptions of the Nucleon EM form factors in the Space- and Time-like regions

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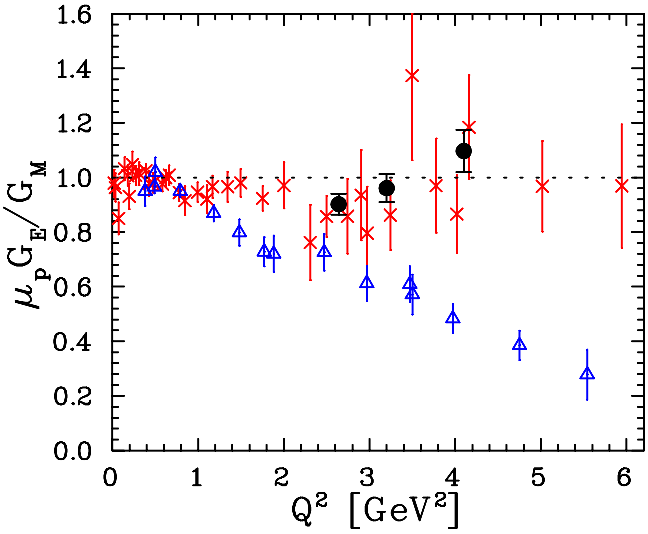
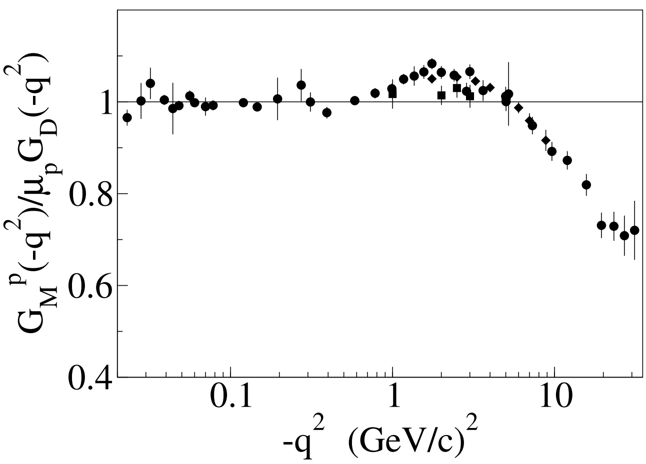
Outline

- A flash on the Exp. status. SL & TL
- Challenges for the Theory; impact of a possible zero. TL neutron ff.
- Lattice calculations for SL momentum transfers: a first answer from npQCD
- Phenomenology I: Dispersion relations and SL & TL data
- Phenomenology II: CQM + $q\bar{q}$ pairs and SL & TL data
- Conclusion & Perspectives

Nucleon Electromagnetic Form Factors in the space-like region

Proton EM Form factors

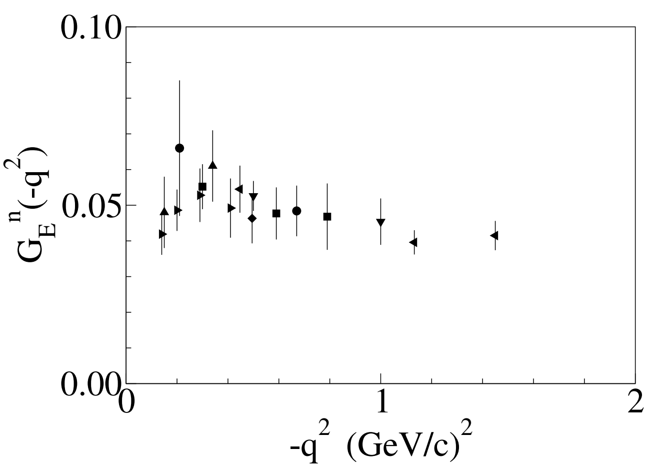


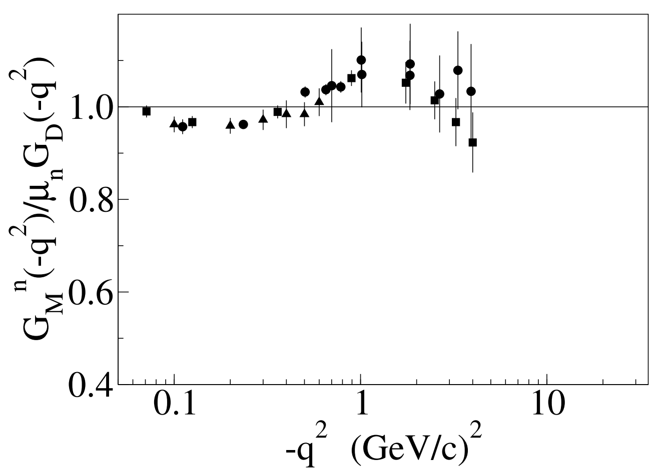


in $\mu_p\, G_E^p/G_M^p$: Blue triangles: TJLAB polarization data (2000-2002) ; Red stars: SLAC Rosenbluth-separation data; Black dots: TJLAB Rosenbluth-separation data (2005)

1) N.B. $Q^2 = -q^2$ 2) $G_D(Q^2) = 1/\left[1 + Q^2/0.71\right]^2$

Neutron EM Form factors

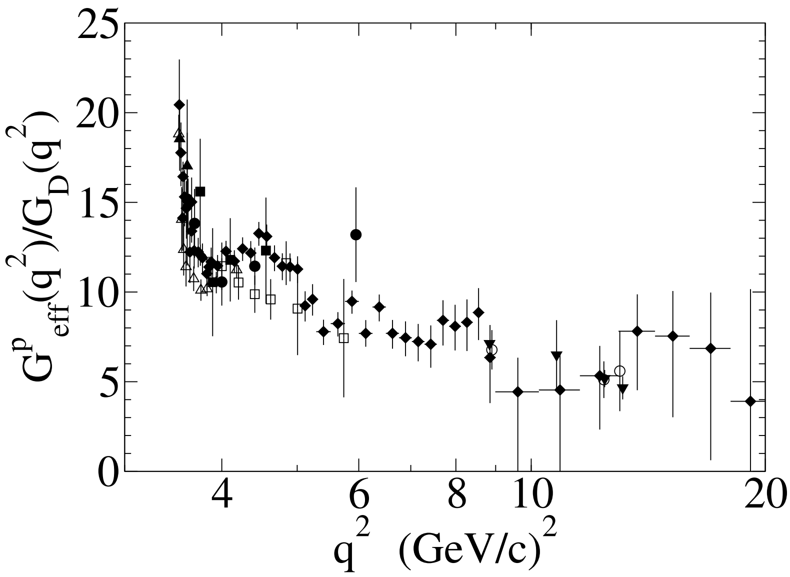




Forthcoming exp. at TJLAB: $\mu_p G_E/G_M$ in the range of $Q^2 < 0.7\; GeV^2$ and $Q^2 = 4.6,\; 7.1,\; 8.6\; GeV^2$

Nucleon Electromagnetic Form Factors in the time-like region

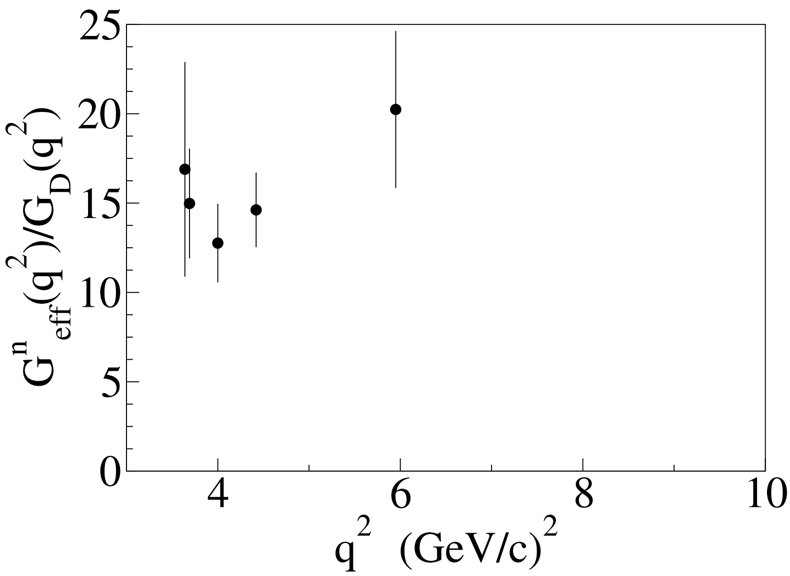
Proton EM Form factors



Diamonds: BABAR ; Full Dots: Frascati - Fenice; Full squares: DM1 - DM2; Open dots: E760; Down-triangles: E835; Up-triangles: ELPAR - PS170

1) N.B. $G^{p(n)}_{eff}(q^2) = \sqrt{|G^{p(n)}_M(q^2)|^2 + \eta|G^{p(n)}_E(q^2)|^2 /(1 + \eta)}$ with $\eta = 2M_N^2/q^2$ 2) $G_D(q^2) = 1/\left[1 + q^2/0.71\right]^2$

Neutron EM Form factors



Frascati - Fenice

Challenges for the Theory

- In the SL, the possible zero for $Q^2 > 7\ GeV^2$ could open a new perspective. The dipole form for the nucleon form factors is supported by the pQCD, therefore a sharp difference between the charge distribution of the current quarks and the charge distribution of the "constituent" quarks appears a clear-cut signature of npQCD effects in act.
- Even low-Q^2 ff's (cf forthcoming analysis from TJLAB) could give stringent constraints for builders of phenomenological models
- In the TL, the neutron data around $q^2 \sim 4GeV^2$ show a behavior $|G^n_{eff}| \sim |G^p_{eff}|$ sharply different from the one suggested by pQCD, naively $|G^n_{eff}| \sim |e_d/e_u|\ |G^p_{eff}|$
- TL proton data show many interesting structures, that could be analyzed in combination with the pion data !

$$\text{Synopsis} = \begin{cases} Lattice \rightarrow SL\ region \\ \\ Phenomenology \rightarrow SL\ \&TL\ region \\ {\scriptstyle (I)\ DR's \quad (II)\ CQ+q\bar{q}} \\ AdS/CFT\ correspondence\ \rightarrow SL\ \&TL \\ {\scriptstyle Pion\ done,\ proton\ preliminary} \\ Dyson - Schwinger\ approach\ \rightarrow SL\ \&TL \\ {\scriptstyle limited\ range\ in\ q^2} \end{cases}$$

Lattice ”Data”

In principle, Lattice QCD allows to achieve a non perturbative description of the EM ff’s.

i) Algorithmic advances, leading to a better description of the fundamental symmetries of QCD, ii) improvements of the available computing power, and last, but not least, iii) theoretical refinements of the chiral extrapolation techniques, have allowed to gain more and more accuracy in the predictions.

Main obstacle to be overcome: Lattice Data are obtained with sizable quark masses and for shrinking the size to the current quark masses $\longrightarrow$ i) down-sizing the spacing, or ii) exploiting more refined extrapolation formulas obtained from the low-energy regime of the QCD, namely chiral ”non analytic” (log-type) extrapolations.

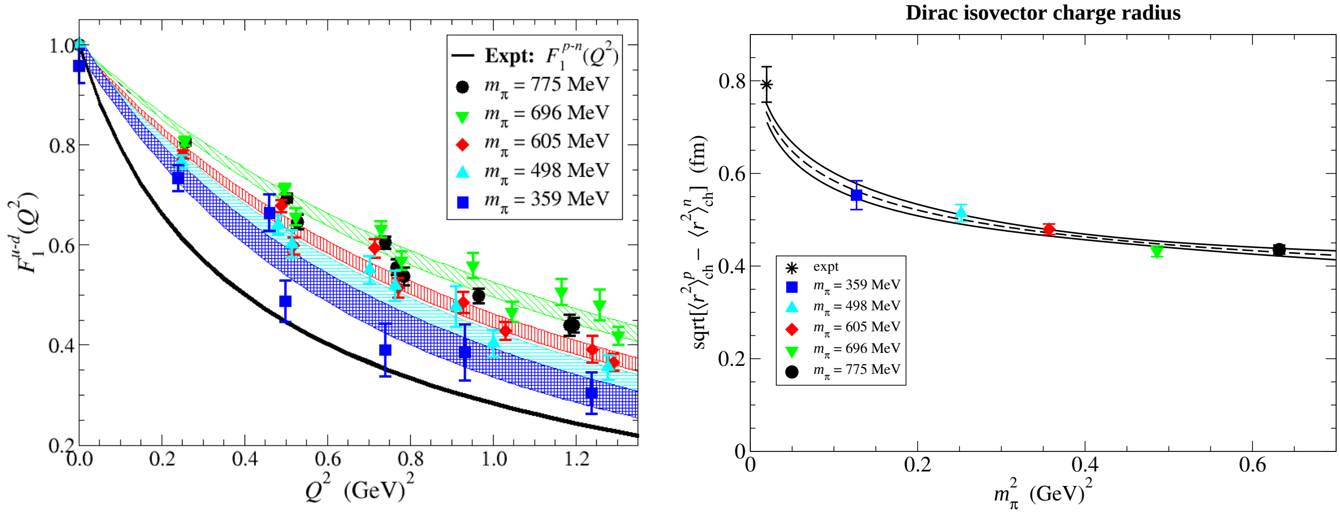
- Euclidean Lattice: no direct evaluation of the matrix elements of the EM current. Then a careful study of the proper ratio between three- and two-point functions, defined on the Lattice. It is necessary to get ”large values” of the Euclidean time in order to filter the contribution from hadronic states with the nucleon quantum numbers. This filtering prevents the extension to TL, no exponential fall-off in time.
- Small momentum transfers can be investigated on Lattices with large spacing, a, namely $q \sim 2\pi/a$, modulo volume effects.
- Large momenta are accessible from nowadays Lattice spacing, but Fourier transforms of two- and three-point functions become noise-dominated for square momentum transfers beyond 2 $(GeV/c)^2$.

- *Quenched approximation*: the sea-quark loop contributions are disregarded, since they are considered largely perturbative. However, nowadays results predominantly unquenched.

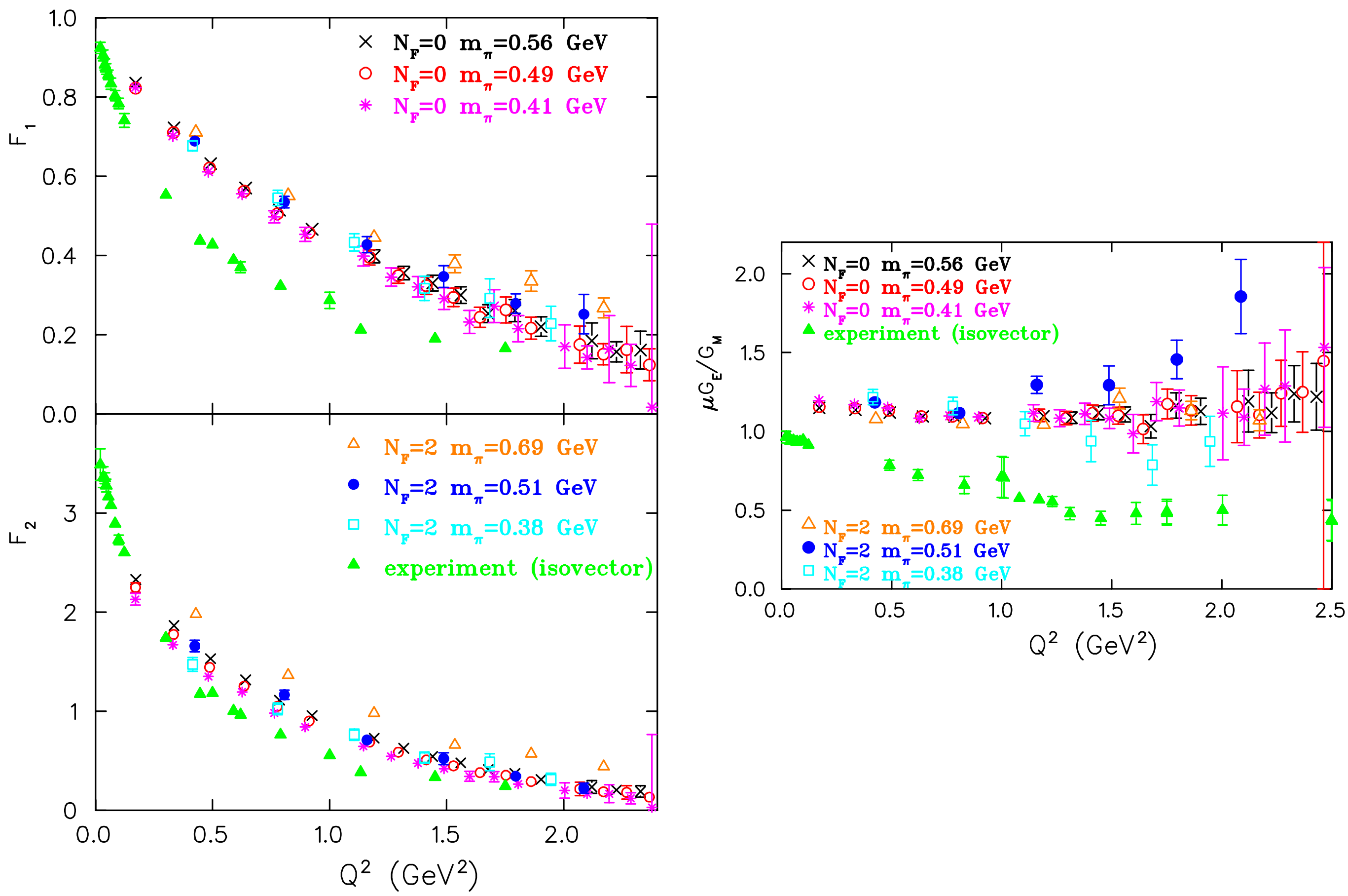
- In general, quark masses in Lattice simulations are very large, and it is customary to give the pion mass as a measure of the size of the quark masses on the Lattice. Indeed, the range of pion masses in the present calculations for the nucleon can be hardly lowered below $\sim\ 350\ MeV$.

- It is necessary to extrapolate to the observed pion mass. This is done by exploiting extrapolation formulas (with non analytic behavior, $log\ m_\pi^2$, as predicted by Chiral Perturbation Theory). The range of applicability is $Q^2 < 0.4\ GeV^2$, not always reached by the present Lattices (for LHPC $Q^2_{lower} \sim 0.23\ GeV^2$, for QCDSF $Q^2_{lower} \sim 0.4\ GeV^2$ and for Cyprus-MIT $Q^2_{lower} \sim 0.15\ GeV^2$, quenched). The presence of terms proportional to $log\ m_\pi^2$ makes the extrapolation critical for values of m_π^2 close to the actual mass of the pion.

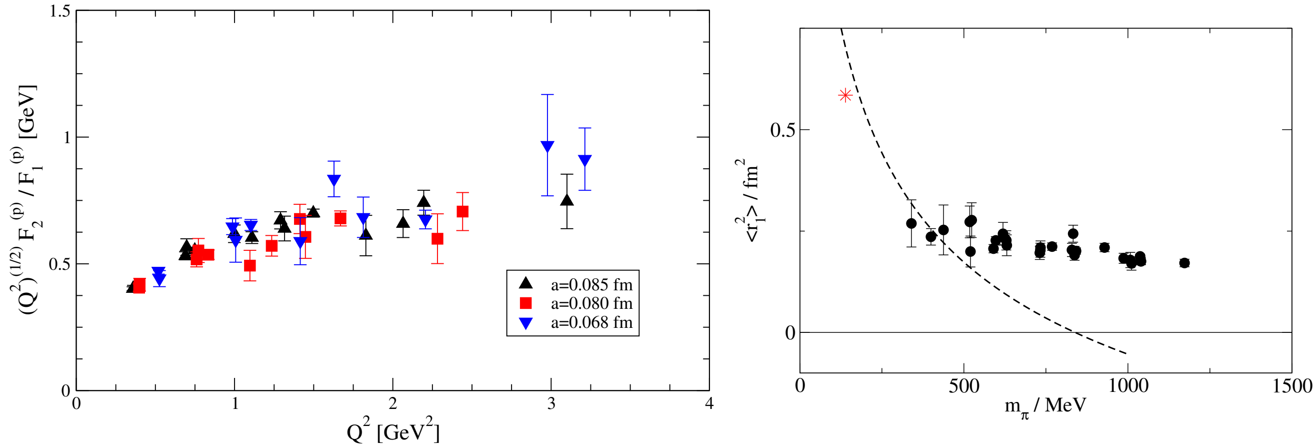
- Finite volume effects...



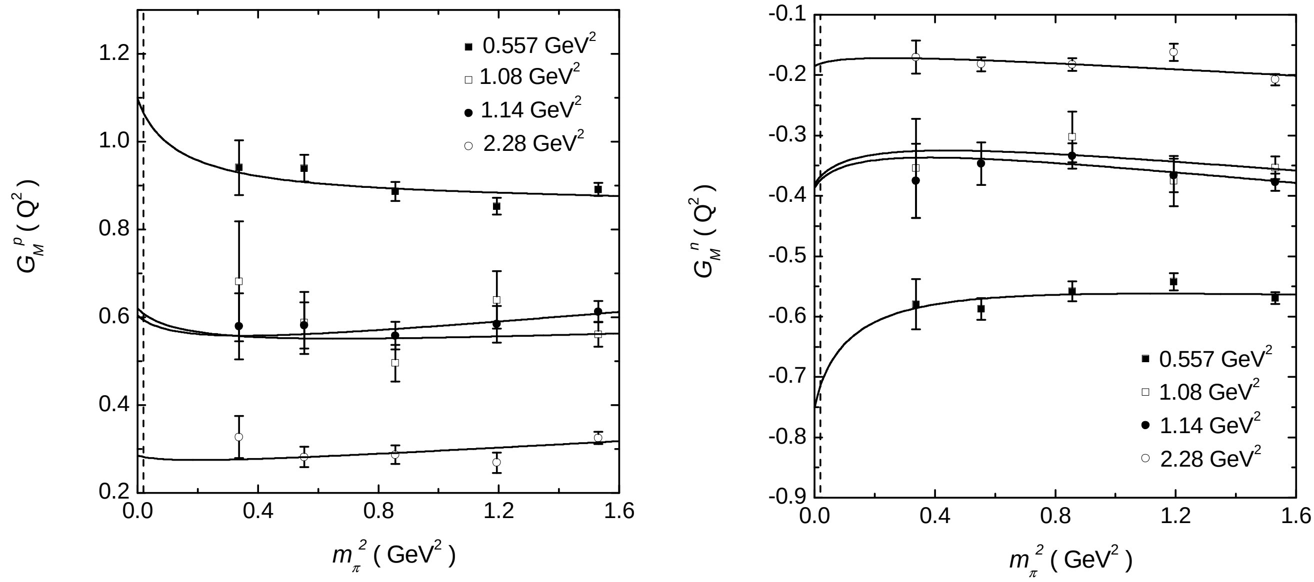
Left panel: LHP Collaboration results for the "isovector" contribution to the Dirac ff $F_1(Q^2)$ ($\langle proton|\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d|proton\rangle$). Right panel: the same as the left panel, but for the isovector radius. Solid and dashed lines: different extrapolation formulas. (After LHPC, arXiv:0711.2048).



Cyprus-MIT collaboration: results for "isovector" contribution to the Nucleon ff's. (After PRD 74 (2006)).



QCDSF-UKQCD Collaboration: results for the ratio of ”isovector” contributions to the Dirac and Pauli ff’s, corresponding to 3 different spacings and $m_\pi \sim 600 MeV$. Right panel: the same as the left panel, but for the isovector radius; dashed curve: chiral extrapolation from the Small Scale Expansion approach. (After arXiv:0709.337).



Adelaide + TJLAB: Nucleon magnetic form factors, extrapolated by using the most refined chiral formula introduced by the Adelaide group, for 3 values of Q^2, $Q^2 = 0.557,\ 1.08 \sim 1.14,\ 2.28\ GeV^2$. (After PRD 75 (2007)).

Phenomenology I: Dispersion Relations

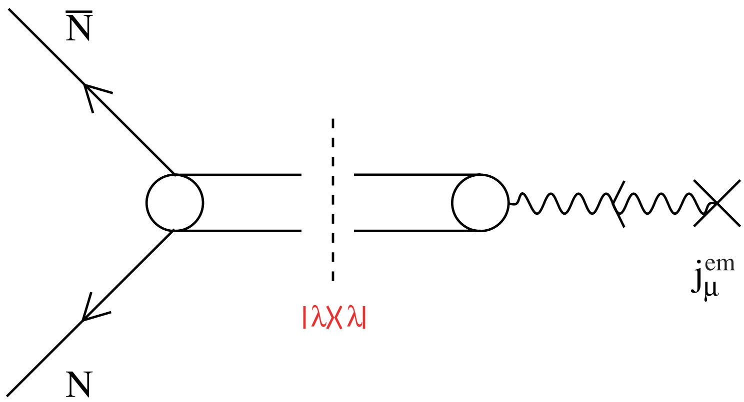
Dispersion relation (DR) approaches have a very long story! Within such a framework, one can relate real and imaginary parts of the EM form factors, once the unitarity (probability conservation) and analyticity (causality) properties are implemented. The analysis based on DR allows one to investigate both SL and TL regions, even to analyze the unphysical region in the TL region: at the threshold, there is a possible bound state $p\bar{p}$.

$$F(q^2) = \frac{1}{\pi} \int_{q_0^2}^{\infty} \frac{ImF(q'^2)}{q'^2 - q^2 - i\varepsilon} \, dq'^2$$

or a once subtracted version, where $F(0)$ appears.

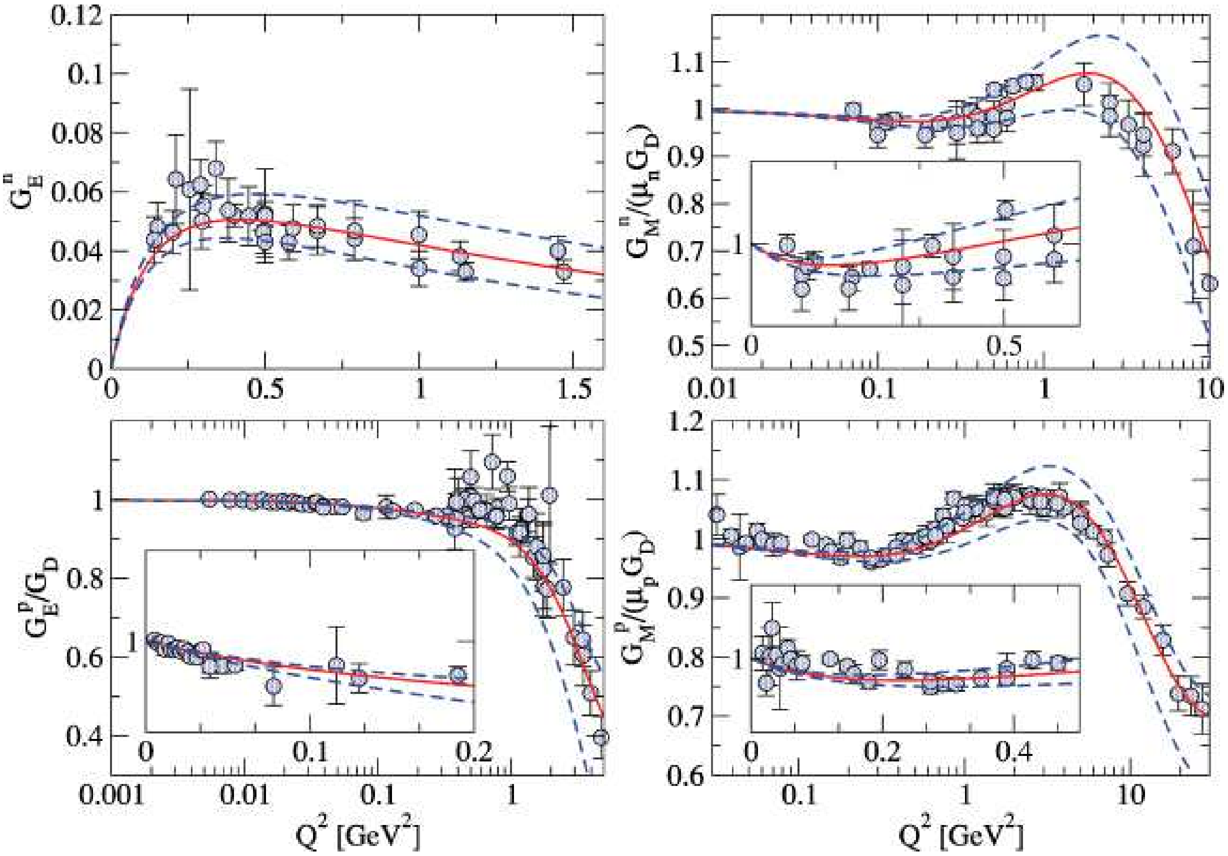
$$F(q^2) = F(0) + \frac{q^2}{\pi} \int_{q_0^2}^{\infty} \frac{ImF(q'^2)}{q'^2 \ (q'^2 - q^2 - i\varepsilon)} \, dq'^2$$

Ansatzes, **physically motivated**, **for** $ImF(t')$ **(e.g.** $\rho \to \pi\pi$**,** $\phi \to K\bar{K}$**,** $\omega \to \rho\pi$**, and related continua)+ constrains from pQCD**

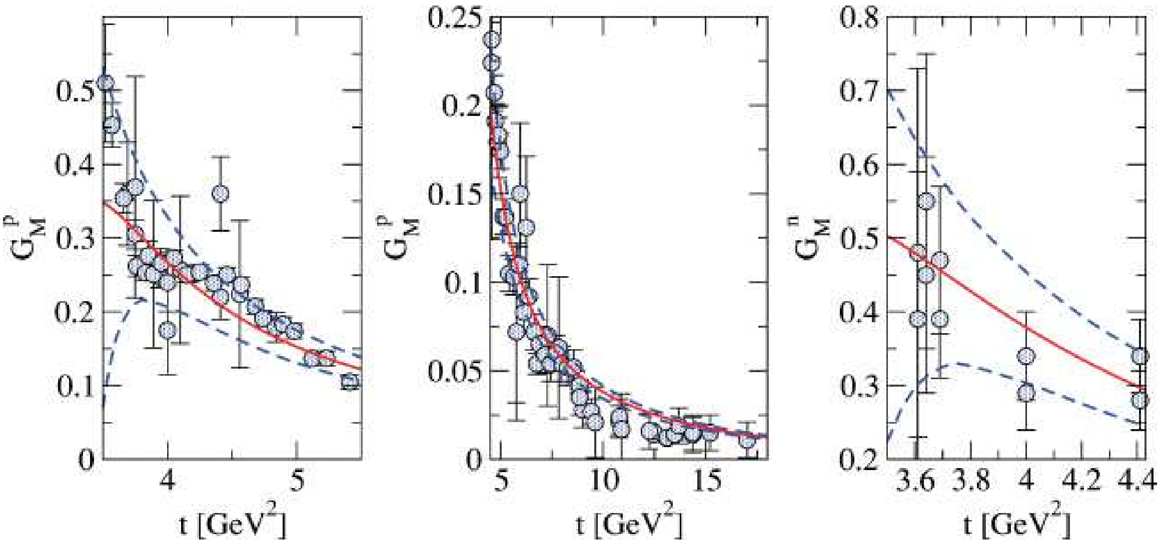


The spectral decomposition of the nucleon matrix element of the electromagnetic current j_μ^{em}**;** $|\lambda\rangle \langle\lambda|$ **denotes a hadronic intermediate state, namely** $\langle N\bar{N}|j_\mu^{em}|e^+e^-\rangle = \sum \langle N\bar{N}|T|\lambda\rangle \langle\lambda|j_\mu^{em}|e^+e^-\rangle$**. (After Belushkin-Hammer-Meissner PRC 75, (2007))**

In the Bonn approach (Belushkin - Hammer - Meissner PRC **75** (2007)), the fit has 17 free parameters and a total $\chi^2/datum$ of 1.8, (in the fit: proton and neutron SL data + proton TL data). Beside the 2π continuum, also $K\bar{K}$ and $\rho\pi$ continua !!



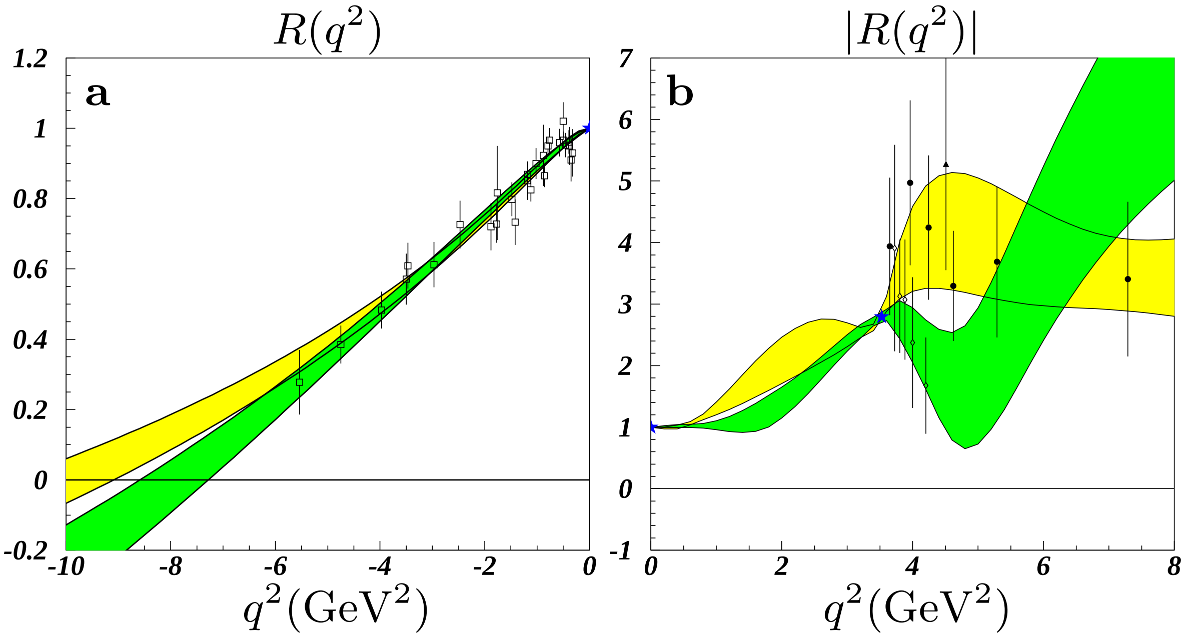
Nucleon ff's for SL momentum transfers, in the Super Constrained approach. Solid line: Bonn best fit; dashed lines: 1σ error bands.



Nucleon ff's for TL momentum transfers.

Pacetti (EPJA **32**, (2007)) has directly applied DR to the ratio $R(q^2) = \mu_p G_E^p(q^2)/G_M^p(q^2)$, both in SL and TL regions. A very general form for $Im\ R(q^2)$, with six parms, has been adopted.

$\chi^2/datum \sim= 1.3$



The band represents the error. Green band: retaining LEAR data in the TL region. Yellow band: retaining BABAR data in the TL region. (After S. Pacetti, EPJA **32**, (2007), see also Baldini et al, EPJC **11** (1999) for the analysis of all the ff's) .

Two Predictions

The descriptions obtained with the BABAR and Lear data predict a SL zero at:

$q_0^2(BABAR) = -10 \pm 1\ GeV^2$,
$q_0^2(LEAR) = -7.9 \pm 0.7\ GeV^2$

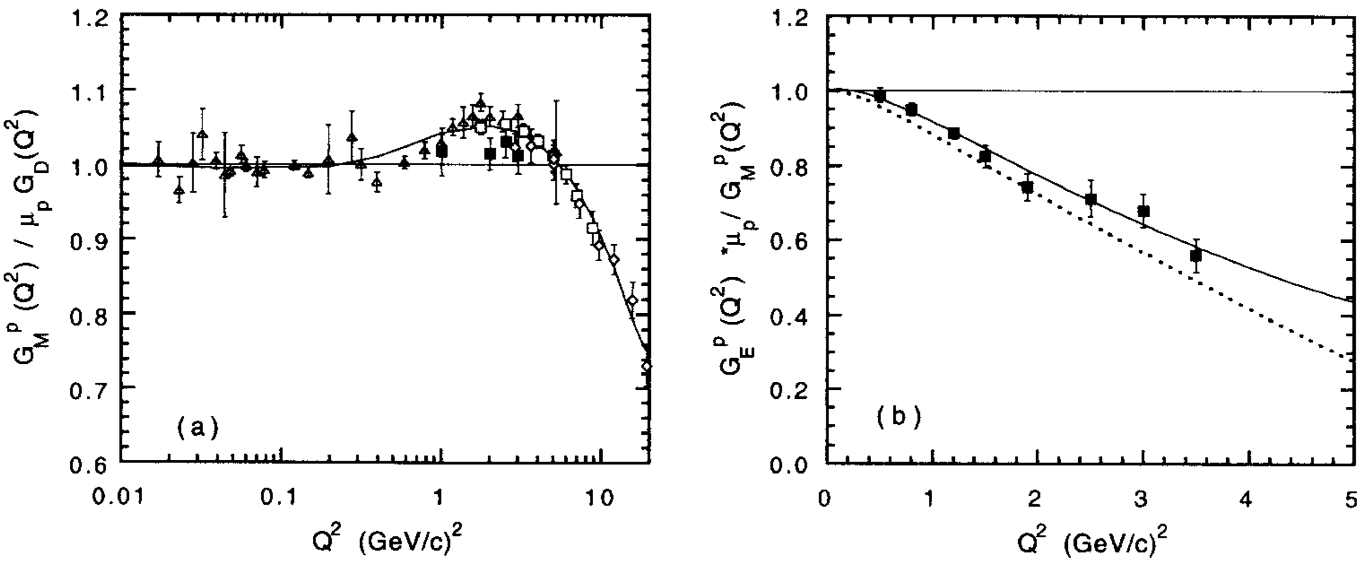
$$\lim_{q^2 \to \pm\infty} \left| \frac{G_E^p(q^2)}{G_M^p(q^2)} \right| = \begin{cases} 0.95 \pm 0.20\ BABAR \\ 2.3 \pm 0.7\ LEAR \end{cases}$$

Phenomenology II:

CQM + $q\bar{q} \rightarrow$ Microscopic Approaches

Relativistic CQ Models have been widely adopted for obtaining a microscopic description of the EM properties of the nucleon.

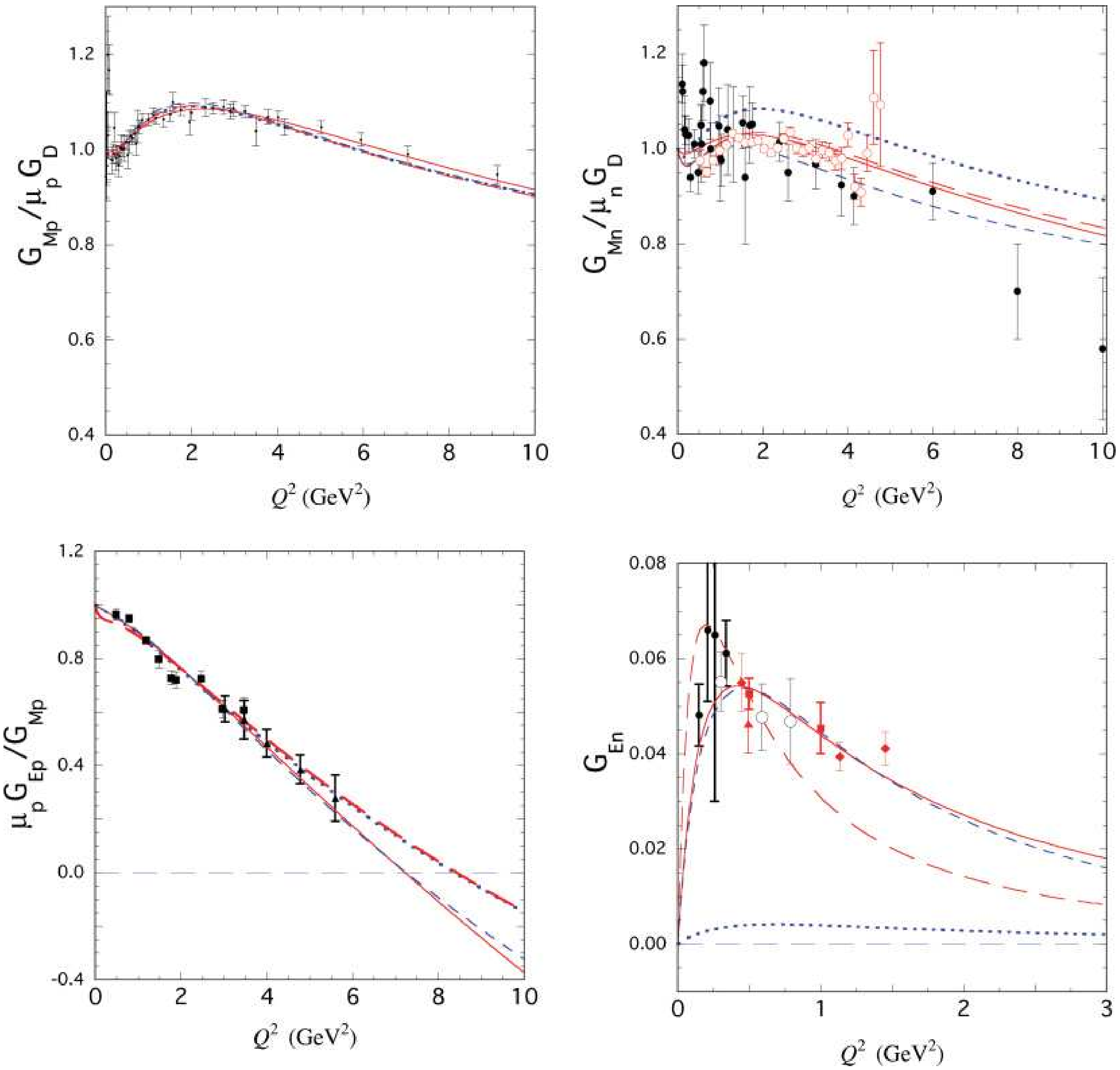
Since '90, it was recognized the relevance of dressing the CQM, e.g. by suitable form factors, with a spatial extension of the order of .4 - 0.5 fm, (a scale different from a naive pion-clod model), in order to obtain very accurate description of the Nucleon and Pion ff's in the SL region (Cardarelli et al PLB 557 (1995)).



Left panel: Proton magnetic ff. Right panel: Proton ratio. Solid line: results with CQ ff's; dashed line: without CQ ff's. The CQ ff's have 12 parms, but Capstick-Isgur w.f. for the nucleon. (After Cardarelli et al NPA **666** (1999)).

A recent application in the covariant Spectator Model (Gross - Ramalho -Peña PRC 77 (2008)) has achieved a very accurate description of the Nucleon ff's in the SL region.

The Nucleon is composed of three valence CQ's with form factors. An Ansatz for the Nucleon w.f., consistent with the properly symmetrized covariant Spectator formalism, in S wave. Two-pion cut added for improving the fit at low momentum transfers.

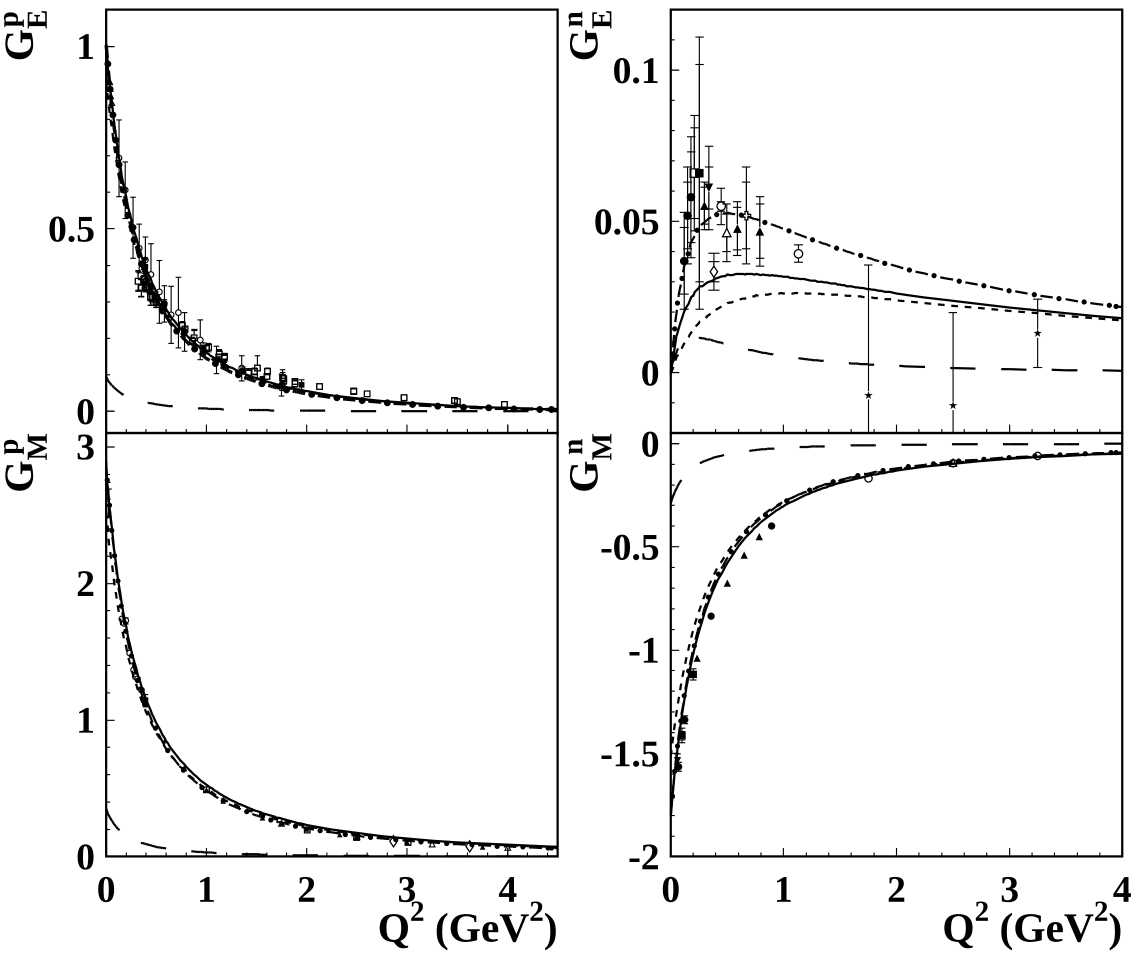


Dotted line: 4 parms and isospin symm. in the CQ ff's ($m_\rho \neq m_\omega$); short dashed line: 5 parms. isospin maximally broken in Dirac CQ ff; long dashed line: 6 parms. isospin symm. maximally broken in Dirac CQ ff's + two-pion cut; solid line: 9 parms, isospin symm. fully broken + two-pion cut. Static em properties OK for model II and IV. (After Gross et al PRC **77** (2008)).

CQM + Meson Cloud

Within the Hamiltonian Light-front dynamics, Pasquini - Boffi (PRD **76** (2007)) have developed a " meson-cloud" + CQM approach, based on a Fock expansion: $|qqq\rangle + |qqq; q\bar{q}\rangle$. The nucleon w.f. is an Ansatz.

Meson cloud significant only for $Q^2 < 0.5\ GeV^2$. Three parameters + percentage of S', necessary to improve the neutron. Two cut-of parms chosen from DIS. With no S': $\mu_p = 2.87$, $\mu_n = -1.80$, $r_p = 0.877\ fm$ and $r_n^2 = -0.064\ fm^2$

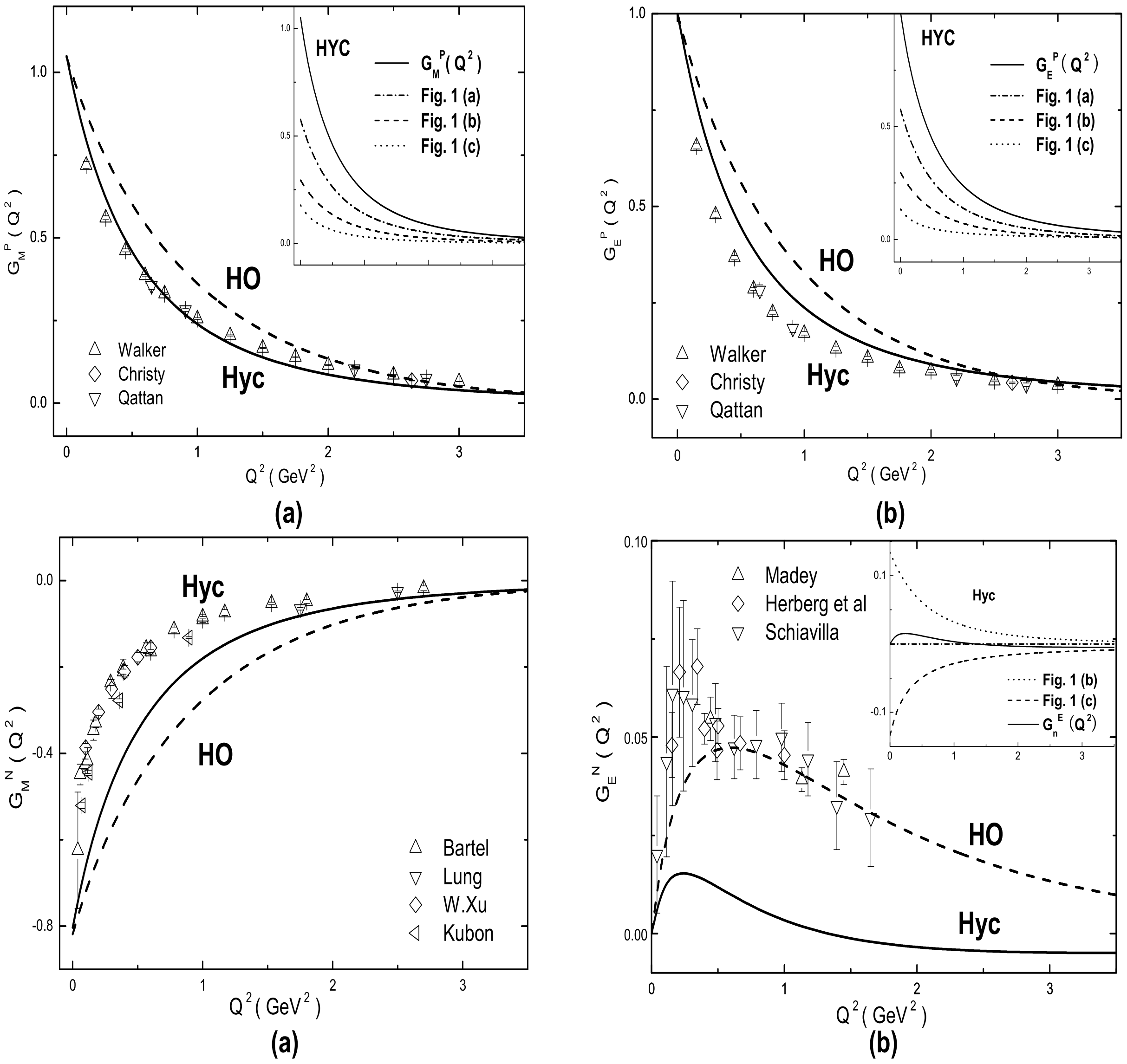


Long-dashed curve: the contribution of the meson cloud; dotted curve: the valence-quark contribution with SU(6) instant-form nucleon wave function; solid curve: the sum of the two contributions; dash-dotted curve: the total result with a 1% mixed-symmetry S'-state in (After B. Pasquini and S. Boffi, arXiv:0711.0821 and PRD **76** (2007)).

The Genoa group has performed calculations of the Nucleon ff's taking into account the pion-cloud effects and adopting the framework of the Hypercentral Constituent Quark Model, namely the nucleon w.f. is a solution of an eigenequation, without the spin-spin interaction, but with the explicit inclusion of the pion-cloud corrections. No relativistic effects in the kinematics.

$$|N\rangle = \sqrt{Z^N}\left[1+\frac{H_{int}}{(E_N - H_0 - \Lambda\, H_{int}\, \Lambda)}\right]|N_0\rangle$$

where H_{int} describes the process of emission and absorption of pions and Λ projects all the components of $|N\rangle$, with least one pion.



Dot-dashed curve: valence contr.; dashed curve: em coupling to a virtual baryon; dotted curve: em coupling to a pion. (After Chen , Dong, Giannini and Santopinto, Nucl.Phys. **A782** (2007) 62)

Relativistic CQ + microscopical VMD

The S. Josè-Rome Collaboration has extended the analysis performed for the SL & TL pion ff (PLB B 581 (2004) and PRD 73 (2006)) to the Nucleon (NPA 782 (2007) and submitted to PRL).

$$|meson\rangle = |q\bar{q}\rangle + |q\bar{q}q\bar{q}\rangle + |q\bar{q}\ g\rangle.....$$

$$|baryon\rangle = \underbrace{|qqq\rangle}_{\text{valence}} + \underbrace{|qqq\ q\bar{q}\rangle + |qqq\ g\rangle.....}_{\text{nonvalence}}$$

★ A meaningful Fock expansion within Light-Front framework, due to the properties of the LF vacuum.

★★ The Mandelstam covariant expression for the matrix elements of the em current. E.g., in the SL region

$$j^{\mu} = -\imath e \int \frac{d^4k}{(2\pi)^4}\, Tr\left[S_Q(k-P_h)\bar{\Lambda}_h(k-P'_h, P'_h)\ S(k-q) \times \Gamma^{\mu}(k,q)\ S(k)\Lambda_h(k,P_h)\right]$$

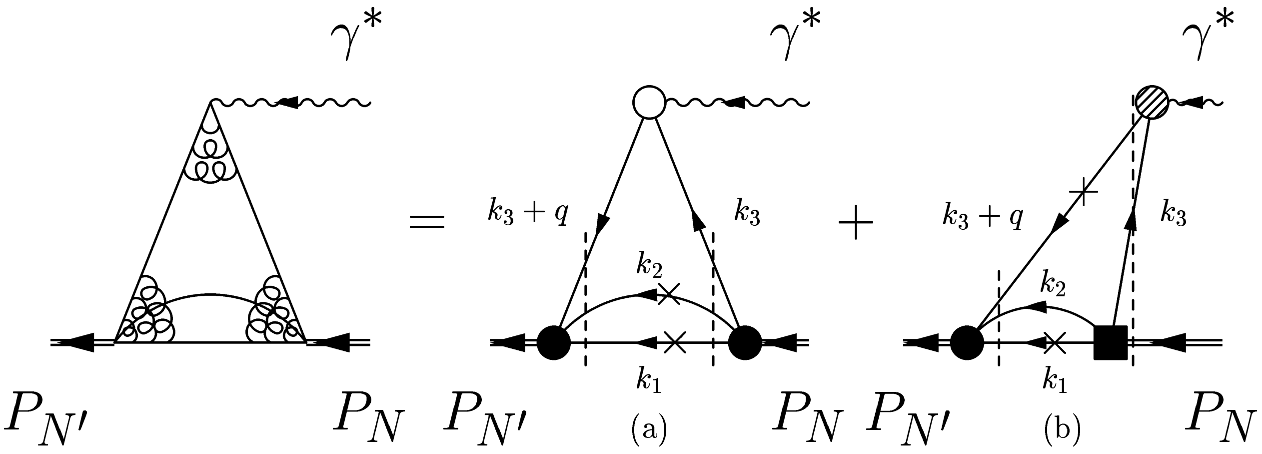
- $S(p) = \dfrac{1}{\not{p} - m + \imath\epsilon}$, with m the mass of the CQ struck by γ^*
- $S_Q(p)$: propagator(s) of the spectator CQ('s) for a meson (baryon)
- $\Lambda_h(k, P_h)$ is the hadron vertex function; P_h^{μ} and $P_{h'}^{\mu}$ are the hadron momenta. Very important: it contains a Dirac structure, i.e. a proper combination of Dirac matrices.
- $\Gamma^{\mu}(k,q)$ is the quark-photon vertex (q^{μ} the virtual photon momentum)

Let us integrate on k^- (i.e. projecting on the Light front), retaining the contribution from only the Dirac poles

★★★ In the chosen frame $q^+ \neq 0 \quad \rightarrow$ SL & TL

Space-like

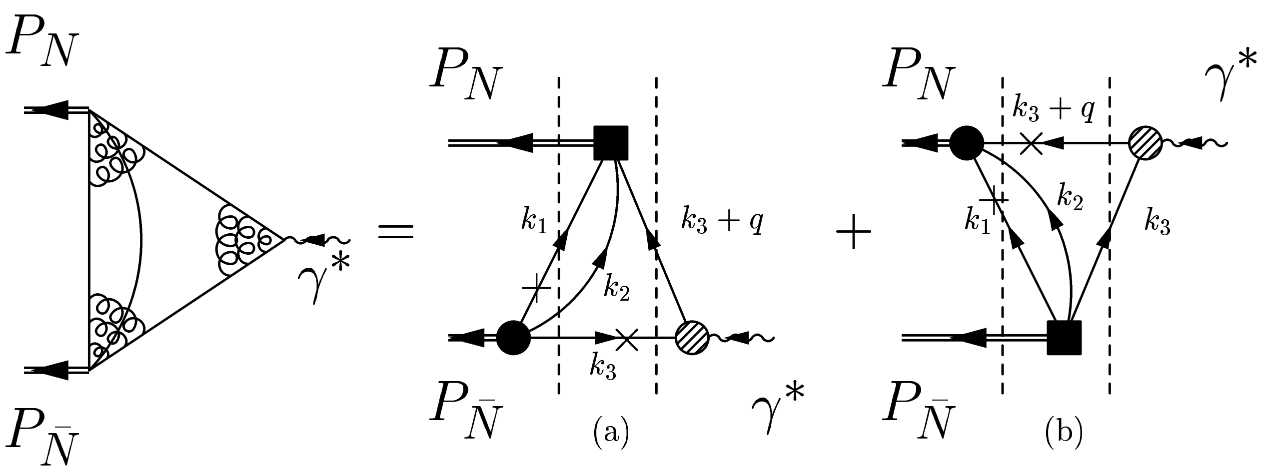
Triangle cont. Pair cont. (Z-diag.)



(val.) $0 < k^+_{1(2)} < P^+_N$ $\quad P^+_N < k^+_1 < q^+$ (non-val.)

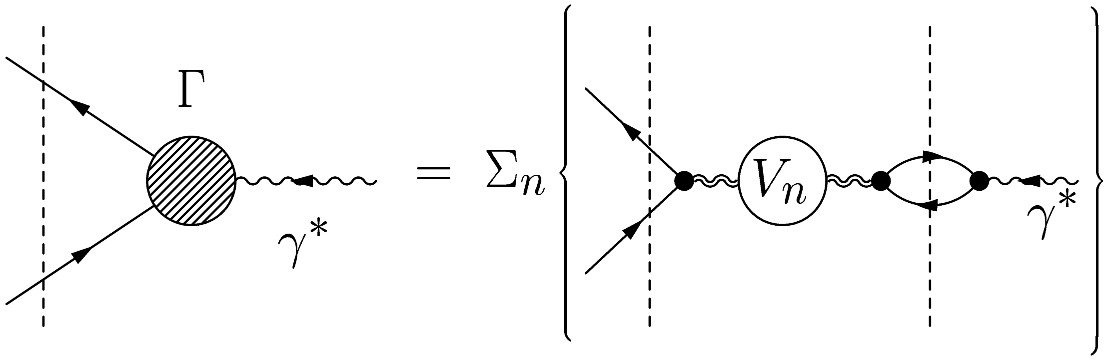
$\times \quad \Rightarrow \quad k_1$ on the mass shell : $k^-_{1on} = (m^2 + k^2_{1\perp})/k^+_1$

Time-like



The non valence contribution of the photon is involved: $|q\bar{q}, q\bar{q}, q\bar{q}\rangle$

★★★ Em current operator: bare + hadronic component of the photon w.f. (VMD). It needs a Microscopic model for VMD



⋆ A Vector Meson Dominance approximation has been applied to the quark-photon vertex, when a $q\bar{q}$ pair is produced

$$\Gamma^{+}(k,q) = \sqrt{2}\sum_{n,\lambda}\left[\epsilon_\lambda \cdot \widehat{V}_n(k,k-q)\right]\frac{\Lambda_n(k,P_n)\,[\epsilon^{+}_\lambda]^{*} f_{Vn}}{(q^2 - M_n^2 + \imath M_n\Gamma_n(q^2))}$$

- f_{Vn} is the decay constant of the n-th vector meson into a virtual photon (to be calculated in our model !), M_n the mass, $\Gamma_n(q^2) = \Gamma_n q^2/M_n^2$ (for $q^2 > 0$) the corresponding total decay width and ϵ_λ is the VM polarization
- $\left[\epsilon_\lambda \cdot \widehat{V}_n(k,k-q)\right] \equiv$ Dirac structure of the VM Bethe-Salpeter amplitude.

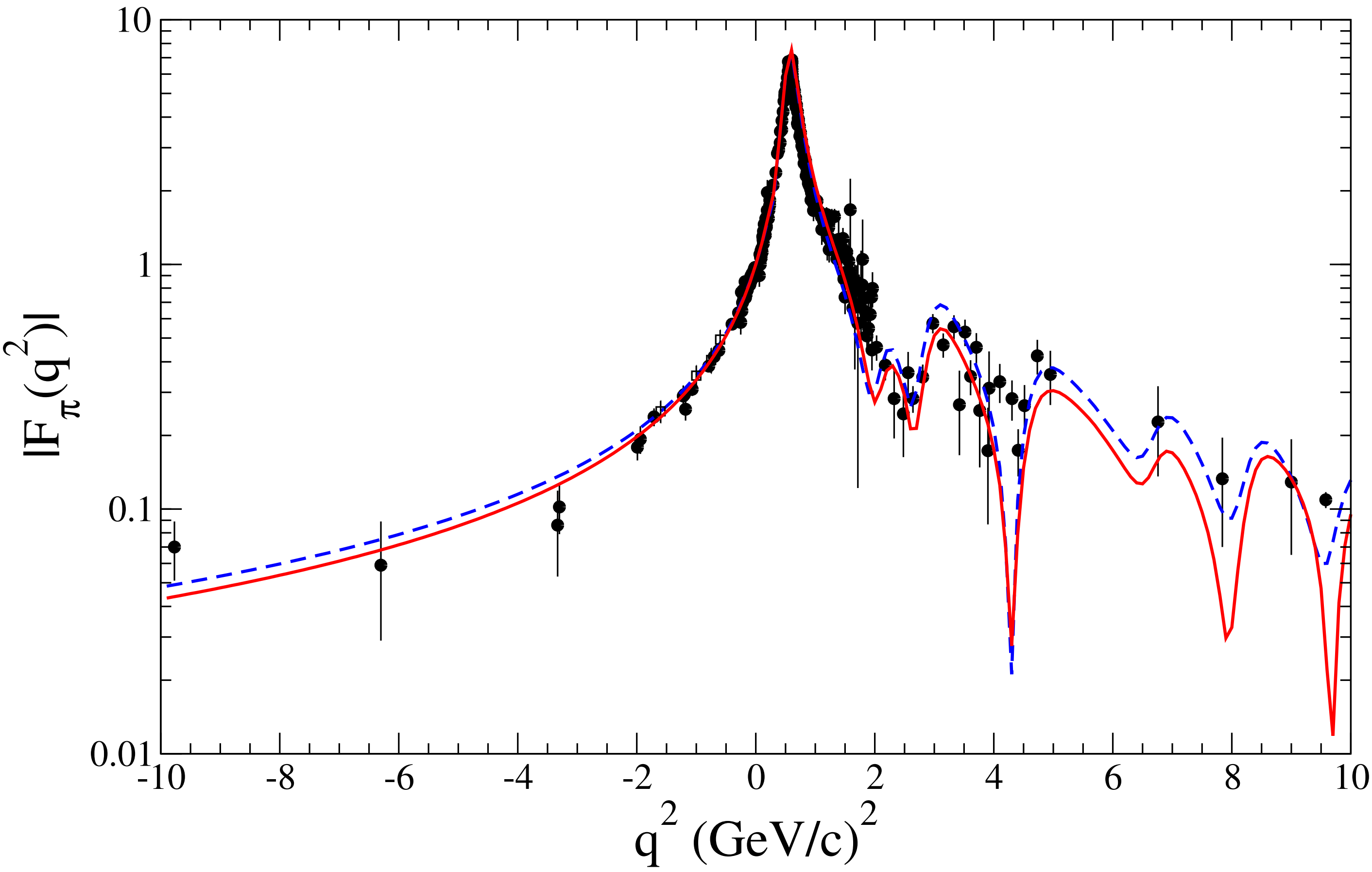
  $\Lambda_n(k,q) \equiv$ momentum-dependent part of the BS amplitude (to be approximated in our approach).

VM's eigenfunctions from a relativistic CQ square mass operator (Frederico, Pauli & Zhou, PRD 66 (2002) 116011)

Adjusted parm from pion ff: the width, $\Gamma_n = 0.15\ GeV$, of the VM's with mass $> 2.150\ GeV$. The chosen value∼ to the width of the first four VM's

Pion EM Form Factor in the SL and TL regions

Comparison with Exp. data



●: Data, R. Baldini et al. (EPJ. C11 (1999) 709, and Refs. therein.)

Solid line: calculation with the pion w.f. from the FPZ model for the Bethe-Salpeter amplitude in the valence region ($w_{VM} = -0.7$).

Dashed line: the same as the solid line, but with the asymptotic pion w.f. $(\Lambda_\pi(k; P_\pi) = 1)$

$$\psi_\pi(k^+, \mathbf{k}_\perp; P_\pi^+, \mathbf{P}_{\pi\perp}) = \frac{m}{f_\pi} \frac{P_\pi^+}{[M_\pi^2 - M_0^2(k^+, \mathbf{k}_\perp; P_\pi^+, \mathbf{P}_{\pi\perp})]}$$

Our microscopical VMD depends upon one parameter and now is fixed !

Quark-Photon Vertex for The Nucleon ff's

$$\mathcal{I}^{\mu} = \mathcal{I}^{\mu}_{IS} + \tau_z \mathcal{I}^{\mu}_{IV}$$

each term contains a contribution corresponding to a purely valence sector (Space-like only) and a contribution corresponding to the pair production (or Z-diagram).

In turn, the Z-diagram contribution can be decomposed in a bare term + a Vector Meson Dominance term (according to the decomposition of the photon state in bare, hadronic [and leptonic] contributions), viz

$$\mathcal{I}^{\mu}_{IS(IV)}(k,q) = \mathcal{N}_{IS(IV)}\theta(P^+_N - k^+)\,\theta(k^+)\gamma^{\mu} \;+$$
$$+\theta(q^+ + k^+)\,\theta(-k^+)\;\left\{Z_b\,\mathcal{N}_{IS(IV)}\gamma^{\mu} + Z_{VM}\,\Gamma^{\mu}_V[k,q,IS(IV)]\right\}$$

with $\mathcal{N}_{IS} = 1/6$ and $\mathcal{N}_{IS} = 1/2$. The constant Z_b (bare term) and Z_{VM} (VMD term) are unknown renormalization constants to be extracted from the phenomenological analysis of the data.

Momentum Dependence of the Bethe-Salpeter Amplitude

In the valence sector, where the spectator quarks are on their-own k^--shell and the struck one is a quark, the momentum dependence of the Nucleon Bethe-Salpeter amplitude reduces to a 3-momentum dependence, due to the LF projection we have applied.

It is approximated through a Nucleon Wave Function a la Brodsky (PQCD inspired)

$$\mathcal{W}_N = \mathcal{N} \, \frac{1}{(\xi_1 \xi_2 \xi_3)^p} \, \frac{1}{\left[\beta^2 + M_0^2(1,2,3)\right]^{7/2}}$$

★ β fixed through anomalous magnetic moments

Proton: 2.878 (Exp. 2.793) Neutron : -1.859 (Exp. -1.913)

★ ★ The powers, $p = 0.13$ and $7/2$ allow a falloff a little bit faster than $1/Q^4$ for the triangle contribution .

In the non-valence sector, relevant for evaluating the Z-diagram contribution, the momentum dependence is approximated by

$$\Lambda^{SL} = [g_{12}]^{5/2} \, \frac{(k_1^+ + k_2^+)}{P_N'^+} \, g_{N\bar{3}} \, \left(\frac{P_N^+}{k_{\bar{3}}^+}\right)^r$$

where $g_{AB} = (m_A \, m_B)/\left[\beta^2 + M_0^2(A,B)\right]$ and $r = 0.37$ for obtaining correctly both $r_p^2 \sim 0.81 \; fm^2$ and $r_n^2 \sim -0.114 \; fm^2$ (cf C. E. Hyde-Wright and K. de Jager, A. Rev. Nucl. and Part. Sci. **54**, 217 (2004)).

An analogous expression holds in the TL.

Fixed parms

- quark mass adopted: 200 MeV
- VMD, up to 20 mesons for achieving a good convergence in the q^2-range investigated. The isovector part obtained from the Pion analysis. The isoscalar part is an extension of the isovector one, with eigenvectors and eigenvalues from the Frederico, Pauli and Zhou model.

Four adjusted parms

from the fit of the SL ff's, (even disregarding the proton ratio!) with a $\chi^2/datum \sim 1.7$

i) Two renormalization constants for the pair production terms: $Z_b^{IS} = Z_b^{IV} = Z_{VM}^{IV}$, and Z_{VM}^{IS}, close each others $\sim 10\%$:

ii) Two powers, p and r, in the valence and non valence vertex functions, respectively

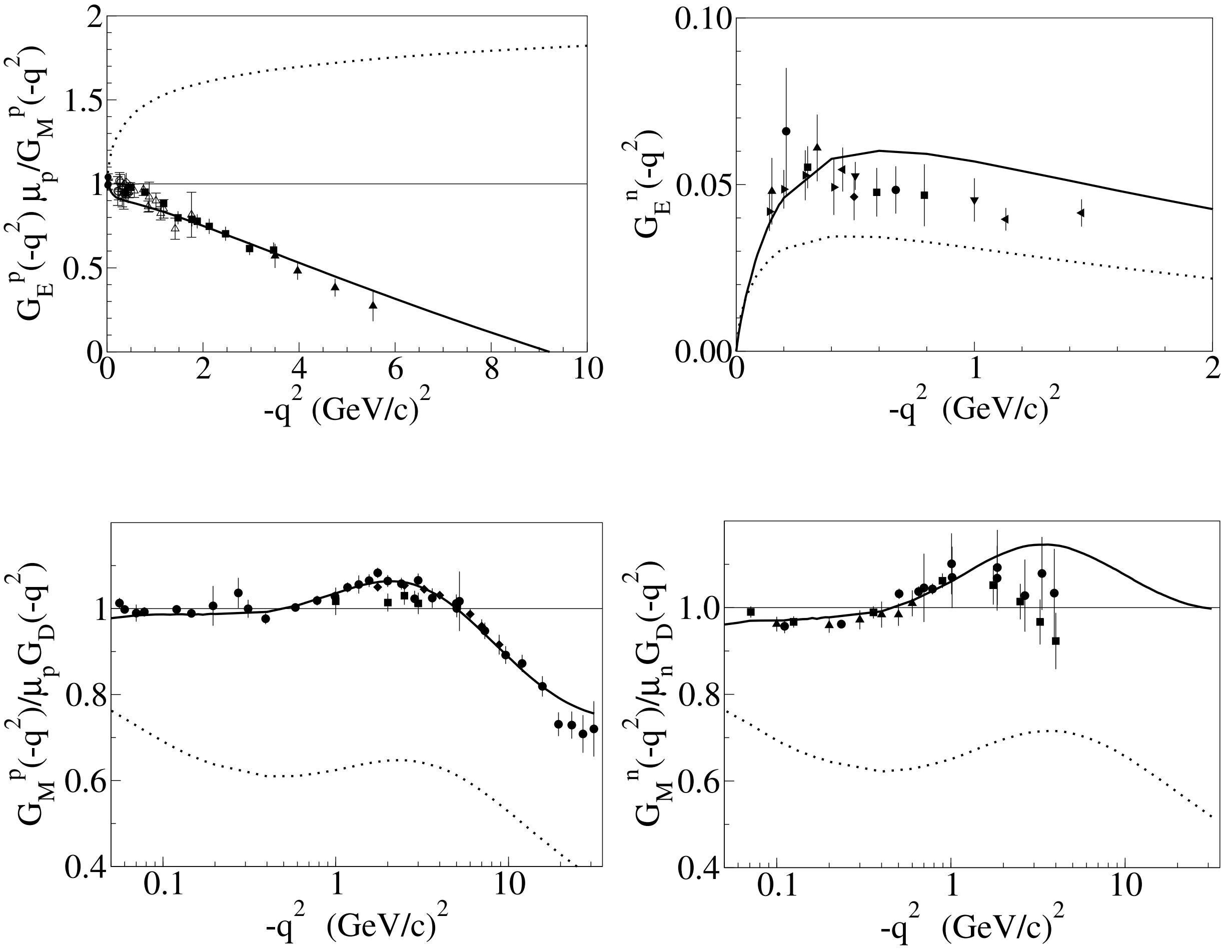
★ The model shows that the possible zero in G_E^p is due to the interference between the valence term and the Z-diagram contribution, i.e. higher Fock components of the proton state.

★ ★The model puts in evidence that some strength is lacking in our model for $q^2 \sim 4.5$ and $\sim 8\ (GeV/c)^2$ (as for the pion). Resonances?

But, the neutron TL data remains a challenge

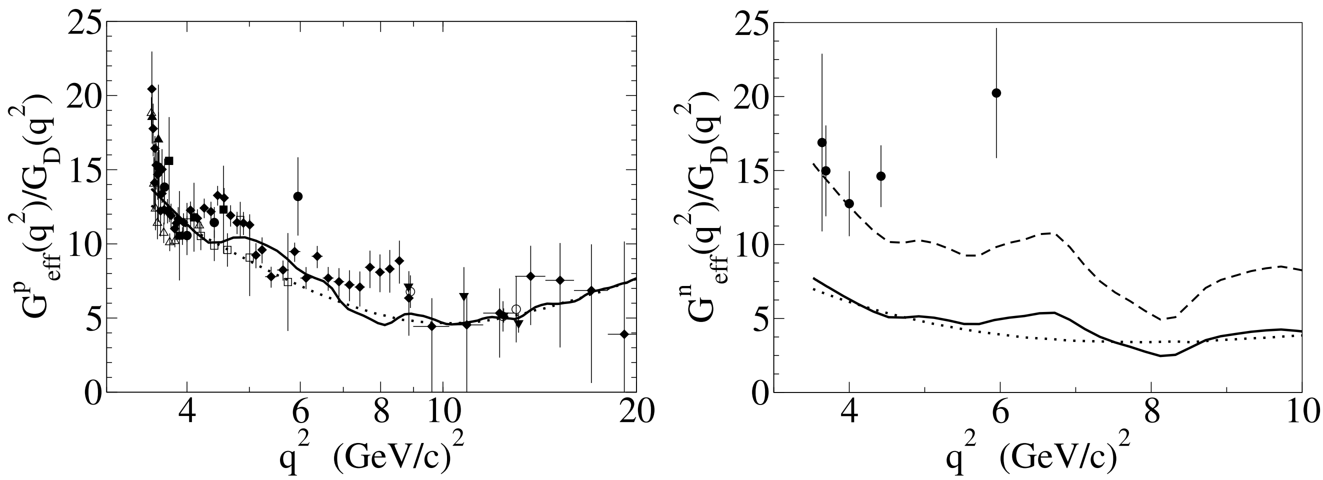
Notation: $G_D = 1/(1 + |q^2|/0.71)^2$

Spacelike



Solid line: full calculation $\equiv \mathcal{F}_{val} + Z_b\mathcal{F}_{bare} + Z_{VM}\mathcal{F}_{VMD}$;
Dotted line: $\mathcal{F}_{val}$ (elastic contribution only)

Timelike



Solid line: full calculation $\equiv Z_b\mathcal{F}_{bare} + Z_{VM}\mathcal{F}_{VMD}$; Dotted line: $Z_b\mathcal{F}_{bare}$ (bare contribution only); Dashed line: full calculation $\times$ 2 (a guide for the eyes)

Conclusions & Perspectives

- Lattice data are becoming more and more accurate, but only SL q^2 (Euclidean lattice...) and a limited range.
- DR's (powerful and predictive) yield accurate fits of the present SL & TL data, and can allow to embed a wide set of physical constrains. To make contact with the CQ one should calculate some relevant transition matrix elements...
- Models based on CQ + $q\bar{q}$ effects are able, with different accuracy, to describe the present data in the SL region. A suggestion: in view of a better analysis of the physics contents, theorists should adopt the same kind of plots for comparing theory and data: in the SL $G^p_E \mu_p / G^p_M$, G^p_M / G_D and G^n_M / G_D, otherwise all the cows are black...as in the Hegel's moonless night.
- The CQ + microscopical VMD model, as DR's, can investigate SL & TL on the same footing. The Z-diagram is essential for explaining the zero in the proton ratio.
- Accurate data for the Nucleon ff's, in both SL and TL regions, are essential for Non Perturbative studies of QCD