Electron Model: Lattice and Performance

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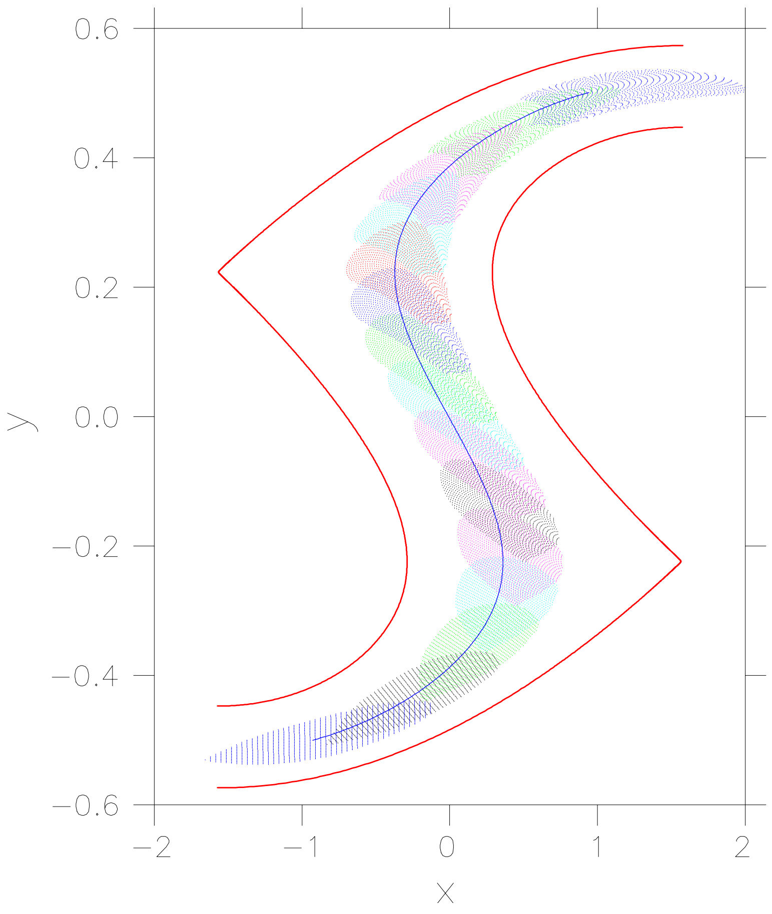
`http://keil.home.cern.ch/keil/MuMu/Doc/NuFact05/model.pdf`

Aim and Scope of Electron Model

- Simulate rapid acceleration of relativistic muons in non-scaling FFAG ring in a small relativistic electron ring
- Use magnetic elements with field and gradient
- Avoid field components of orders higher than gradient
- Do not consider models with nonlinear fields
- Do not study slow acceleration of non-relativistic protons and ions

Acceleration outside Buckets

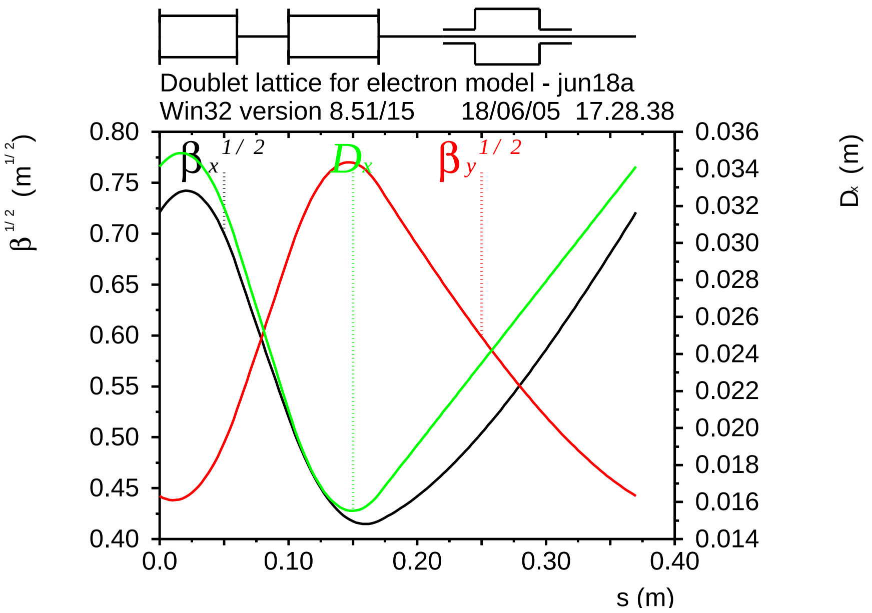


Bunch motion in longitudinal phase space

- RF system runs at constant frequency ≈ 1300 MHz
- Two strings of stable and unstable fixed points at $y \approx \pm 0.2$, caused by two zeroes of time of flight due to quadratic variation
- Abscissa is phase $-\pi \leq x \leq \pi$
- Ordinate y is energy scale, $y = \pm 0.5$ correspond to 10 and 20 MeV
- Gutter boundaries in red, symmetrical around $x = \pm\pi/2$, period 2π
- Acceleration of bunch centres from $(x, y) = (-1, -0.5)$ to $(x, y) = (+1, +0.5)$ along blue line
- $A_b = 10^{-4}$ eVs, $\ell_t = \pm 0.125$ ns, $\Delta E = \pm 0.25$ MeV

Lattice Cells of Electron Model

- Prefer doublets over triplets
- Space for 1300 MHz RF cavity
- Aperture for electron momenta from 10 to 20 MeV/c
- Magnet have field and gradient, but no higher components
- Length of magnets from pole tip field at $B \approx 0.2$ T
- Rectangular magnets have smaller tune spread ΔQ
- Magnets resemble offset quadrupoles more than dipoles

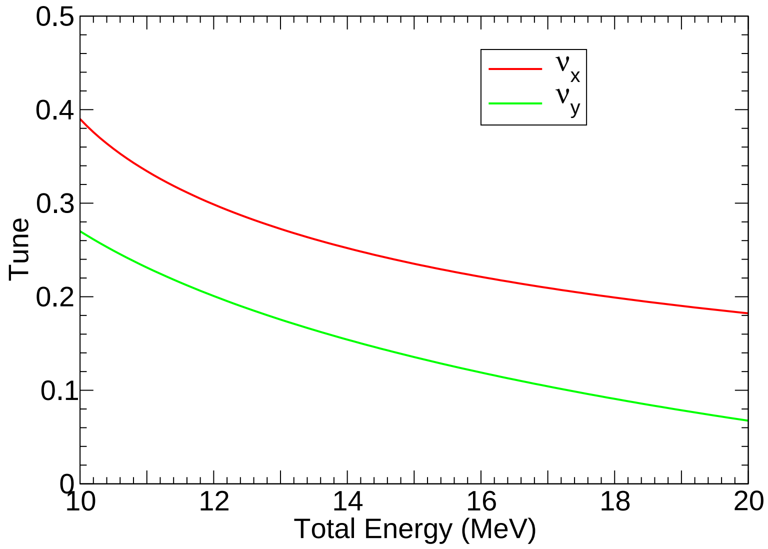


Schematic layout and orbit functions

- Reversed field in F magnet reduces D_x and ΔT
- Cell length $L_p \approx 370$ mm
- Fringe field effects due to aperture $\approx$ magnet length under study

Tunes in a Lattice Cell

- Achieve small D_x and ΔT by letting ν_x at 10 MeV/c approach 0.5 from below, avoiding stopband at $\nu_x = 0.5$ and associated rise in β_x
- Improve spread ΔT by using $\nu_y < \nu_x$
- Essentially $\nu_x \propto \nu_y \propto 1/E$ because of linear lattice
- Control of tune variation $\nu(E)$ only by rectangular or sector shape of magnets, without nonlinear field components



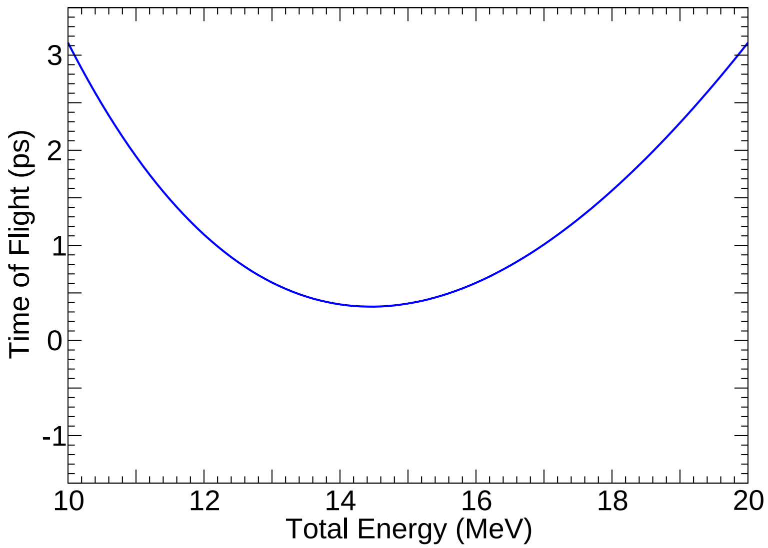
- Total tune variation in 42 cells is many units $\Rightarrow$ during acceleration Q_x and Q_y cross many resonant values with $n_x Q_x + n_y Q_y = k$, where $|n_x| + |n_y|$ is order of resonances and k is harmonic around circumference
- Resonances driven only by errors in field shape, excitation, and alignment

Circumference C of Electron Model

- Dimension-less parameters a and b with time of flight spread ΔT

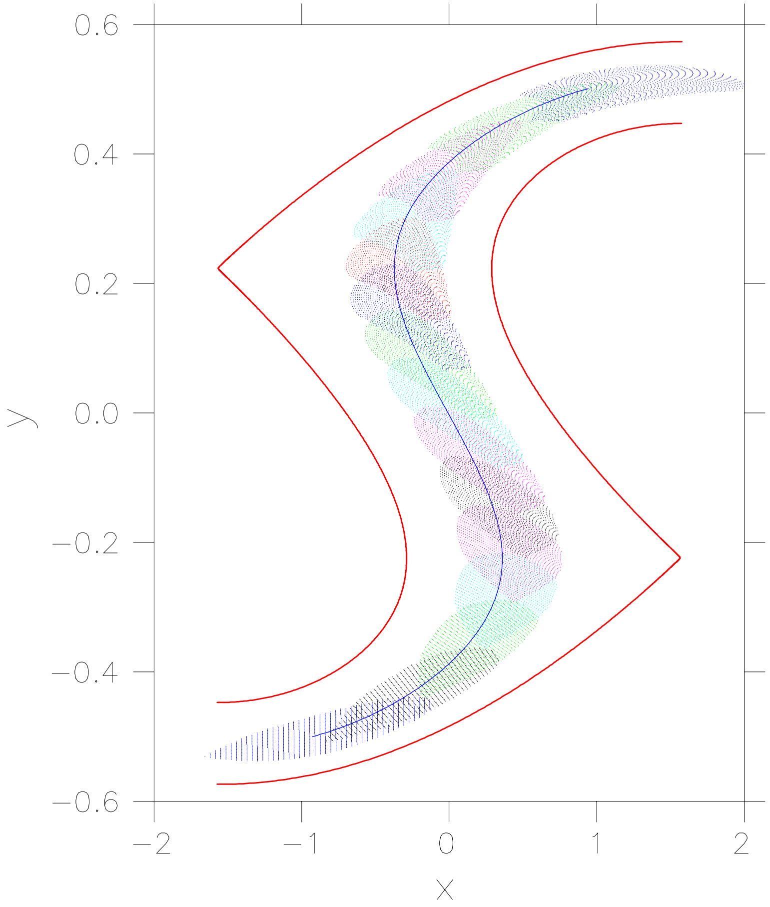
$$a = \frac{e\hat{V}}{2\pi f_{\mathrm{RF}} \Delta T \Delta E} \qquad b = \frac{T_0}{\Delta T}$$

- Number of lattice cells N_p and circumference C determined by parameters $a = 1/12$, $b = 1/5$, acceleration ΔE from 10 to 20 MeV/c, frequency 1300 MHz, cell length $L_p = 370$ mm, and number of cell traversals M_c
- Ordinate in graph of $\Delta T \propto 1/N_p$ vs. $\delta p/p$ has arbitrary origin
- Adjust T_0 and b by choice of f_{RF}
- Design model for $M_c \approx 500$ and divergences x' and y', similar to values in FFAG rings for accelerating muons
- Number of lattice cells $N_p \propto \sqrt{M_c}$
- Pick $N_p = 42$, a multiple of 6



- Choice of parameters well advanced, simulations to be continued and/or launched

Simulations in Progress



Bunch distortion in longitudinal phase space

- Confirm feasibility of model parameters by further simulation
- Limits on emittance increase due to crossing resonances yield alignment and excitation tolerances
- Acceleration with finite longitudinal emittance results in bunch distortion and particle loss
- Adding finite transverse emittance will make things how much worse?
- Finite fringe fields may drive systematic nonlinear resonances
- Kinematic terms due to divergences x' and y'