Electron- and Neutrino-Nucleus Scattering in the Impulse Approximation Regime

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work done in collaboration with

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Outline

- Motivation
- Many body theory of electroweak interactions with nuclei
- Results for (e, e') and comparison to data
- Results for (ν, ℓ)
- Conclusions and prospects

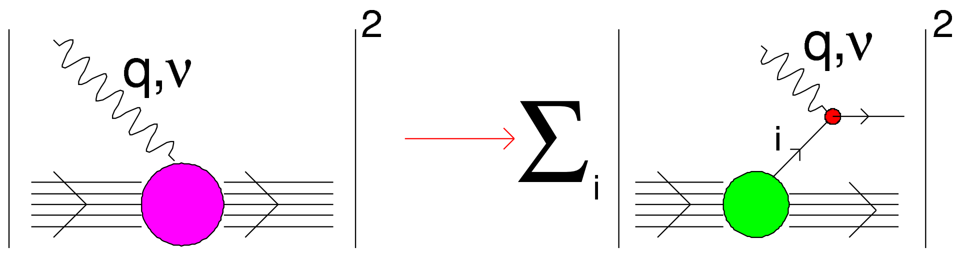
Motivation

- Quantitative understanding of the weak nuclear response at $E_\nu \sim 0.5 - 3$ GeV required for the analyses of many neutrino oscillation experiments.

- Need to develop a theoretical framework
  - applicable to a wide range of kinematical conditions and targets
  - easily implementable in Monte Carlo simulations

Many-body theory of $e + A \to e' + X$

In the Impulse Approximation scheme (IA) the scattering process off a nucleus reduces to the incoherent sum of the elementary processes involving individual nucleons:



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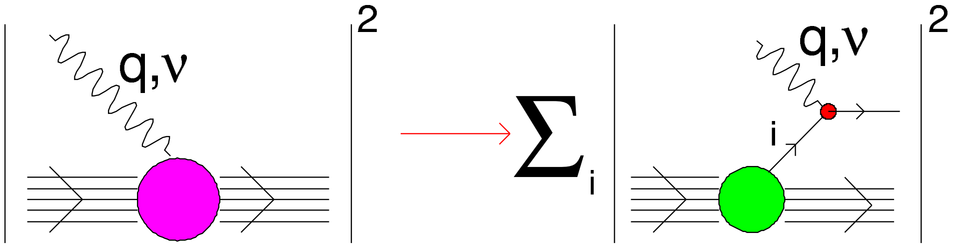

Cross section can be written:

$$\frac{d\sigma_A}{d\Omega_{e'} dE_{e'}} = \int d^4p \, P(p) \left(\frac{d\sigma_N}{d\Omega_{e'} dE_{e'}} \right)$$

Ingredients of IA calculations

- $e + N \rightarrow\ e' + X$ elementary cross-section

$$\frac{d\sigma_{eN}}{d\Omega_{e'} dE_{e'}} = \frac{\alpha^2}{Q^4} \frac{E'_e}{E_e}\ \frac{m}{E_{\mathbf{p}}} L_{\mu\nu} w_N^{\mu\nu}$$

$$w_N^{\mu\nu} = w_1^N \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + \frac{w_2^N}{m^2} \left(p^\mu - \frac{(pq)}{q^2} q^\mu \right) \left(p^\nu - \frac{(pq)}{q^2} q^\nu \right)$$

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- Target spectral function $P(\mathbf{p}, E)$: probability of removing a nucleon of momentum $\mathbf{p}$ from the target, leaving the residual spectator system with excitation energy E

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- exact or highly accurate calculations feasible for A=2,3,4 and ∞
- spectral functions for Carbon, Oxygen, Iron and Gold have been obtained using Local Density Approximation (LDA)

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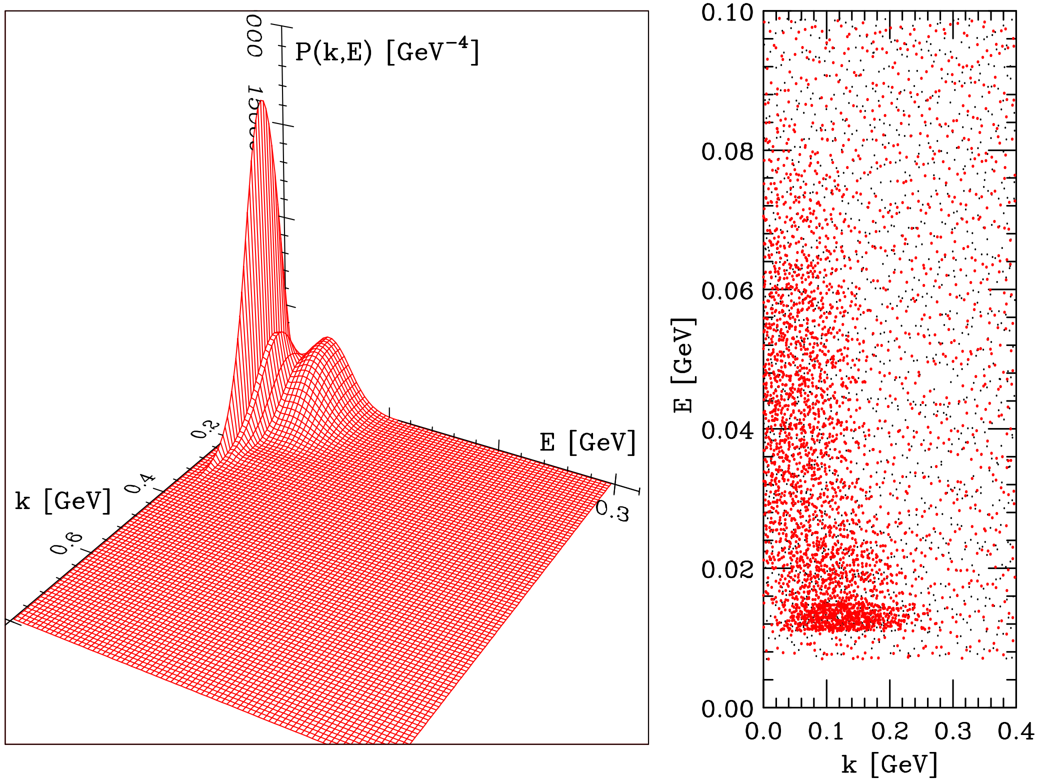
$$P_{corr}(\mathbf{p}, E) = \int d^3r \, \rho_A(r) \, P_{corr}^{NM}(\mathbf{p}, E; \rho = \rho_A(r))$$



- the shell model contribution $P_{MF}(\mathbf{p}, E)$ accounts for $\sim 80\%$ of the strenght;
- the remaining $\sim 20\%$, accounted for by $P_{corr}(\mathbf{p}, E)$, is located at high momentum *and* large removal energy $(\mathbf{p} \gg p_F, E \gg e_F)$

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$$\frac{d\sigma_{eN}}{d\Omega_{e'} dE_{e'}} = \frac{\alpha^2}{Q^4}\frac{E'_e}{E_e}\frac{m}{E_{\mathbf{p}}} L_{\mu\nu} w_N^{\mu\nu}$$

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$$w_i^{\mu\nu} = \sum_x \langle \mathbf{p}, \mathrm{N} | j_i^{\mu} | x, \mathbf{p} + \mathbf{q} \rangle \langle \mathbf{p} + \mathbf{q}, x | j_i^{\nu} | \mathrm{N}, \mathbf{p} \rangle$$

$$\times \quad \delta(\widetilde{\nu} + \sqrt{\mathbf{p}^2 + m^2} - E_x) \ .$$

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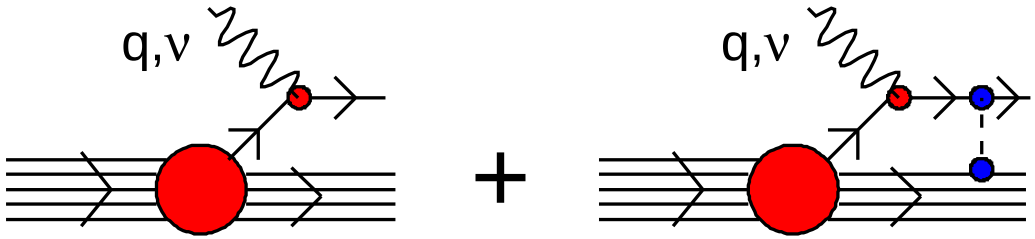
Binding of the struck nucleon is taken into account by replacing:

$$\begin{aligned} q &\equiv (\nu, \mathbf{q}) \to \widetilde{q} \equiv (\widetilde{\nu}, \mathbf{q}) \\ \widetilde{\nu} &= \nu - E + m - \sqrt{\mathbf{p}^2 + m^2} = \nu - \delta\nu \end{aligned}$$

Beyond IA



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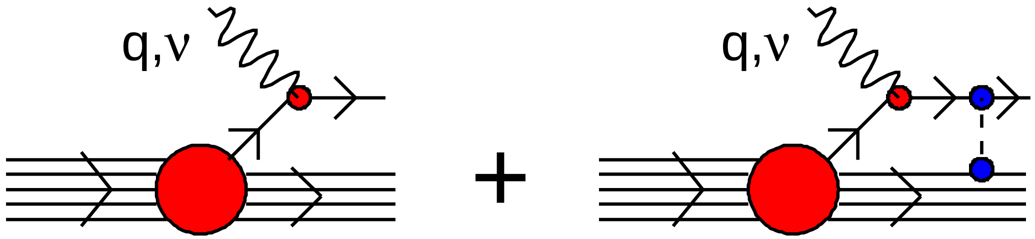

- FSI (Final State Interactions)

  A) energy shift $\longrightarrow$ mean field of the spectators

  B) redistributions of the strenght $\longrightarrow$ coupling of $1p1h$ final state to $np-nh$

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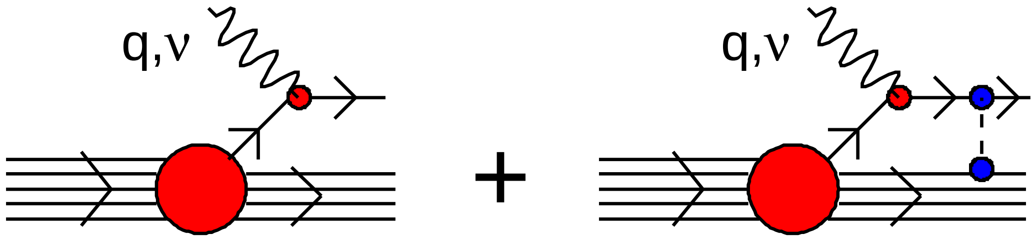

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  A) energy shift → mean field of the spectators

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High energy approximation:

i) the struck nucleon moves along a straight trajectory with constant velocity;

ii) the fast struck nucleon "sees" the spectator system as a collection of fixed scattering centers.

FSI described by:

$$\bar{U}^{FSI}_{\mathbf{p}+\mathbf{q}}(t) = \langle 0| \frac{1}{A} \sum_{i=1}^{A} \mathrm{e}^{i \sum_{j\neq i} \int_0^t dt' \Gamma_{\mathbf{p}+\mathbf{q}}(|\mathbf{r}_{ij}+\mathbf{v}t'|)} \, |0\rangle \ ,$$

Γ_{if} is the Fourier Transform of NN scattering amplitude.

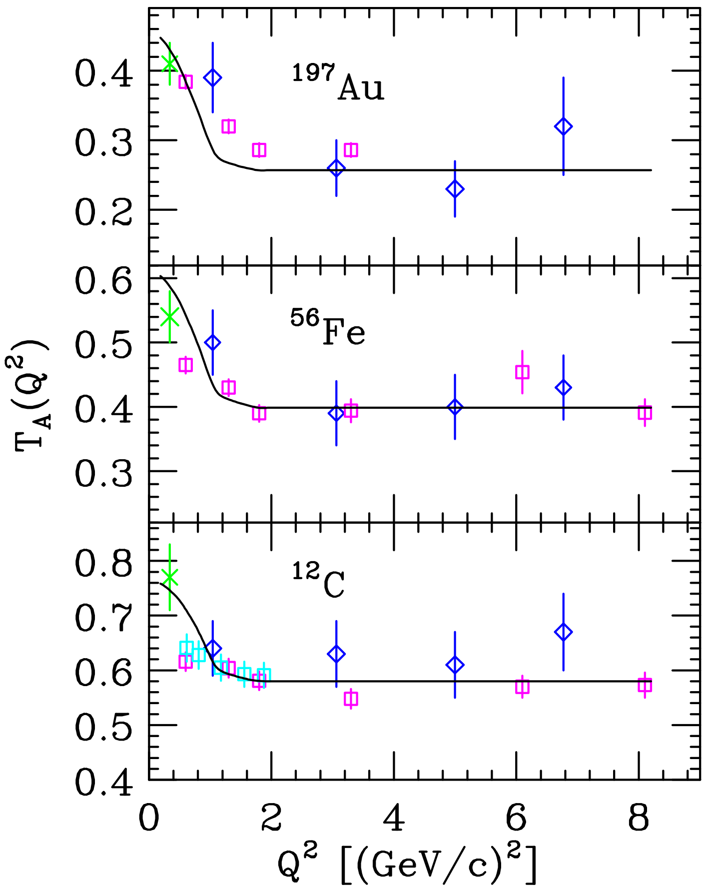
A) $\longrightarrow$ Re Γ_{ij} B) $\longrightarrow$ Im Γ_{ij}

At high energy B) dominates.

$$\frac{d\sigma}{d\Omega_{e'} d\nu} = \int d\nu' \ f_{\mathbf{q}}(\nu - \nu') \ \left(\frac{d\sigma}{d\Omega_{e'} d\nu'} \right)_{IA}$$

$$f_{\mathbf{q}}(\nu) = \delta(\nu) \sqrt{T_A} + \int \frac{dt}{2\pi} \ \mathrm{e}^{i\nu t} \left[U^{FSI}_{\mathbf{q}}(t) - \sqrt{T_A} \right]$$

$$T_A = \lim_{t\to\infty} \langle 0| |U^{FSI}_{\mathbf{q}}(R;t)|^2 |0\rangle$$



Calculated transparency compared to MIT-Bates, SLAC and JLAB data (D. Rohe et al., (JLAB E97-006) - nucl-ex/0506007)

Beyond IA (continued)

Statistical FSI $\longrightarrow$ Pauli blocking effects.

A rather crude prescription: modify the spectral function

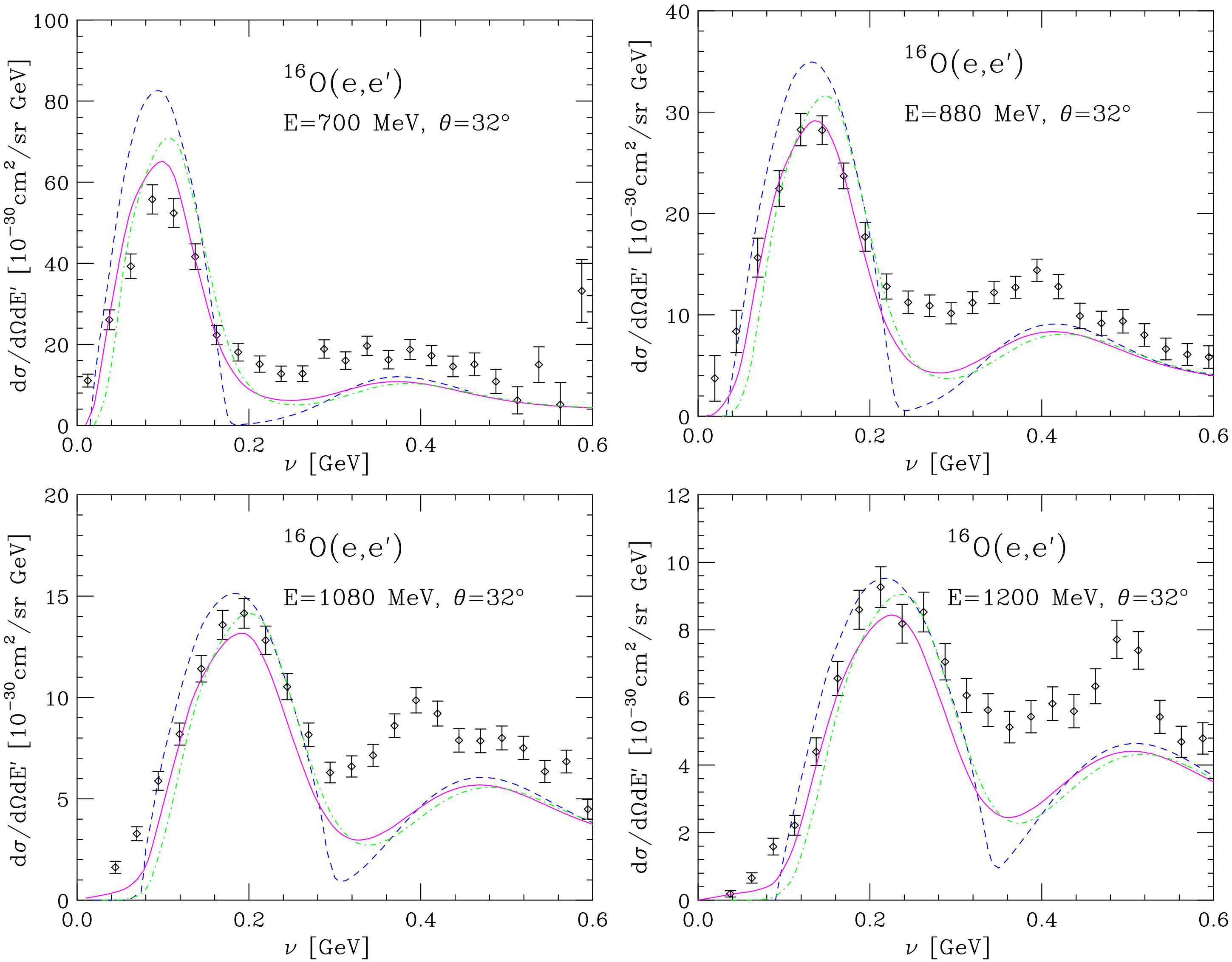
$$P(\mathbf{p}, E) \longrightarrow \; P(\mathbf{p}, E)\, \theta(|\mathbf{p} + \mathbf{q}| - \overline{p}_F)$$

Average nuclear Fermi $\overline{p}_F$ momentum defined as:

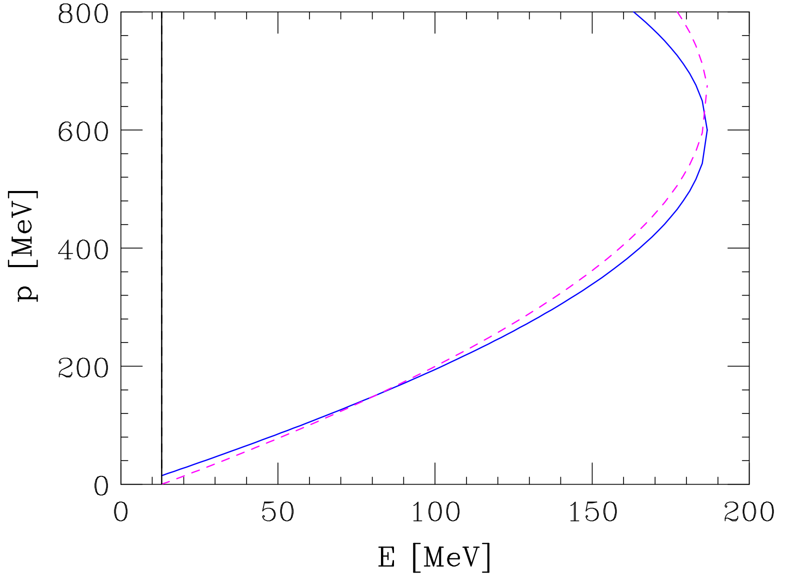
$$\overline{p}_F = \int \; d^3r \; \rho_A(\mathbf{r}) p_F(\mathbf{r}) \quad , \quad p_F(\mathbf{r}) = \frac{3}{2}\pi^2 \rho_A(\mathbf{r})$$

Inclusion of Pauli blocking is hardly visible in the lepton energy distribution at a fixed angle, but becomes dominant in the Q^2 distribution in the low Q^2 region.

Comparison to Frascati ^{16}O (e, e') data

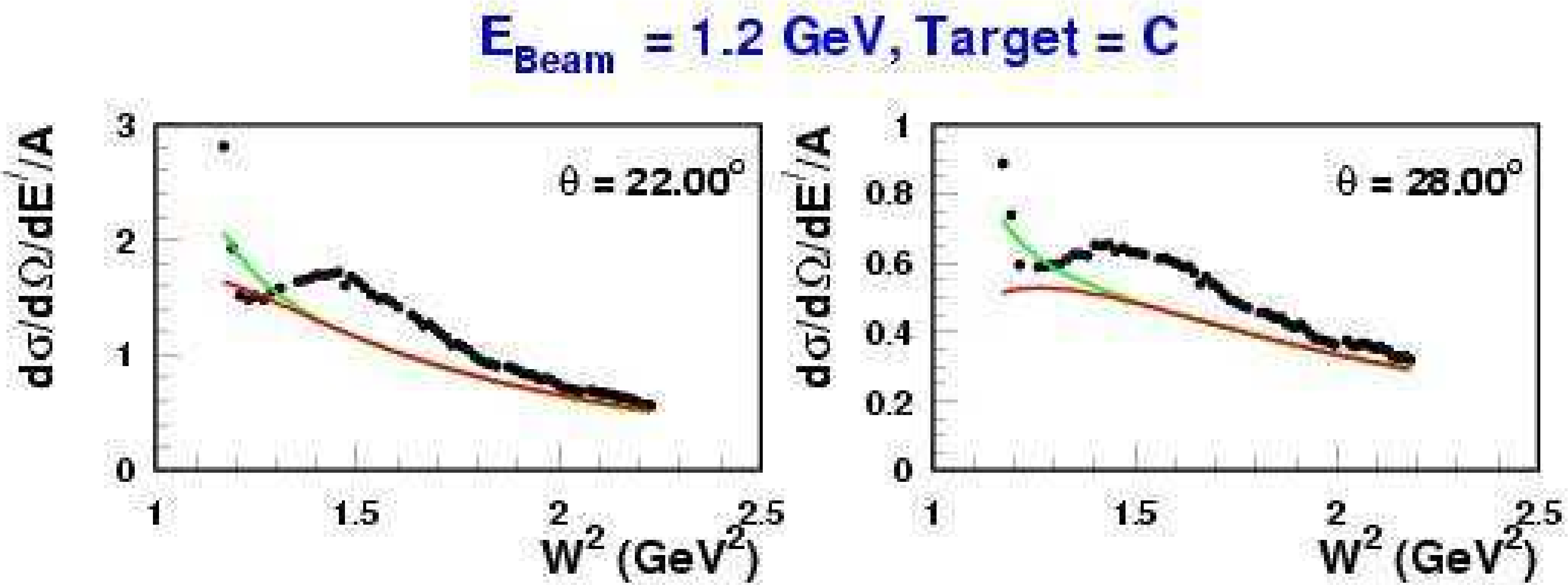


- OK in the region of quasi elastic peak
- data significantly underestimated above π production threshold
- deficiencies of the SF unlikely



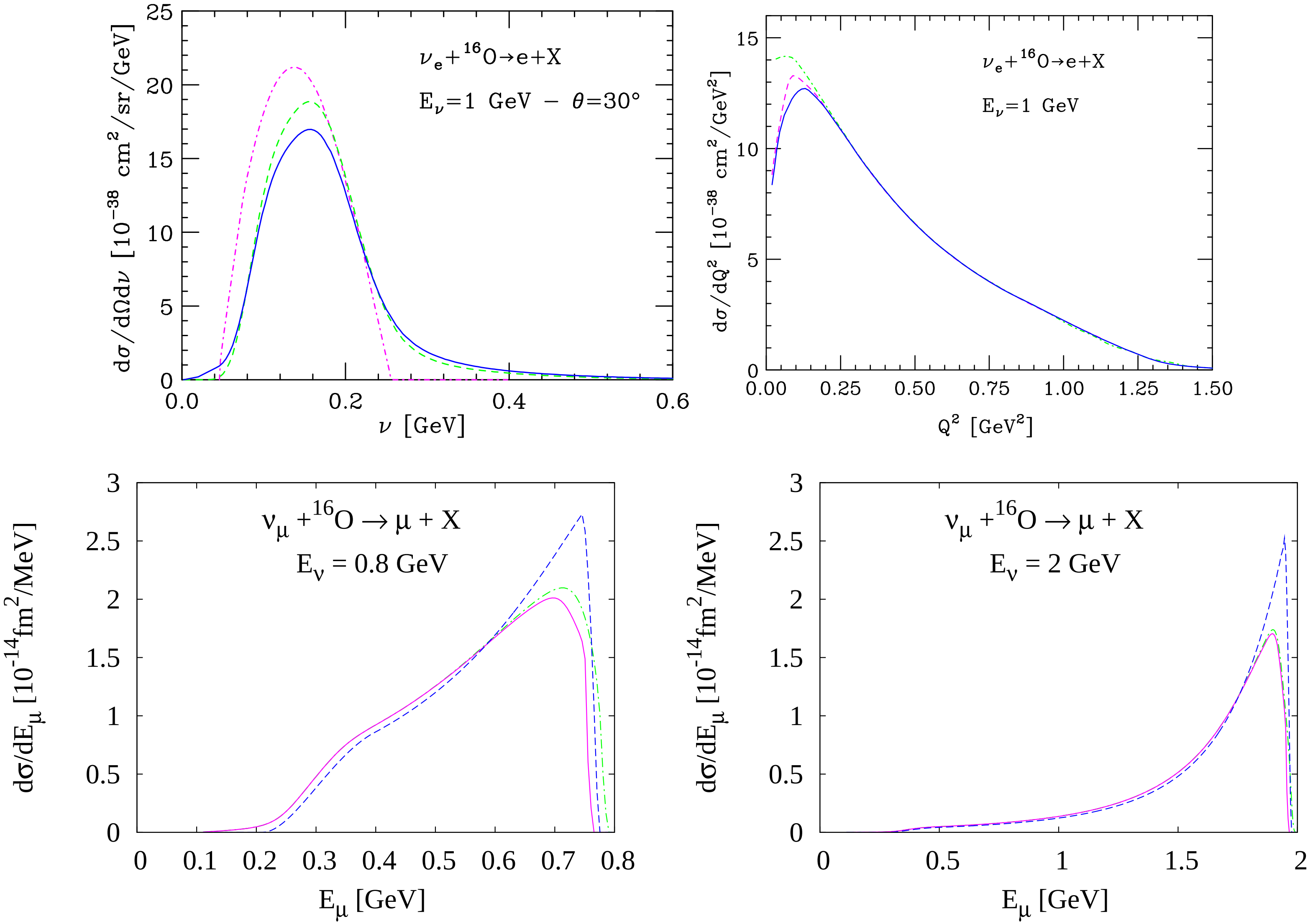
- low Q^2 nucleon structure functions in the Δ production region poorly known

New data at low Q^2 (JLAB E04-001, courtesy of C. Keppel)



PRELIMINARY

Results for $^{16}O\ (\nu_e, e)$ scattering



Conclusions and prospects

- Using nuclear many-body theory quantitative predictions of the inclusive cross sections can be made for broad range of targets and kinematical conditions
- Data in the region of quasi elastic peak reproduced with accuracy better than 10%
- Problems in the region of quasi-free Δ production, likely to be ascribed to the uncertainty associated with the nucleon structure functions.
- Pauli blocking important at Q^2<0.2 GeV^2
- Extension to semi-inclusive processes is under way