Using Electron Scattering Superscaling to predict Charge-changing Neutrino Cross Sections in Nuclei

Maria B. Barbaro

University of Torino (Italy) and INFN

Collaboration

- J.E. Amaro (Granada)
- M.B. B. (Torino)
- J.A. Caballero (Sevilla)
- T.W. Donnelly (MIT)
- C. Maieron (Catania)
- A. Molinari (Torino)
- I. Sick (Basel)

Phys. Rev. C **71** (2005) 015501 [arXiv:nucl-th/0409078]

Motivation

- The interpretation of neutrino experiments requires good control of nuclear physics
- From (e, e') we know that nuclear effects can be large
- Can we extract predictions for ν-A in the QEP and Δ regions from e-A experimental cross sections?

General strategy

- [1.] Extract scaling function from (e, e') data, embodying the nuclear physics content of the problem
- [2.] Invert the scaling approach to predict (ν_l, l) cross sections in the few-GeV region

Summary

- General formalism for charge-changing ν-nucleus scattering
- y-scaling in inclusive electron scattering:
  - the quasi-elastic peak (QEP)
  - the Δ-resonance region
- Predictions for (ν, μ) and $(\bar{\nu}, \mu)$ cross sections

Charge-changing neutrino cross section

- Cross section in lab. frame

$$\left[\frac{d^2\sigma}{d\Omega dk'}\right]_\chi \equiv \sigma_0 \mathcal{F}_\chi^2 \qquad \begin{cases} \chi = + & \text{for neutrinos} \\ \chi = - & \text{for antineutrinos} \end{cases}$$

$$\sigma_0 \equiv \frac{(G\cos\theta_c)^2}{2\pi^2}\left[k'\cos\widetilde{\theta}/2\right]^2 \qquad \begin{cases} \text{Fermi } G = 1.16639\text{x}10^{-5} \text{ GeV}^{-2} \\ \text{Cabibbo } \cos\theta_c = 0.9741 \end{cases}$$

$\widetilde{\theta}$=generalized scattering angle

$$\tan^2\widetilde{\theta}/2 \equiv \frac{|Q^2|}{v_0}, \qquad v_0 \equiv (\epsilon+\epsilon')^2 - q^2 = 4\epsilon\epsilon' - |Q^2|$$

Weak responses

- Generalized Rosenbluth decomposition (q along z)

$$\mathcal{F}_\chi^2 = \left[\widehat{V}_{CC} R_{CC} + 2\widehat{V}_{CL} R_{CL} + \widehat{V}_{LL} R_{LL} + \widehat{V}_T R_T\right]$$
$$+ \chi \left[2\widehat{V}_{T'} R_{T'}\right]$$

- Hadronic tensor:

$$W^{\mu\nu} = \overline{\sum}_{fi} \delta(E_f - E_i - \omega) \langle f | J^{\mu *}(Q) | i \rangle \langle i | J^\nu(Q) | f \rangle$$

$$R_{CC} = W^{00} \, , \quad R_{CL} = -W^{03} \, , \quad R_{LL} = W^{33}$$
$$R_T = W^{11} + W^{22} \, , \quad R_{T'} = -iW^{12}$$

- Nuclear weak current: $J^\mu = J^\mu_V - J^\mu_A$
- Vector and axial-vector contributions

$$R_i = R_i^{VV} + R_i^{AA} \,, \quad i = CC, CL, LL, T$$

$$R_{T'} = R_{T'}^{VA}$$

$$R_{CL}^{VV} = -\frac{\omega}{q} R_{CC}^{VV} \qquad R_{LL}^{VV} = \frac{\omega^2}{q^2} R_{CC}^{VV}$$ CVC

$$\Rightarrow \widehat{V}_{CC} R_{CC}^{VV} + 2\widehat{V}_{CL} R_{CL}^{VV} + \widehat{V}_{LL} R_{LL}^{VV} = \widehat{V}_L R_L^{VV} \equiv X_L^{VV}$$ Longitudinal

$$\widehat{V}_{CC} R_{CC}^{AA} + 2\widehat{V}_{CL} R_{CL}^{AA} + \widehat{V}_{LL} R_{LL}^{AA} \equiv X_{C/L}^{AA}$$ Collapse does not occur for the AA terms.

$$\widehat{V}_T \left[R_T^{VV} + R_T^{AA} \right] \equiv X_T$$ Transverse

$$2\widehat{V}_{T'} R_{T'}^{VA} \equiv X_{T'}$$ V/A interference

- Full response: $\mathcal{F}_\chi{}^2 = X_L^{VV} + X_{C/L}^{AA} + X_T + \chi X_{T'}$

Relativistic Fermi Gas

- On-shell nucleons move freely (except for Pauli blocking) inside the nucleus with $E_p = \sqrt{p^2 + m_N^2}$ and are described by free Dirac spinors $u(\mathbf{p}, s)$
- One free parameter: k_F
- Lorentz covariance: $W^{\mu\nu}$ is a Lorentz tensor
- Gauge invariance: $Q_\mu J^\mu = 0$ vector current conservation
- Analytic expressions for response functions

- Analytic integration yields

$$W^{\mu\nu}_{X(RFG)}(q,\omega) \;=\; \mathcal{N}\frac{m_N T_F |Q^2|}{k_F^3 q} U^{\mu\nu}_X(q,\omega) f_{RFG}(\psi_X)$$

$X = QE,\ \Delta;\quad \mathcal{N} = Z, N;\quad T_F = \epsilon_F - m_N$

- $U^{\mu\nu}_X$ "single nucleon" tensor
- RFG superscaling function

$$f_{RFG}(\psi) = \frac{3}{4}(1-\psi^2)\theta(1-\psi^2)$$

is universal: when plotted against ψ no dependence is left either on momentum transfer (scaling of the I kind) or on nuclear species via k_F (scaling of the II kind)

- I + II kind scaling $\Rightarrow$ superscaling
- RFG scaling variables ($X = QE, \Delta$)

$$\psi_X(q,\omega) = \sqrt{\frac{T_0^X}{T_F}} \times \begin{cases} +1 & \omega \geq {\omega_X}^P \\ -1 & \omega \leq {\omega_X}^P \end{cases}$$

$$\omega_X^P = \sqrt{q^2 + m_X^2} - m_N,$$

$$T_0^X = \frac{q}{2}\sqrt{\rho_X^2 + \frac{|Q^2|}{4m_N^2}} - \frac{\omega}{2}\rho_X - m_N, \qquad \rho_X = 1 + \frac{m_X^2 - m_N^2}{|Q^2|}$$

- RFG response region $-1 \leq \psi_X \leq 1$
- QE and Δ peaks: $\psi_X = 0$
- Energy shift: $\omega \rightarrow \omega' = \omega - E_{shift} \Rightarrow \psi_X \rightarrow \psi'_X$

QE CC single nucleon responses

- Vector and axial-vector currents: $j^\mu = j^\mu_V - j^\mu_A$

$$
\begin{aligned}
j^\mu_V &= \bar{u}(P')\left[F_1^{(1)}\gamma^\mu + \frac{i}{2m_N}F_2^{(1)}\sigma^{\mu\nu}Q_\nu\right]u(P), \\
j^\mu_A &= \bar{u}(P')\left[G_A\gamma^\mu + \frac{1}{2m_N}G_P Q^\mu\right]\gamma^5 u(P),
\end{aligned}
$$

- s.n. response functions (at leading order in k_F/m_N)

$$R_L^{VV} = \frac{q^2}{|Q^2|}\left[G_E^{(1)}\right]^2 \qquad R_T^{VV} = \frac{|Q^2|}{2m_N^2}\left[G_M^{(1)}\right]^2$$

$$R_{CC}^{AA} = \frac{\omega^2}{|Q^2|}\left[G_A'^{(1)}\right]^2 \qquad R_{CL}^{AA} = -\frac{\omega q}{|Q^2|}\left[G_A'^{(1)}\right]^2$$

$$R_{LL}^{AA} = \frac{q^2}{|Q^2|}\left[G_A'^{(1)}\right]^2 \qquad R_T^{AA} = 2\left(1+\frac{|Q^2|}{4m_N^2}\right)\left[G_A^{(1)}\right]^2$$

$$R_{T'}^{VA} = \frac{1}{m_N}\sqrt{|Q^2|\left(1+\frac{|Q^2|}{4m_N^2}\right)}G_M^{(1)}G_A^{(1)}$$

$$G_A' \equiv G_A - \frac{|Q^2|}{4m_N^2}G_P$$

Δ single-nucleon responses

- Elementary reactions

$$\nu_\mu p \to \mu^- \Delta^{++} \ , \quad \nu_\mu n \to \mu^- \Delta^{+} \ , \quad \bar{\nu}_\mu p \to \mu^+ \Delta^{0} \ , \quad \bar{\nu}_\mu n \to \mu^+ \Delta^{-}$$

- Associated currents

$$J^\mu_{N\to\Delta}(q) = \mathcal{T} \bar{u}^{(\Delta)}_\alpha(p',s')\Gamma^{\alpha\mu} u(p,s)$$

$\mathcal{T} = \sqrt{3}$ for Δ^{++} and Δ^- production and $=1$ for Δ^+ and Δ^0 production, $u^{(\Delta)}_\alpha(p',s')$:

Rarita-Schwinger spinor

- Vertex tensor [Alvarez-Ruso, Singh, Vicente Vacas, PRC 59, 3368 (1999)]

$$
\begin{aligned}
\Gamma^{\alpha\mu} & \\
= & \left[\frac{C_3^V}{m_N}\left(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu\right) + \frac{C_4^V}{m_N^2}\left(g^{\alpha\mu} q\cdot p' - q^\alpha p'^\mu\right)\right. \\
+ & \left.\frac{C_5^V}{m_N^2}\left(g^{\alpha\mu} q\cdot p - q^\alpha p^\mu\right)\right]\gamma_5 \\
+ & \left[\frac{C_3^A}{m_N}\left(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu\right) + \frac{C_4^A}{m_N^2}\left(g^{\alpha\mu} q\cdot p' - q^\alpha p'^\mu\right)\right. \\
+ & \left. C_5^A g^{\alpha\mu} + \frac{C_6^A}{m_N^2} q^\alpha q^\mu\right]
\end{aligned}
$$

- CVC $\Rightarrow C_6^V = 0$
- PCAC $\Rightarrow C_6^A = C_5^A \; m_N^2/(m_\pi^2 + |Q^2|)$

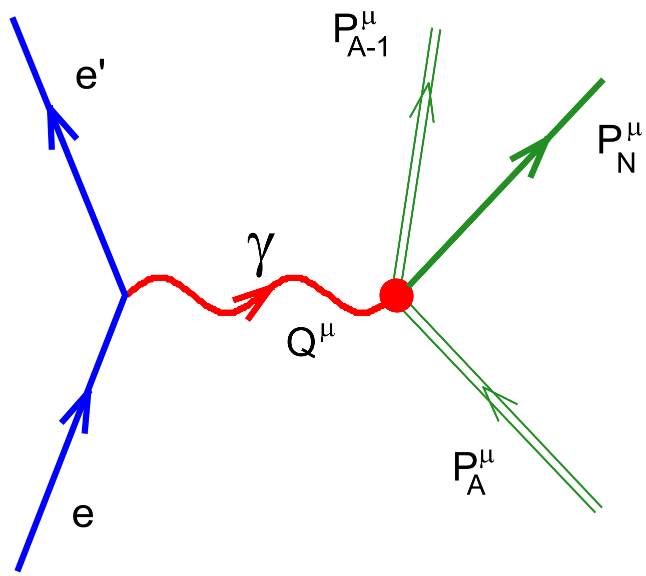
- Δ hadronic tensor

$$
\begin{aligned}
w^{\mu\nu} &= \frac{m_\Delta}{m_N} \mathrm{Tr}\left\{ J^{\mu\dagger}_{N\to\Delta}(q) J^{\nu}_{N\to\Delta}(q) \right\} \\
&= w^{\mu\nu}_{VV} + w^{\mu\nu}_{AA} + w^{\mu\nu}_{VA} \\
w^{\mu\nu}_{VV} &= -w_{1V}(g^{\mu\nu} + \frac{q^\mu q^\nu}{|Q^2|}) + w_{2V}(k^\mu + 2\rho_\Delta q^\mu)(k^\nu + 2\rho_\Delta q^\nu) \\
w^{\mu\nu}_{AA} &= -w_{1A}(g^{\mu\nu} + \frac{q^\mu q^\nu}{|Q^2|}) + w_{2A}(k^\mu + 2\rho_\Delta q^\mu)(k^\nu + 2\rho_\Delta q^\nu) \\
&\quad -u_{1A}\frac{q^\mu q^\nu}{|Q^2|} + u_{2A}(2q^\mu k^\nu + 2k^\mu q^\nu) \\
w^{\mu\nu}_{VA} &= 4i w_3 \epsilon^{\alpha\beta\mu\nu} k_\alpha q_\beta
\end{aligned}
$$

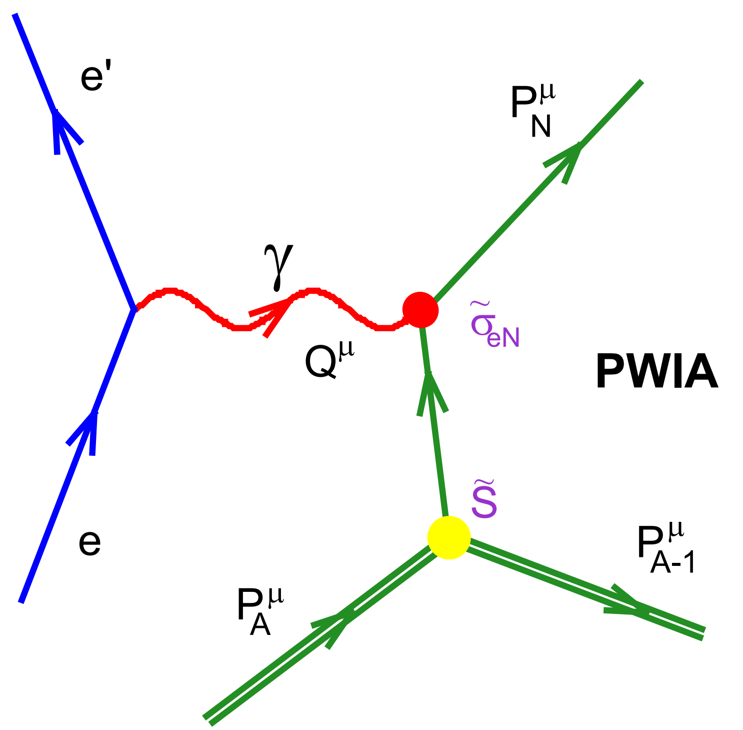
$w_i \equiv w_i(C_K^{V,A})$ are obtained by performing the traces

Scaling in quasielastic (e, e')

- Basic assumption: the QEP is dominated by free ejection of one nucleon



$\Rightarrow$



- Guided from the RFG result, define an experimental QE scaling function as

$$F(q, \psi'_{QE}) \equiv \frac{[d^2\sigma/d\Omega_e d\omega]_{QE}}{S^{QE}}$$

- Dividing function

$$S^{QE} \equiv \sigma_M \left[v_L G_L^{QE} + v_T G_T^{QE} \right]$$

$$\begin{aligned} G_L^{QE} &= \frac{q}{|Q^2|} \left[Z G_{Ep}^2 + N G_{En}^2 \right] + \mathcal{O}(\eta_F^2) \\ G_T^{QE} &= \frac{|Q^2|}{2q} \left[Z G_{Mp}^2 + N G_{Mn}^2 \right] + \mathcal{O}(\eta_F^2) \end{aligned}$$

- Scaling of first kind

$$F(q,\psi') \xrightarrow{q\to\infty} F(\psi') \equiv F(\infty,\psi')$$

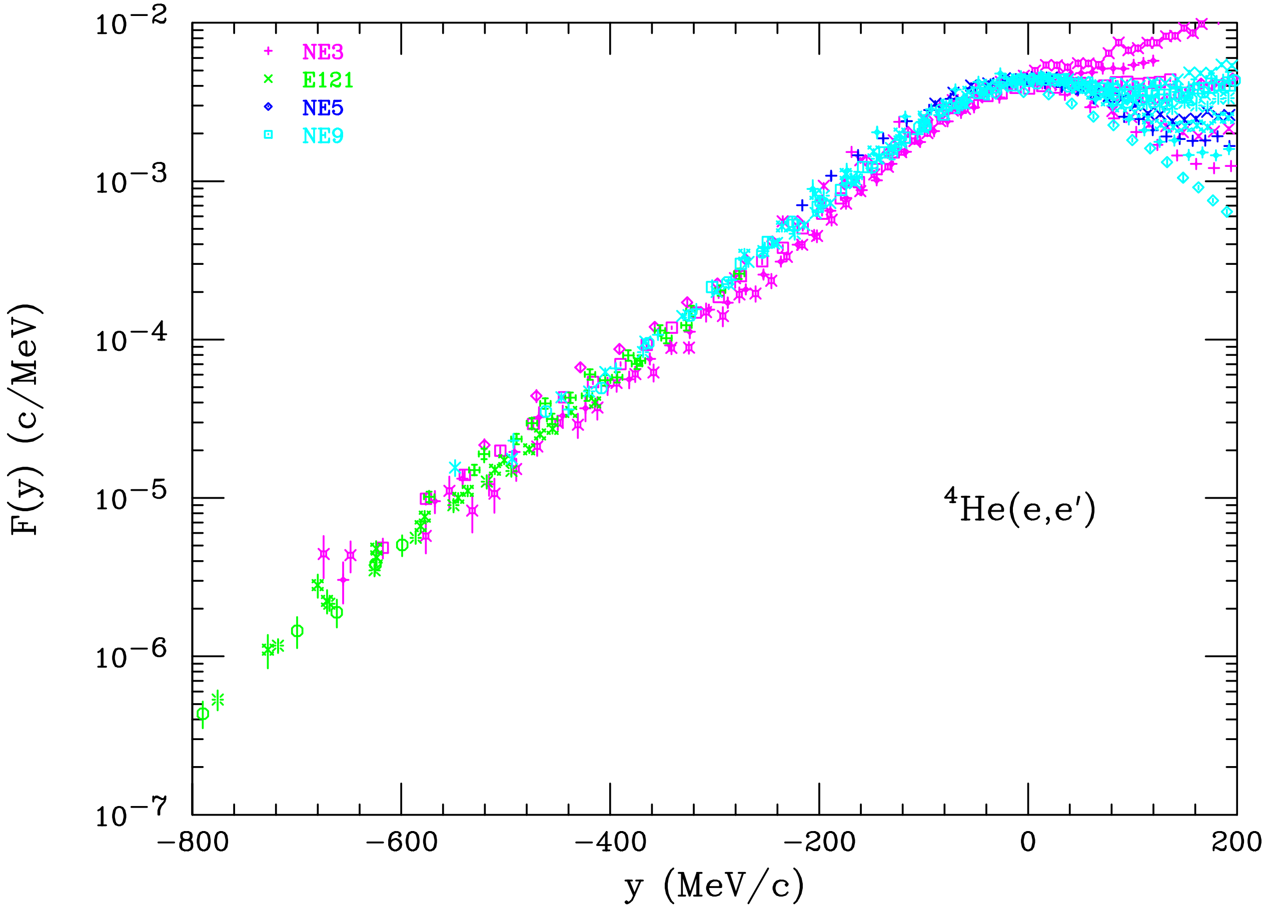
- Superscaling function

$$f^{QE}(q,\psi') \equiv k_F \times F^{QE}(q,\psi')$$

- Extensively studied in Day et al., Phys.Rev.Lett.59(1987)427; Day, McCarthy, Donnelly & Sick, Ann.Rev.Part.Sci.40 (1990) 357; Donnelly & Sick, Phys.Rev.C60,065502 (1999); Phys.Rev.Lett.82, 3212 (1999); Maieron, Donnelly & Sick, Phys.Rev.C65,025502 (2002).

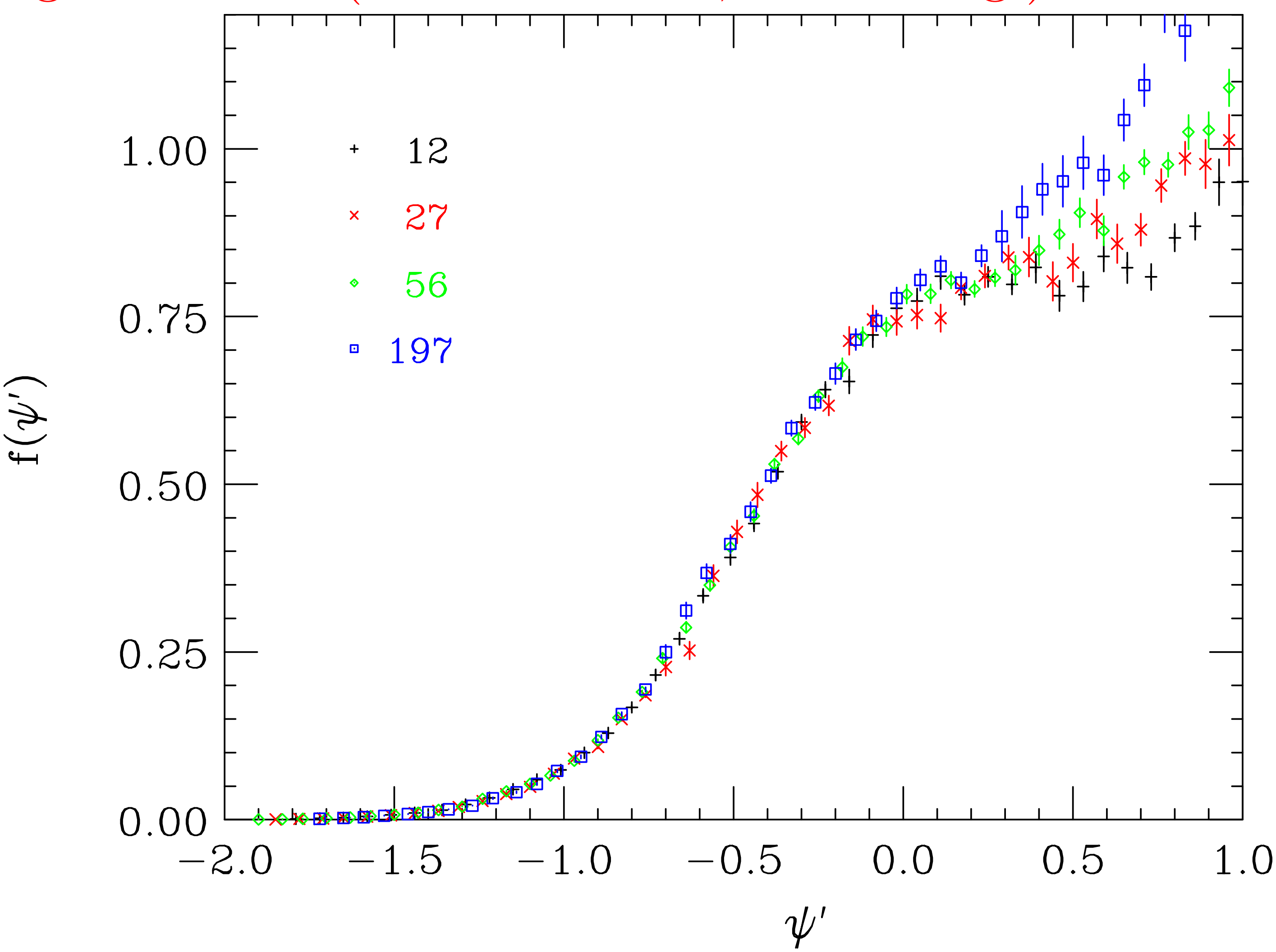
Donnelly & Sick, Phys.Rev.C60,065502 (1999); Phys.Rev.Lett.82, 3212 (1999)

Scaling og I kind



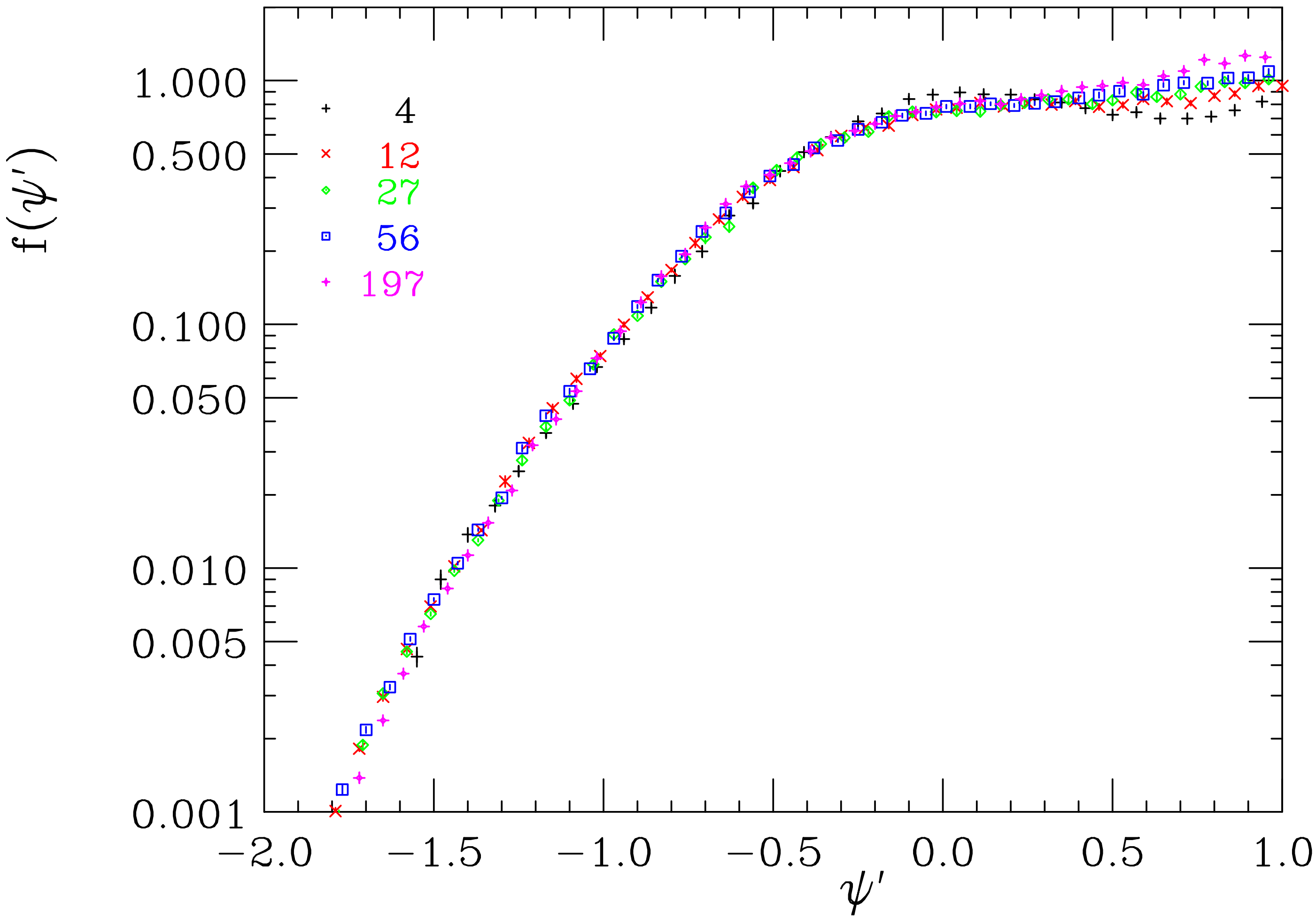
Donnelly & Sick, Phys.Rev.C60,065502 (1999); Phys.Rev.Lett.82, 3212 (1999)

Scaling of II kind ($E_e = 3.6$ GeV, θ_e=16 deg.) - linear scale



Donnelly & Sick, Phys.Rev.C60,065502 (1999); Phys.Rev.Lett.82, 3212 (1999)

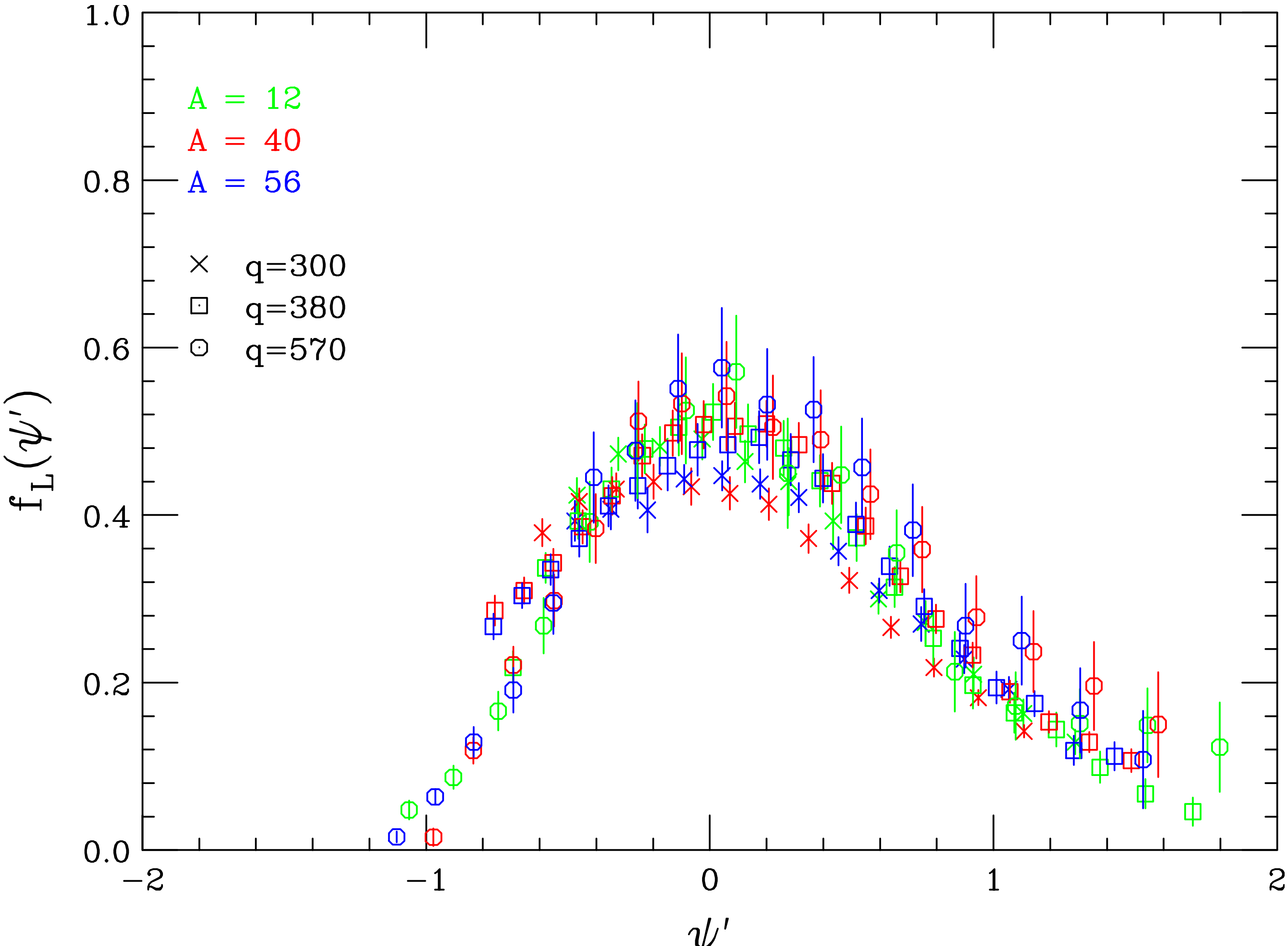
Scaling of II kind ($E_e = 3.6$ GeV, θ_e=16 deg.)-semilog scale



Donnelly & Sick, Phys.Rev.C60,065502 (1999); Phys.Rev.Lett.82, 3212 (1999)

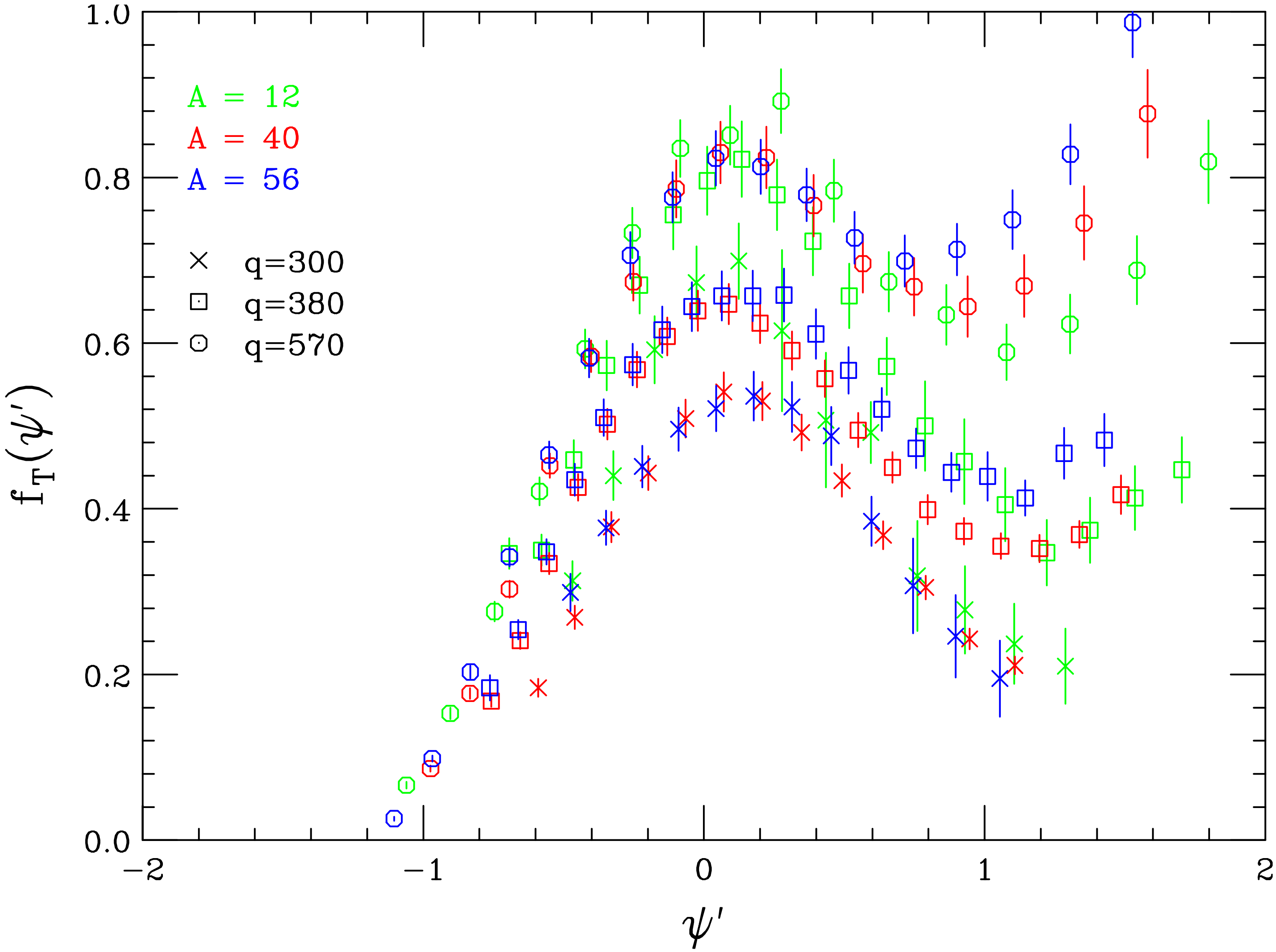
Donnelly & Sick, Phys.Rev.C60,065502 (1999); Phys.Rev.Lett.82, 3212 (1999)

Longitudinal superscaling function



Donnelly & Sick, Phys.Rev.C60,065502 (1999); Phys.Rev.Lett.82, 3212 (1999)

Transverse superscaling function

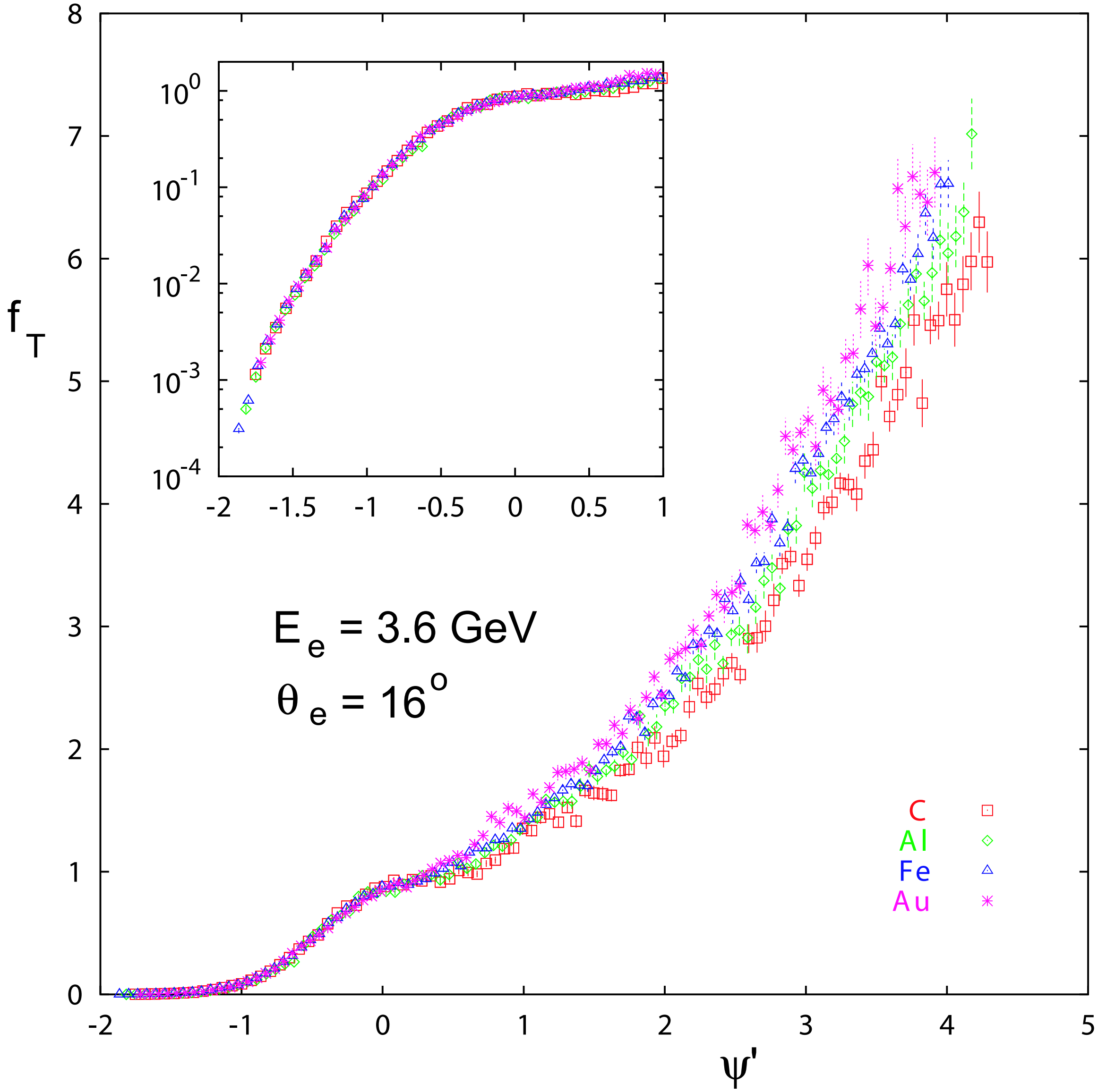


Donnelly & Sick, Phys.Rev.C60,065502 (1999); Phys.Rev.Lett.82, 3212 (1999)

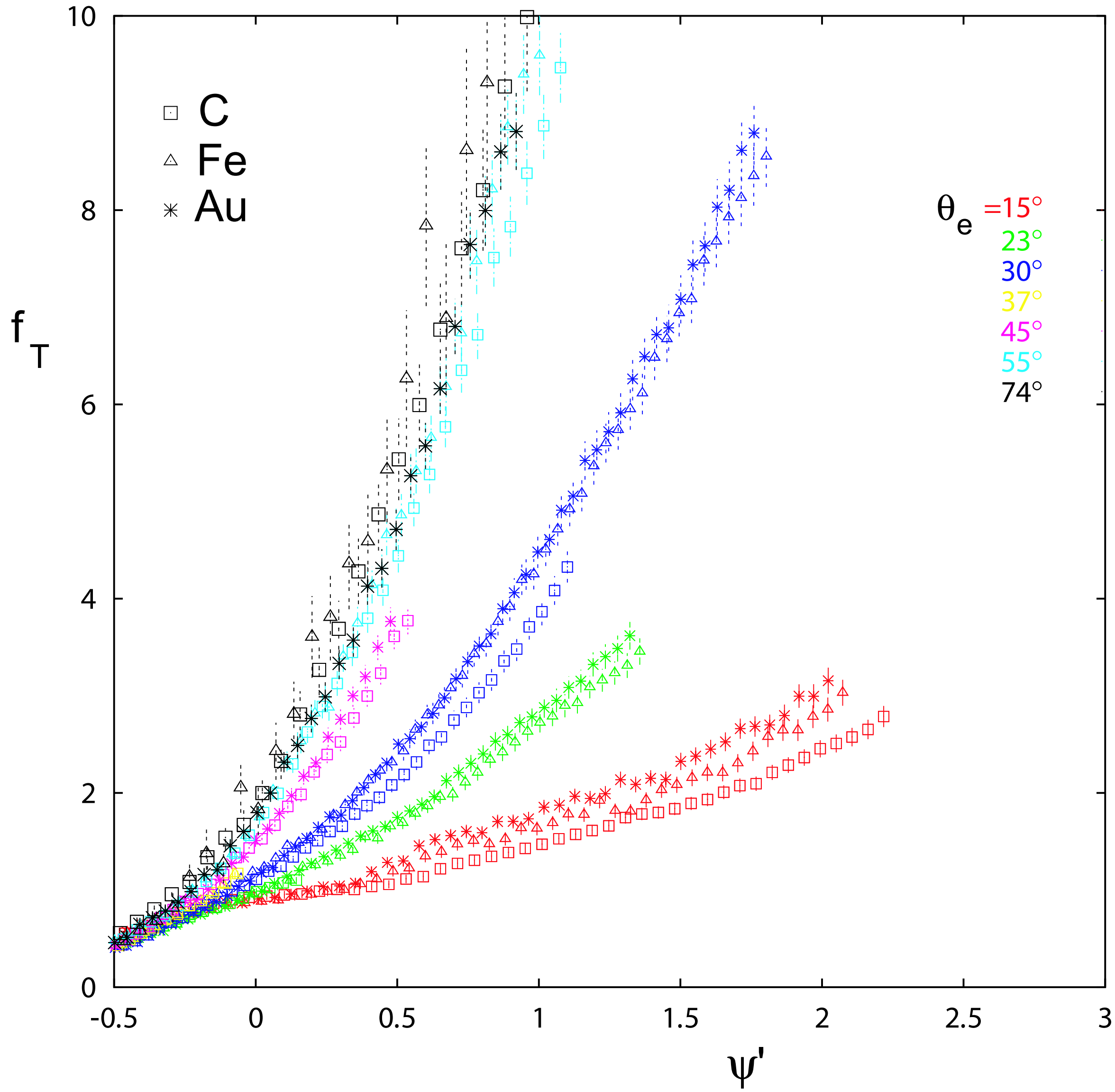
- A "transverse scaling function" can be constructed where no LT separation is available. Assumption: f_L is *universal*. Then

  1. $\Sigma_L = \frac{1}{k_F} f_L \sigma_M v_L G_L$,
  2. $\Sigma_T = \frac{d^2\sigma}{d\Omega_e d\omega} - \Sigma_L$
  3. $f_T(\psi') = \frac{\Sigma_T}{\sigma_M v_T G_T}$

Maieron, Donnelly & Sick, Phys.Rev.C 65, 025502 (2002)



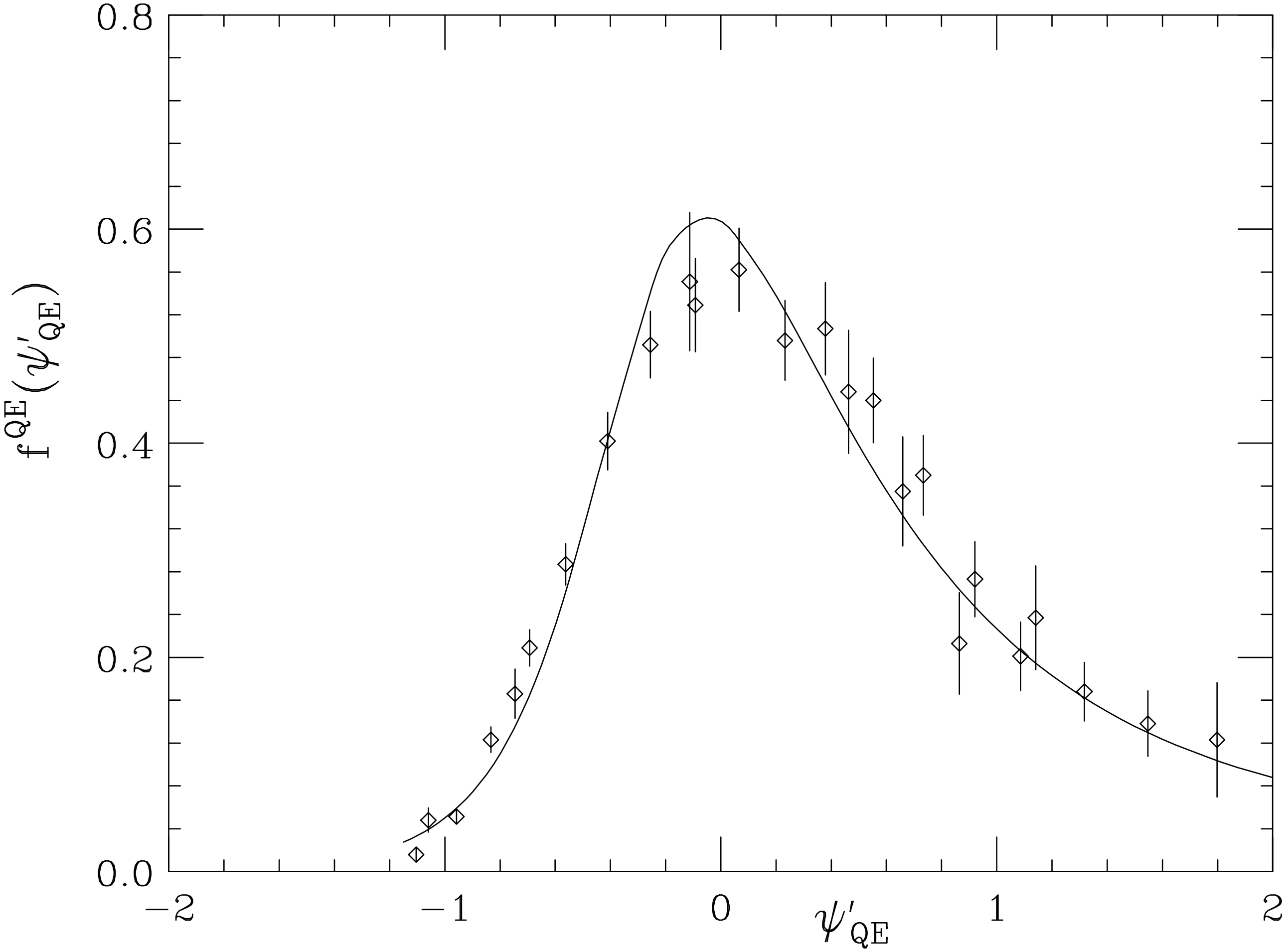
Maieron, Donnelly & Sick, Phys.Rev.C 65, 025502 (2002)



Maieron, Donnelly & Sick, Phys.Rev.C 65, 025502 (2002)

- the longitudinal part of the cross section *superscales*
- the transverse part does not : reasonably good scaling of the second kind, but something is breaking the scaling of the first kind, especially above the QE peak:
  - inelastic scattering from the nucleons
  - MEC and their associated correlations (required by gauge invariance) in both 1p-1h and 2p-2h channels
  - initial and final state interaction effects

Fit of QE Superscaling Function



Scaling in the Δ region

- The scaling analysis performed in the QEP can be extended to the Δ-resonance peak
- To isolate the contributions in the Δ region, we use the following procedure:
  1. $\left[\frac{d^2\sigma}{d\Omega_e d\omega}\right]_{QE} = \frac{1}{k_F} f^{QE}(\psi'_{QE}) \sigma_M (v_L G_L + v_T G_T)$
  2. $\left[\frac{d^2\sigma}{d\Omega_e d\omega}\right]_{\Delta} = \left[\frac{d^2\sigma}{d\Omega_e d\omega}\right]_{tot} - \left[\frac{d^2\sigma}{d\Omega_e d\omega}\right]_{QE}$
  3. Δ superscaling function $\Rightarrow$

$$f_\Delta(q,\psi'_\Delta) \equiv k_F \frac{\left[d^2\sigma/d\Omega_e d\omega\right]_\Delta}{\sigma_M\left[v_L G_L^\Delta + v_T G_T^\Delta\right]}$$

- Dividing factors

$$G_L^\Delta = \frac{\kappa}{2\tau}\left[\mathcal{N}\left\{\left(1+\tau\rho_\Delta^2\right) w_2^\Delta(\tau) - w_1^\Delta(\tau)\right\}\right] + \mathcal{O}(\eta_F^2)$$

$$G_T^\Delta = \frac{1}{\kappa}\left[\mathcal{N}\left\{w_1^\Delta(\tau)\right\}\right] + \mathcal{O}(\eta_F^2)$$

- $N \to \Delta$ single-baryon responses:

$$\begin{aligned} w_1^\Delta(\tau) &= \nu_1^\Delta \left[G_{M,\Delta}^2(\tau) + 3G_{E,\Delta}^2(\tau)\right] \\ w_2^\Delta(\tau) &= \nu_2^\Delta \left[G_{M,\Delta}^2(\tau) + 3G_{E,\Delta}^2(\tau) + \frac{4\tau}{\mu_\Delta^2} G_{C,\Delta}^2(\tau)\right] \end{aligned}$$

- Electric, magnetic and Coulomb $N \to \Delta$ form factors

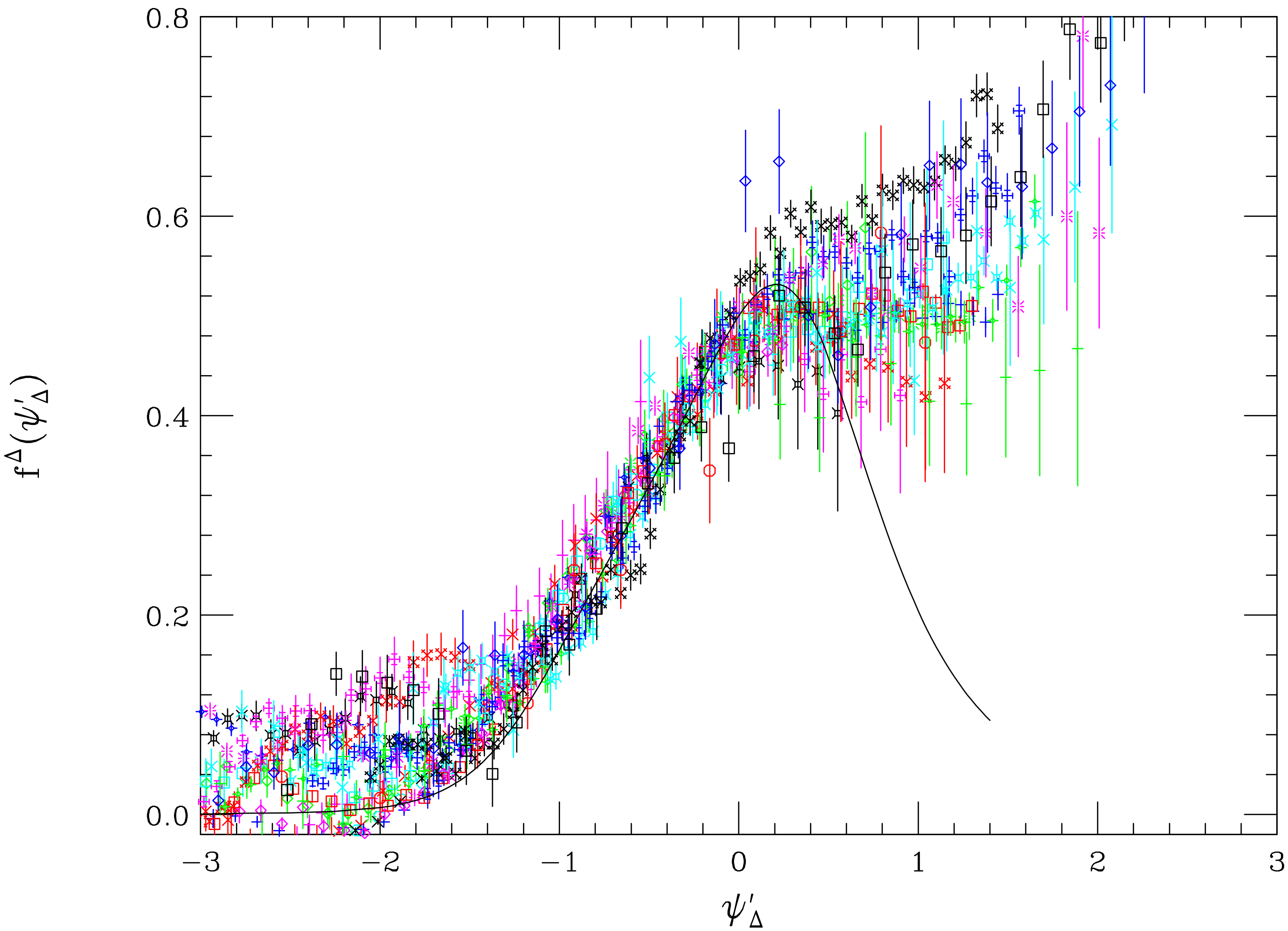
$$\begin{aligned} G_{M,\Delta}(\tau) &= 2.97 g_\Delta(Q^2) \\ G_{E,\Delta}(\tau) &= -0.03 g_\Delta(Q^2) \\ G_{C,\Delta}(\tau) &= -0.15 G_{M,\Delta}(\tau) \end{aligned}$$

$$g_\Delta(Q^2) \equiv \frac{G_{Ep}(\tau)}{\sqrt{1+\tau}} \; ; \qquad G_{Ep}(\tau) = \frac{1}{[1+4.97\tau]^2}$$

- By doing this subtraction, we remove the impulsive L and T contributions that arise from elastic eN scattering, leaving (at least)
  - MEC effects with their associated correlations ($\sim$ 10-20%)
  - impulsive contributions arising from inelastic eN scattering
- This approach can work only at $\psi'_\Delta < 0$, since at $\psi'_\Delta > 0$ other resonances and the tail of DIS start contributing
- Plot f_Δ vs ψ'_Δ $\Rightarrow$

Amaro et al., Phys.Rev.C71, 015501 (2005) [^{12}C and ^{16}O data, energies spanned from 0.30 to 4 GeV, angles from 12 to 145 deg.]

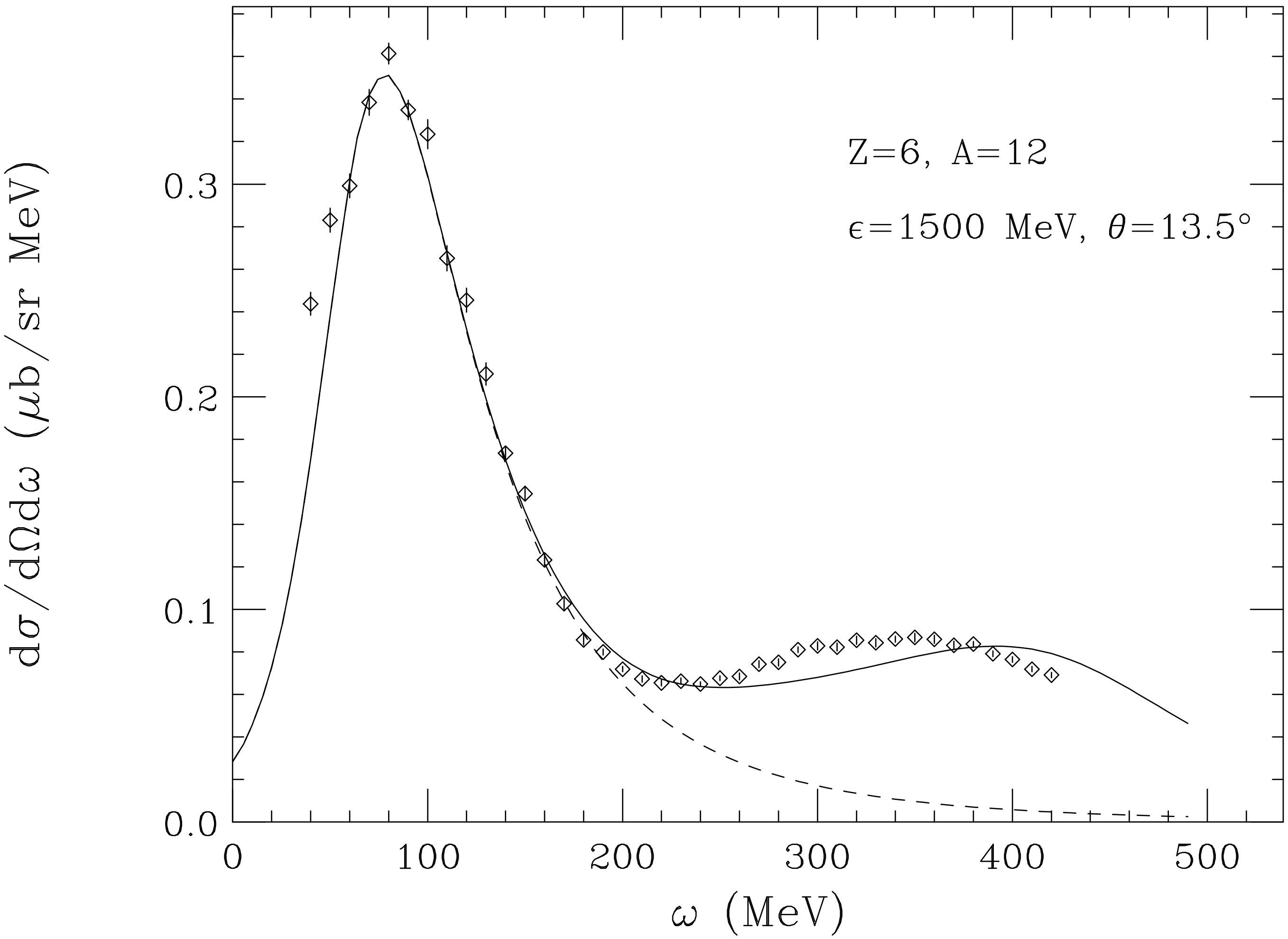
Δ Superscaling Function



Amaro et al., Phys.Rev.C71, 015501 (2005) [^{12}C and ^{16}O data, energies spanned from 0.30 to 4 GeV, angles from 12 to 145 deg.]

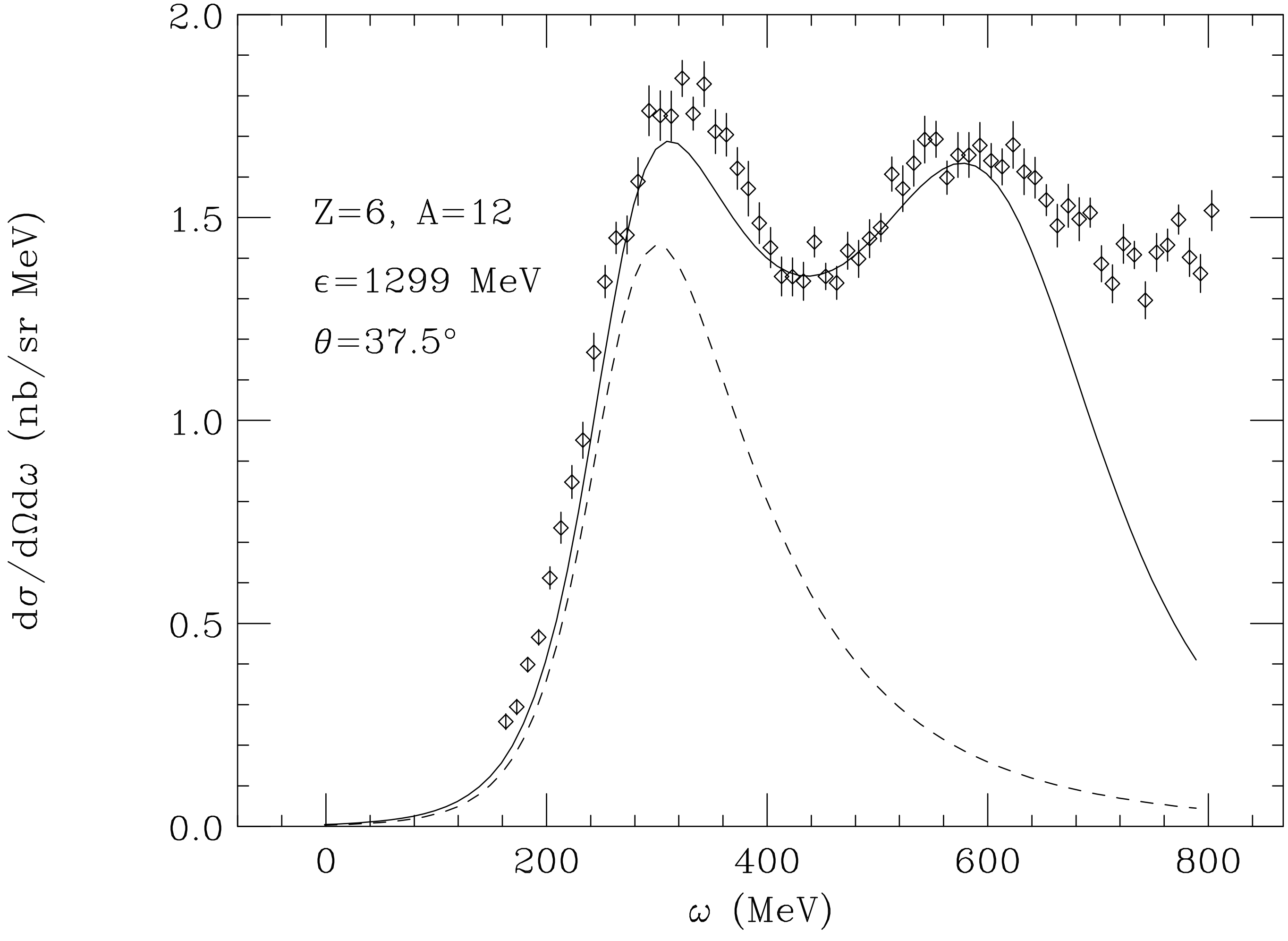
Amaro et al., Phys.Rev.C71, 015501 (2005)

Test of Superscaling Functions



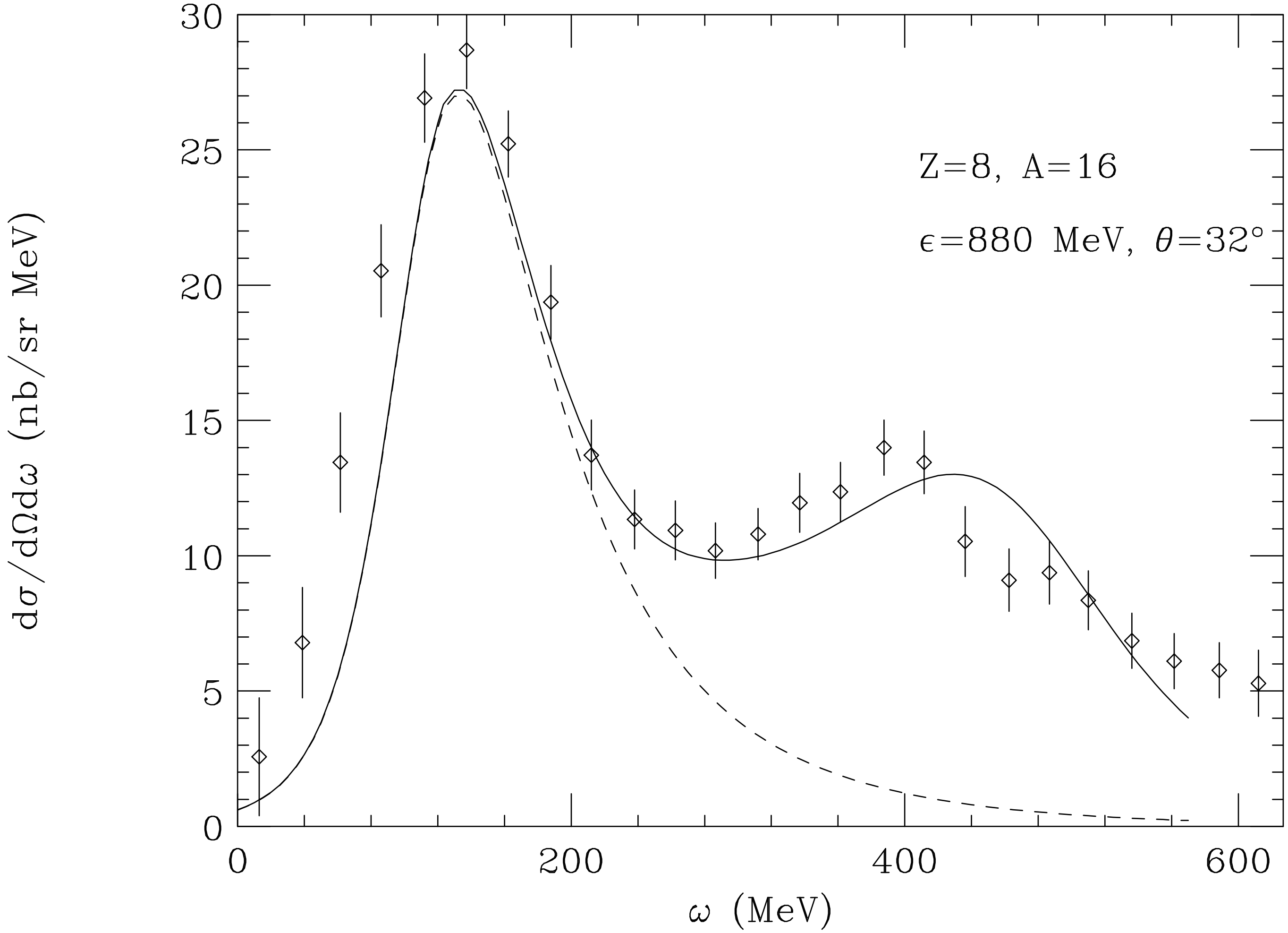
Amaro et al., Phys.Rev.C71, 015501 (2005)

Test of Superscaling Functions



Amaro et al., Phys.Rev.C71, 015501 (2005)

Test of Superscaling Functions

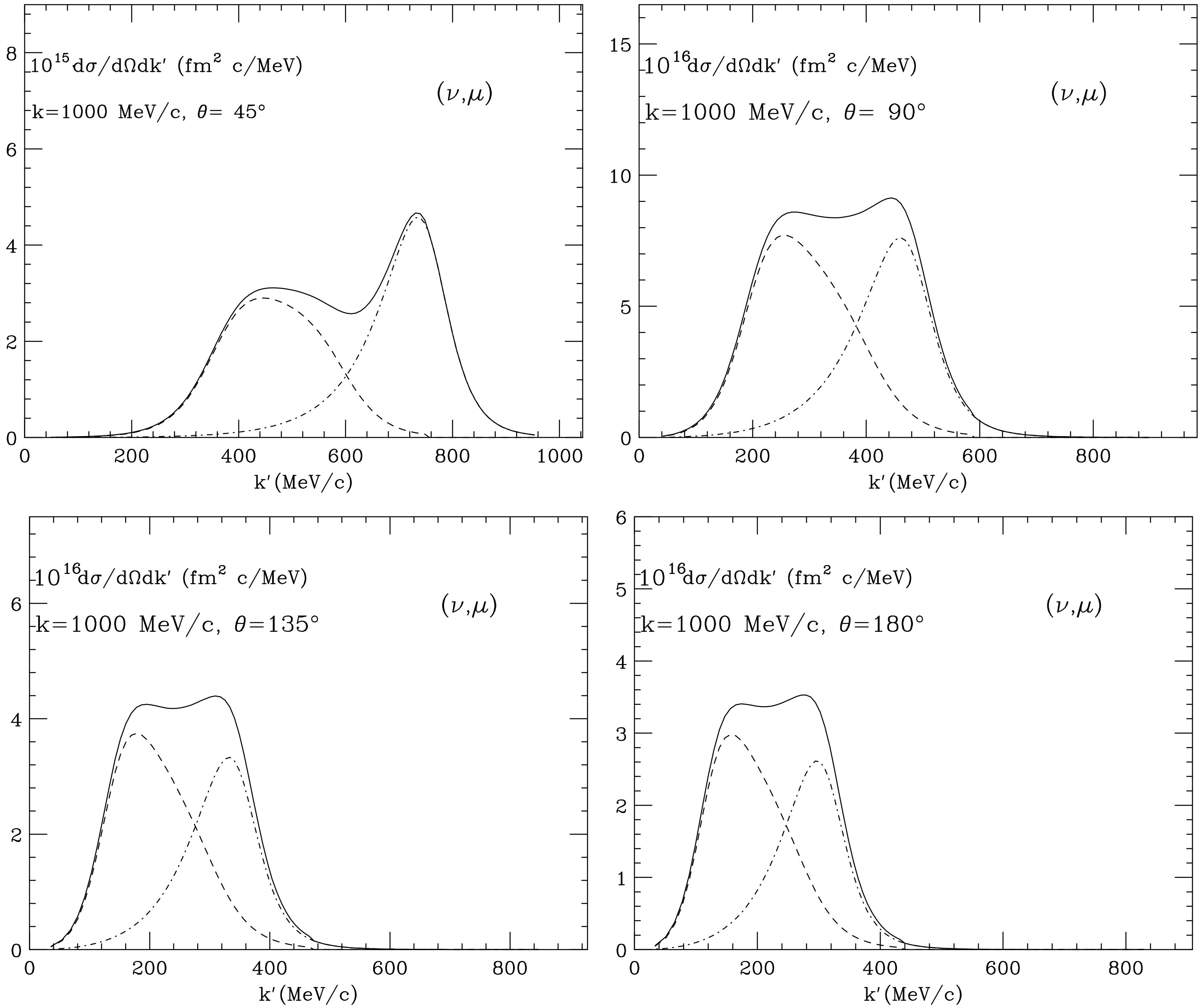


Predictions for (ν, μ)

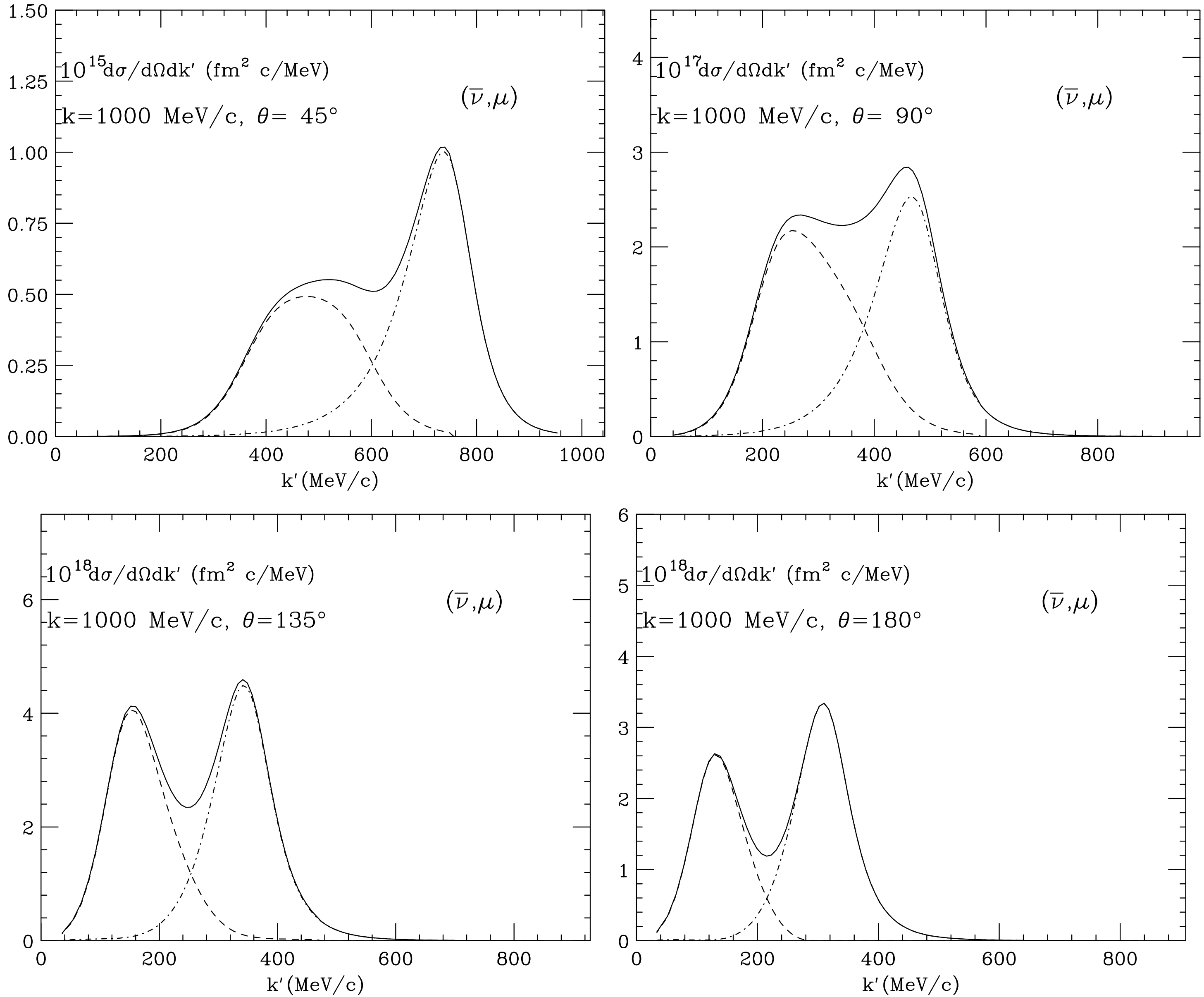
$$f_{RFG}(\psi_{QE}) \quad \Rightarrow \quad f_{QE}(\psi_{QE})$$

$$f_{RFG}(\psi_{\Delta}) \quad \Rightarrow \quad f_{\Delta}(\psi_{\Delta})$$

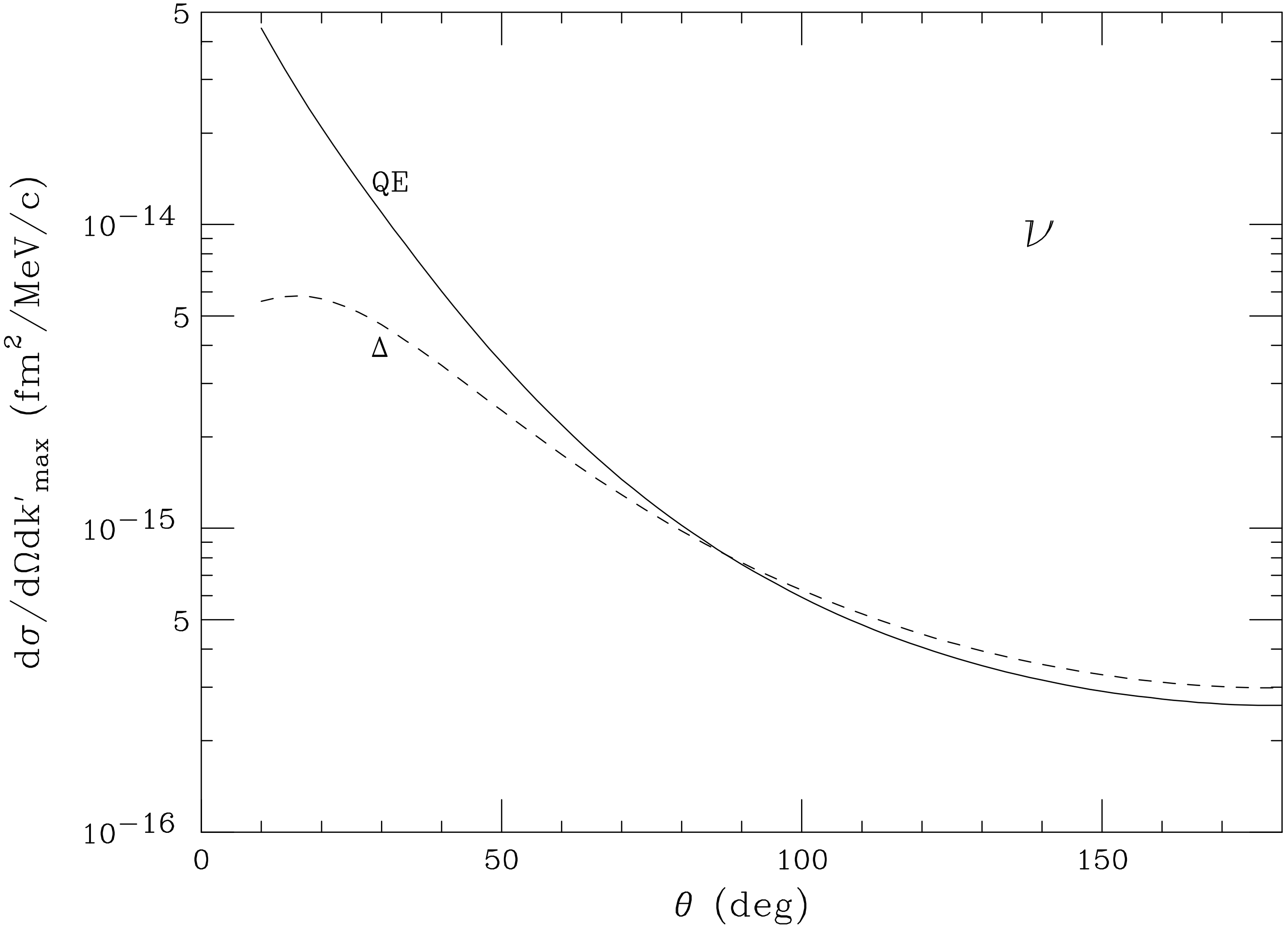
(ν, μ) cross sections; ^{12}C



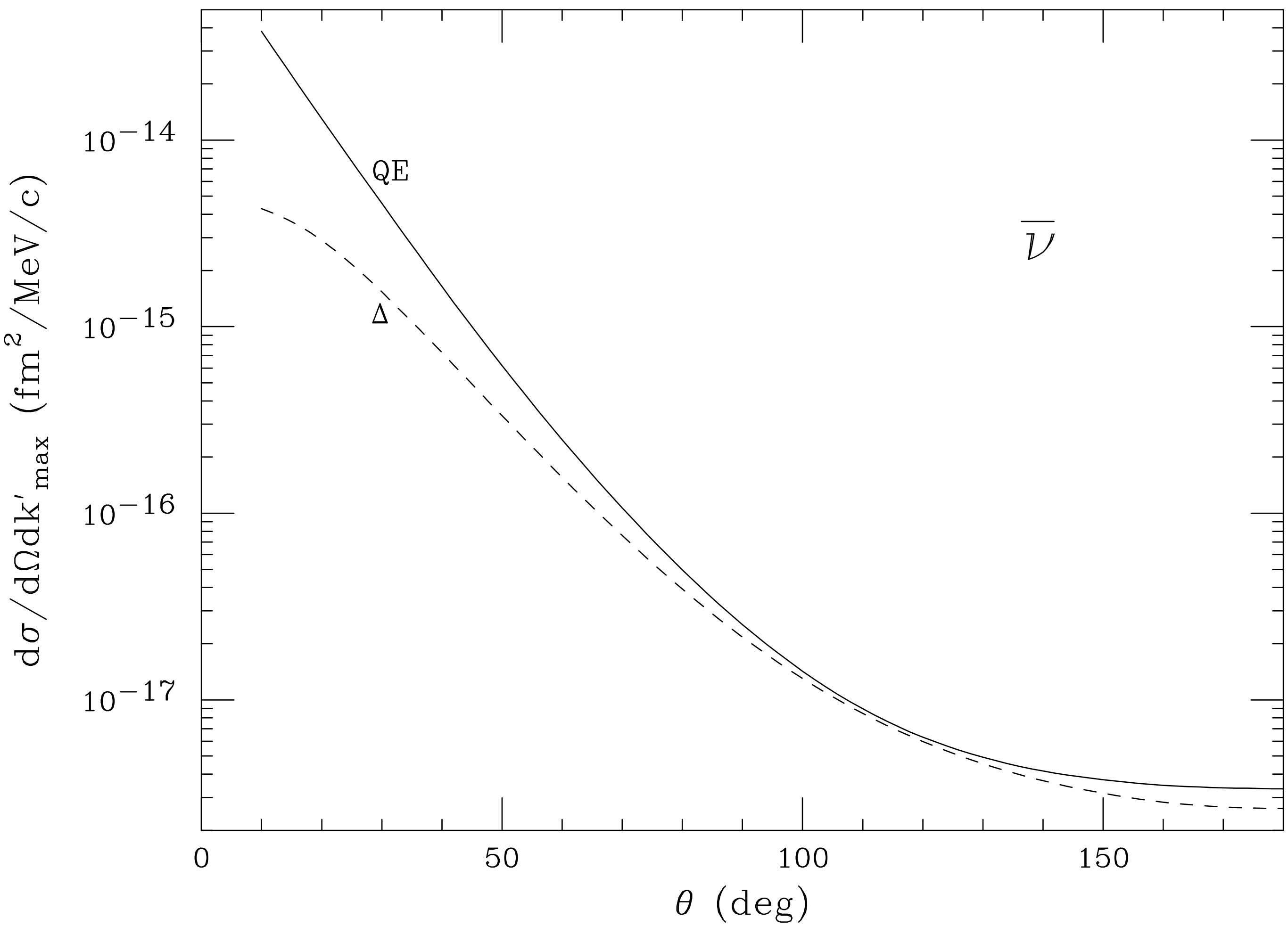
$(\bar{\nu}, \mu)$ cross sections; ^{12}C



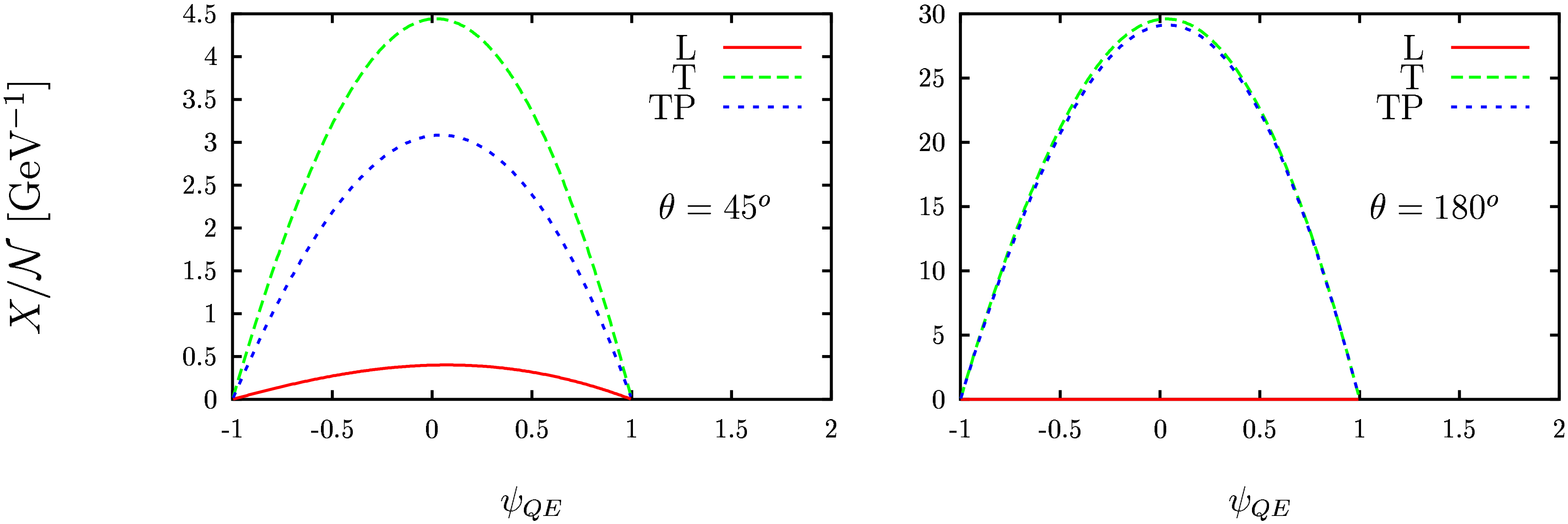
Angular dependence of ν cross section at the peak



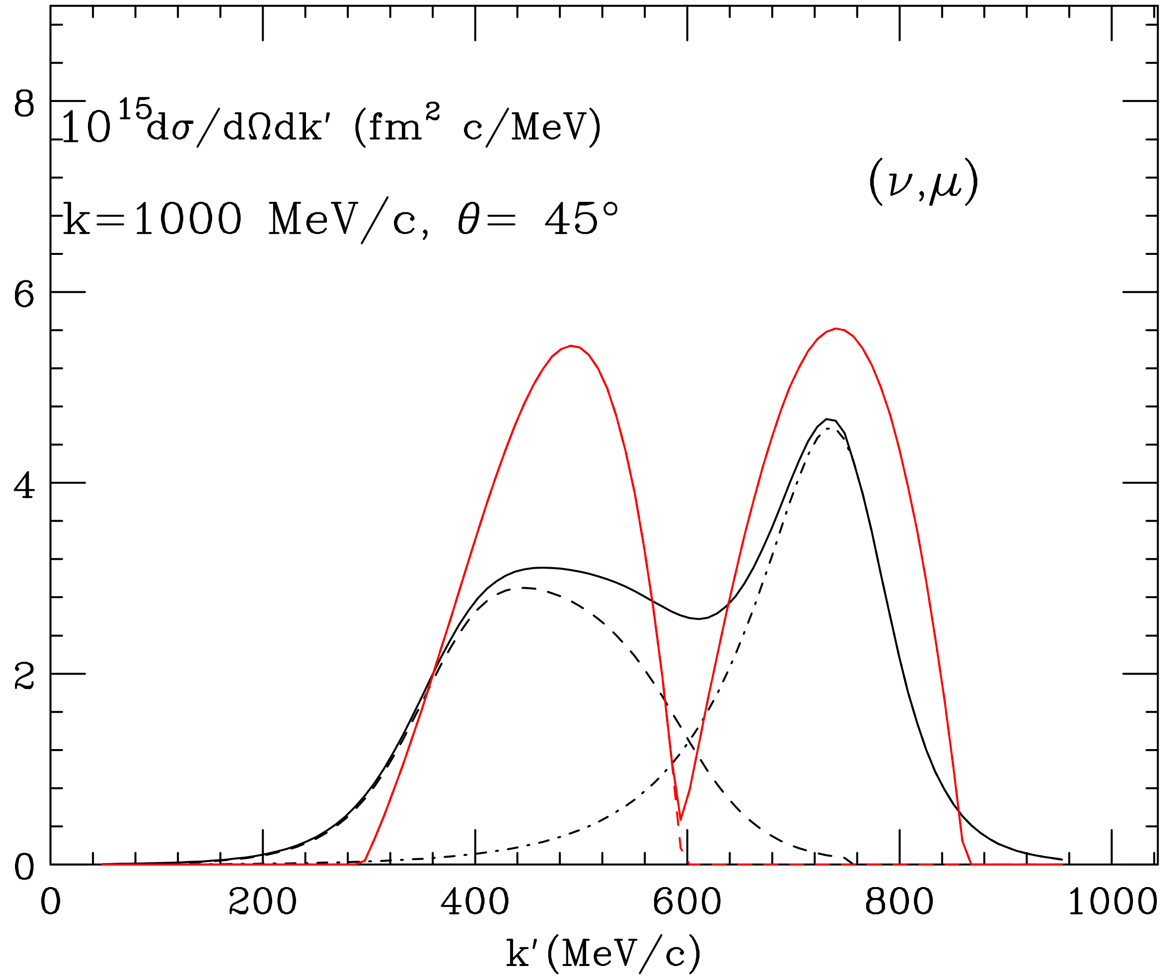
Angular dependence of $\bar{\nu}$ cross section at the peak



QE responses $E_\nu = 1$ GeV, RFG



Comparison of superscaling with RFG



Results

- Dramatic difference between ν and $\bar{\nu}$ at backward angles, since $X_T \simeq X'_T$ (constructive interference in ν scattering, cancellation in $\bar{\nu}$): small changes in the model could have large effects on the predictions for $\bar{\nu}$
- In the ν case the T and T' responses dominate (CC, CL, $LL < 5\%$) in both $QE$and Δ regions
- The QE and Δ contributions are comparable for $\theta \sim 90^o$
- RFG (and mean field) predictions differ significantly from the scaling predictions

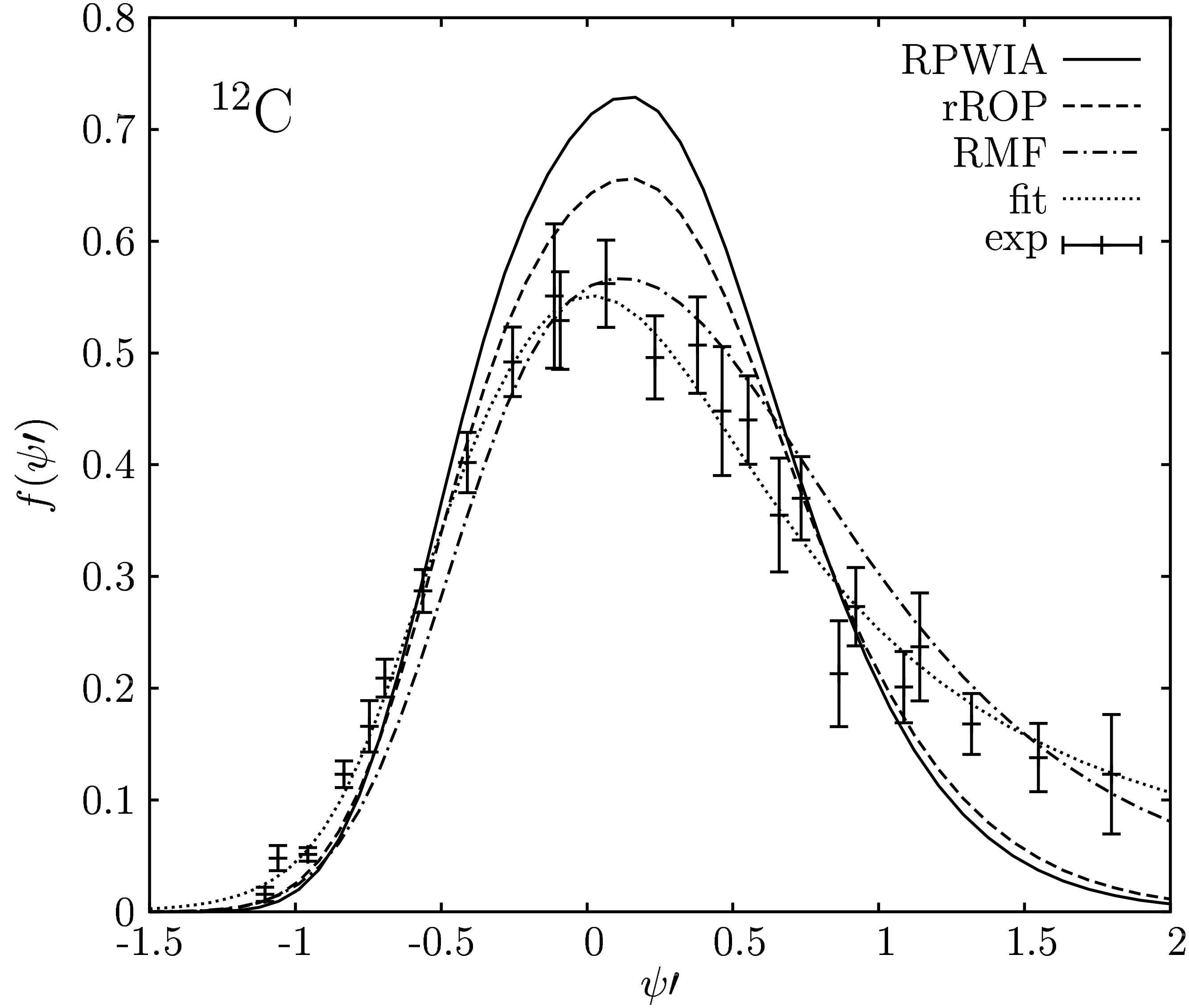
Conclusions

- (e, e')
  - Scaling is observed in inclusive electron scattering for high energies (several hundred MeV to a few GeV) from the scaling region up to the Δ peak.
  - For the longitudinal contribution it is possible to find a scaling function f^{QE} which, when plotted versus ψ'_{QE}, is seen to superscale. This universal scaling function embodies the basic nuclear dynamics in the problem.
  - The scaling function f^{QE} is used to obtain the impulsive part of the transverse response

- The residual is analyzed in terms of a scaling function f^{Δ} by dividing by the elementary $N \to \Delta$ cross section and plotted versus a scaling variable ψ'_{Δ}
- Very good representation of inclusive electron scattering at relatively high energies up to the Δ peak.

- (ν, l)
  - Invert the scaling procedure: use the empirical scaling functions and the $N \to N$ and $N \to \Delta$ weak amplitudes to predict charge-changing νA and $\overline{\nu} A$ cross sections for the same range of kinematics.

- Future work (in progress):
  1. extend the scaling approach to neutral current neutrino and antineutrino scattering
  2. microscopic model: role of MEC and associated correlations in ph and 2p2h channels
  3. explaining the specific nature of the scaling functions (in particular the asymmetric shape)



^{12}C
RPWIA
rROP
RMF
fit
exp
$f(\psi')$
ψ'
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-1.5
-1
-0.5
0
0.5
1
1.5
2

FINE