Hadronic Light by Light Contribution to Muon $g-2$: Status and Prospects

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PHIPSI08 Workshop, 7-10 April 2008, Frascati

Plan

- Introduction

Plan

- Introduction
- “Old” Calculations: 1995-2001

Plan

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- New Short-Distance Constraints: 2003-2004

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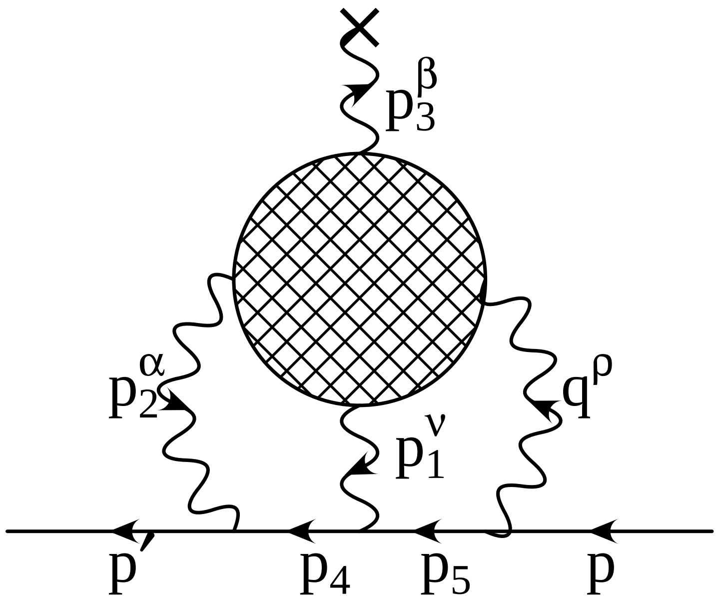
- Introduction
- “Old” Calculations: 1995-2001
- New Short-Distance Constraints: 2003-2004
- Comparison
- Conclusions and Prospects

Introduction

Hadronic light-by-light contribution to muon $g-2$



$$\mathcal{M} = |e|^7 A_\beta \int \frac{d^4 p_1}{(2\pi)^4} \int \frac{d^4 p_2}{(2\pi)^4} \frac{1}{q^2\, p_1^2\, p_2^2 (p_4^2 - m^2)\,(p_5^2 - m^2)}$$
$$\times \underline{\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}\, \bar{u}(p')\gamma_\alpha(\not{p}_4 + m)\gamma_\nu(\not{p}_5 + m)\gamma_\rho u(p)$$

Introduction

Need

$$\Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3) = i^3 \int \mathrm{d}^4x \int \mathrm{d}^4y \int \mathrm{d}^4z \, \exp^{i(p_1\cdot x + p_2\cdot y + p_3\cdot z)} \times$$
$$\times \langle 0|T[V^\rho(0)V^\nu(x)V^\alpha(y)V^\beta(z)]|0\rangle$$

with $V^\mu(x) = [\overline{q}\,\widehat{Q}\gamma^\mu q](x)$ and $\widehat{Q} = \frac{1}{3}\mathrm{diag}(2,-1,-1)$

full four-point function with $p_3 \to 0$ •

Using gauge invariance

$$\Pi^{\rho\nu\alpha\lambda}(p_1,p_2,p_3) = -p_{3\beta}\frac{\delta\Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3)}{\delta p_{3\lambda}}$$

one just needs derivatives at $p_3 = 0$ •

Introduction

★ Many scales involved: Impose low energy and several OPE limits ➪ Not full first principle calculation at present •

(Two lattice groups just starting: Not clear final uncertainty) •

Large N_c and CHPT counting:

Organizes different degrees of freedom contributions •

E. de Rafael

- Goldstone boson exchange: $\mathcal{O}(N_c)$ and $\mathcal{O}(p^6)$ •
- Quark Loop and non-Goldstone boson exchange: $\mathcal{O}(N_c)$ and $\mathcal{O}(p^8)$ •
- Goldstone bosons Loop: $\mathcal{O}(1)$ in $1/N_c$ and $\mathcal{O}(p^4)$ •

Introduction

Based on this counting:

- Two full calculations
  J. Bijnens, E. Pallante, J.P. (BPP)
  M. Hayakawa, T. Kinoshita, A. Sanda (HKS)

- Dominant pseudo-scalar exchange: Extensive analytic analysis •
  M. Knecht, A. Nyffeler (KN)

  Found sign mistake ✔
  M. Knecht, A. Nyffeler, M. Perrottet, E. de Rafael

Introduction

★ New four-point form factor short-distance constraint:

K. Melnikov, A. Vainshtein

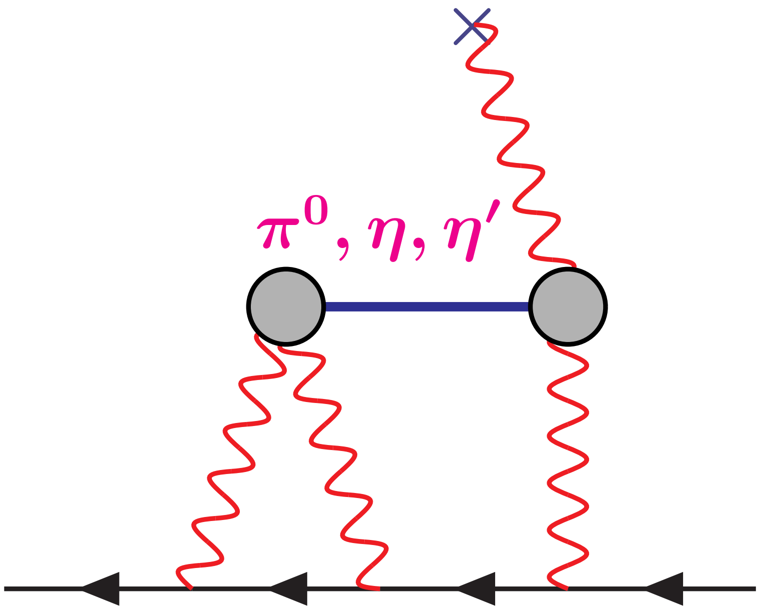
(see also M. Knecht, S. Peris, M. Perrottet, E. de Rafael)

Model:

Full light-by-light saturated by pseudo-scalar and pseudo-vector pole exchanges •

“Old” Calculations: Pseudo-Scalar Exchange

Dominant contribution ➾ pseudo-scalar exchange •



Here, I discuss work in J. Bijnens, E. Pallante, J.P. •

“Old” Calculations: Pseudo-Scalar Exchange

We used a variety of $\pi^0\gamma^*\gamma^*$ form factors

$$\mathcal{F}^{\mu\nu}(p_1,p_2) = \frac{N_c}{6\pi}\frac{\alpha}{f_\pi}\, i\varepsilon^{\mu\nu\alpha\beta}\, p_{1\alpha}p_{2\beta}\, \underline{\mathcal{F}(p_1^2,p_2^2)}$$

fulfilling as many as possible QCD constraints •
(Short-distance, data, U$_A$(1) normalization and slope at the origin). In particular,

$$\begin{aligned}\mathcal{F}(Q^2,Q^2) &\to \frac{A}{Q^2}\\ \mathcal{F}(Q^2,0) &\to \frac{B}{Q^2}\end{aligned}$$

for Q^2 Euclidean and very large

"Old" Calculations: Pseudo-Scalar Exchange

All form factors we used converge for $\mu \sim (2-4)$ GeV and the numerical difference between them is small ✔

Somewhat different $\pi^0\gamma^*\gamma^*$ form factors used in
M. Hayakawa, T. Kinoshita, A. Sanda and M. Knecht, A. Nyffeler •

Results agree very well (after correcting a mistake in the sign of the phase space) •

Adding π^0, η and η' contributions

	$10^{10} \times a_\mu$
BPP	(8.5 ± 1.3)
HKS	(8.3 ± 0.6)
KN	(8.3 ± 1.2)

"Old" Calculations: Pseudo-Vector Exchange

Need $a_1^0\gamma\gamma^*$ and $a_1^0\gamma^*\gamma^*$ form factors •

➩ related to $\pi^0\gamma\gamma^*$ and $\pi^0\gamma^*\gamma^*$ by anomalous Ward identities ✔

Pseudo-vector exchange

	$10^{10} \times a_\mu$
BPP	(0.25 $\pm$ 0.10)
HKS	(0.17 $\pm$ 0.10)

“Old” Calculations: Scalar Exchange

Need $S^0\gamma\gamma^*$ and $S^0\gamma^*\gamma^*$ form factors •

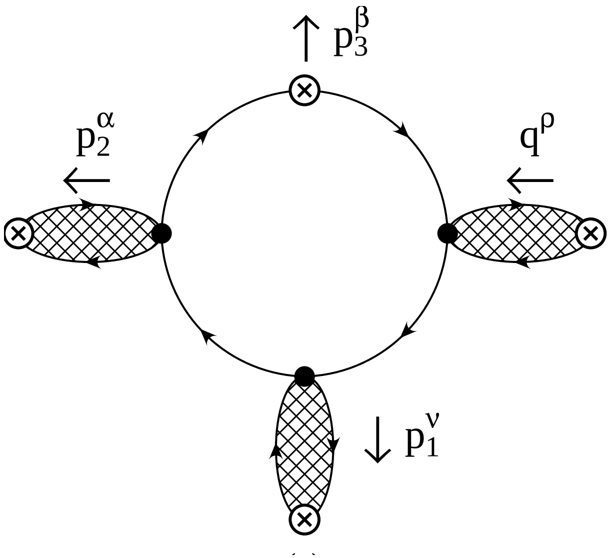
They are constrained by CHPT at $\mathcal{O}(p^4)$: L_i’s reproduced ✔

Within ENJL: Ward identities impose relations between Quark loop and Scalar exchange •

$$a_\mu(\mathrm{Scalar}) = -(0.7 \pm 0.2) \cdot 10^{-10}$$

Not included by M. Hayakawa, T. Kinoshita and A. Sanda nor by K. Melnikov and A. Vainshtein •

“Old” Calculations: Non-Meson Exchange “Quark-Loop”



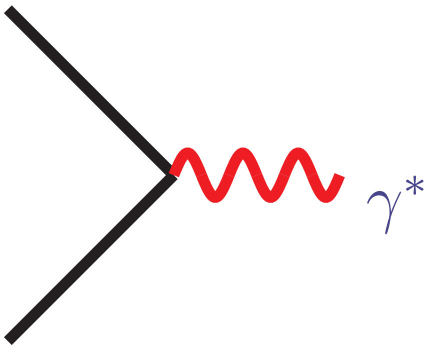
Λ [GeV]	$10^{10} \times a_\mu$
0.7	2.2
1.0	2.0
2.0	1.9
4.0	2.0

- Low Energy (0 to Λ): ENJL model •
- High Energy (Λ to ∞): Bare heavy quark loop with $m_Q = \Lambda$ •
- Numerical matching ✔

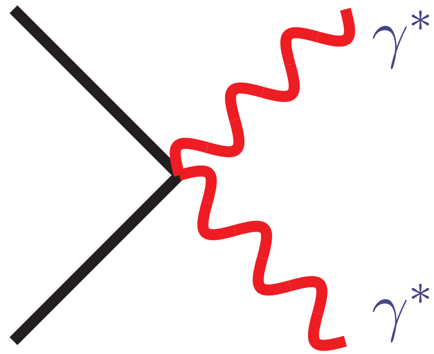
"Old" Calculations: Pion- and Kaon-Loop

Leading contribution in chiral counting, suppressed by $1/N_c$

OK:



Problem:



No $\gamma^*\gamma^* \to \pi\pi$ data available: Models needed !

Model for $\pi\pi\gamma(\gamma)$	$10^{10} \times a_\mu$
BPP (Full VMD)	-1.8
HKS (HGS)	-0.4

Kaon loop is much smaller: -0.05×10^{-10} •

New Short Distance Constraints

K. Melnikov and A. Vainshtein

New short-distance constraint on four-point function form factor

$$\langle 0 | T[V^\nu(p_1) V^\alpha(p_2) V^\rho(-(p_1+p_2+p_3))] | \gamma(p_3 \to 0)\rangle$$

using OPE with $-p_1^2 \simeq -p_2^2 >> -(p_1+p_2)^2$ Euclidean and large,

$$T[V^\nu(p_1) V^\alpha(p_2)] \sim \frac{1}{\hat{p}^2}\, \varepsilon^{\nu\alpha\mu\beta}\, \hat{p}_\mu \, [\overline{q}\, \widehat{Q}^2 \gamma_\beta \gamma_5 q](p_1+p_2)$$

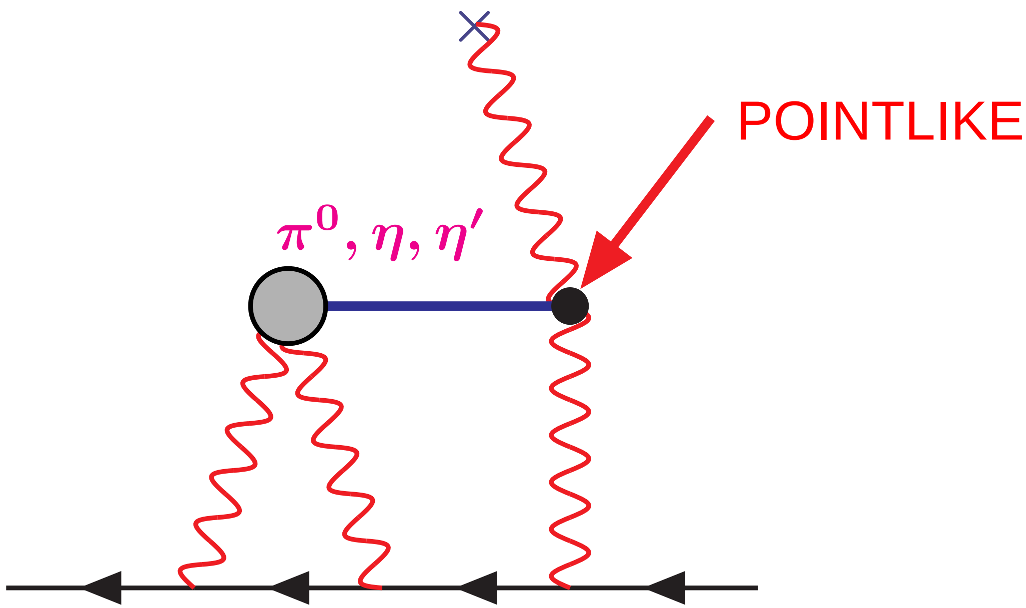
with $\hat{p} = (p_1 - p_2)/2 \simeq p_1 \simeq -p_2$

New OPE Constraint: Pseudo-scalar exchange

New OPE constraint saturated by pseudo-scalar exchange

➪ Model uses a point-like vertex when $p_3 \to 0$ •

Not all OPE constraints satisfied: Negligible numerically •



New OPE Constraint: Axial-Vector exchange

Axial-Vector exchange depends very much on the resonance mass mixing •

K. Melnikov and A. Vainshtein:
Ideal mixing for $f_1(1285)$ and $f_1(1420)$ •

Mass mixing	$10^{10} \times a_\mu$
No New OPE (Nonet symmetry)	0.3 $\pm$ 0.1
M=1.3 GeV (Nonet symmetry)	0.7
M=M$_\rho$ (Nonet symmetry)	2.8
Ideal mixing	2.2 $\pm$ 0.5

Comparison: Leading order in N_c

Leading order in N_c:

Quark Loop + Pseudo-Scalar + Pseudo-Vector + Scalar Exchanges •

Total at $\mathcal{O}(N_c)$	$10^{10} \times a_\mu$
BPP (Nonet symmetry)	(10.9 $\pm$ 1.9) + -(0.7 $\pm$ 0.1) = (10.2 $\pm$ 1.9)
HKS (Nonet symmetry)	(9.4 $\pm$ 1.6) + ??Scalar??

MV: Hadronic model saturated by pole exchanges:
Cannot compare individual contributions •

Total at $\mathcal{O}(N_c)$	$10^{10} \times a_\mu$
MV (Nonet symmetry)	(12.1 $\pm$ 1.0) + ??Scalar??
MV (Ideal mass mixing)	(13.6 $\pm$ 1.5) + ??Scalar??

Masses produce main difference in pseudo-vector exchange •

Comparison: Momenta Regions for π^0

Study of momenta regions contribution for π^0 exchange

$$P_1^2 = -p_1^2, \quad P_2^2 = -p_2^2, \quad Q^2 = -(p_1+p_2)^2$$

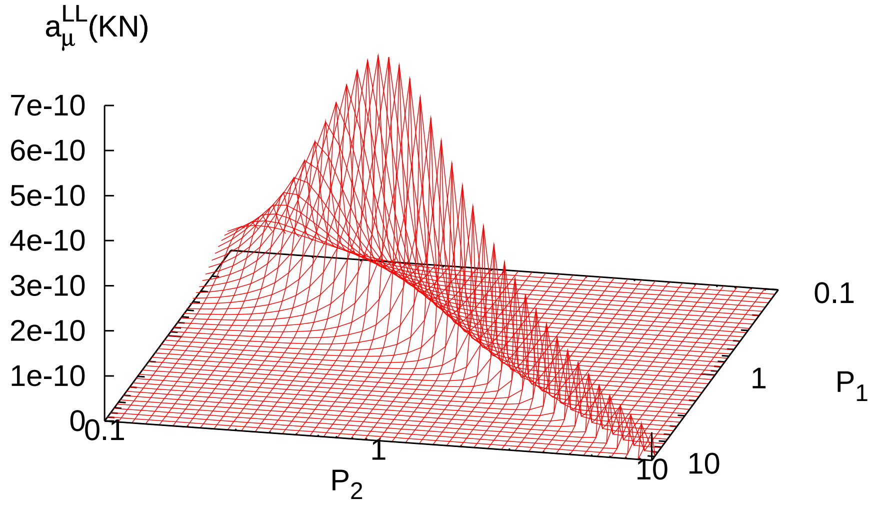
$$a_\mu^{\mathrm{lbl}} = \int \mathrm{d}P_1\,\mathrm{d}P_2\ \ a_\mu^{PP}(P_1,P_2)$$

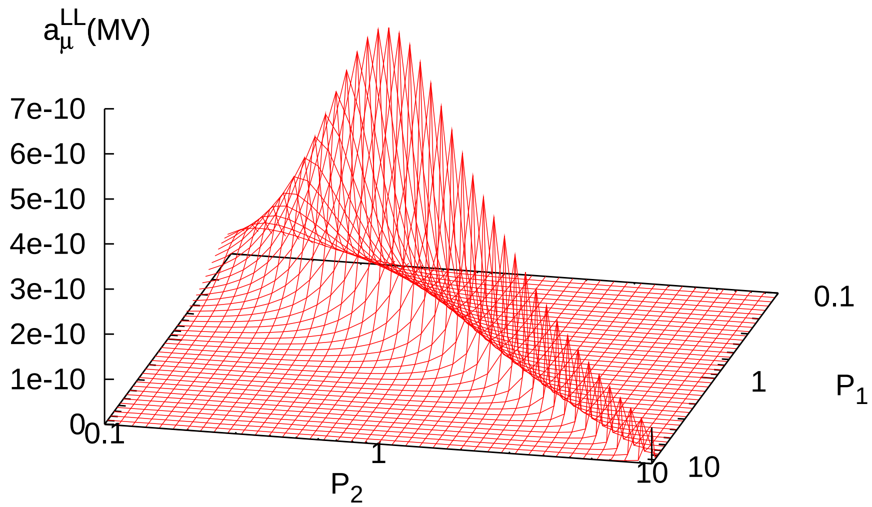
$$= \int \mathrm{d}l_1\,\mathrm{d}l_2\ \ a_\mu^{LL}(l_1,l_2)$$

$$= \int \mathrm{d}l_1\,\mathrm{d}l_2\mathrm{d}q\ \ a_\mu^{PPQ}(l_1,l_2,q)$$

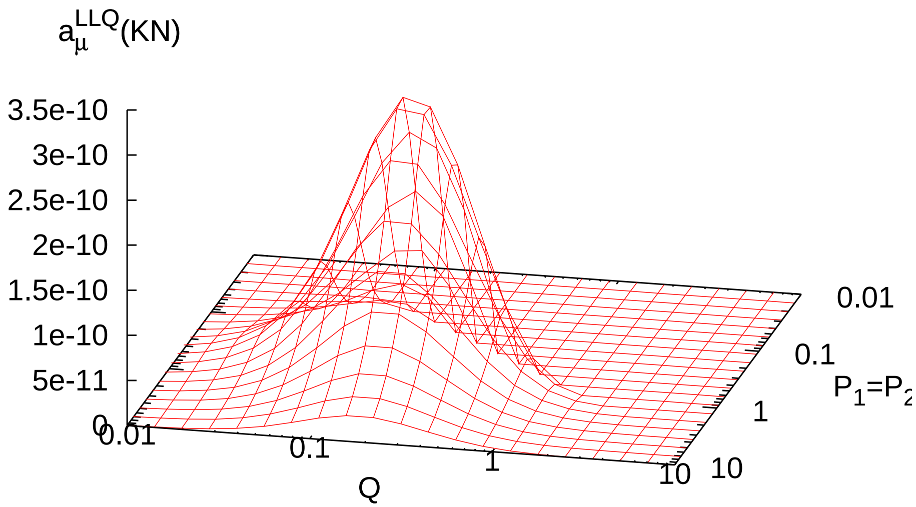
with $l_1 \equiv \ln(P_1/\mathrm{GeV})$, $l_2 \equiv \ln(P_2/\mathrm{GeV})$ and $q \equiv \ln(Q/\mathrm{GeV})$

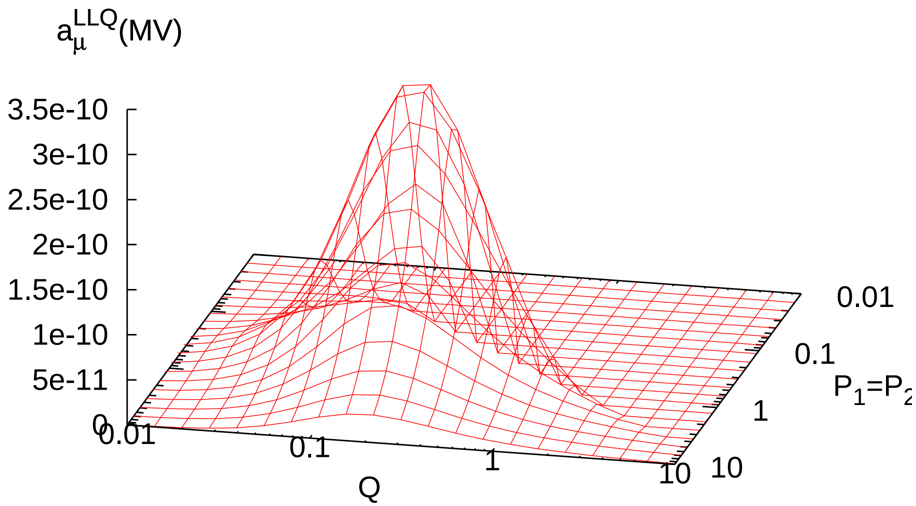
Comparison: Momenta Regions for π^0





Comparison: Momenta Regions for π^0





Comparison: Momenta Regions for π^0

Conclusions of this comparison:

Cut-off in Q ➪ 58 % of numerical difference come from Melnikov-Vainshtein OPE constraint violating regions •

Fixing $P1 = P2$ ➪ Numerical difference comes from low values of Q and moderate values of $P_1 = P_2$ •

➪ Important to control energy regions below 2 GeV •

Main ENJL quark-loop contribution is from that region •

Comparison: NLO in $1/N_c$

Next to leading order in $1/N_c$ contributions:

Charged Pion and Kaon Loop •

Model for $\pi\pi\gamma(\gamma)$	$10^{10} \times a_\mu$
BPP (Full VMD)	-1.9 ± 0.5
HKS (HGS)	-0.45 ± 0.8

K. Melnikov and A. Vainshtein:

Full NLO in $1/N_c$ estimate

$$a_\mu = (0 \pm 1) \cdot 10^{-10}$$

Comparison

BPP vs HKS:

Full	$10^{10} \times a_\mu$
BPP	8.3 $\pm$ 3.2
HKS	8.9 $\pm$ 1.7

No scalar exchange, different quark loop and different pion and kaon loops almost compensate •

Comparison

BPP vs MV:

Full	$10^{10} \times a_\mu$
BPP	8.3 ± 3.2
MV	13.6 ± 2.5

Several order $1.5 \cdot 10^{-10}$ differences,
in addition to new OPE effects •

$-1.5 \cdot 10^{-10}$ (Different pseudo-vector mass mixing)
$-0.7 \cdot 10^{-10}$ (No scalar exchange)
$-1.9 \cdot 10^{-10}$ (No pion+kaon loop)
= $-4.1 \cdot 10^{-10}$ •

Final [BPP-MV] difference: $-5.3 \cdot 10^{-10}$ •

Conclusions and Prospects

Unsatisfactory situation: Needs new evaluation(s) of the full hadronic light-by-light contribution •

★ At $\mathcal{O}(N_c)$: Study relevant reduced full four-point function with large N_c techniques •

☛ First step: Analytic expressions • Granada-Marseille

☛ Second step: Implement as many short-distance and low energy constraints as possible •

(possible problems J. Bijnens, E. Gámiz, E. Lipartia, J.P.)
Granada-Marseille-Lund-València

Conclusions and Prospects

★ At NLO in $1/N_c$:

⇨ Goldstone bosons at one loop: Need $\gamma^*\gamma^* \to \pi^+\pi^-$ vertex • Experiment can help here •

⇨ Non-Goldstone bosons at one loop: Little is known • (recent related work by A. Pich, I. Rosell, J.J. Sanz-Cillero) •

More theoretical work is clearly needed here •

Conclusions and Prospects

At present, large N_c results agree within 1 σ ✔

$$a_\mu^{\rm lbl} = (11.0 \pm 4.0) \times 10^{-10}$$

More work needed to have the hadronic light-by-light contribution to muon $g-2$ with reduced uncertainty •

Goal: To have under control model dependences •