Muon $g-2$ update

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Outline of Talk:

① **Hadronic VP: e^+e^-–data updates**

② **Updates for $a_\mu^{\rm had}$**

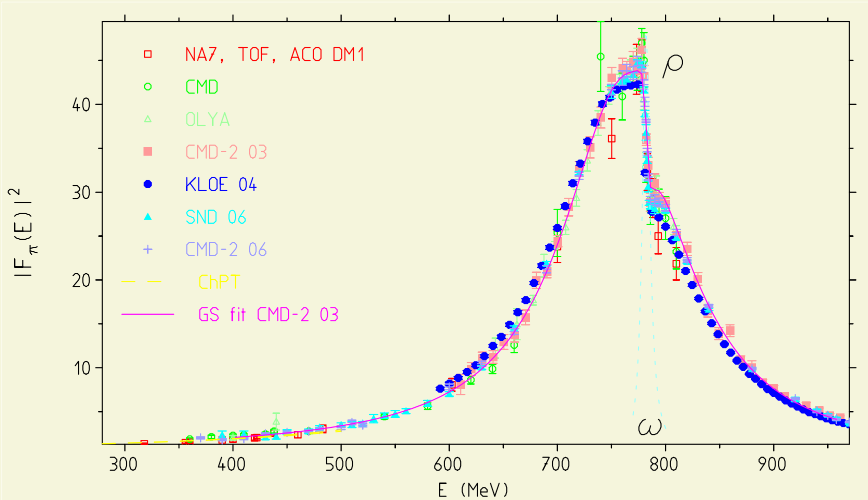
③ **Evaluation of $a_\mu^{\rm LbL}$ in the large-N_c framework**

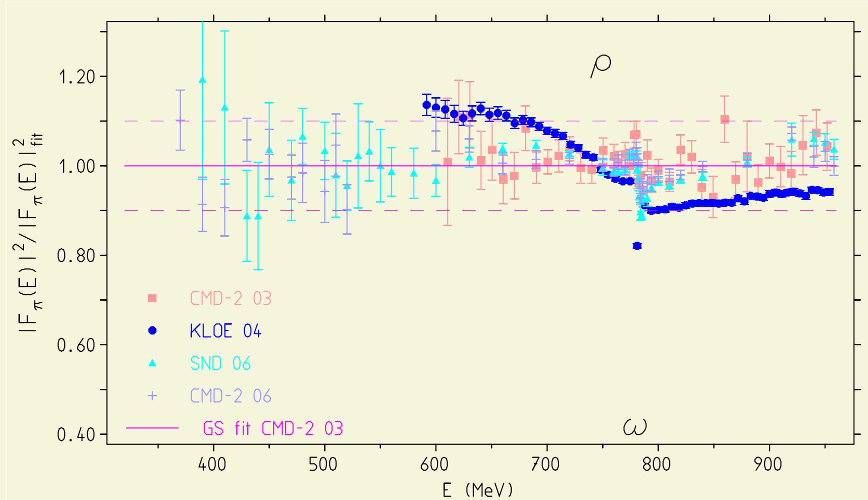
④ **Remarks on τ vs. e^+e^- data**

Abstract: We present an update of the theoretical prediction of the muon $g-2$. Mainly new BaBar data required a new update of the hadronic contribution, although no substantial change results. We also recalculated the hadronic light-by-light contribution in the large-N_c framework.

① Hadronic VP: the e^+e^-–data

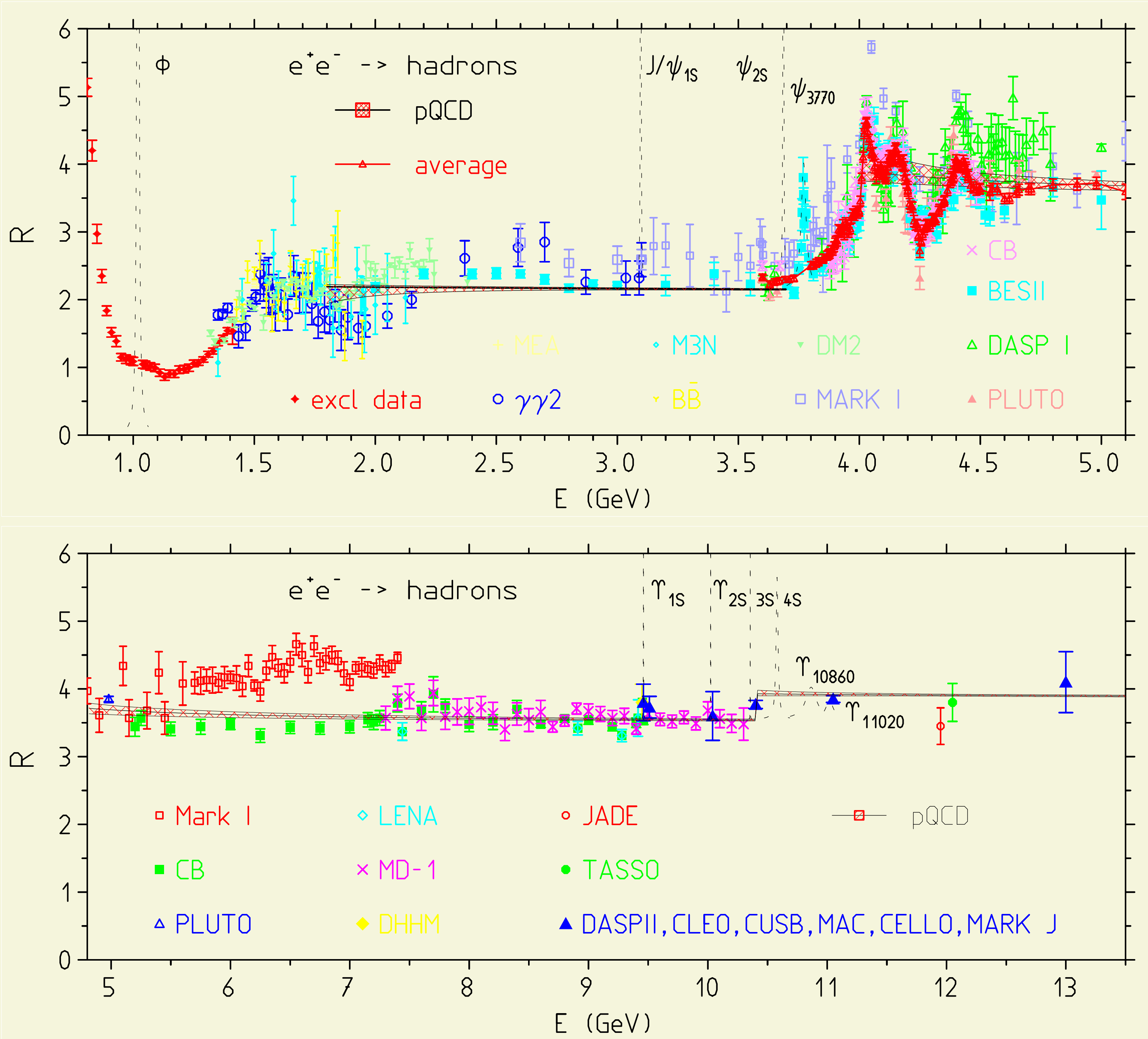
Low energy region:

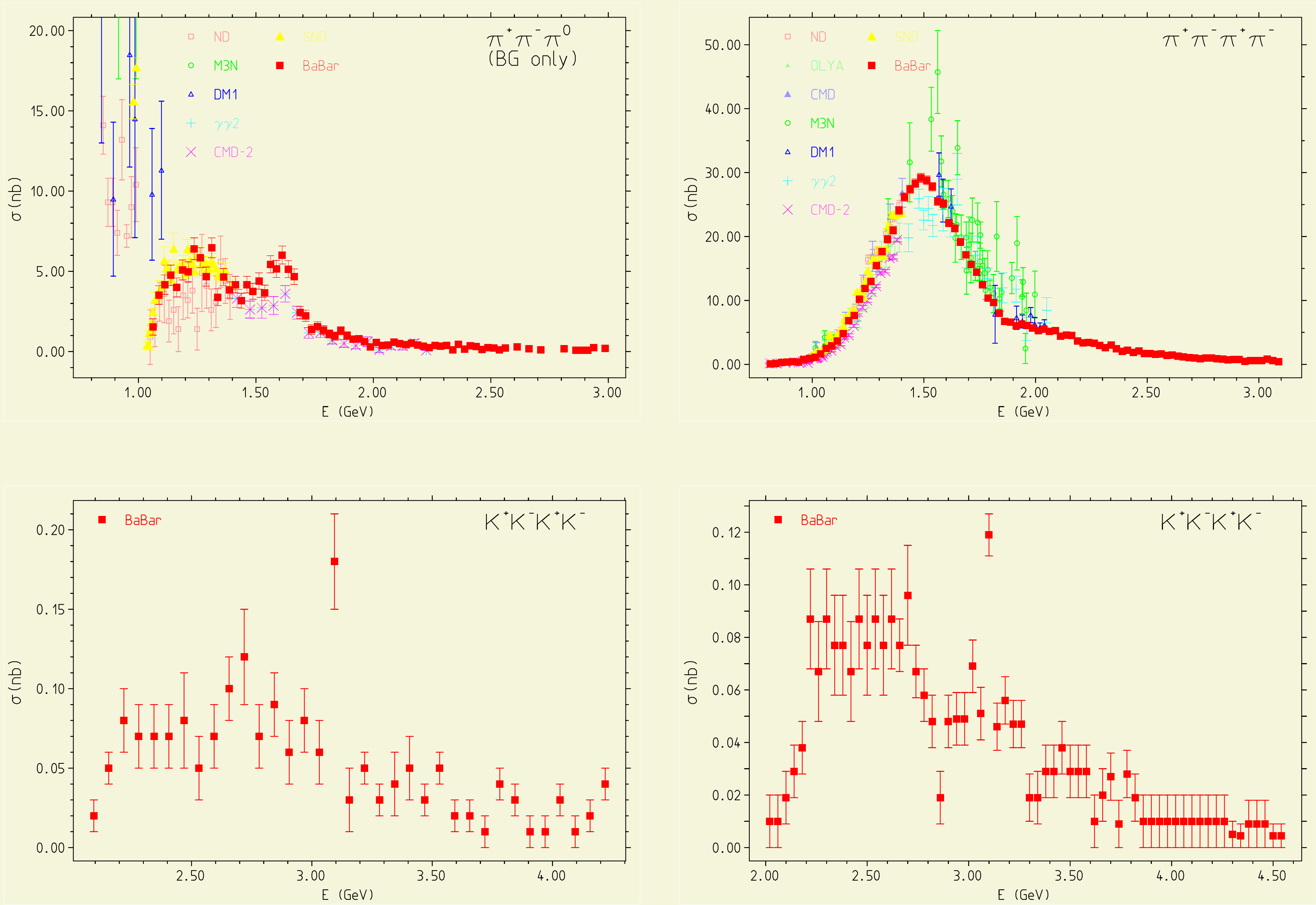




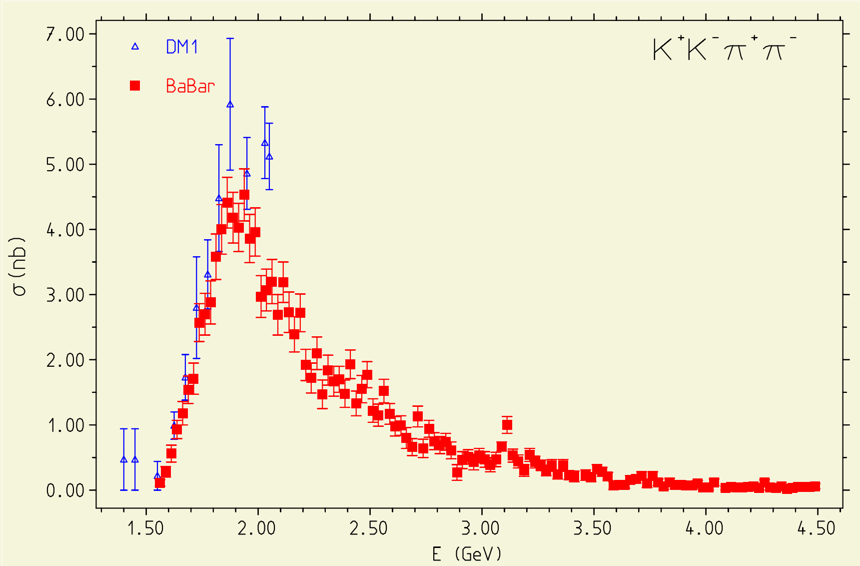
KLOE	$a_\mu^{\pi\pi}[0.35, 0.95]$ GeV^2 $\times 10^{10}$		$a_\mu^{\pi\pi}[630, 958]$ MeV $\times 10^{10}$
2001 data publ.	$388.7 \pm 1.8 \pm 4.9$	CMD-2	$361.5 \pm 1.7 \pm 2.9$
2001 data upd.	$384.7 \pm 1.8 \pm 4.9$	SND	$361.0 \pm 2.0 \pm 4.7$
2002 data prel.	$386.3 \pm 0.6 \pm 3.9$	KLOE	$355.5 \pm 0.5 \pm 3.6$

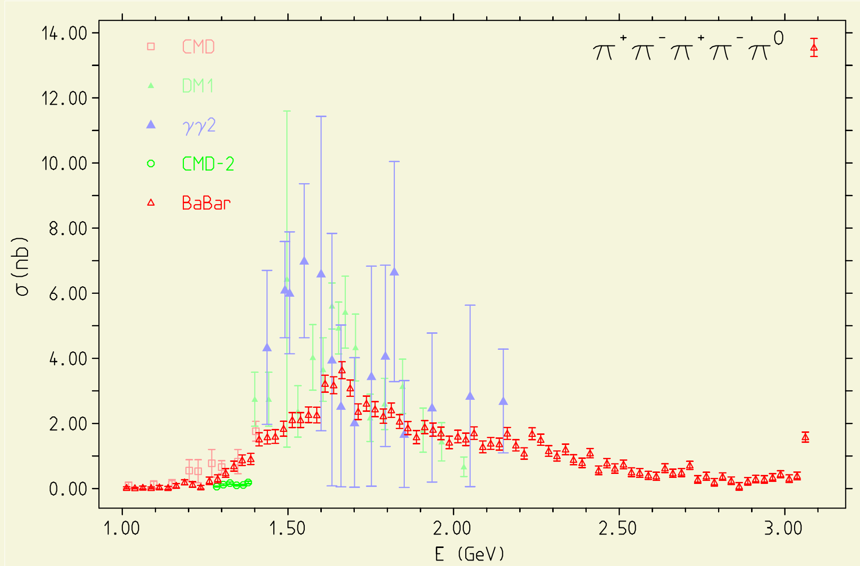
KLOE prel.	$a_\mu^{\pi\pi}[0.50, 0.85]$ GeV^2 $\times 10^{10}$
2002 small angle	$255.4 \pm 0.4 \pm 2.5$
2002 large angle	$252.5 \pm 0.6 \pm 5.1$

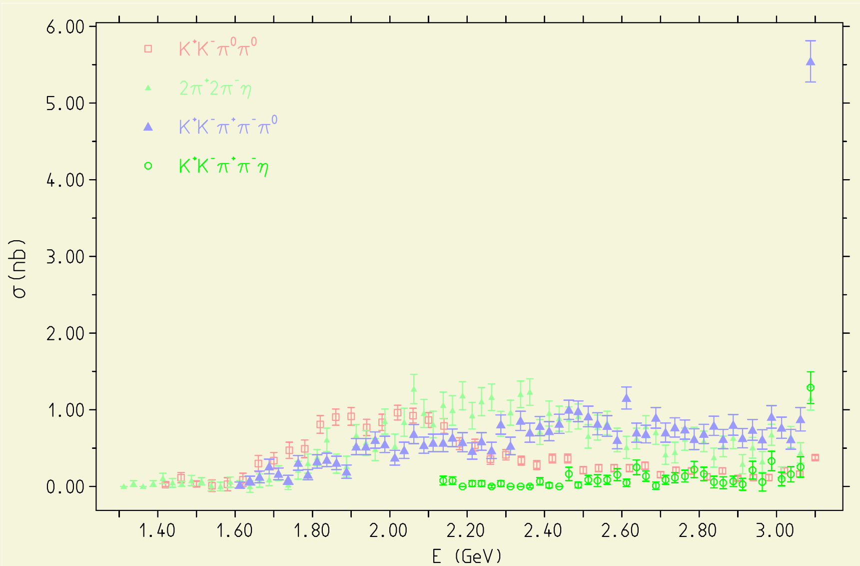

e+e- -> hadrons
φ
J/ψ1S
ψ2S
ψ3770
pQCD
average
excl data
γγ2
MEA
M3N
DM2
DASP I
BB̄
MARK I
PLUTO
CB
BESII
R
E (GeV)
e+e- -> hadrons
ϒ1S
ϒ2S
3S
4S
ϒ10860
ϒ11020
Mark I
LENA
JADE
pQCD
CB
MD-1
TASSO
PLUTO
DHHM
DASPII, CLEO, CUSB, MAC, CELLO, MARK J
R
E (GeV)



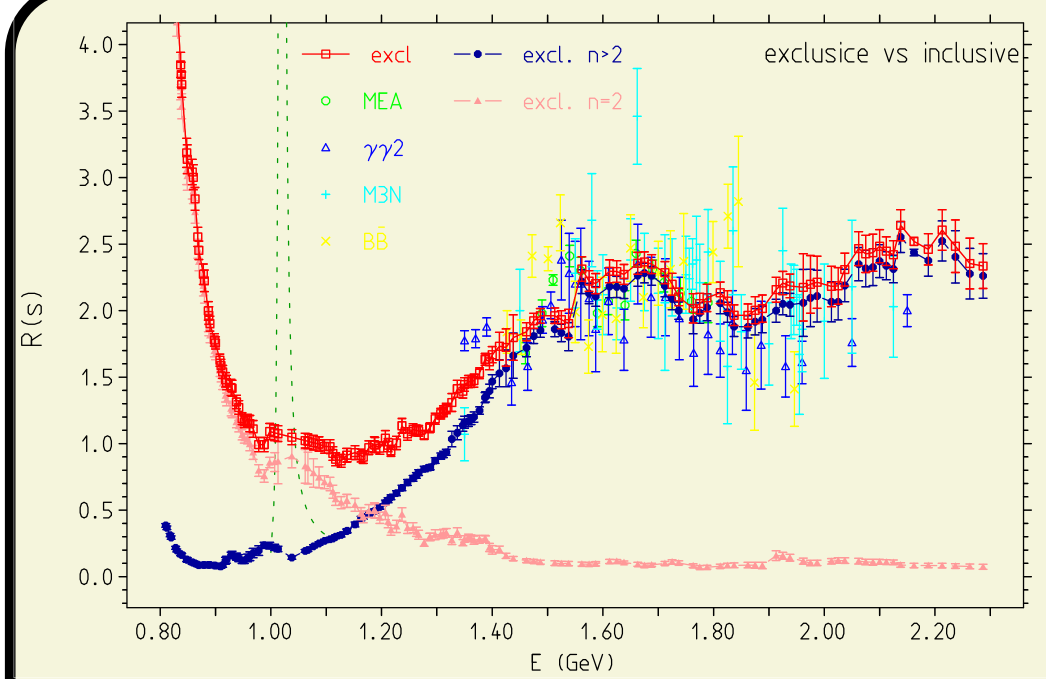
BaBar radiative return measurements

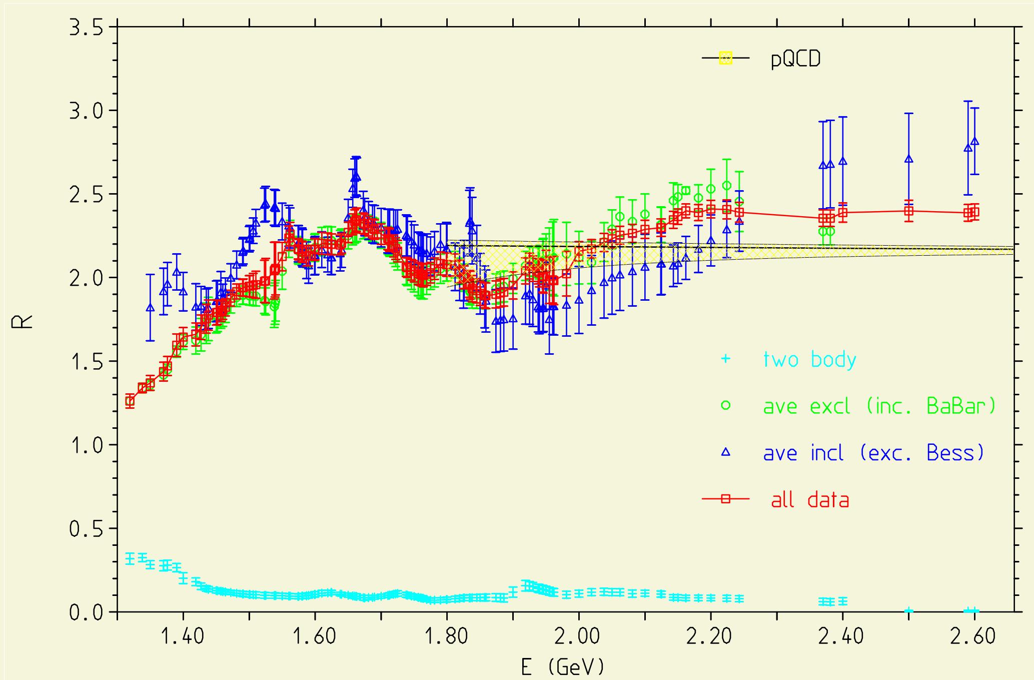






BaBar radiative return measurements


exclusice vs inclusive
excl
excl. n>2
MEA
excl. n=2
γγ2
M3N
BB
R(s)
E (GeV)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.80
1.00
1.20
1.40
1.60
1.80
2.00
2.20


pQCD
two body
ave excl (inc. BaBar)
ave incl (exc. Bess)
all data
R
E (GeV)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
1.40
1.60
1.80
2.00
2.20
2.40
2.60

② Updates for $a_\mu^{\rm had}$

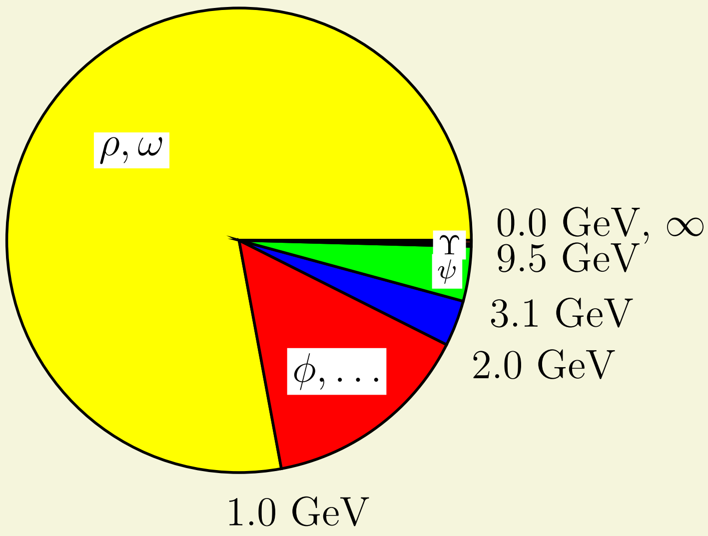
Energy range	$a_\mu^{\rm had}[\%](\rm error) \times 10^{10}$	rel. err.	abs. err.
ρ,ω ($E < 2M_K$)	539.35 [77.9](3.67)	0.7 %	37.9 %
$2M_K < E < 2$ GeV	101.48 [14.7](4.49)	4.4 %	56.8 %
2 GeV $< E < M_{J/\psi}$	22.13 [3.2](1.23)	5.6 %	4.3 %
$M_{J/\psi} < E < M_\Upsilon$	26.40 [3.8](0.59)	2.2 %	1.0 %
$M_\Upsilon < E < E_{\rm cut}$	1.40 [0.2](0.09)	6.2 %	0.0 %
$E_{\rm cut} < E$ pQCD	1.53 [0.2](0.00)	0.1 %	0.0 %
$E < E_{\rm cut}$ data	690.77 [99.8](5.96)	0.9 %	100.0 %
total	692.30 [100.0](5.96)	0.9 %	100.0 %

in red range relevant for VEPP-2000 and DAFNE2! minor changes [old 692.10$\pm$5.64]

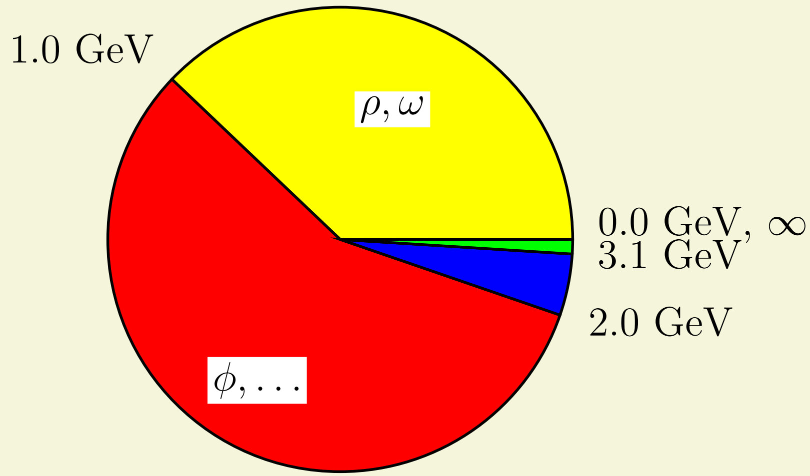
- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced: $\sim 67\%$ of error on $a_\mu^{\rm had}$ comes from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$

$$a_\mu^{\rm had(1)} = (692.3 \pm 6.0)\ 10^{-10}$$

e^+e^-–data based



contributions



error^2

present distribution of contributions and errors

③ Evaluation of $a_\mu^{\rm LbL}$ in the large-N_c framework

- Knecht & Nyffeler and Melnikov & Vainshtein were using pion-pole approximation together with large-N_c $\pi^0\gamma\gamma$–formfactor
- FJ & Nyffeler: relax from pole approximation, using KN off-shell LDM+V formfactor

$$\begin{aligned}
\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(p_\pi^2,q_1^2,q_2^2) &= \frac{F_\pi}{3}\,\frac{\mathcal{P}(q_1^2,q_2^2,p_\pi^2)}{\mathcal{Q}(q_1^2,q_2^2)} \\
\mathcal{P}(q_1^2,q_2^2,p_\pi^2) &= h_7 + h_6\,p_\pi^2 + h_5\,(q_2^2+q_1^2) + h_4\,p_\pi^4 + h_3\,(q_2^2+q_1^2)\,p_\pi^2 \\
&\quad + h_2\,q_1^2\,q_2^2 + h_1\,(q_2^2+q_1^2)^2 + q_1^2\,q_2^2\,(p_\pi^2+q_2^2+q_1^2)) \\
\mathcal{Q}(q_1^2,q_2^2) &= (q_1^2-M_1^2)\,(q_1^2-M_2^2)\,(q_2^2-M_1^2)\,(q_2^2-M_2^2)
\end{aligned} \tag{1}$$

all constants are constraint by SD expansion (OPE), except for $h_3+h_4=2\,c_{VT}$ with $c_{VT}=M_{V_1}^2\,M_{V_2}^2\,\chi/2$ and $\Pi_{VT}(0)=-(\langle\bar\psi\psi\rangle_0)/2\,\chi$ with evaluations of χ[GeV^{-2}]

-2.7 (Ball et al. '03) -3.3 (LMD) -8.2 (Ioffe&Smilga '84) -8.9 (Vainshtein '03)

New estimate together with A. Nyffeler: $h_3 \in [-10, 10]$ GeV^{-2}

X	$a_\mu(\mathrm{LbL};X)\times 10^{11}$				
π^0,η,η'	96.63 ± 4.47	a_1, f_1', f_1	$28.13 \pm 20\%$	a_0, f_0', f_0	$-5.98 \pm 20\%$

$$\Rightarrow \quad a_\mu(\mathrm{LbL}) \simeq (118.76 \pm 40)\times 10^{-11}$$

Contribution	Value	Error	Reference
QED incl. 4-loops+LO 5-loops	11 658 471.81	0.02	Remiddi et al., Kinoshita et al. ...
Leading hadronic vacuum polarization	692.3	6.0	2008 update
Subleading hadronic vacuum polarization	-10.0	0.2	2006 update
Hadronic light–by–light	11.9	4.0	new evaluation (J&N)
Weak incl. 2-loops	15.32	0.22	CMV06
Theory	11 659 181.3	7.2	–
Experiment	11 659 208.0	6.4	BNL
The. - Exp. 2.8 standard deviations	-26.7	9.6	–

Standard model theory and experiment comparison [in units 10^{-10}]

④ Remarks on τ vs. e^+e^- data

- Unknown isospin violations in parameters: $m_{\rho^+} - m_{\rho^0}, m_{\rho'^+} - m_{\rho'^0}, m_{\rho''^+} - m_{\rho''^0}$; same for widths, mixing parameters; largely not established (theor. and exper.)

- Needed what is measured in e^+e^-: $|A_{I=1}(s) + A_{I=0}(s)|^2 < |A_{I=1}(s)|^2 + |A_{I=0}(s)|^2$;

- τ evaluations based on $|A^{\tau}_{I=1}(s)|^2 + |A^{e^+e^-}_{I=0}(s)|^2$ which may overestimate the effects; separation of $|A^{e^+e^-}_{I=0}(s)|^2$ using Gounaris-Sakurai fit of the $\rho - \omega$ [$\varepsilon_{\rho\omega} = (2.02 \pm 0.1) \times 10^{-3}$]; see HLS model calculation of Benayoun et al. (next talk) suggests large diminution by interference. also Colangelo talk

Look at $\delta_m = (m_{\rho^+} - m_{\rho^0})/\bar{m}_\rho$ etc.

δ	ρ	ρ'	ρ''
m	0.2%	7.4 %	[0.0] %
Γ	2.9%	4.8 %	[0.0] %
γ	–	45.3 %	65.7 %
ϕ_γ	–	21.5 %	[0.0] %

Davier 2003

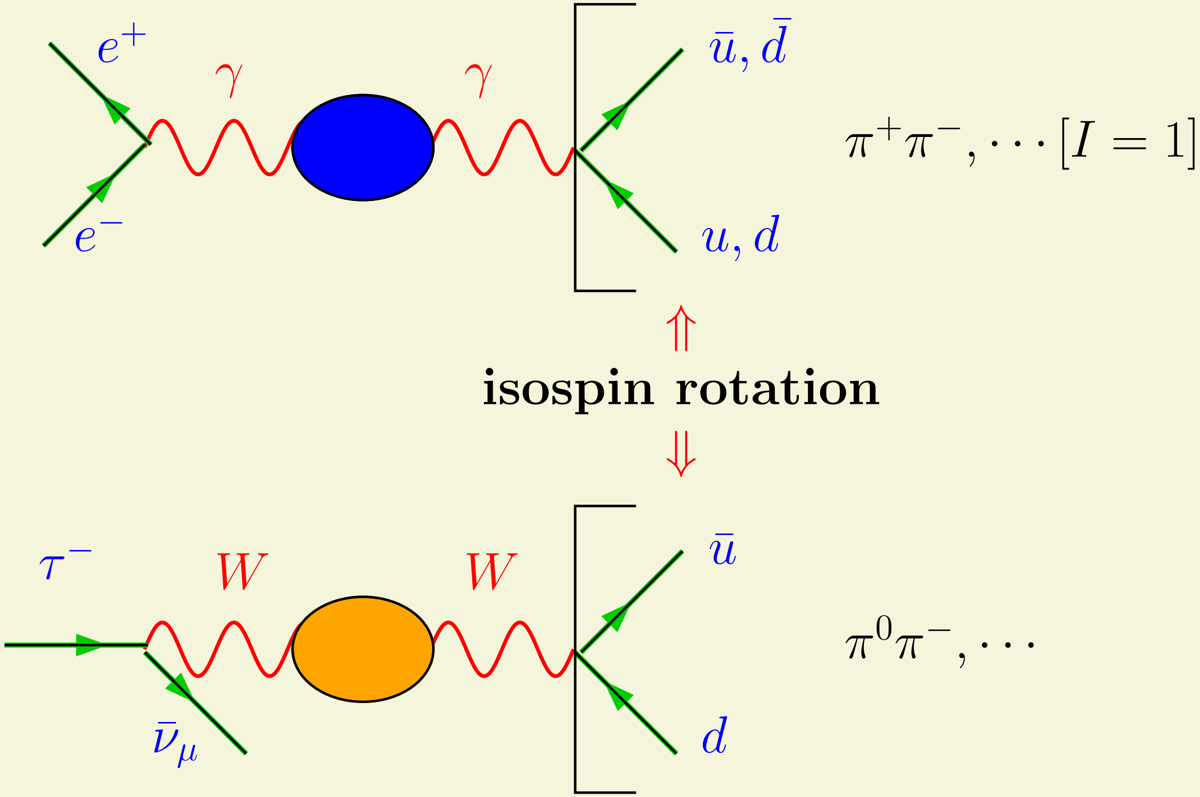
Cottingham formula calculating $m^2_{\pi^-} - m^2_{\pi^0}$ very successfully suggests $\Delta m^2_\rho = \Delta m^2_\pi \Rightarrow$ $m_{\rho^+} - m_{\rho^0} \simeq 0.88$ MeV ~ 1 MeV

Also: $\Gamma_{\rho^0} = \left(\frac{m_{\rho^0}}{m_{\rho^-}}\right)^3 \left(\frac{\beta^0}{\beta^-}\right)^3 \Gamma_{\rho^-} + \Delta\Gamma_{\rm em} \Rightarrow \Gamma_{\rho^-} - \Gamma_{\rho^0} \simeq 2.1 \pm 0.5$ MeV

	τ	e^+e^-
m_{ρ^0}	-	773.3 $\pm$ 0.6
Γ_{ρ^0}	-	145.2 $\pm$ 1.3
m_{ρ^-}	775.0 $\pm$ 0.6	-
Γ_{ρ^-}	149.5 $\pm$ 1.1	-
$\alpha_{\rho\omega}$	-	$(2.02 \pm 0.10)\ 10^{-3}$
β	0.195 $\pm$ 0.028	0.123 $\pm$ 0.011
ϕ_β	173.0 $\pm$ 7.0	139.4 $\pm$ 6.5
$m_{\rho'}$	1440 $\pm$ 34	1337 $\pm$ 35
$\Gamma_{\rho'}$	597 $\pm$ 102	569 $\pm$ 81
γ	0.095 $\pm$ 0.029	0.048 $\pm$ 0.008

Results of fits to the pion form factor squared to τ and e^+e^- data (ALEPH and CLEO) separately, then combined. The parametrization of the ρ line shapes is by Gounaris-Sakurai. All mass and width values are in MeV and the phase ϕ_β in degrees. $\phi_\gamma = 0^\circ$, $m_{\rho''} = 1713 \pm 15$ MeV and $\Gamma_{\rho''} = 235$ MeV kept fix. (table from Davier 2003, see also Ghozzi & FJ 2004)

Besides possible experimental problems, unaccounted (quantitatively not well established) isospin breakings and missing interference information do not allow us to include isospin rotated τ–data in calculations of $g-2$.



- e^+e^-: renormalized (running) charge $\alpha_{\mathrm{QED}}(s)$

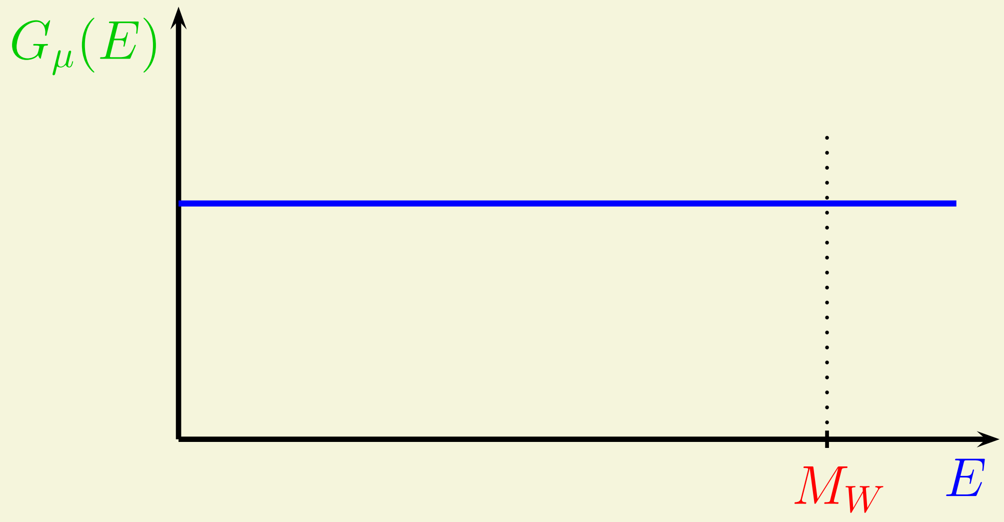
$$e^2 \to e^2(s) = \frac{e^2}{1+(\Pi'_\gamma(s)-\Pi'_\gamma(0))}$$

need photon vacuum polarization $\Pi'_\gamma(s)$

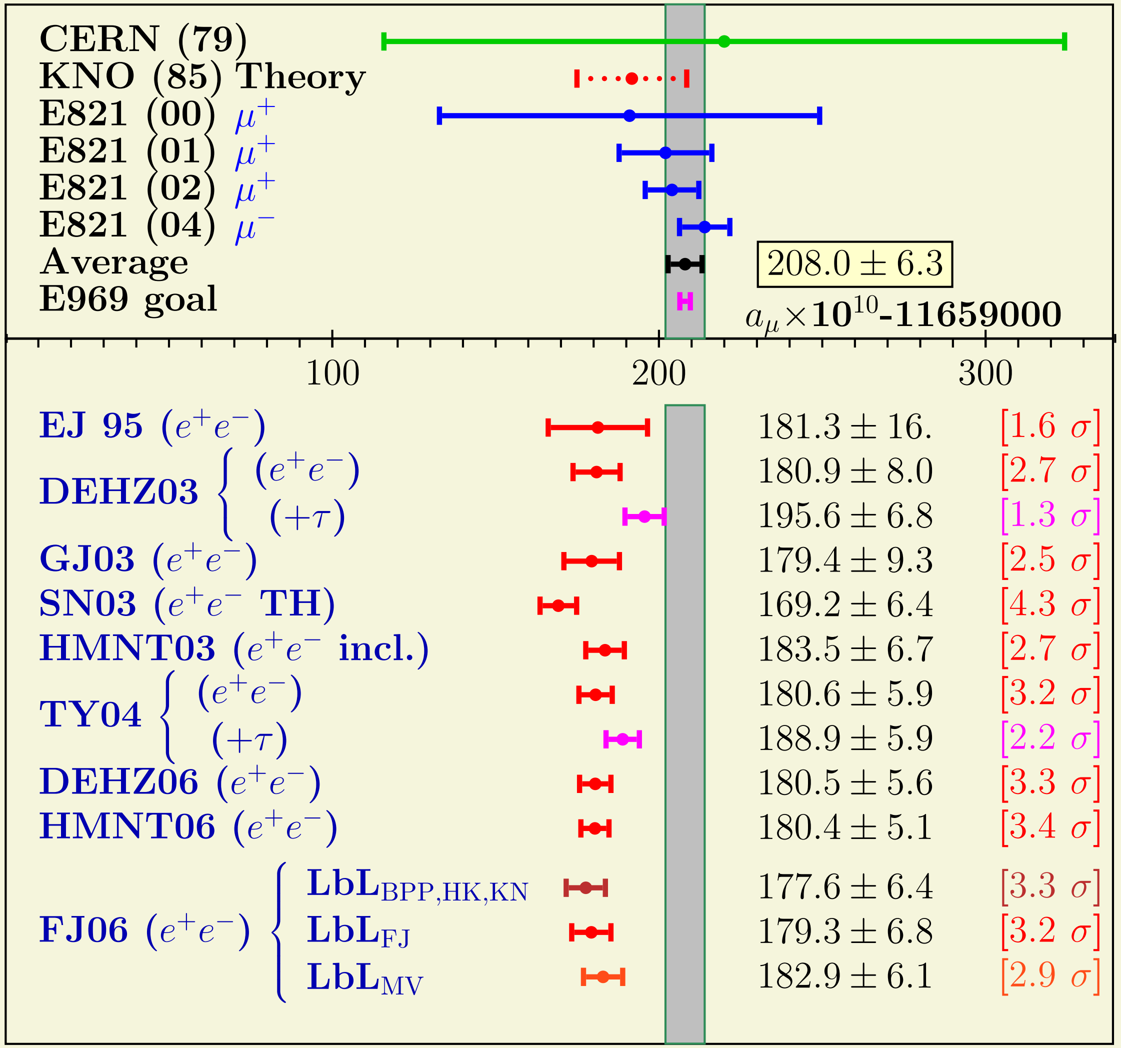
- τ–decay: Fermi constant G_μ

$$\frac{G_\mu}{\sqrt{2}} = \frac{g_0^2}{8M_{W0}^2}\left\{\frac{1}{1-\frac{\Pi_W(0)}{M_W^2}}+\cdots\right\}$$

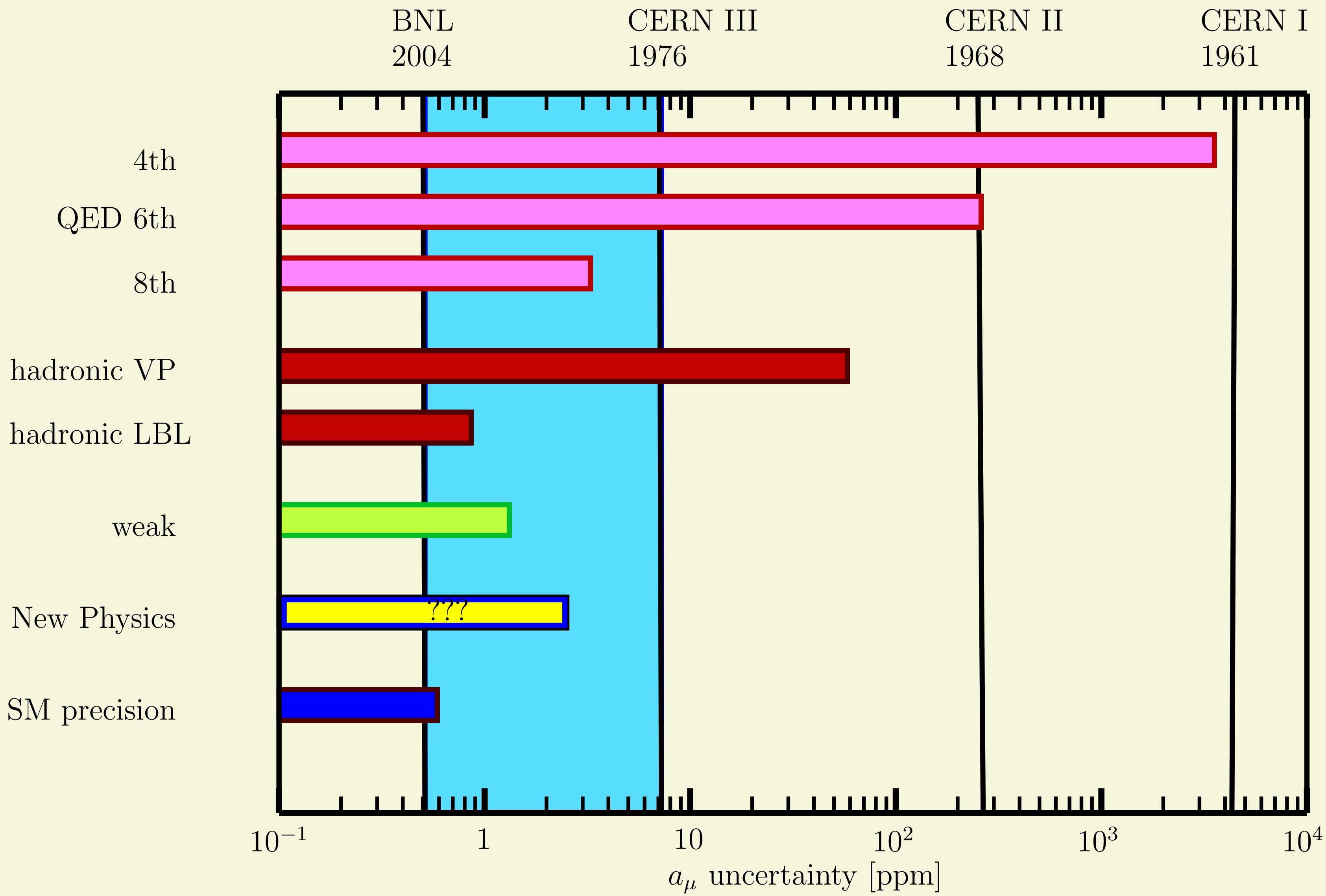
calculable subtraction (renormalization) constant $\Pi_W(0)$ [$m_\tau \ll M_W$]



⑤ Backup slides



differ by Hadronic VP only: data used, fitting data, pQCD etc



History and sensitivity to different effects