# Study of Resonance Crossing in FFAG

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# Introduction

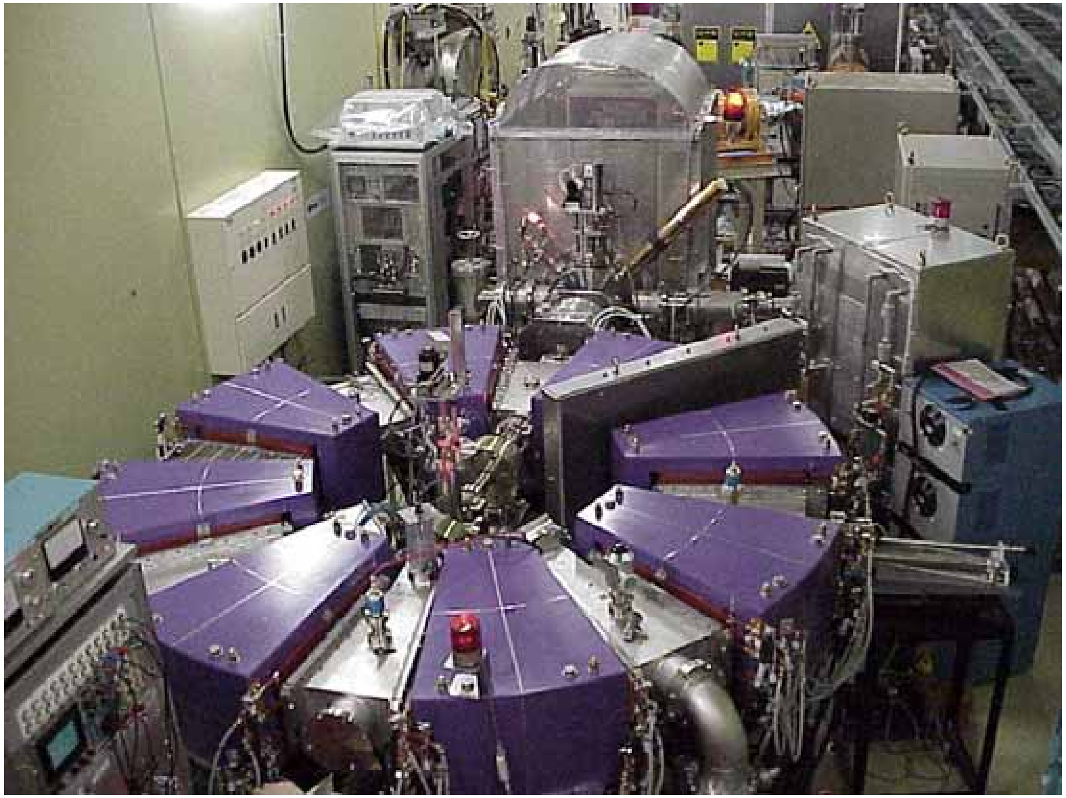
- FFAG accelerator:
  - For proton driver
  - For muon acceleration
- Resonance crossing
  - In scaling FFAG: tune variation due to imperfection of scaling
  - In non-scaling FFAG: tune variation in wide range

- Dynamics of resonance crossing is important

➡ Experimental study at PoP FFAG and HIMAC

# Experiment at PoP FFAG

PoP FFAG: radial sector type scaling FFAG

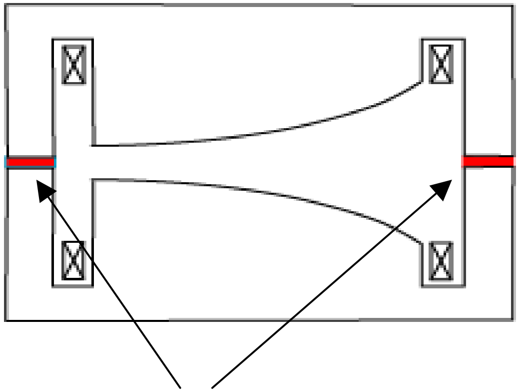


Parameter list

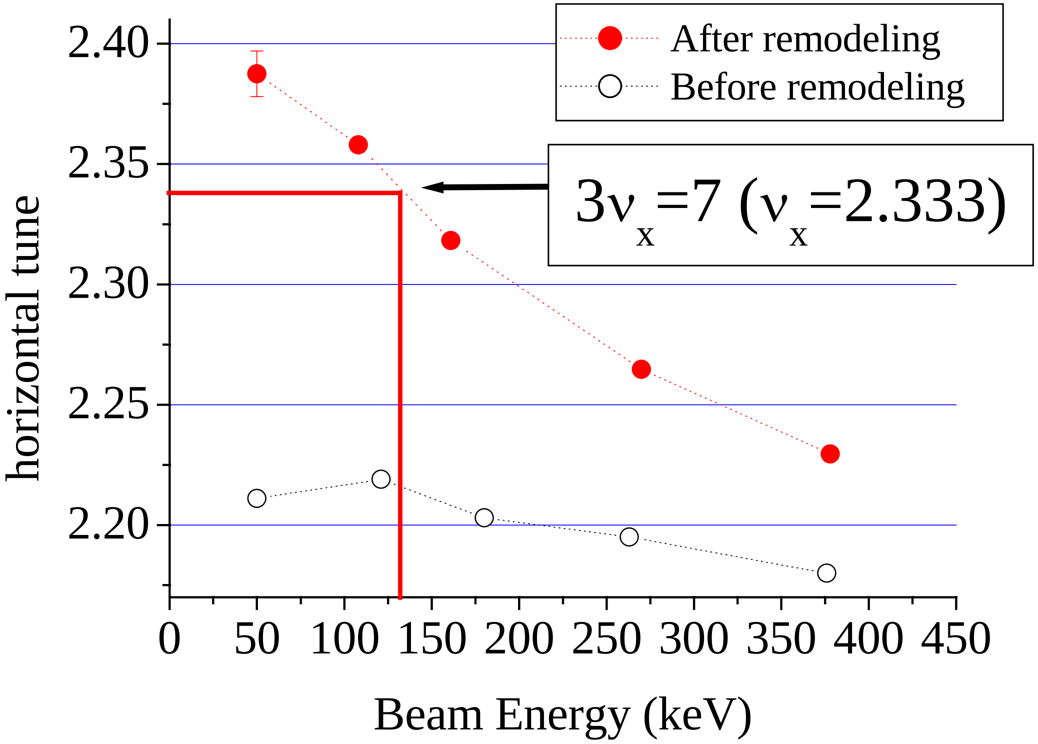
| | |
|---|---|
| sector number | 8 (DFD triplet) |
| k value | 2.5 |
| kinetic energy | 50-500keV |
| fmagnetic field | 0.14-0.32T(F) |
| | 0.04-0.13T(D) |
| average radius | 0.81-1.14m |
| betatron tune | 2.22-2.16(Hor.) |
| | 1.26-1.23(Ver.) |
| revolution freq. | 0.61-1.40MHz |
| RF voltage | 5kVpp |

Resonance crossing with
various driving term and crossing speed

# Remodeling magnets



4mm iron plates



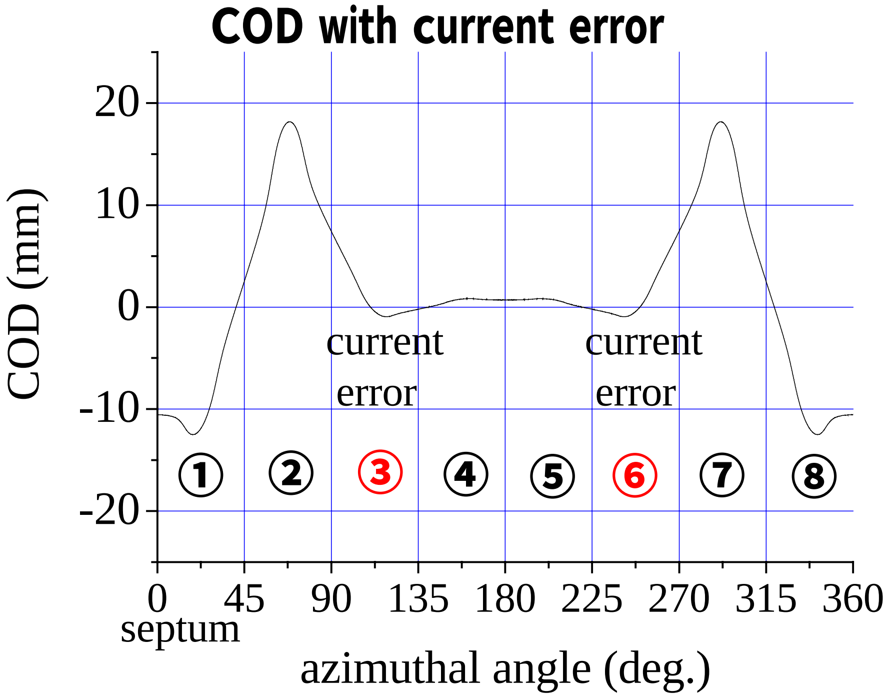

Crossing third order resonance during acceleration

# Driving term

## Driving term with COD

Feed Down: $O(x+D)^3 = O(x^3 + \underline{3x^2D} + 3xD^2 + D^3)$

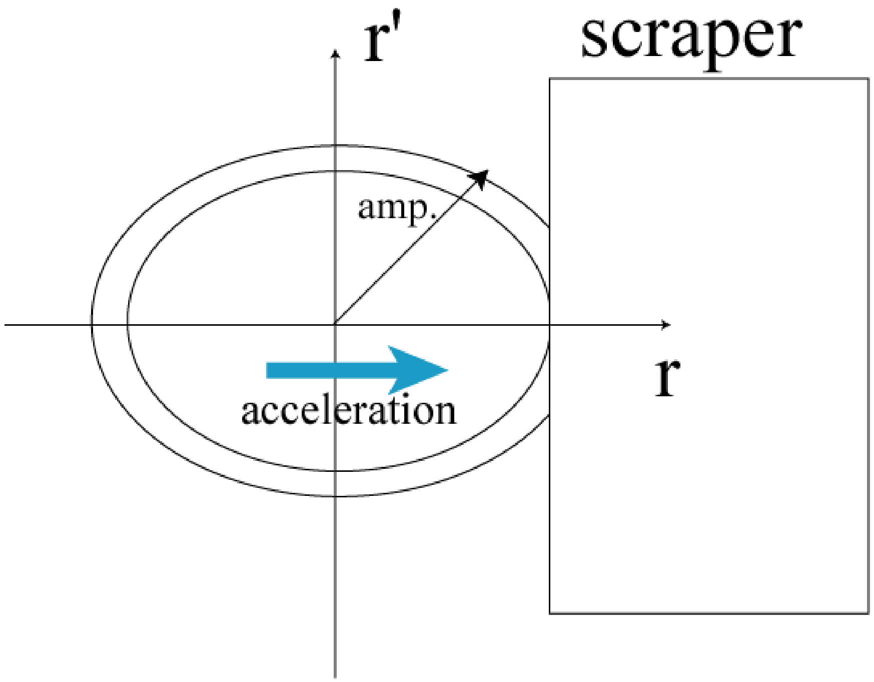
## Controlling COD

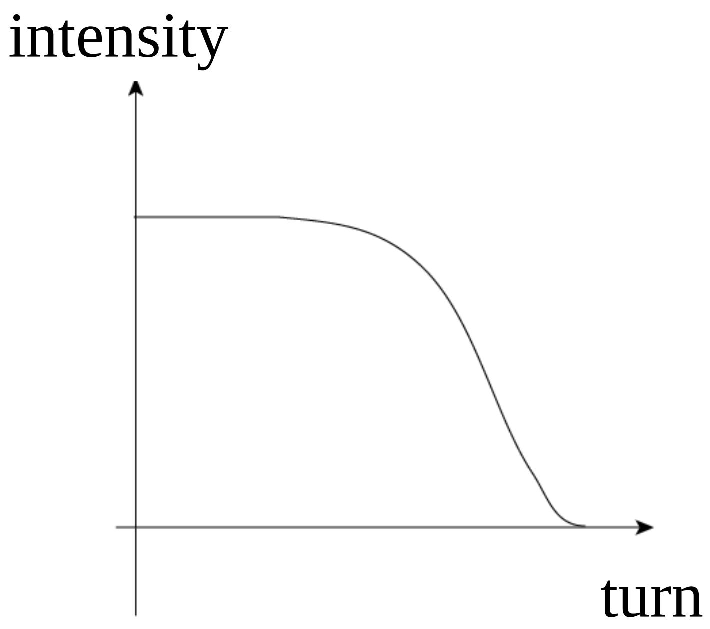



Driving term is varied and controlled

# Beam measurement

## Beam scraping & intensity measurement




Particle distribution in beam emittance was measured before and after crossing.

# Fast crossing

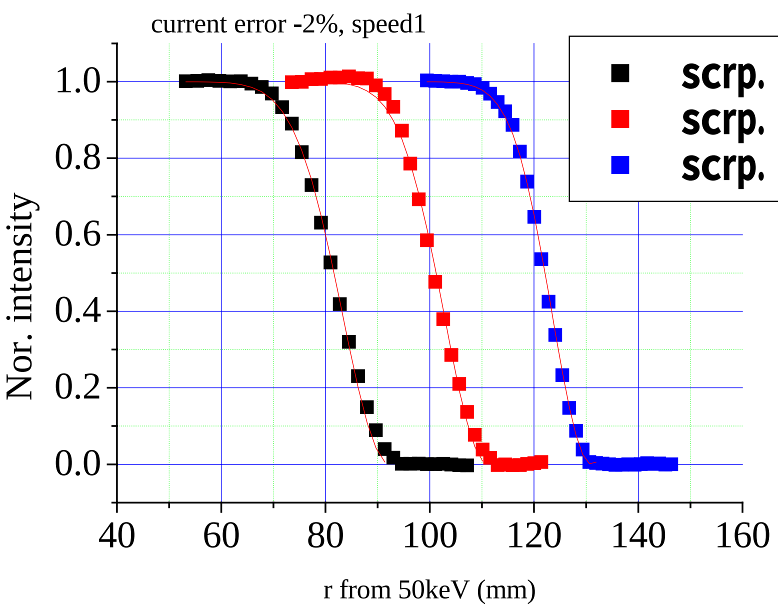
Energy gain 1.6kV/turn = $\Delta\nu_x$ $1.4\times10^{-3}$

Current error –2%

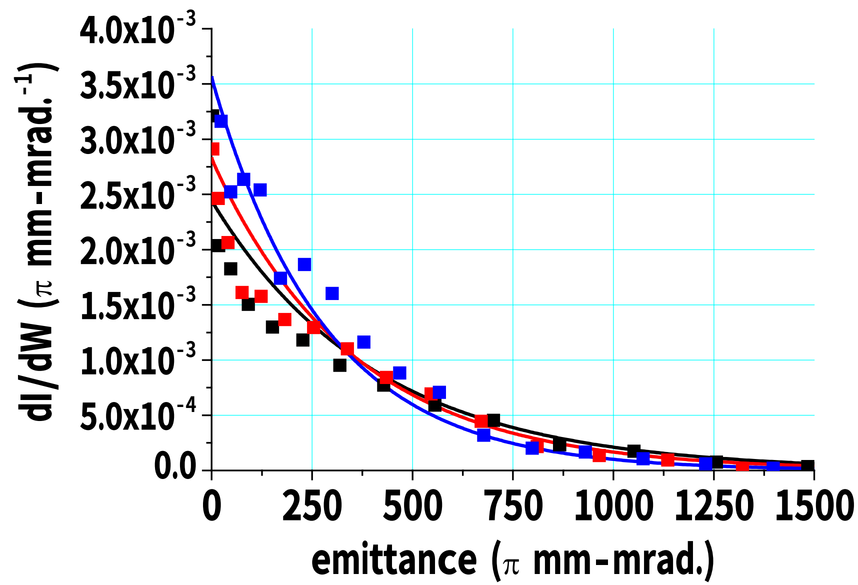
Scraping data

110 130 150 (keV)

BeforeCrossingAfter




Particle distribution in beam emittance




Fast crossing: no clear signal of a damage due to crossing

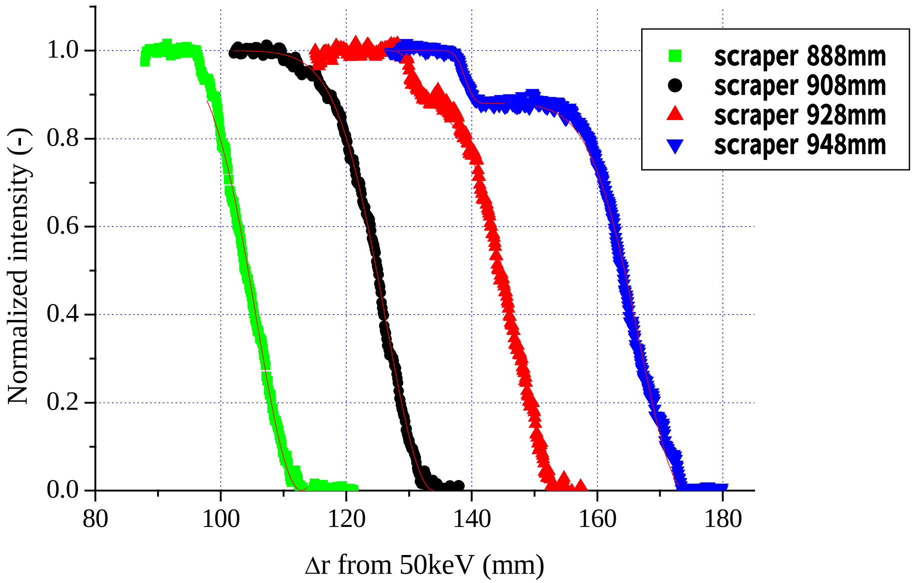
# Slow crossing

Energy gain 0.13kV/turn = $\Delta\nu_x$ 1.2 × $10^{-4}$
Current error –2%

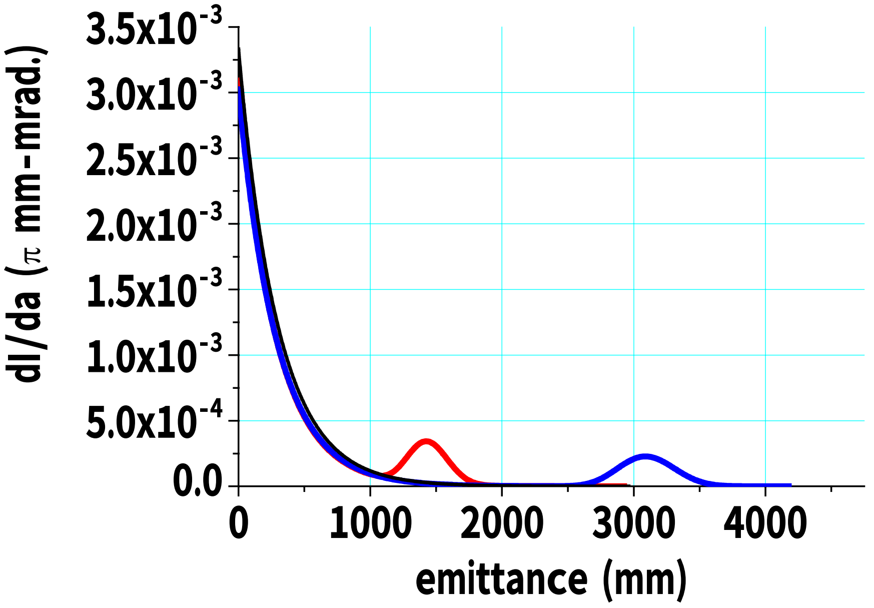
Scraping data

130 150 170 190 (keV)

Crossing After




Particle distribution in beam emittance



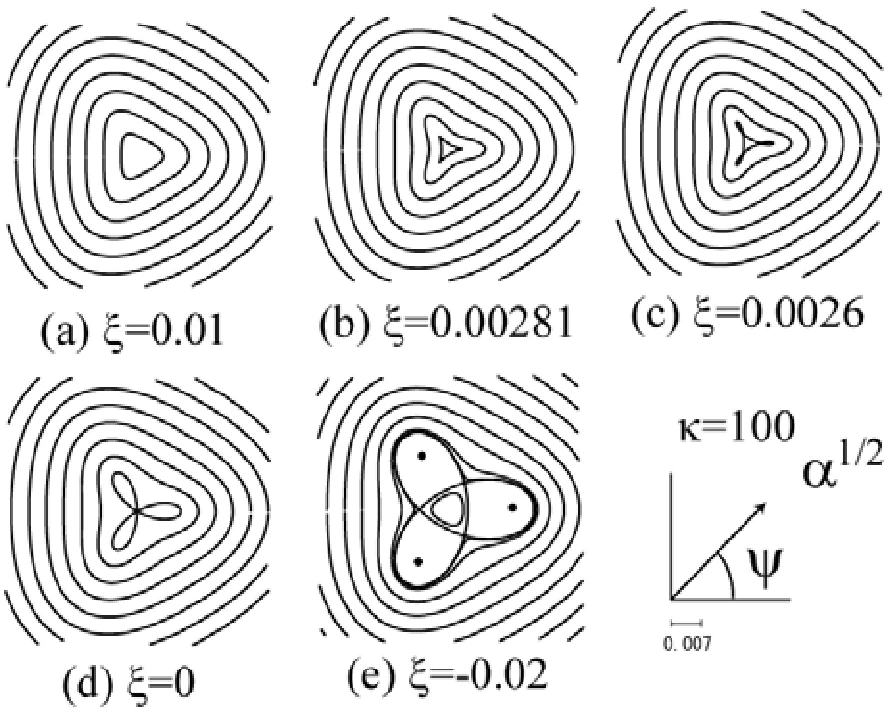

Slow crossing: a part of beam is transported to large amplitude

# "Particle trapping model"

Reference: A.W.Chao and M.Month, NIM 121, P.129 (1974)

"PARTICLE TRAPPING DURING PASSAGE THROUGH A HIGH-ORDER NONLINEAR RESONANCE"

Phase space topology during crossing third order resonance




Assuming:
- nonlinear detuning (octupole)
- driving term (sextupole)

Distance from resonance

$$\xi \propto \frac{1}{3} p - \nu$$

This model supports the experimental result.

# Trapping efficiency

## Trapping efficiency for third order resonance

$$P_T = \frac{A}{\pi \alpha_s} \exp(-\alpha_1) \qquad \alpha_s = \begin{cases} \alpha_1, & if \;\; \alpha_1 > 1, \\ 1, & if \;\; \alpha_1 < 1 \end{cases}$$

$\alpha_s$ : the beam emittance of island center

$A \approx \frac{\pi^2}{\sqrt{2}} \kappa^{-\frac{1}{2}} \alpha_s^{\frac{3}{4}}$ : the total area of islands

$\alpha_1 = \left( \frac{\varepsilon}{4\pi \Delta_{NL} \Delta_e} \right)^{\frac{2}{3}}$ : the adiabatic parameter

*The adiabatic parameter means a speed of islands moving during crossing.*

Crossing speed: $\varepsilon$

Nonlinear detuning: $B_0 = \frac{<\beta>}{16\pi\nu} \int_0^{2\pi} d\theta O(\theta)$

Driving term: $\left| A_p \right| = \frac{<\beta>^{\frac{1}{2}}}{8\pi\nu} \int_0^{2\pi} d\theta e^{-ip\theta} S(\theta)$

Linear tune shift: $\Delta_L = \frac{1}{3} p - \nu$

Nonlinear tune shift: $\Delta_{NL} = -12 B_0 a_0$

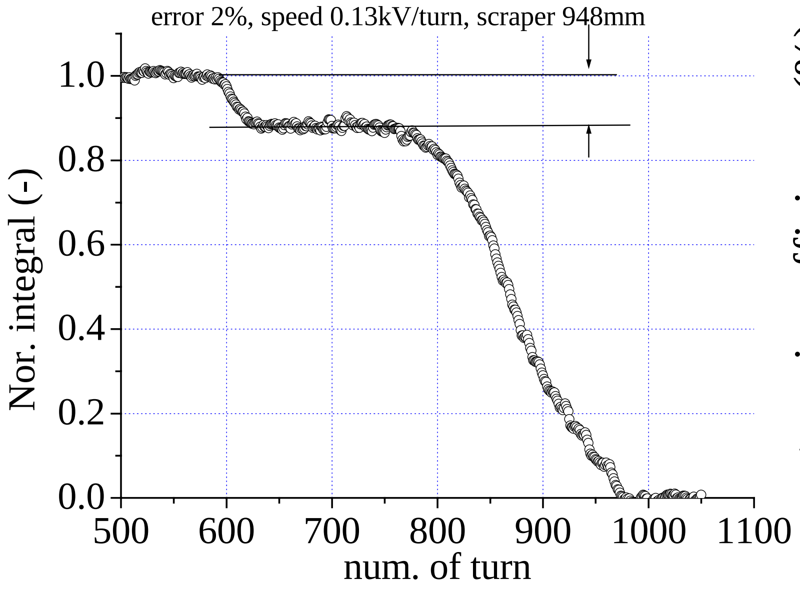
Excitation width: $\Delta_e = -3 \left| A_p \right| a_0^{\frac{1}{2}}$

$\kappa \equiv 3\Delta_{NL} / 4\Delta_e , \qquad \xi \equiv 3\Delta_L / 2\Delta_e$

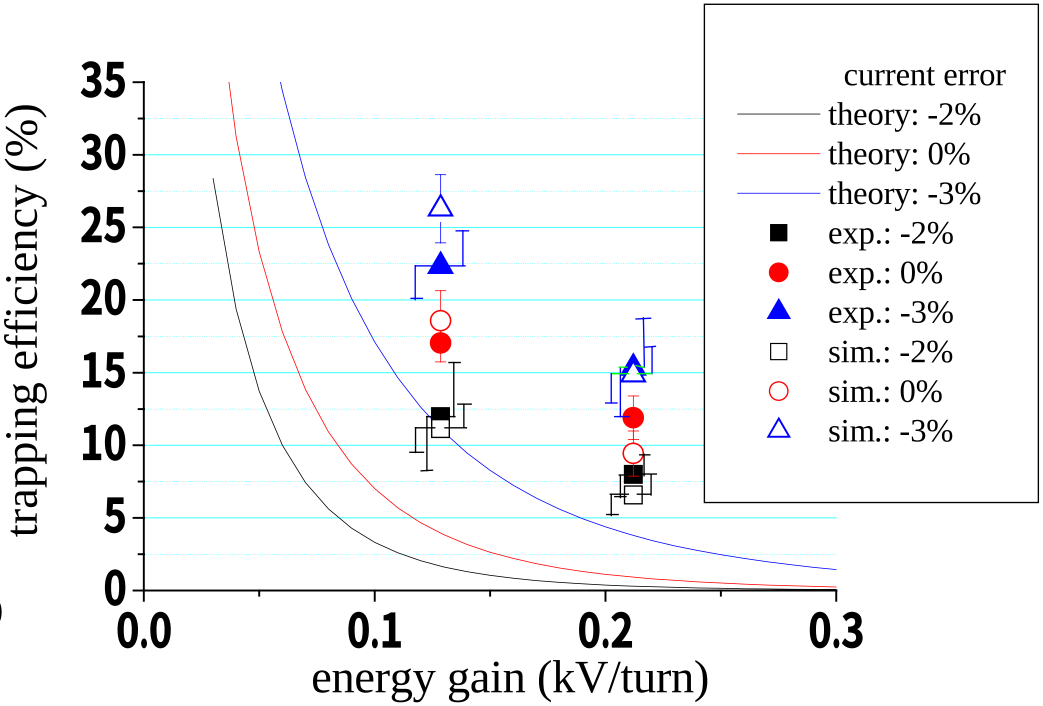
*Assuming k>>1 to derive the trapping efficiency

# Comparison of trapping efficiencies

Efficiency in experiment

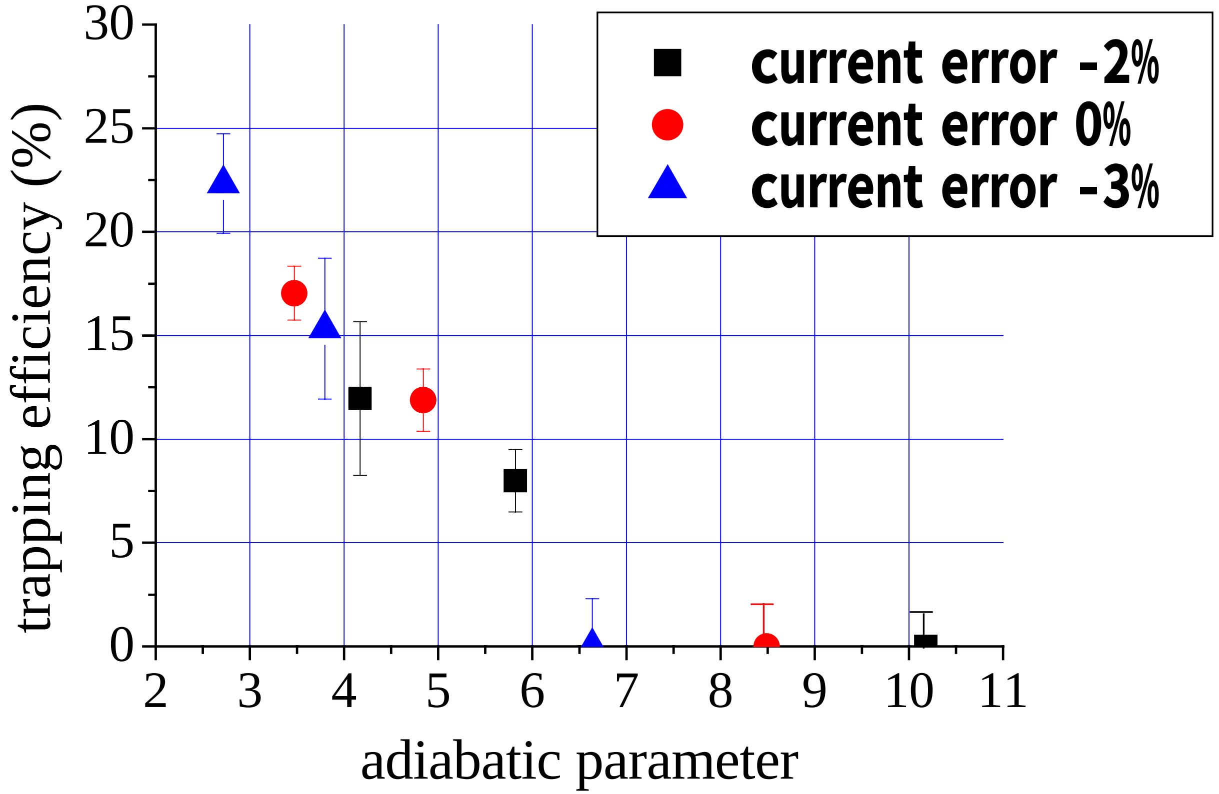



Trapping efficiencies



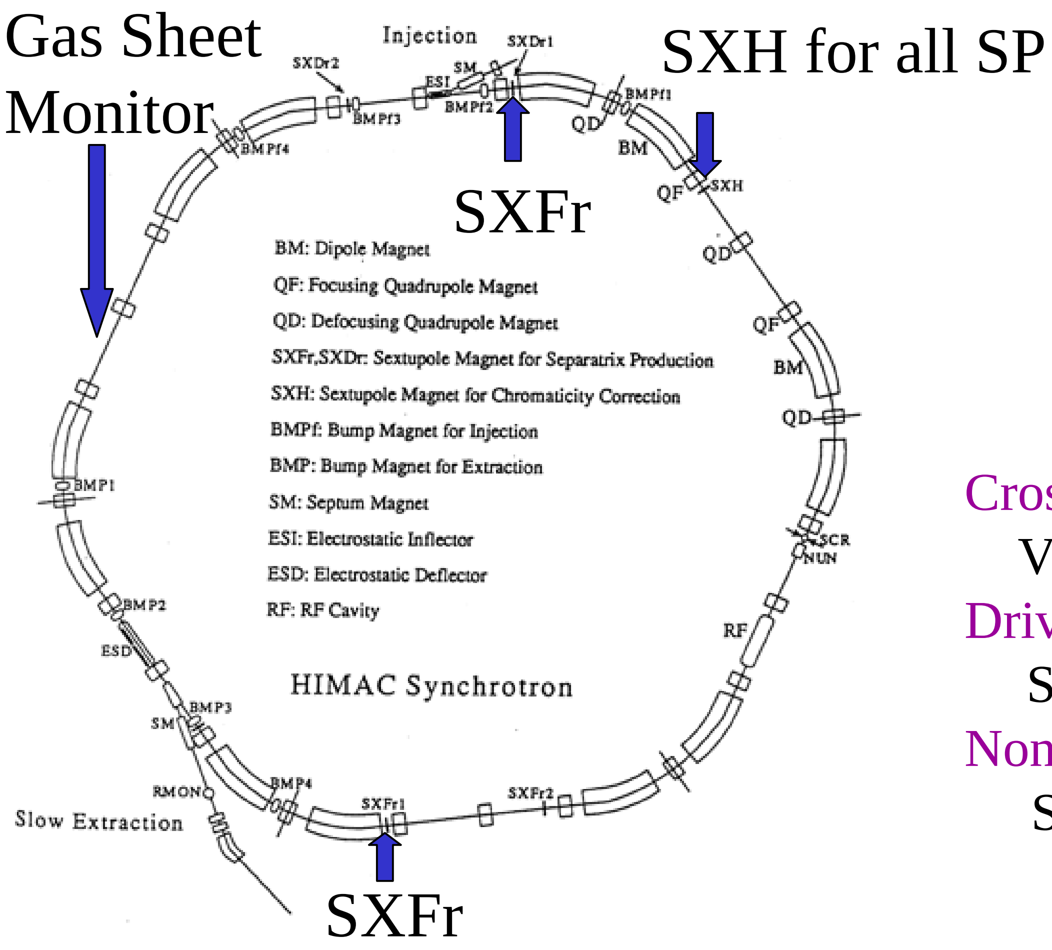

The experiment results are consistent to simulations.

# Criterion to avoid trapping




Adiabatic parameter more than 7 will be harmless.

# Crossing experiment at HIMAC



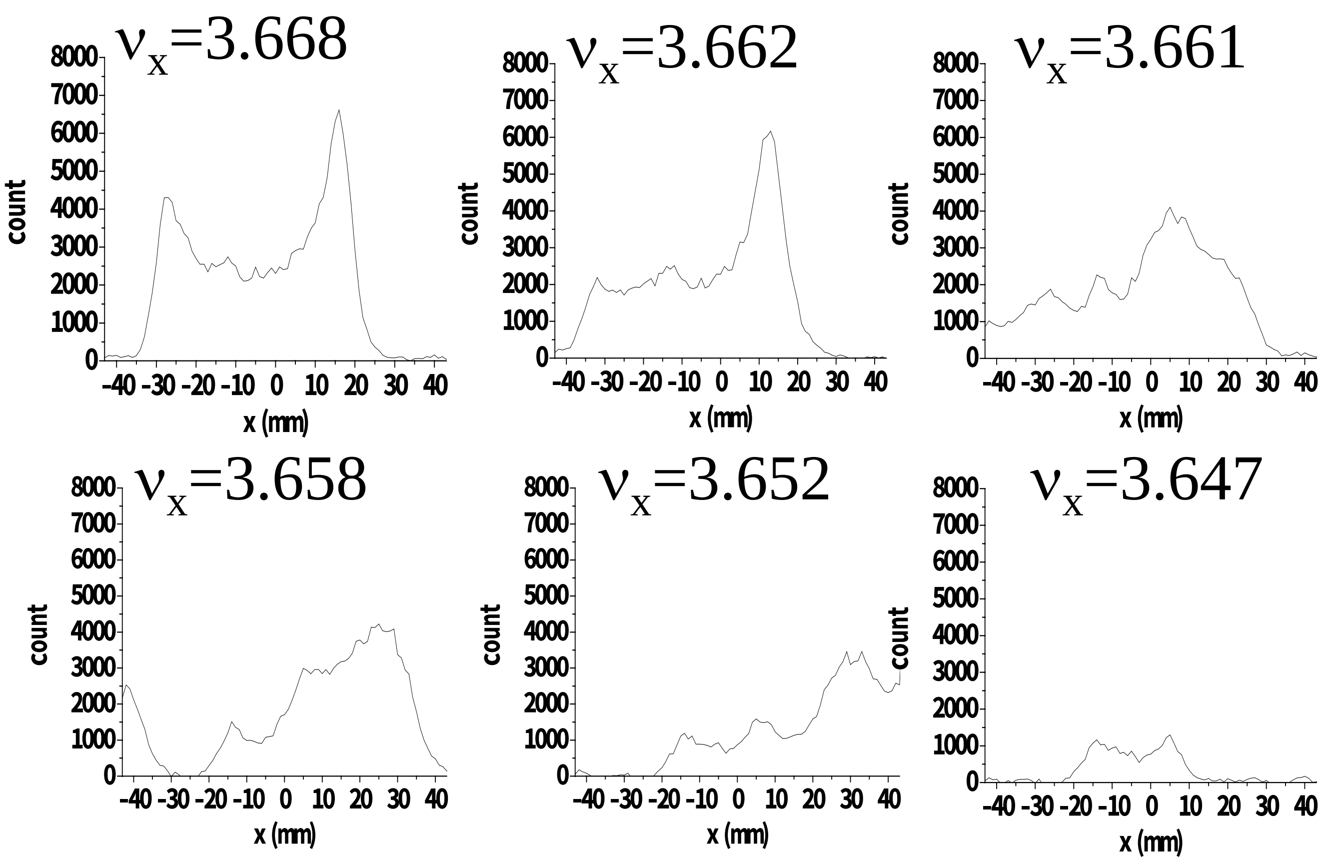

| Flat bottom operation parameter | |
|---|---|
| circumference | 129.6m |
| super period / cell | 6／12 |
| particle | carbon 6+ |
| inj. energy | 6MeV/u |
| operation point | (3.69, 2.13) |

Crossing:
Varying quadrupole strength
Driving term:
SXFr*2 sextupole
Nonlinear detuning:
Second order effect of SXH sextupole

Crossing 3vx=11 in both direction
Observing beam profile with Gas Sheet Monitor directly

# Crossing in a direction of tune decreasing

## Beam profiles during crossing




Crossing speed:

$4.6\times10^{-6}$

Nonlinear detuning:

$2.52\mathrm{m}^{-1}$

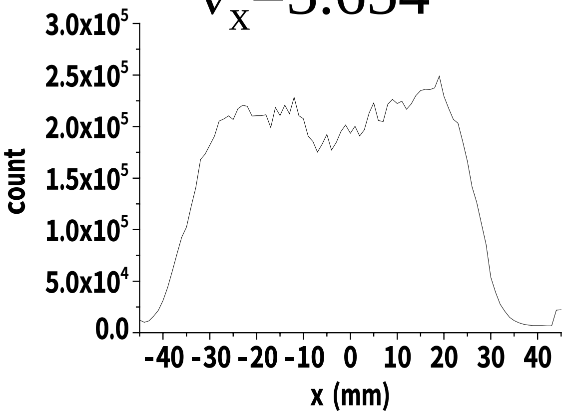
Driving term:

$0.019\mathrm{m}^{-1/2}$

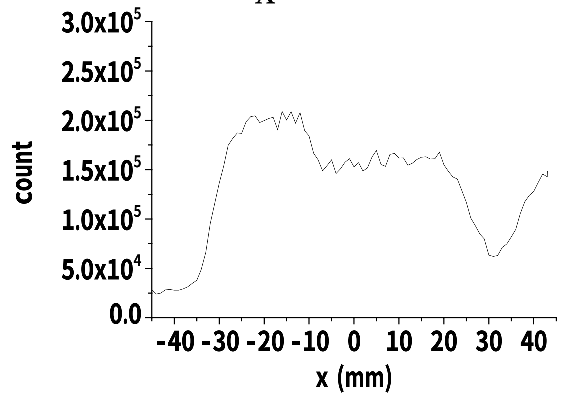
# Crossing in a direction of tune increasing

## Beam profiles during crossing

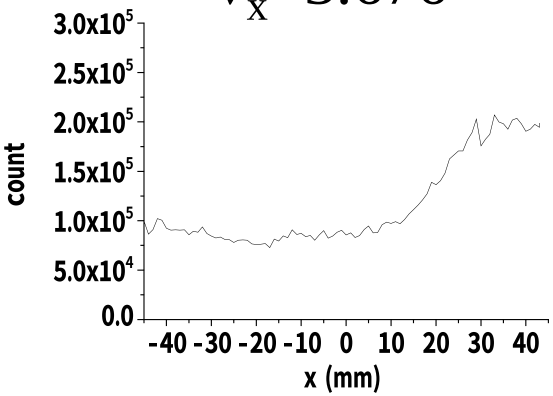
$\nu_x$=3.654



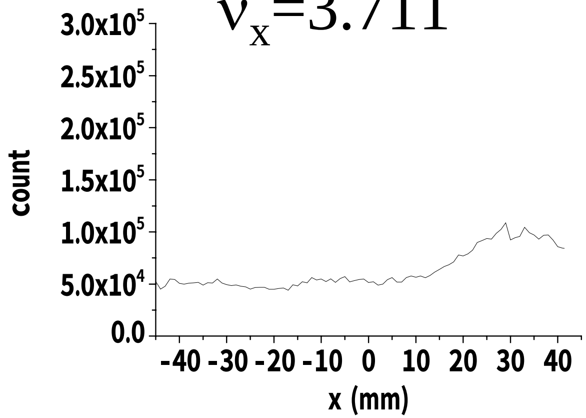

$\nu_x$=3.657




$\nu_x$=3.676




$\nu_x$=3.711




Crossing speed:

$4.6 \times 10^{-6}$

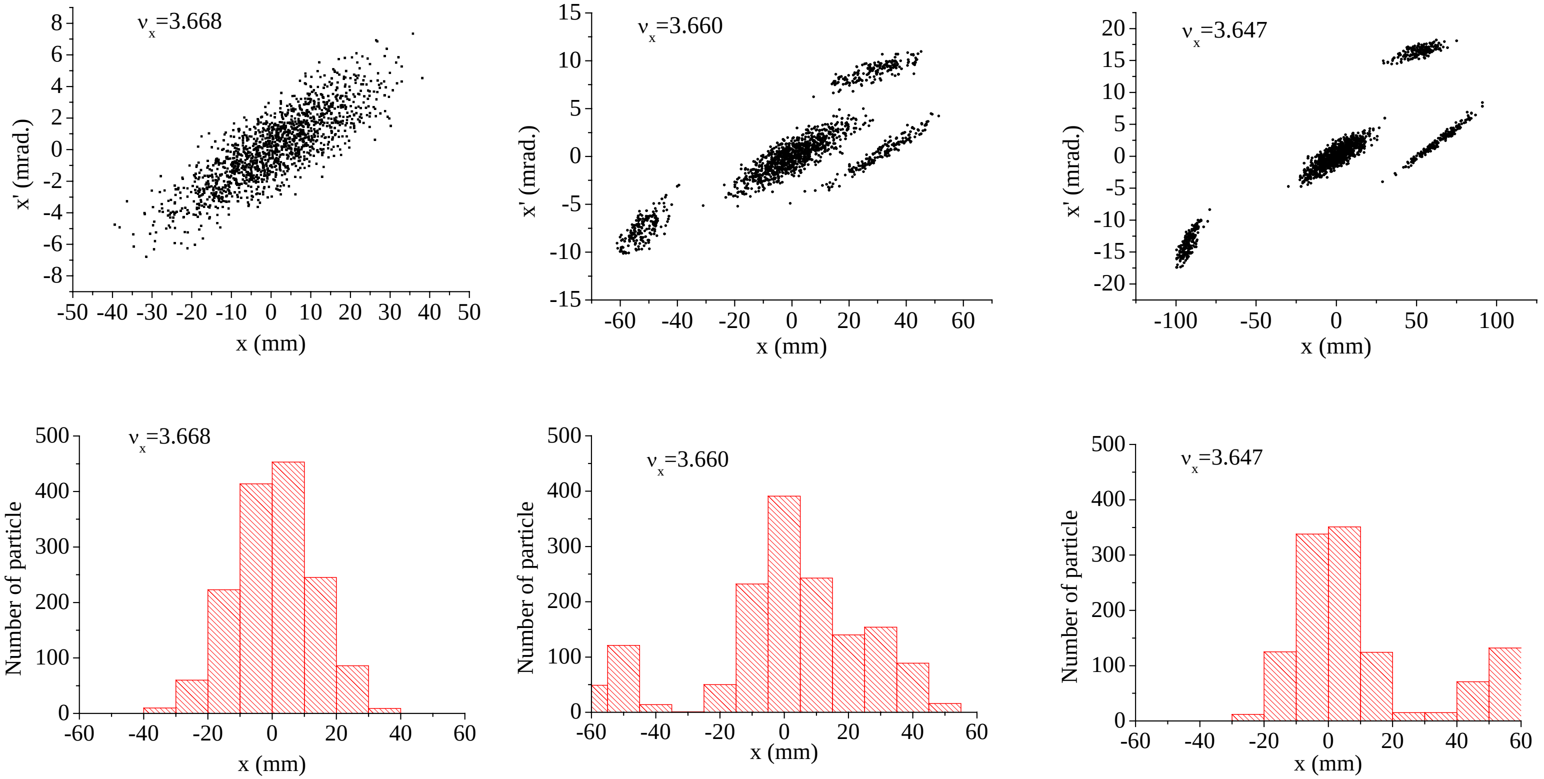
Nonlinear detuning:

$2.52 m^{-1}$

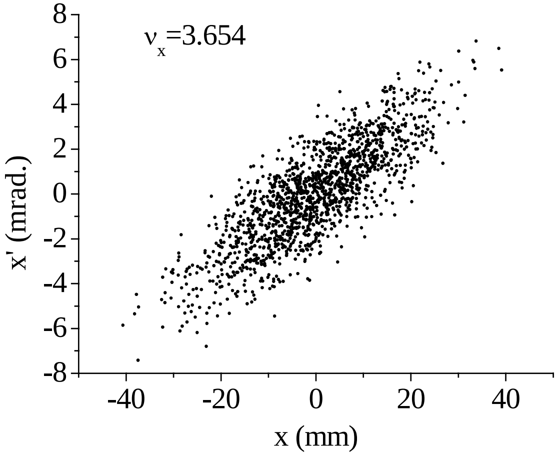
Driving term:

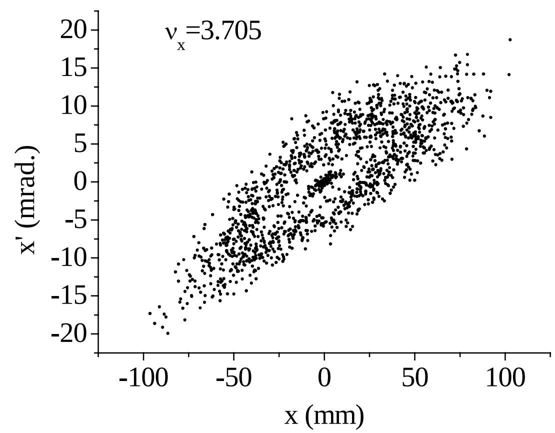
$0.019 m^{-1/2}$

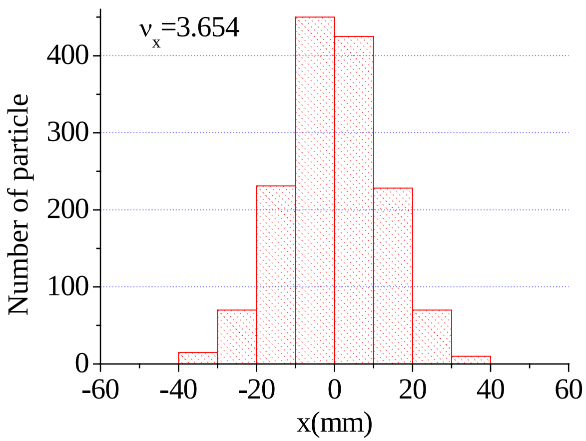
# Simulation – tune decreasing

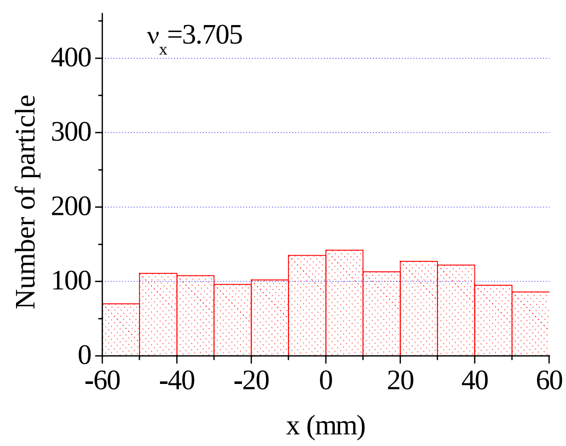


# Simulation – tune increasing



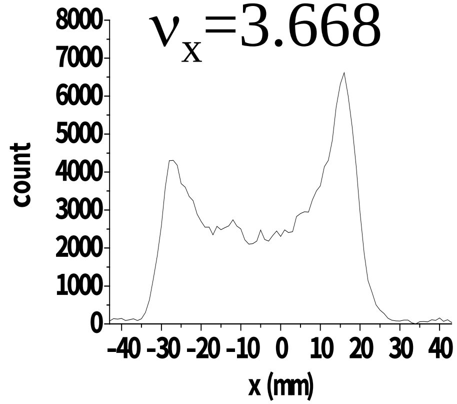



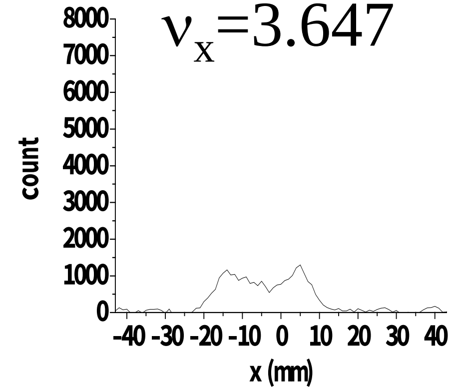
# Difference due to crossing direction

Tune decreasing

$\nu_x=3.668$

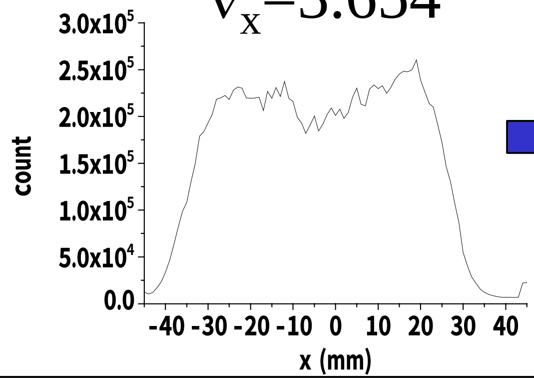



$\nu_x=3.647$

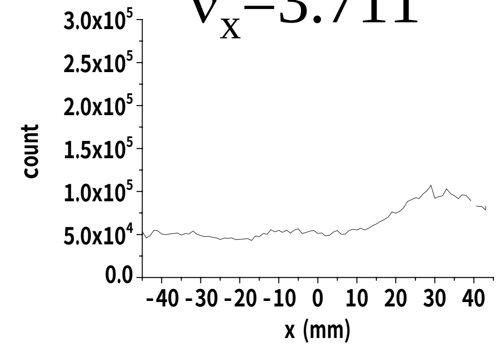



Tune increasing

$\nu_x=3.654$




$\nu_x=3.711$



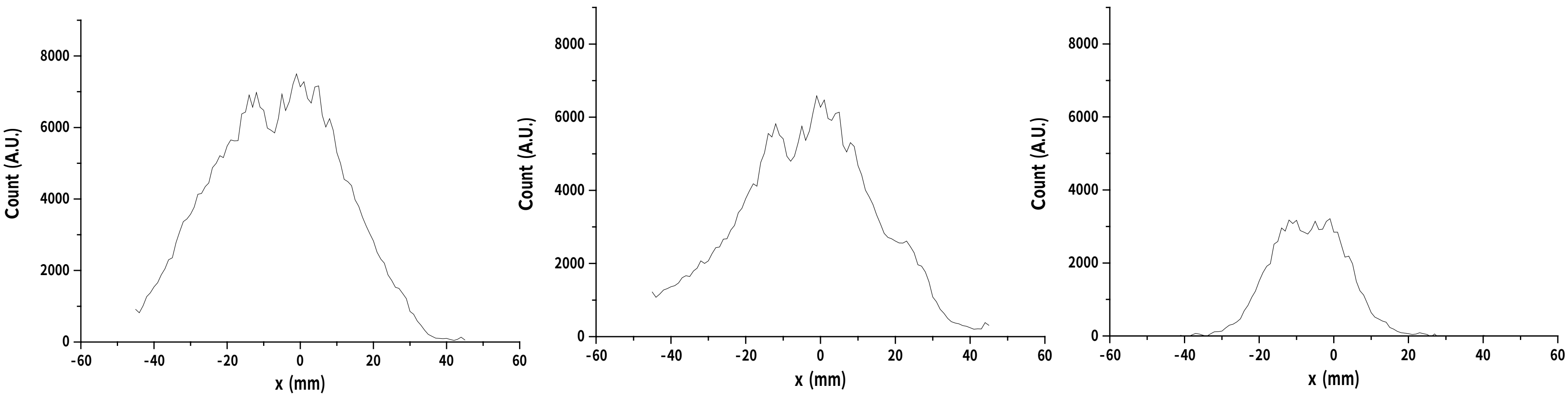

The effect due to crossing depends upon crossing direction.

In one direction: “particle trapping”

In other direction: “emittance growth”

# Crossing without sextupoles

## Beam profiles during crossing (tune decreasing)




"Particle trapping" occurred
even when all magnets are linear elements.

Possible source for nonlinear components:
allowed poles, fringing field …

Crossing speed:
$4.6 \times 10^{-6}$

Nonlinear detuning:
$0\ \mathrm{m}^{-1}$

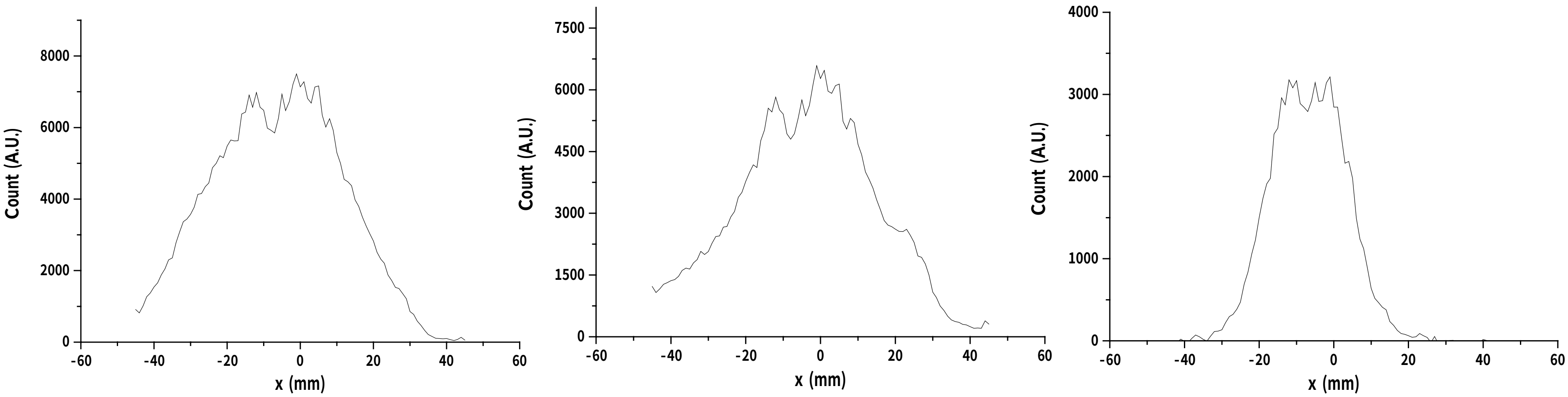
Driving term:
$0\ \mathrm{m}^{-1/2}$

# Summary

- Experiment at PoP FFAG
  - “Particle trapping” due to resonance crossing was observed.
  - Trapping efficiency are understood qualitatively.
  - Adiabatic parameter more than 7 was harmless.
- Experiment at HIMAC
  - Difference due to crossing direction was shown.
  - Even sextupoles are not excited, the effect of crossing was “particle trapping”.

# Crossing without sextupoles

Normalized beam profiles during crossing




Crossing speed:

$4.6 \times 10^{-6}$

Nonlinear detuning:

$0\ m^{-1}$

Driving term:

$0\ m^{-1/2}$