Solving the degeneracy by a neutrino factory with polarized muon beam

Preliminary results ...

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in collaboration with Osamu Yasuda*

Osaka University,

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  - Current status of oscillation parameters
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    — schematic explanation following O. Yasuda hep-ph/0405005
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  - How to resolve the θ_{13}-δ degeneracy only with the neutrino factory?
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Introduction

Degeneracy in the oscillation parameters

Current status of oscillation parameters

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- Current status of oscillation parameters (99% CL):

$$7.2 \times 10^{-5} < \Delta m^2_{21} < 8.9 \times 10^{-5} [\mathrm{eV}^2],$$

$$1.7 \times 10^{-3} < |\Delta m^2_{31}| < 3.3 \times 10^{-3} [\mathrm{eV}^2],$$

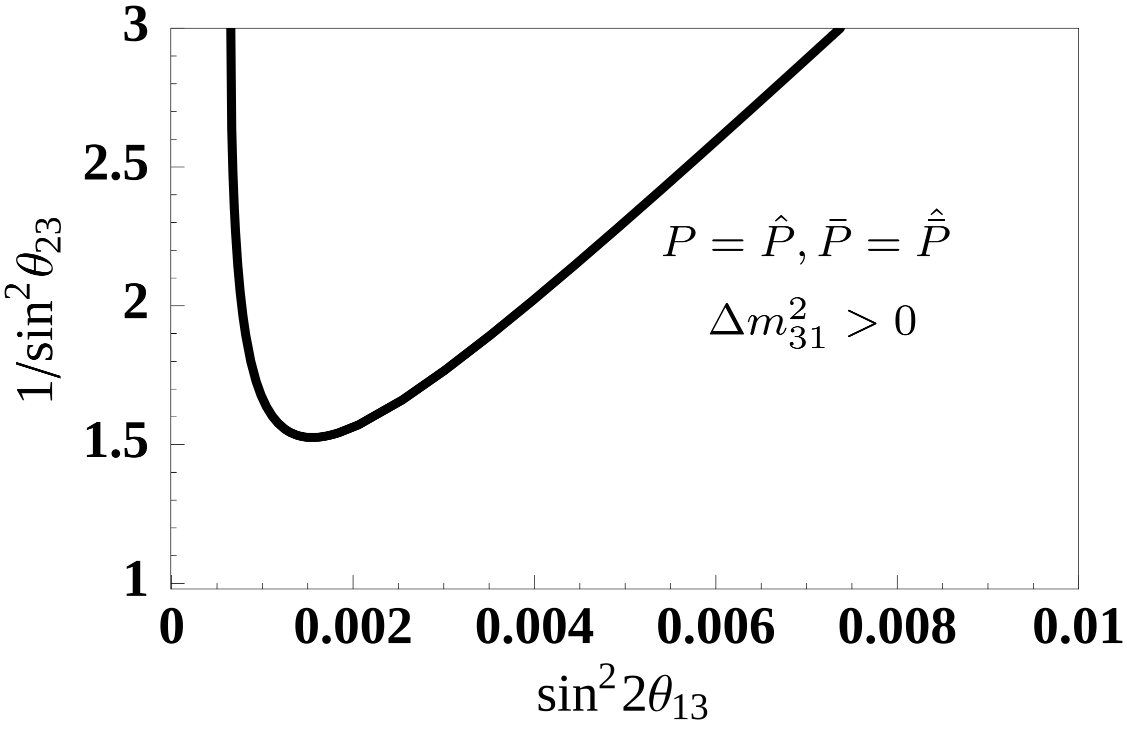
$$30^\mathrm{o} < \theta_{12} < 38^\mathrm{o}, \quad 36^\mathrm{o} < \theta_{23} < 54^\mathrm{o}, \quad \theta_{13} < 10^\mathrm{o},$$

$$\delta = ?.$$

- We want to know ... θ_{13} and δ.
- ... and the sign of Δm^2_{31} and $\theta_{23} < 45^\mathrm{o}$? or $> 45^\mathrm{o}$?.
- When we try to determine a parameter, it is important to know about the other parameters precisely because they are correlated with each other.

Eight-fold degeneracy

- Even if we can measure the oscillation probabilities with high precision, there is the remaining uncertainty for determination of the oscillation parameters — Degeneracy.
- In other words, $P_{\nu_e \to \nu_\mu} = \hat{P}$, $P_{\bar{\nu}_e \to \bar{\nu}_\mu} = \hat{\bar{P}}$ can be reproduced with more than one parameter set.



$$\hat{P} = 4.8 \times 10^{-5},$$
$$\hat{\bar{P}} = 6.8 \times 10^{-5}$$

with
$$\sin^2 2\hat{\theta}_{13} = 0.001, \quad \sin^2 \hat{\theta}_{23} = 0.6, \ \hat{\delta} = -\pi/2$$

Fixed parameters:

$$\Delta m^2_{21} = 8.1 \times 10^{-5} [\mathrm{eV}^2]$$
$$\Delta m^2_{31} = 2.2 \times 10^{-5} [\mathrm{eV}^2]$$
$$\sin^2 2\theta_{12} = 0.84,$$
$$L = 3000[\mathrm{km}], \quad E_\nu = 30[\mathrm{GeV}]$$
$$\rho = 3.32[\mathrm{g/cm}^3]$$

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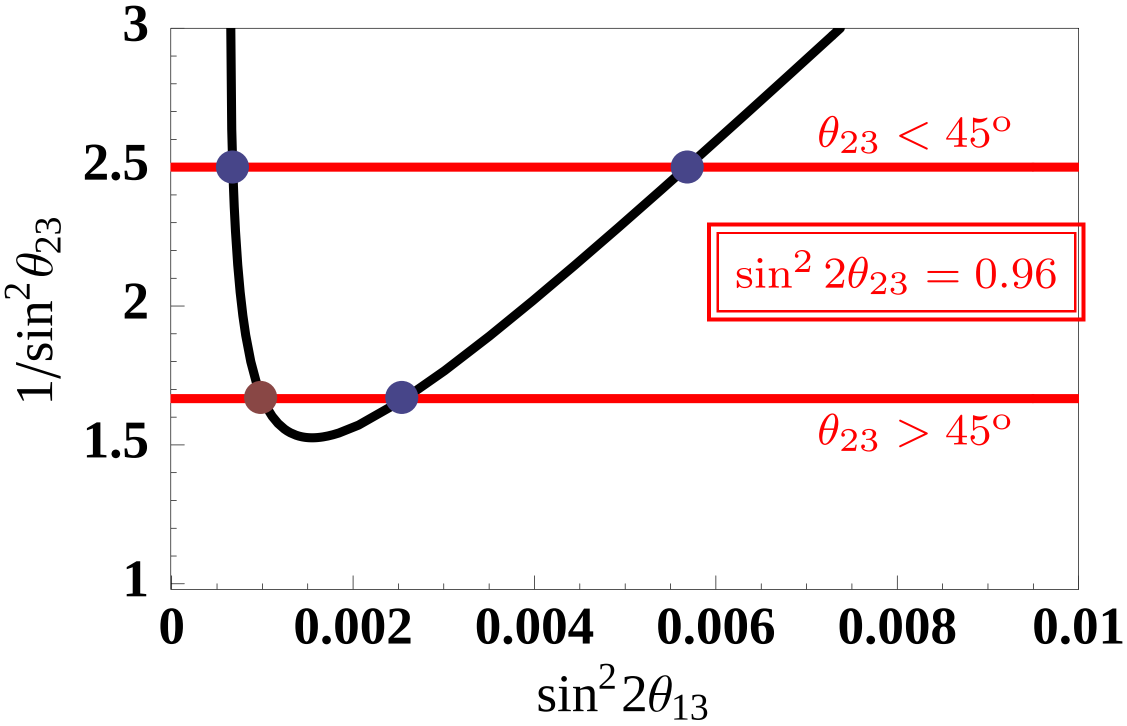

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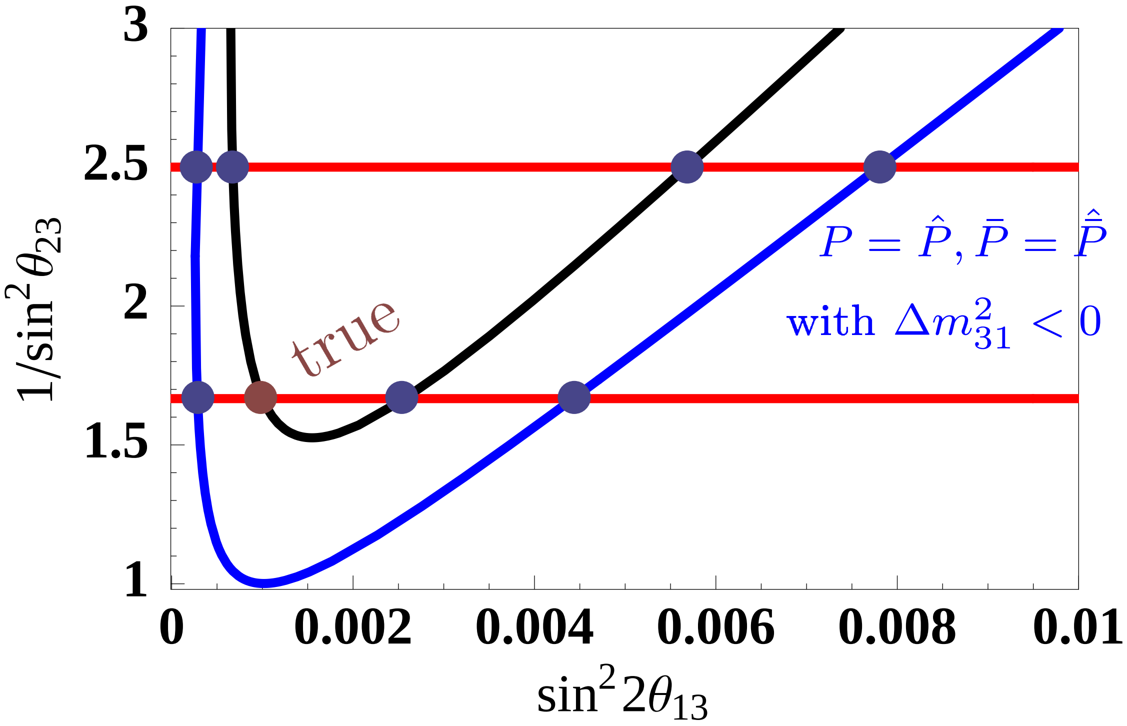

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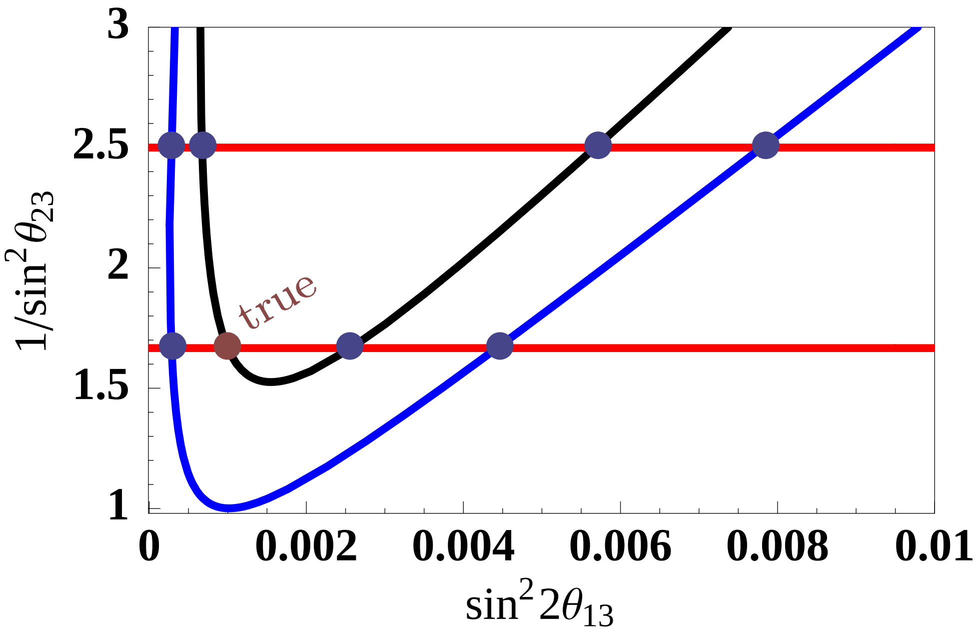
Forthcoming reactor experiment

- Forthcoming reactor experiments can solve this degeneracy.
- They only sensitive to θ_{13}.
- The next generation reactor experiments reach $\sin^2 2\theta_{13} \gtrsim \mathcal{O}(0.01)$.
- If $\sin^2 2\theta_{13} \gtrsim \mathcal{O}(0.01)$, the degeneracy can be resolved.



- If $\sin^2 2\theta_{13} \lesssim \mathcal{O}(10^{-3})$ ... how to resolve the degeneracy?

Next generation experiment

- How to resolve this degeneracy?
  — Synergy, i.e., Golden channel $+\alpha$

Next generation experiment

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      — β-beam etc...
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    - Silver channel $\nu_e \to \nu_\tau$
      — the same baseline but with the different detector
    - $\nu_\mu \to \nu_e$ with polarized muon beam
      — the same baseline, the same detector, but the different accelerator setting

In this talk

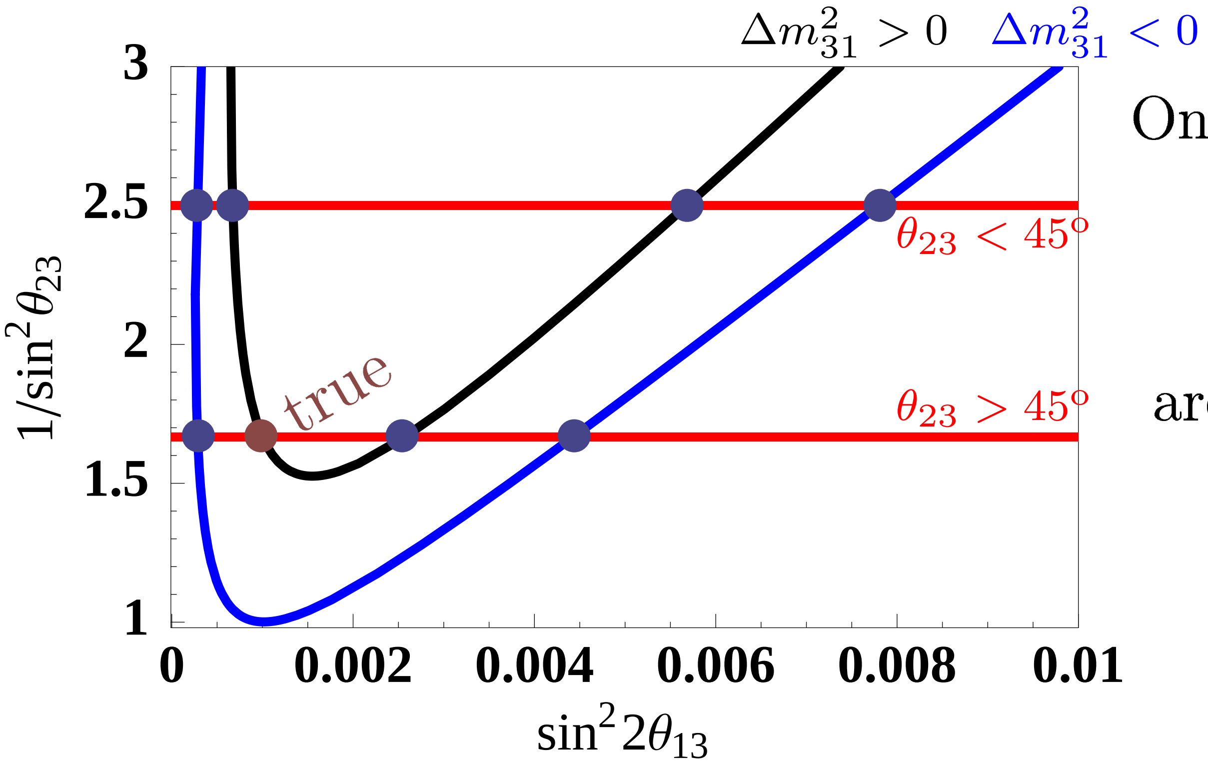
Idea to resolve the degeneracy

using $\nu_\mu \to \nu_e$ channel

- Why $\nu_\mu \to \nu_e$?
- Why polarized beam?

Idea: $P_{\nu_\mu \to \nu_e}$ solve the degeneracy

- $P_{\nu_e \to \nu_\mu} - P_{\bar{\nu}_\mu \to \bar{\nu}_e}$ provides the straight line in the $(\sin^2 2\theta_{13}, 1/\sin^2\theta_{23})$ plane.



On the black and blue curves,

$$P_{\nu_e \to \nu_\mu} = \hat{P}$$
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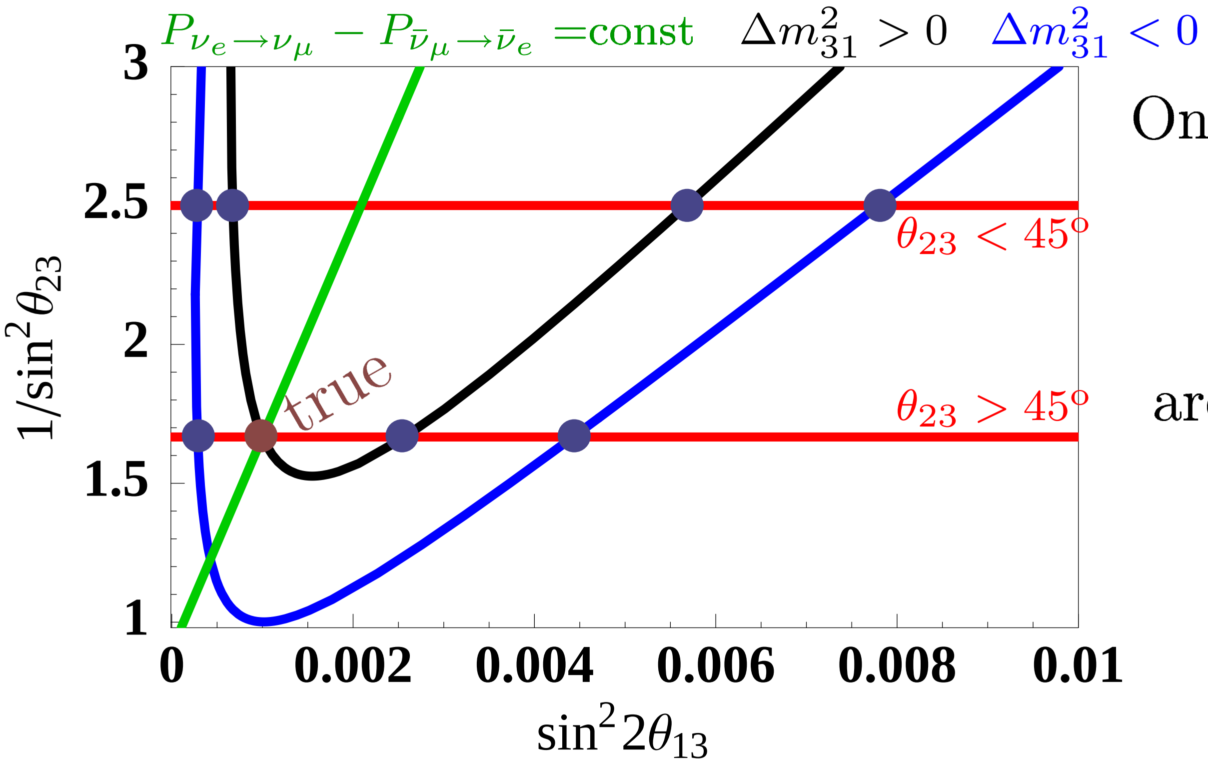

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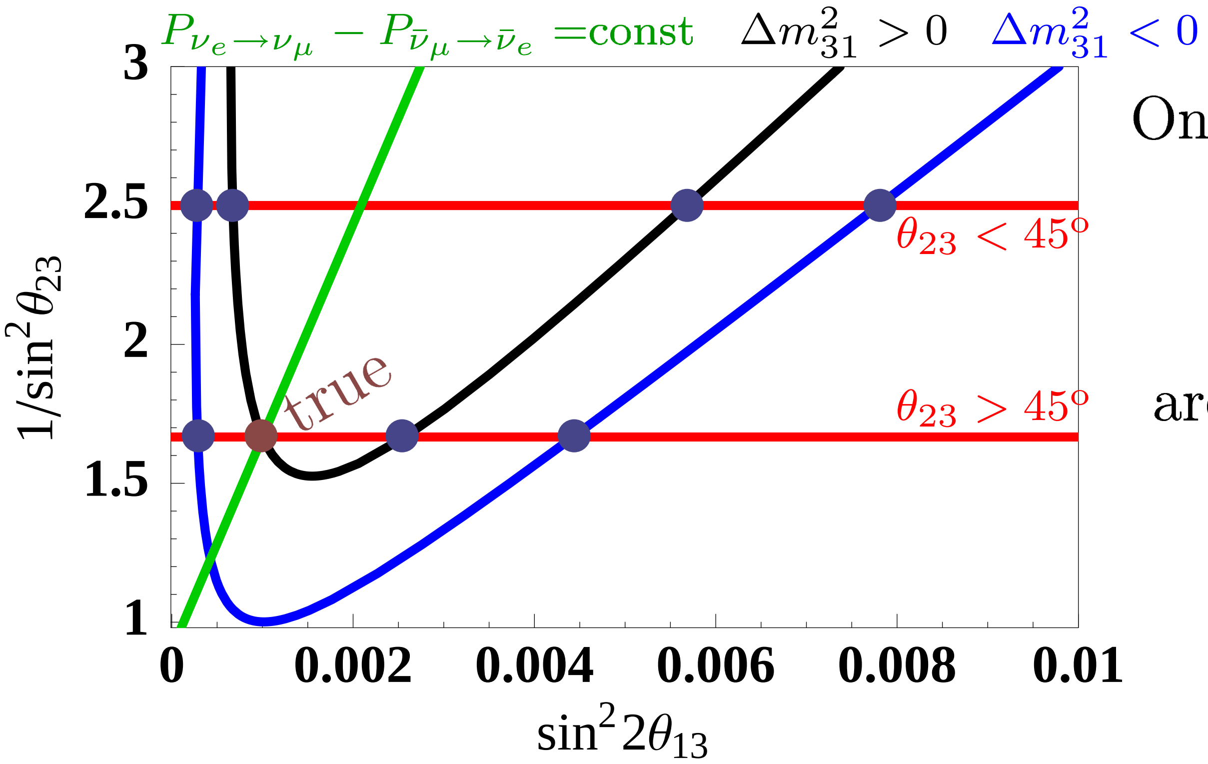

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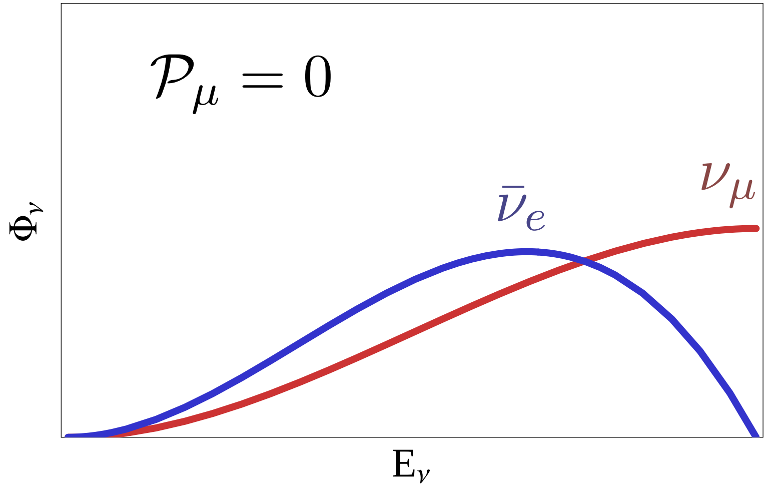
are satisfied.

- Note that the silver channel $P(\nu_e \to \nu_\tau)$ =const provides the curve, and it can also determine the true solution.

Idea: Why polarized muon beam

- Neutrino flux from μ^- with polarisation $\mathcal{P}_\mu$



$\mu^- \rightarrow \nu_\mu \rightarrow \nu_e \rightarrow e^-$ (oscillation event)

$\bar{\nu}_e \rightarrow \bar{\nu}_e \rightarrow e^+$ (background)

charge ID necessary

- The beam with $\mathcal{P}_\mu = -1$ only contains ν_μ.
  - At the detector, the charge identification is not necessary to observe $\nu_\mu \rightarrow \nu_e$ events.
    — Just count the *e-like* events.
  - The background ratio should be quite low.
    — Only mis-ID of $\nu_\mu \rightarrow \nu_\mu$.

Idea: Why polarized muon beam

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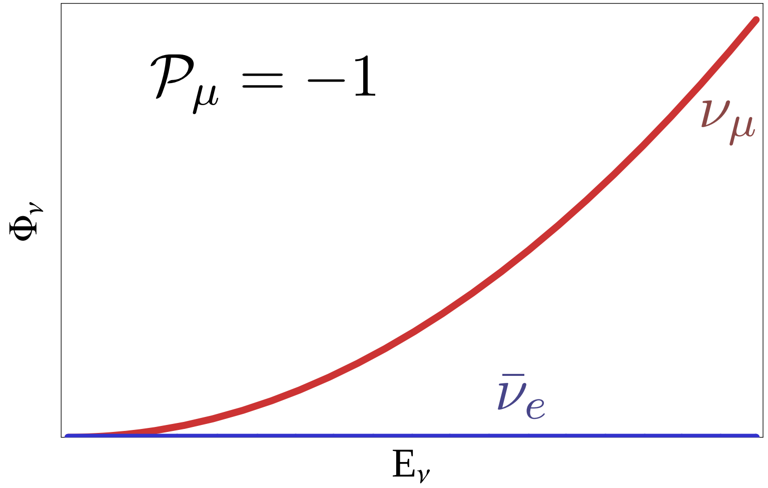

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Parameter fit with $\Delta\chi^2$

- $\Delta\chi^2$ for the golden channel ($\nu_e \rightarrow \nu_\mu$)
- $\Delta\chi^2$ for the $\nu_\mu \rightarrow \nu_e$ channel

Parameter fit with $\Delta\chi^2$ on the θ_{13}-δ plane

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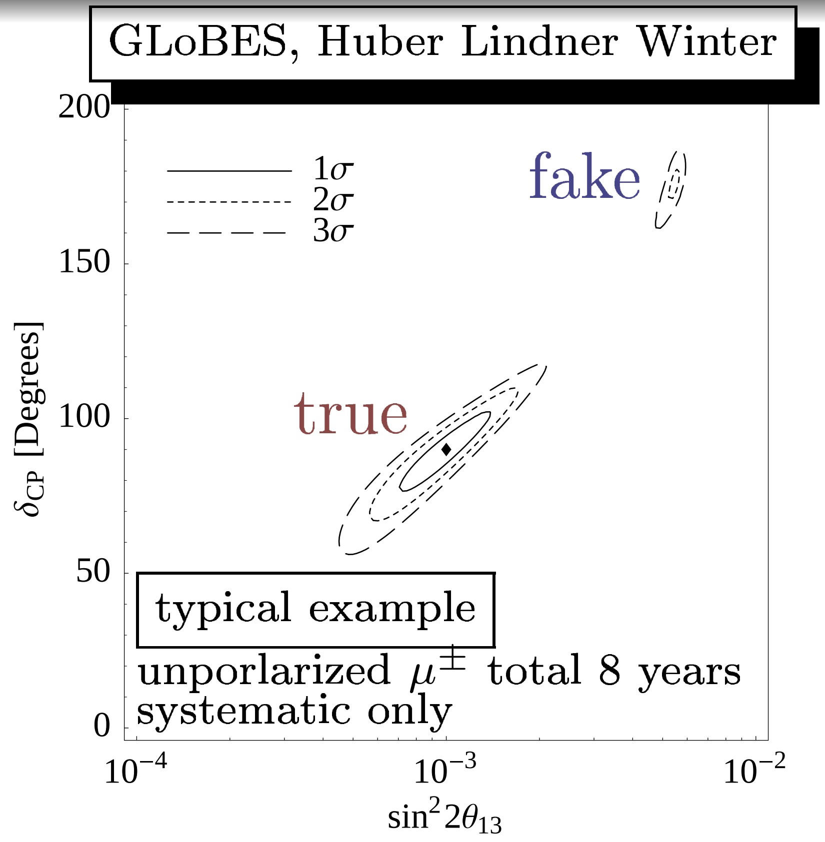

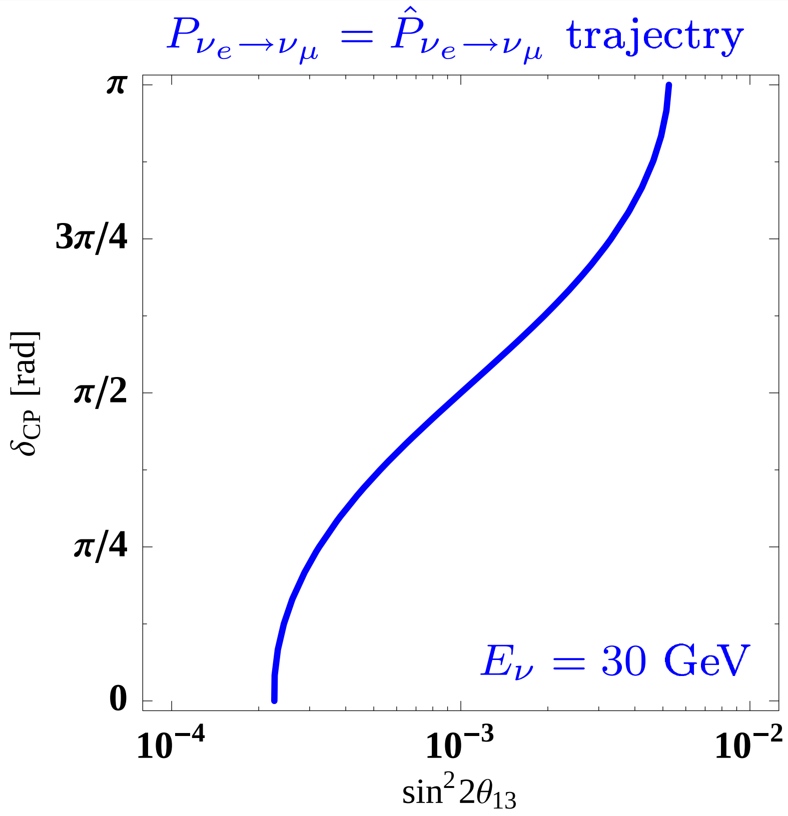

$$\Delta\chi^2 \equiv \sum_j^{\text{channel}} \Delta\chi_j^2 \sim \Delta\chi^2_{\nu_e\to\nu_\mu},$$

$$\Delta\chi^2_{\nu_\alpha\to\nu_\beta} \equiv \sum_i^{E_{\text{bin}}} \frac{|N_i(\hat{\theta}_{13},\hat{\delta}) - N_i(\theta_{13},\delta)|^2}{\sigma_i^2}$$

$$\propto \sum_i^{E_{\text{bin}}} |\hat{P}_{\nu_\alpha\to\nu_\beta} - P_{\nu_\alpha\to\nu_\beta}|^2.$$

- N_i: event number of i-th bin
- σ_i: standard deviation
- $\sin^2 2\hat{\theta}_{13} = 0.001$, $\hat{\delta} = \pi/2$ are taken as the true value.

Parameter fit with $\Delta\chi^2$ on the θ_{13}-δ plane

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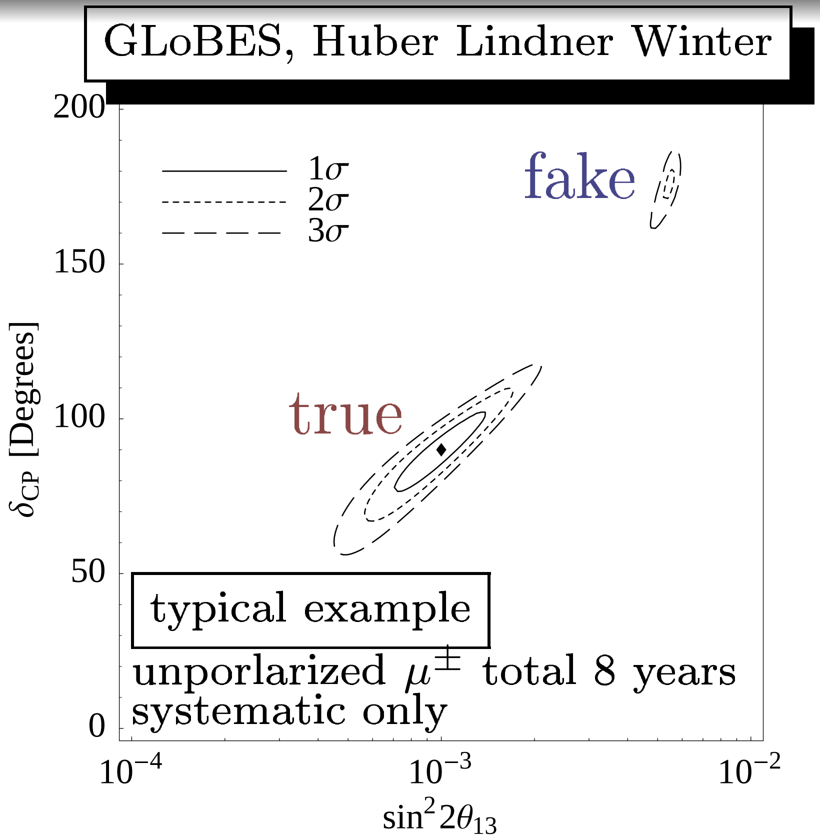

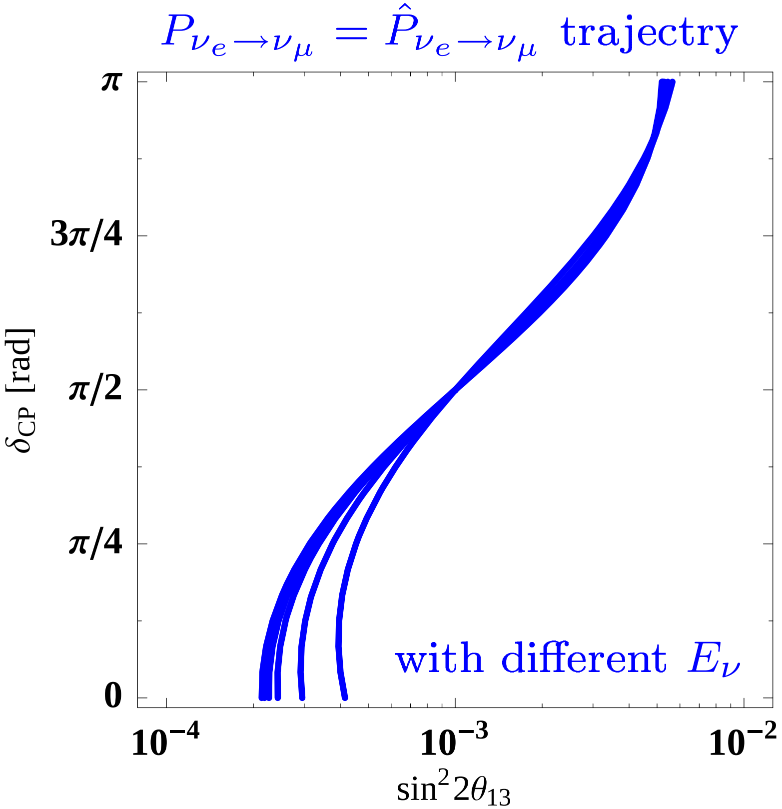

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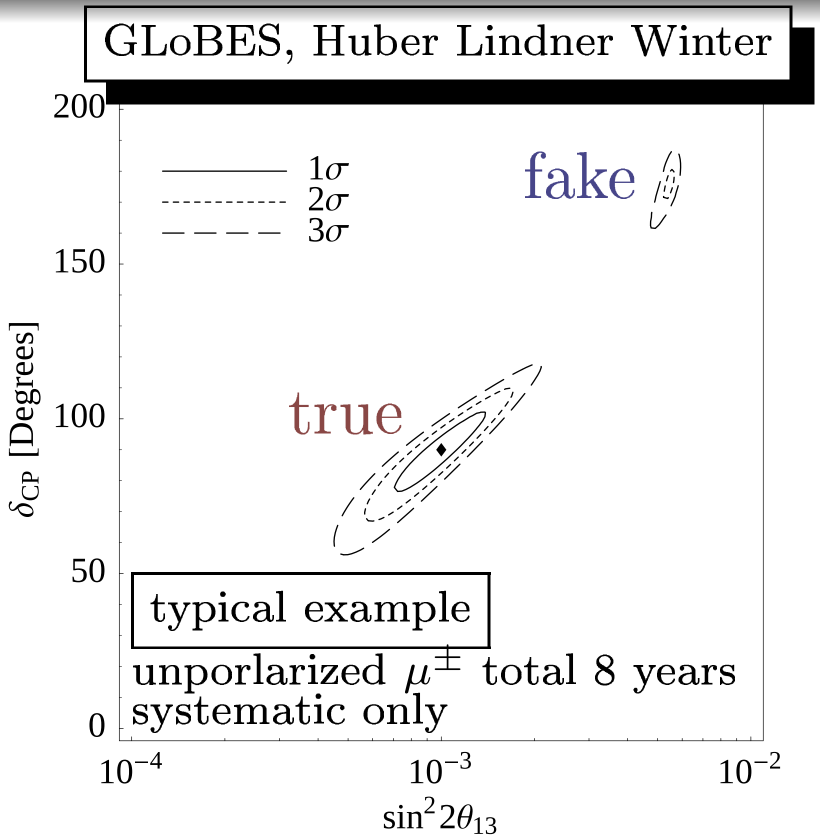

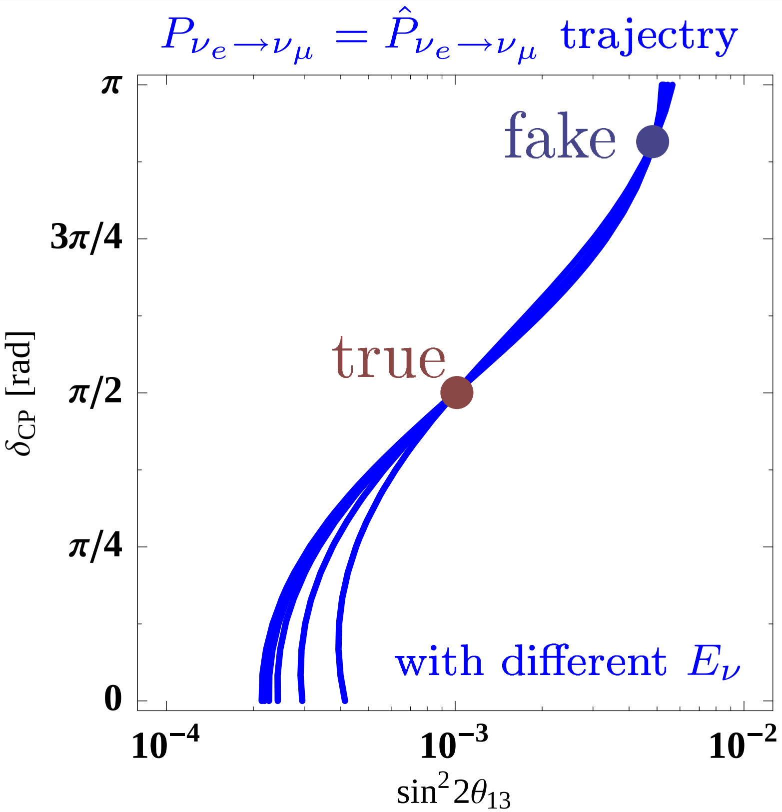

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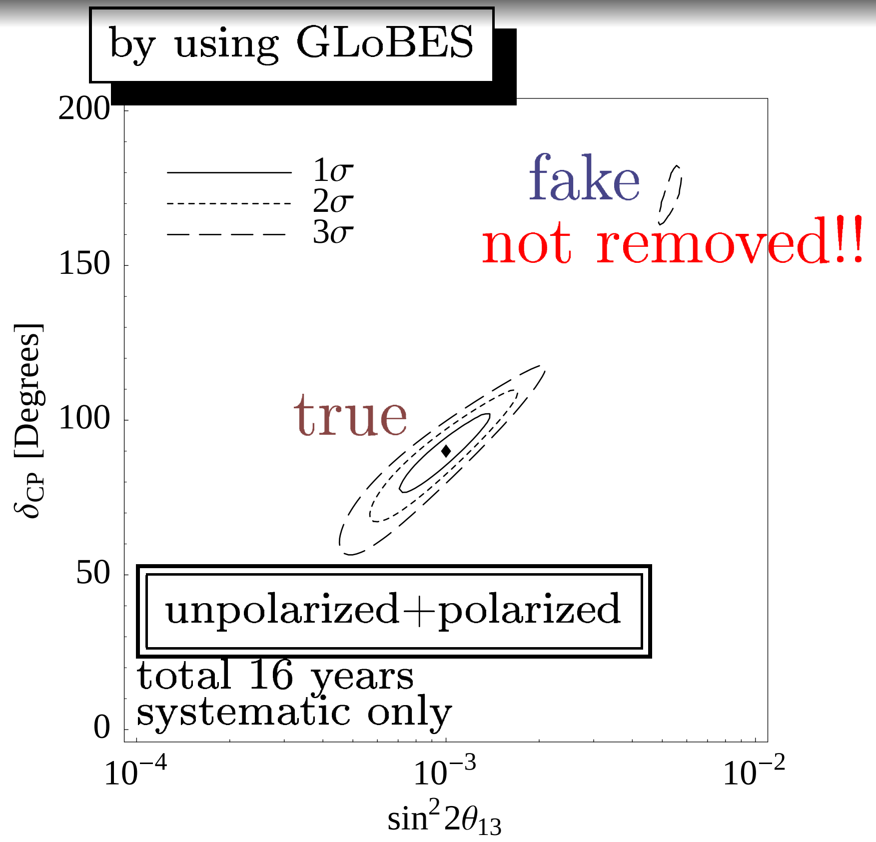

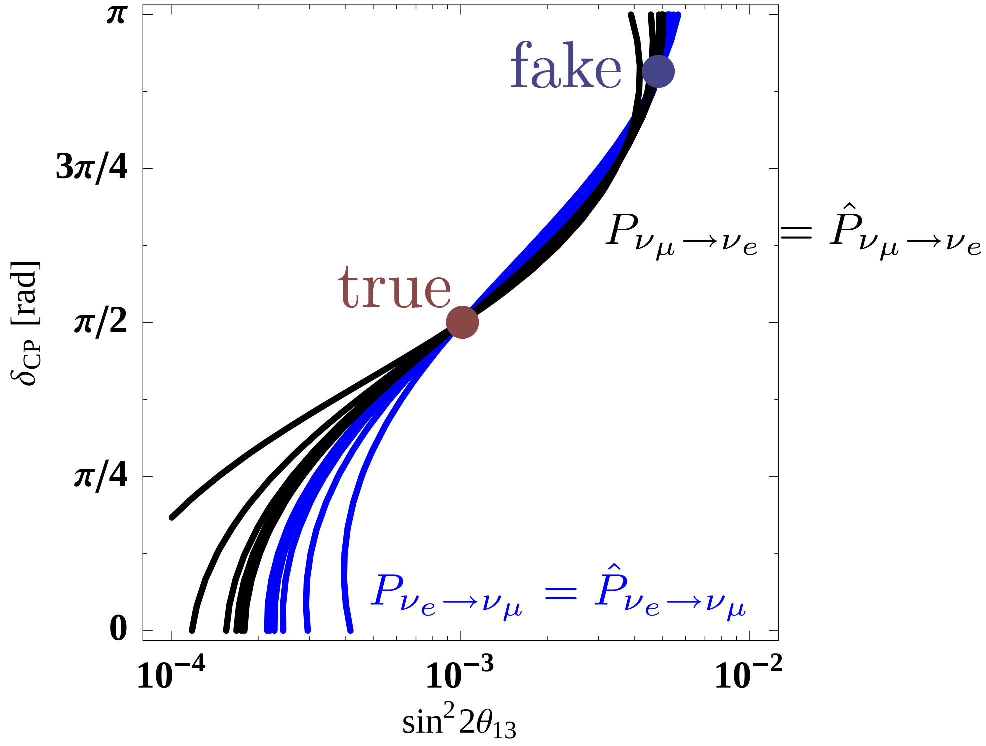

$$\Delta\chi^2 = \Delta\chi^2_{\text{un-pol}} + \Delta\chi^2_{\text{pol}} \sim \sum_i^{E_{\text{bin}}} |\hat{P}_{\nu_e\to\nu_\mu} - P_{\nu_e\to\nu_\mu}|^2 + \sum_i^{E_{\text{bin}}} |\hat{P}_{\nu_\mu\to\nu_e} - P_{\nu_\mu\to\nu_e}|^2$$

- $\nu_\mu \to \nu_e$ channel is not efficient because ...
  - the trajectories are similar to those of $\nu_e \to \nu_\mu$.
  - this is not efficient in the statistical sense.
    - ν_e detection efficiency etc...

Parameter fit with $\Delta\chi^2$ on the θ_{13}-δ plane

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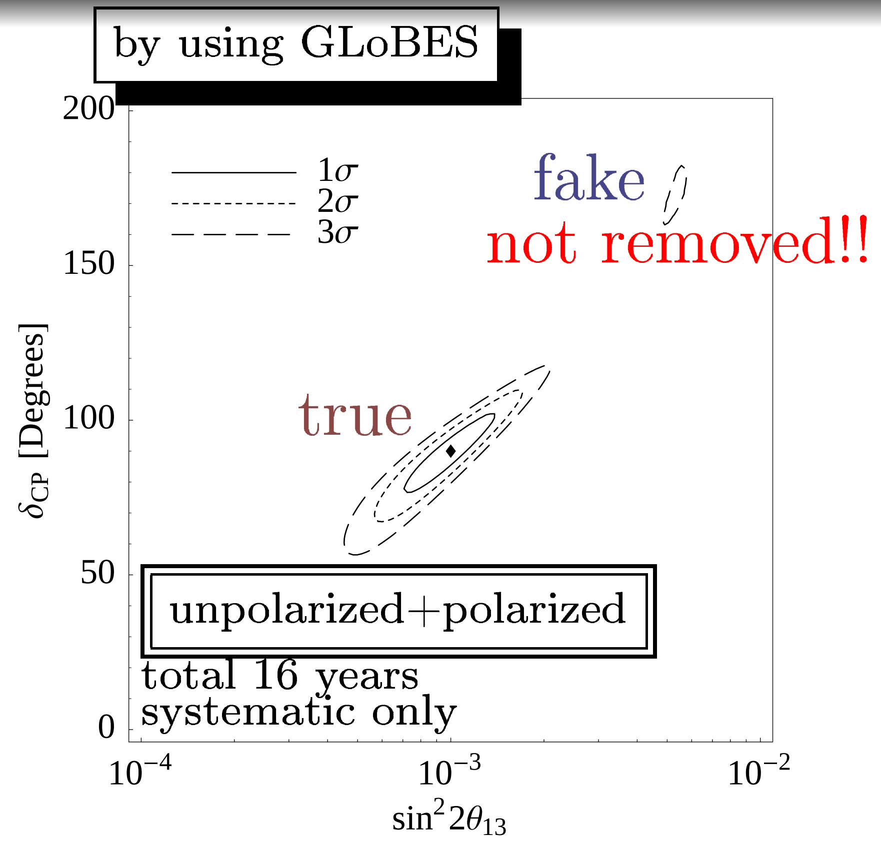

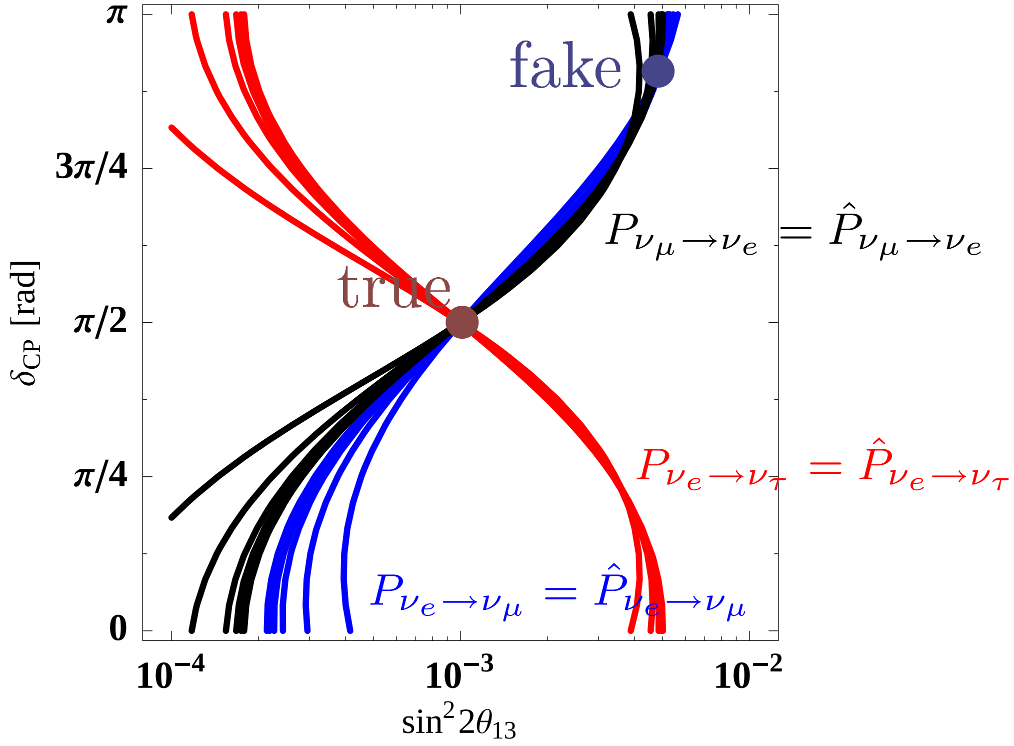

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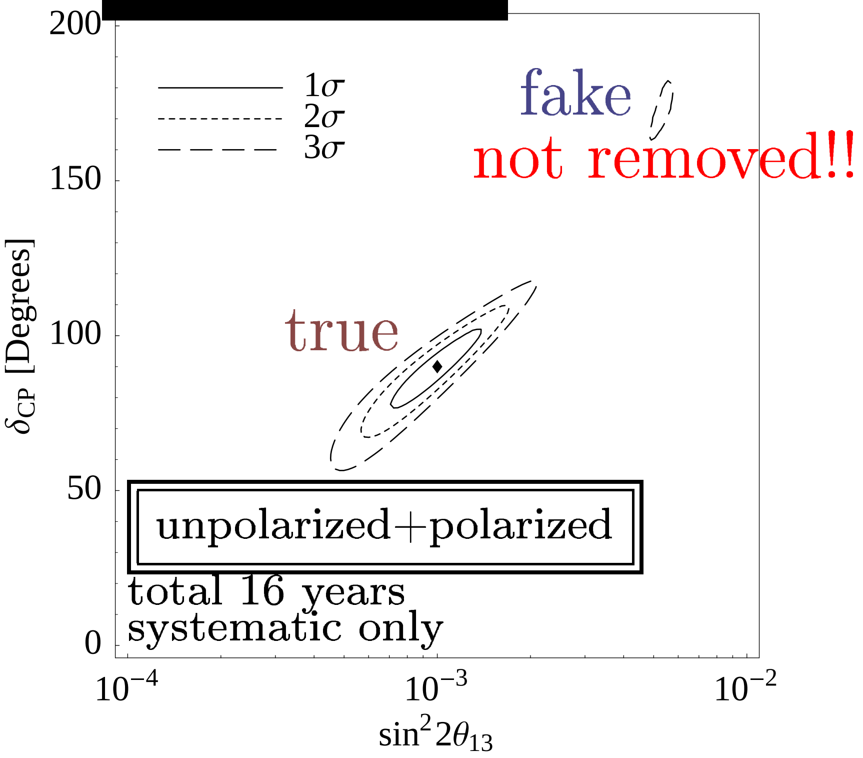
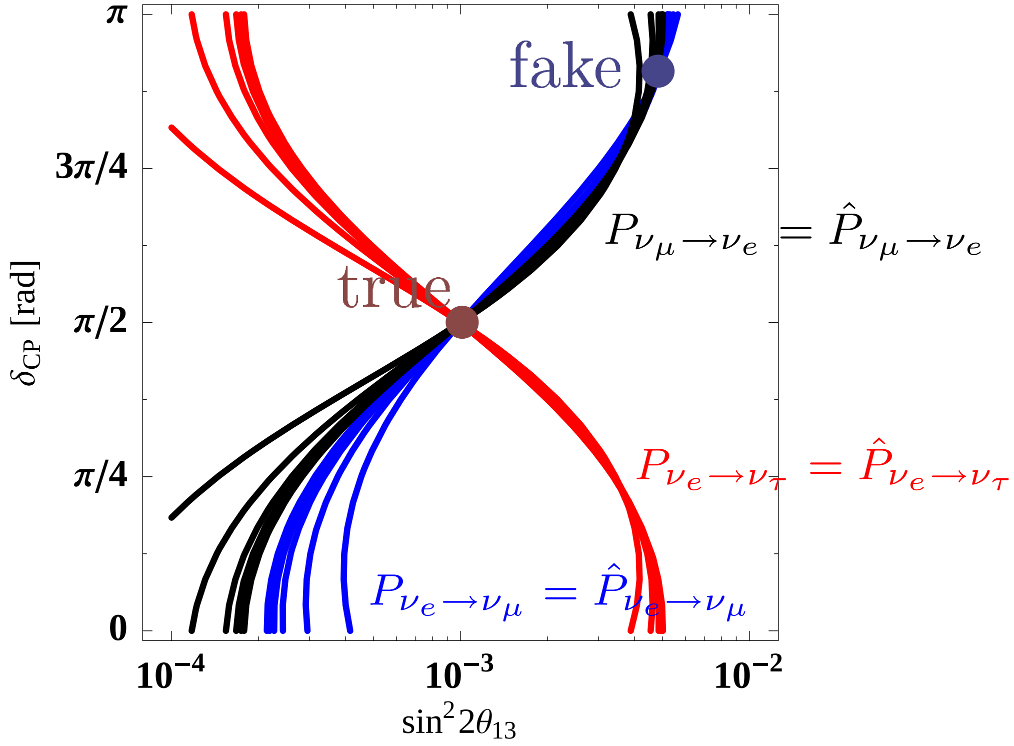
by using GLoBES





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— but why?

Reason why: $\Delta\chi^2$ and $(P_{\nu_e\to\nu_\mu} - P_{\bar{\nu}_e\to\bar{\nu}_\mu})$

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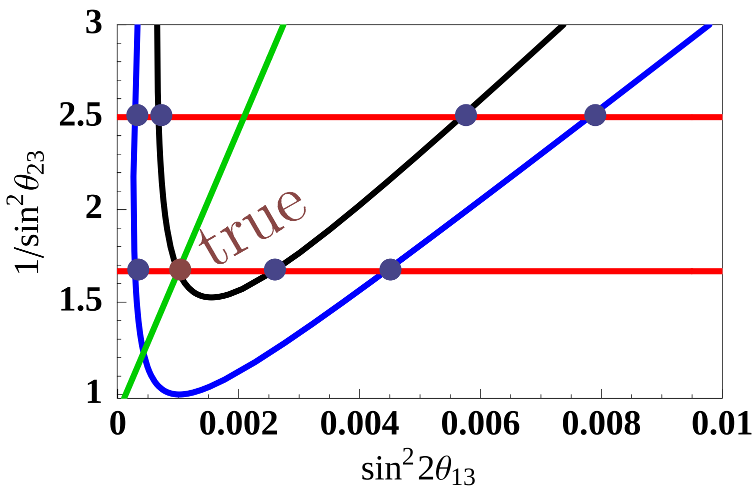
- $\Delta\chi^2$ function can be approximately understood by

$$\Delta\chi^2 = \Delta\chi^2_{\text{un-pol}} + \Delta\chi^2_{\text{pol}},$$

$$\Delta\chi^2_{\text{un-pol}} \sim |\hat{P}_{\nu_e\to\nu_\mu} - P_{\nu_e\to\nu_\mu}|^2, \quad \Delta\chi^2_{\text{pol}} \sim |\hat{P}_{\nu_\mu\to\nu_e} - P_{\nu_\mu\to\nu_e}|^2$$

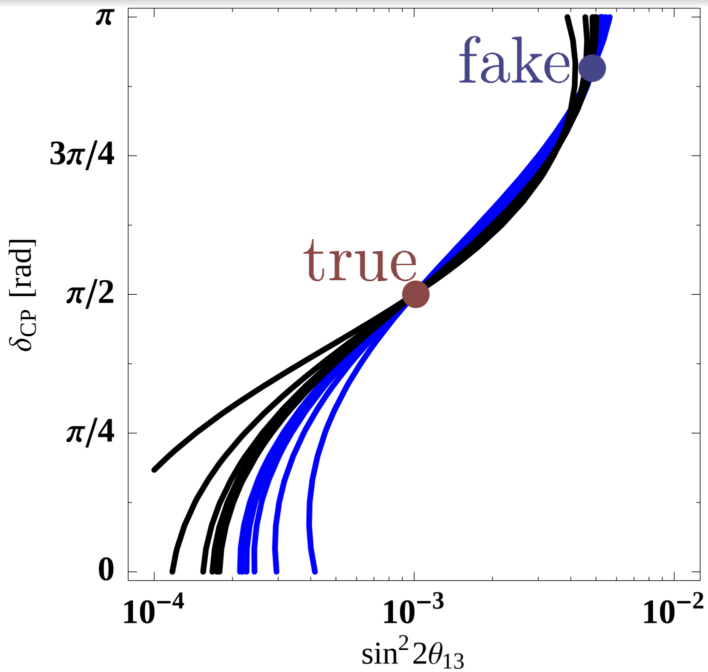


- In the plot, we use the information of $P_{\nu_e\to\nu_\mu} - P_{\bar{\nu}_\mu\to\bar{\nu}_e}$ to resolve the degeneracy ... This is quite different information from $\Delta\chi^2_{\text{pol}}$.

- If we try to introduce this information, then we should construct the statistics like

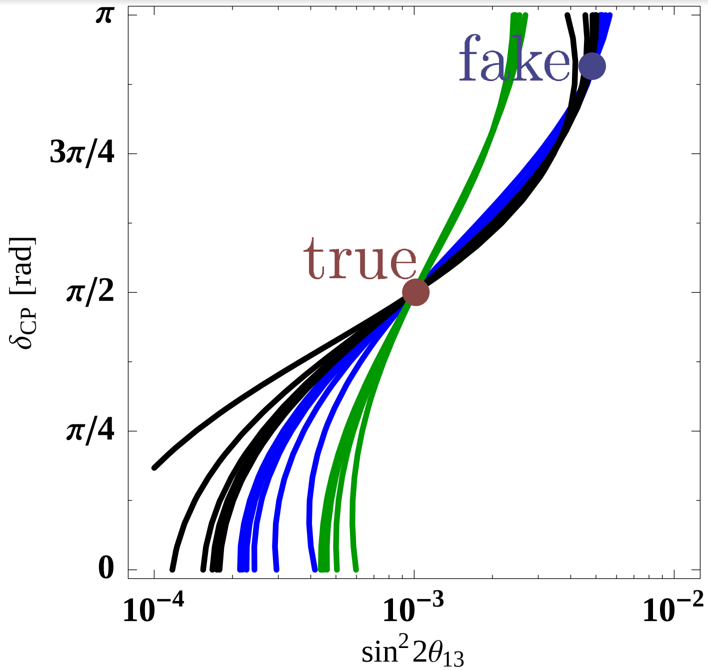
$$\Delta\chi^2_{\text{CPT}} \propto |(\hat{P}_{\nu_e\to\nu_\mu} - \hat{P}_{\bar{\nu}_\mu\to\bar{\nu}_e}) - (P_{\nu_e\to\nu_\mu} - P_{\bar{\nu}_\mu\to\bar{\nu}_e})|^2.$$

Trajectory plot analysis



- Trajectories of $P = \hat{P}$
  - $P_{\nu_e \to \nu_\mu}$ golden channel
  - $P_{\nu_\mu \to \nu_e}$

Trajectory plot analysis



- Trajectories of $P = \hat{P}$
  - $P_{\nu_e \to \nu_\mu}$ golden channel
  - $P_{\nu_\mu \to \nu_e}$
- Trajectories of $P - P' = \hat{P} - \hat{P}'$
  - $P_{\nu_e \to \nu_\mu} - P_{\bar{\nu}_\mu \to \bar{\nu}_e}$ CPT violation

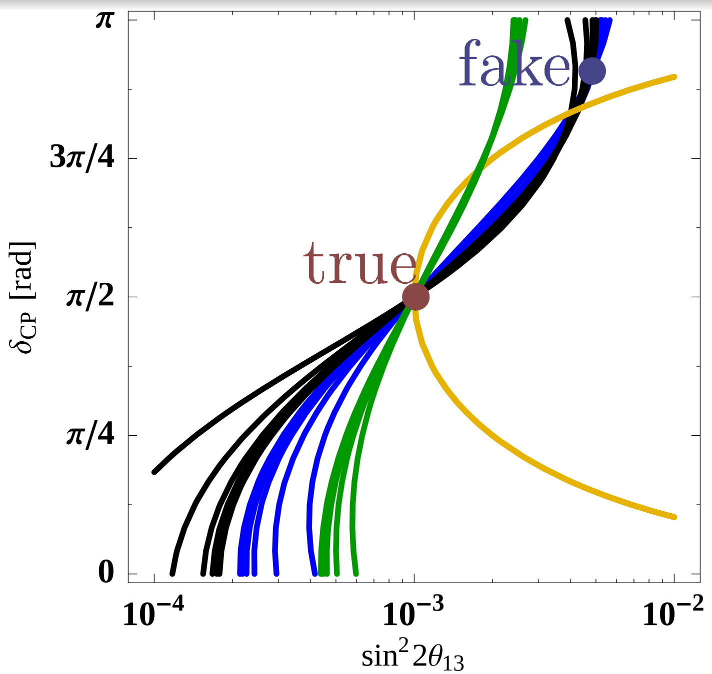
- Information of CPT violation (matter effect) may help to solve the degeneracy

$$\Delta\chi^2 \equiv \Delta\chi^2_{\text{golden}} + \Delta\chi^2_{\text{CPT}},$$

$$\Delta\chi^2_{\text{CPT}} \equiv \sum_i^{E_{\text{bin}}} \frac{|(\hat{N}_i^{\nu_e \to \nu_\mu} - C_i \hat{N}_i^{\bar{\nu}_\mu \to \bar{\nu}_e}) - (N_i^{\nu_e \to \nu_\mu} - C_i N_i^{\bar{\nu}_\mu \to \bar{\nu}_e})|^2}{\sigma_i^2},$$

$$C_i \equiv \Phi_i^{\bar{\nu}_\mu} / \Phi_i^{\nu_e} \; : \; \text{flux normalization factor.}$$

Trajectory plot analysis



- Trajectories of $P = \hat{P}$
  - $P_{\nu_e \to \nu_\mu}$ golden channel
  - $P_{\nu_\mu \to \nu_e}$
- Trajectories of $P - P' = \hat{P} - \hat{P}'$
  - $P_{\nu_e \to \nu_\mu} - P_{\bar{\nu}_\mu \to \bar{\nu}_e}$ CPT violation
  - $P_{\nu_e \to \nu_\mu} - P_{\nu_\mu \to \nu_e}$ T violation

- Trajectories of T violation do not depend on E_ν.
  - Therefore, this is not so efficient in the sense of the parameter fitting.
  - However, this feature is advantageous in the statistical sense.
- This quantity tells us whether T is violated or not —

Summary

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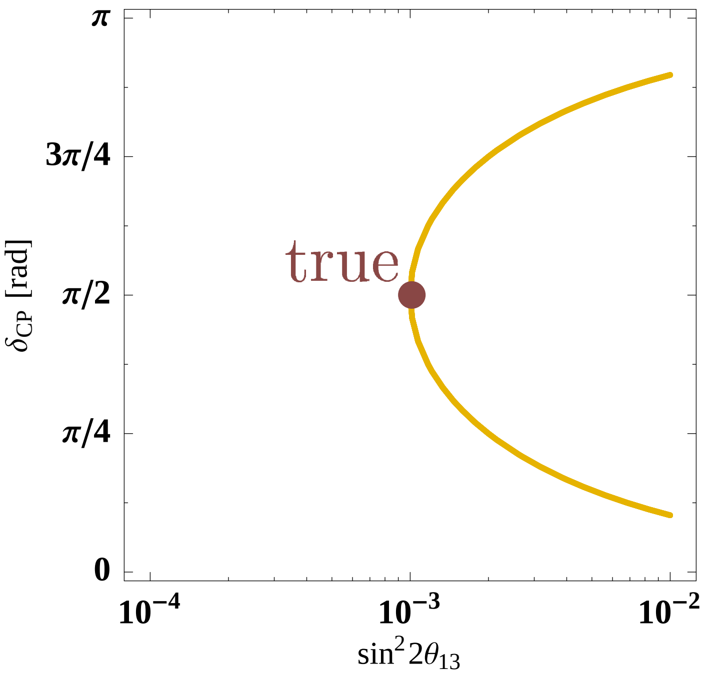
- We propose using the information of $\nu_\mu \to \nu_e$.
- To use $\nu_\mu \to \nu_e$, we assume the neutrino factory with the polarized muon beam.
- This information is not so effective to resolve the degeneracy with the *usual* $\Delta\chi^2$ analysis.
- It may be useful to improve the way to construct the statistics, depending on what we want — $\Delta\chi^2_{\mathrm{CPT,T}}$.
  — The numerical calculation is necessary to check whether $\Delta\chi^2_{\mathrm{CPT}}$ is effective in the statistical sense.
- We can directly (=not parameter fitting sense) obtain the T-violation effect by using $\Delta\chi^2_{\mathrm{T}}$
  —therefore, this channel must be important...

Summary

Note about T violation effect



- $P_{\nu_e \to \nu_\mu} - P_{\nu_\mu \to \nu_e}$

$$\left(\hat{J}_{\rm CP} - J_{\rm CP}\right) f(E_\nu) \propto \left(\sin 2\hat{\theta}_{13} \sin \hat{\delta} - \sin 2\theta_{13} \sin \delta\right)$$

  - Energy dependence is factored out
  - Energy binning cannot help to solve the degeneracy — but advantageous in the statistical sense to extract T-violation effect

- This information cannot determine the parameter but tell us whether T is violated or not.

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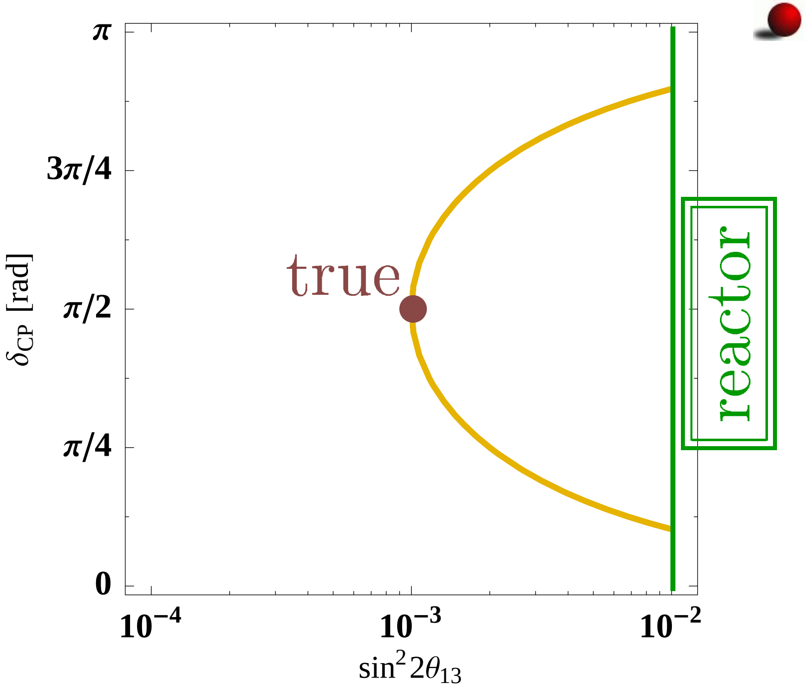

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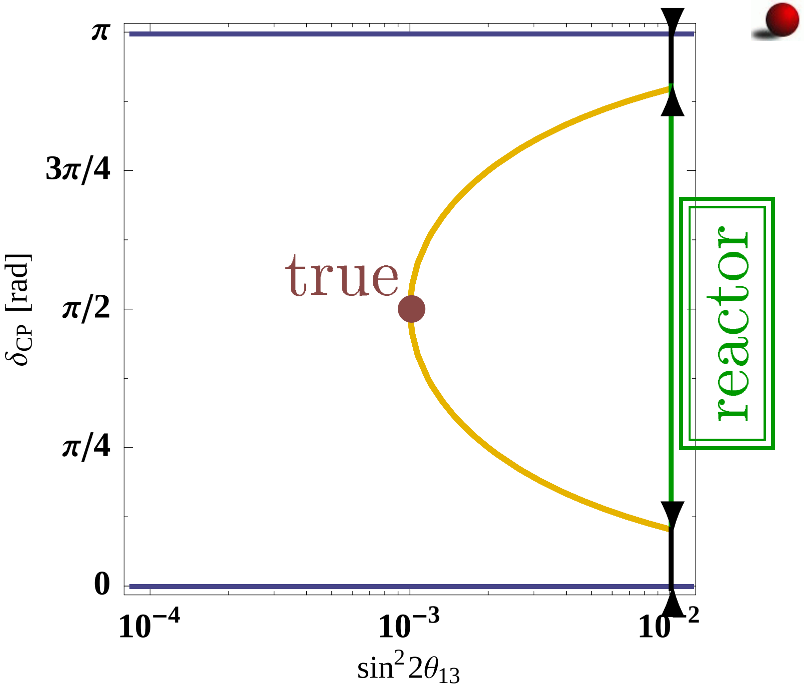

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- If reactor experiments exclude the $\sin^2 2\theta_{13} \lesssim 0.01$, $\delta = 0$ and π can be excluded — T violation is established. goodness of fit between *true* and $\delta = \{0, \pi\}$.