NuFact'05 Frascati, June 21

Neutrino masses and neutrino mixing

Eligio Lisi
INFN, Bari, Italy

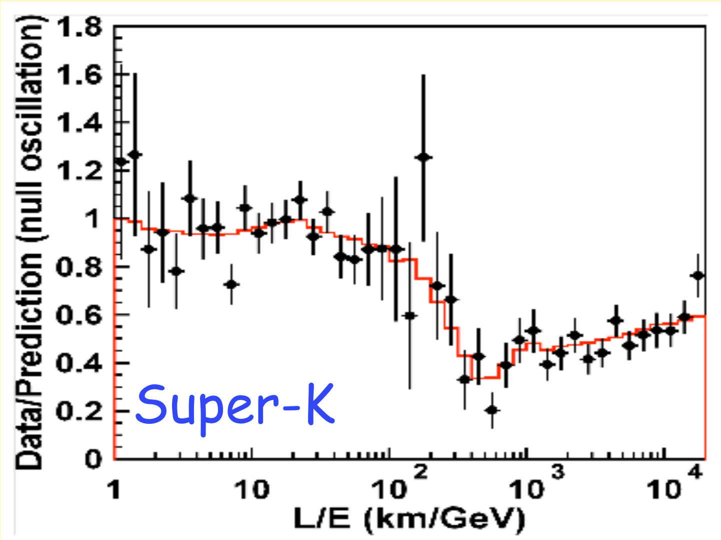
Outline:

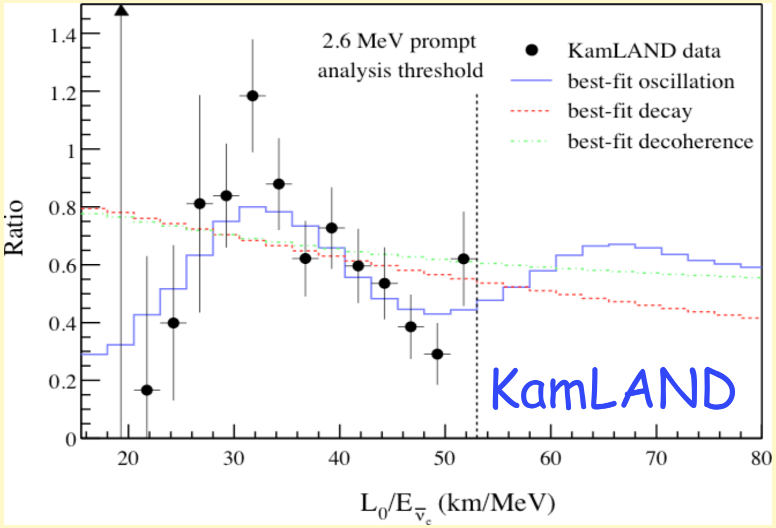
- 3ν framework: Introduction and notation
- Constraints from oscillation searches
- Constraints from non-oscillation searches
- Summary and conclusions

Mainly based on: **hep-ph/0506083 (2005 review),** hep-ph/0505081, hep-ph/0408045; *in collab.with: G.L. Fogli, A. Marrone, A. Melchiorri, A. Palazzo, A.M. Rotunno, P. Serra, J. Silk*
See references therein for credits to experimental and theoretical works in ν physics

3ν framework: Introduction

Neutrino masses, mixing and oscillations are established facts





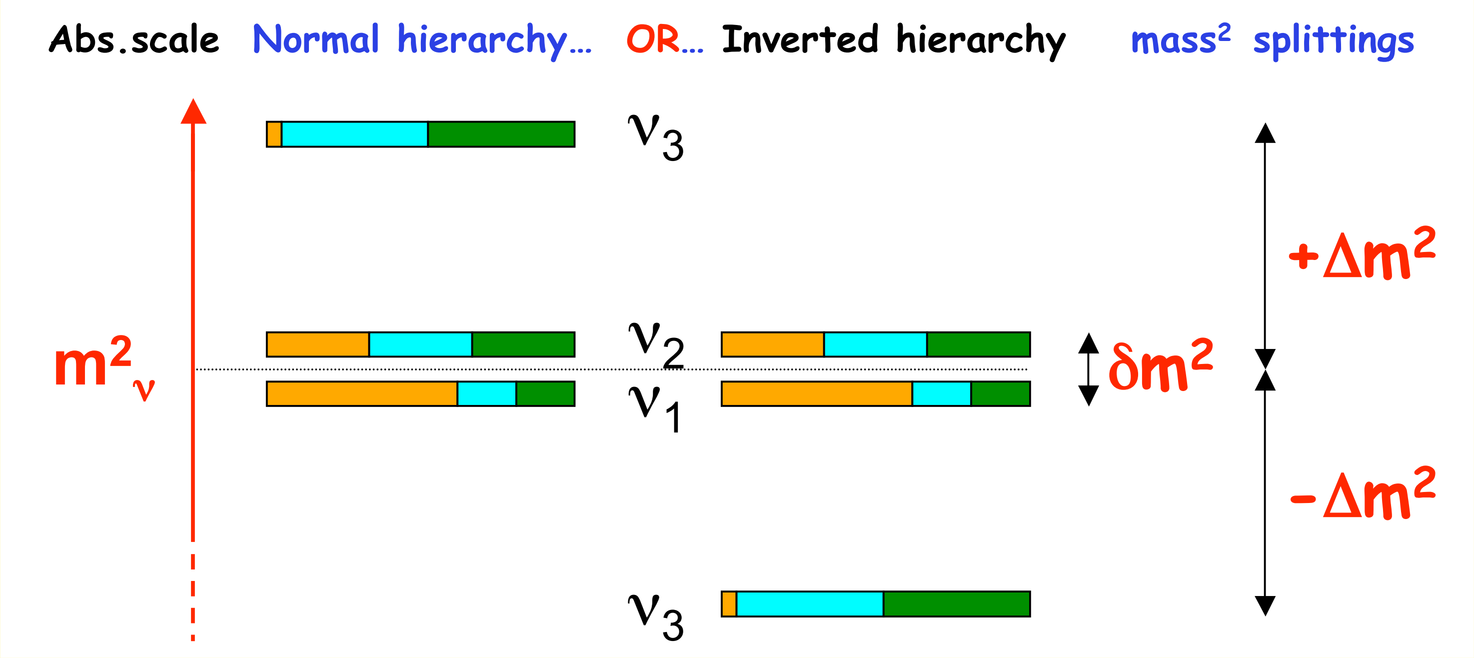
Δm^2-driven oscillations

δm^2-driven oscillations

(about half-period seen in both cases)

Frequencies and amplitudes can be embedded in a **3**ν scenario

For many purposes, a 1-significant-digit summary is enough (flavors = e μ τ):



$\delta m^2 \sim 8 \times 10^{-5}\ \mathrm{eV}^2$ $\quad$ $\sin^2\theta_{12} \sim 0.3$

$\Delta m^2 \sim 3 \times 10^{-3}\ \mathrm{eV}^2$ $\quad$ $\sin^2\theta_{23} \sim 0.5$

$m_\nu < O(1)\ \mathrm{eV}$ $\quad$ $\sin^2\theta_{13} < \mathrm{few}\%$

$\mathrm{sign}(\pm\Delta m^2)$ unknown $\quad$ δ (CP) unknown

At NuFact'05, a more refined summary is appropriate ($\pm 2\sigma$):

$$\delta m^2 = 7.92\,(1 \pm 0.09) \times 10^{-5}\ \mathrm{eV}^2 \qquad \sin^2\theta_{12} = 0.314\,(1^{+0.18}_{-0.15})$$

$$\Delta m^2 = 2.4\,(1^{+0.21}_{-0.26}) \times 10^{-3}\ \mathrm{eV}^2 \qquad \sin^2\theta_{23} = 0.44\,(1^{+0.41}_{-0.22})$$

$$(m_\beta, m_{\beta\beta}, \Sigma) < O(1)\ \mathrm{eV} \qquad \sin^2\theta_{13} < 3.2 \times 10^{-2}$$

Second significant digit may be relevant in some contexts,
e.g., prospective studies of future precision experiments
(and is also necessary for book-keeping progress in estimates)

In such cases, mass-mixing parameters must be precisely defined

Consensus on conventions and notation desirable

3ν framework: Notation

Mixing: No need to change the PDG convention for U

$$U = O_{23}\,\Gamma_\delta\,O_{13}\,\Gamma_\delta^\dagger\,O_{12}$$

with $$\Gamma_\delta = \mathrm{diag}(1, 1, e^{+i\delta})$$

U mixes fields in the CC interaction lagrangian,

$$\nu_{\alpha L} = \sum_{i=1,2,3} U_{\alpha i}\,\nu_{iL} \qquad (\alpha = e, \mu, \tau)$$

and thus U* mixes one-particle states,

$$|\nu_\alpha\rangle = \sum_{i=1,2,3} U^*_{\alpha i}\,|\nu_i\rangle$$ ← PDG

In the following, we shall limit ourselves to the two <u>inequivalent</u> CP-conserving cases (U=U*) with $e^{i\delta}=\pm1$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & \pm s_{13} \\ 0 & 1 & 0 \\ \mp s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

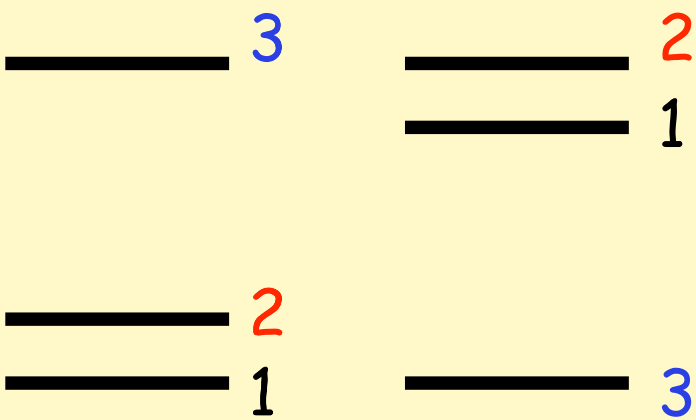
[$\cos\delta = \pm1$ = "CP parity"]

The two cases are transformed into one another through:

$$+s_{13} \rightarrow -s_{13}$$ (CP parity flip)

Masses: labels and splittings

Consensus labels: doublet=(ν_1, ν_2), with ν_2 heaviest in both hierarchies



$$\delta m^2 = m_2^2 - m_1^2 > 0$$

Sign of smallest splitting: conventional.
The relative ν_e content of ν_1 and ν_2 is instead physical (given by MSW effect)

$$\text{Note}: \quad |m_3^2 - m_1^2| = \begin{cases} \text{largest splitting (N.H.)} \\ \text{next–to–largest splitting (I.H.)} \end{cases}$$

$$\Rightarrow \Delta m_{31}^2 \ (\text{or } \Delta m_{32}^2) \text{ change physical meaning from NH to IH}$$

We prefer to define the 2nd independent splitting as:

$$\Delta m^2 = \left| \frac{\Delta m^2_{31} + \Delta m^2_{32}}{2} \right| = \left| m^2_3 - \frac{m^2_1 + m^2_2}{2} \right|$$

so that the largest and next-to-largest splittings, in both NH & IH, are given by: $\Delta m^2 \pm \frac{\delta m^2}{2}$

and only one physical sign distinguishes NH (+) from IH (-), as it should be:

$$(m^2_1, m^2_2, m^2_3) = \frac{m^2_2 + m^2_1}{2} + \left(-\frac{\delta m^2}{2}, +\frac{\delta m^2}{2}, \pm\Delta m^2 \right)$$

sign($\pm\Delta m^2$) can be determined - in principle - by interference of Δm^2-driven oscillations with some Q-driven oscillations, provided that sign(Q) is known. Two ways (barring new neutrino physics):

$Q = V_{MSW}(x) = \pm\sqrt{2}G_F N_e(x)$ (only in matter & for $s_{13}>0$)

$Q = \delta m^2 > 0$ (also in vac. & for $s_{13}=0$, but hard)

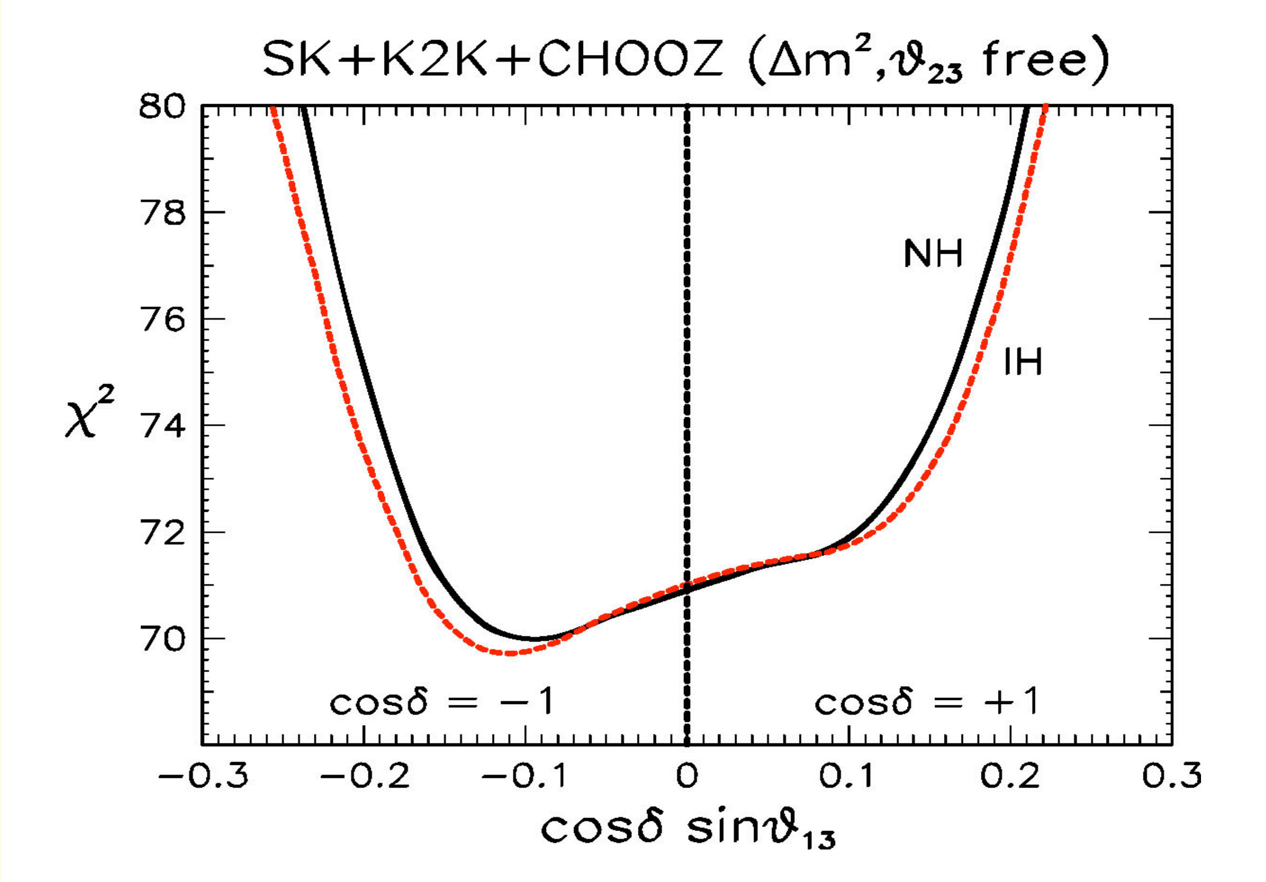
The sensitivity to such interference effects, suppressed by the smallness of s_{13} and/or of $\delta m^2/\Delta m^2$, is very weak within current data.

In the next figure we shall see, e.g., how small is the current effect of δm^2 in "distinguishing" the two hierarchies, within an analysis of SK_{ATM}+ K2K + CHOOZ data with (Δm^2, s^2_{23}, s^2_{13}) unconstrained

Constraints on $(\Delta m^2,\ s_{23},\ s_{13})$ from SK_{ATM}+K2K+CHOOZ

with $(\delta m^2,\ s^2_{12})$ fixed at their best-fit values from solar+KamLAND

Four cases with slight differences: $[\mathrm{sign}(\pm\Delta m^2) = \pm 1] \otimes [\cos\delta = \pm 1]$

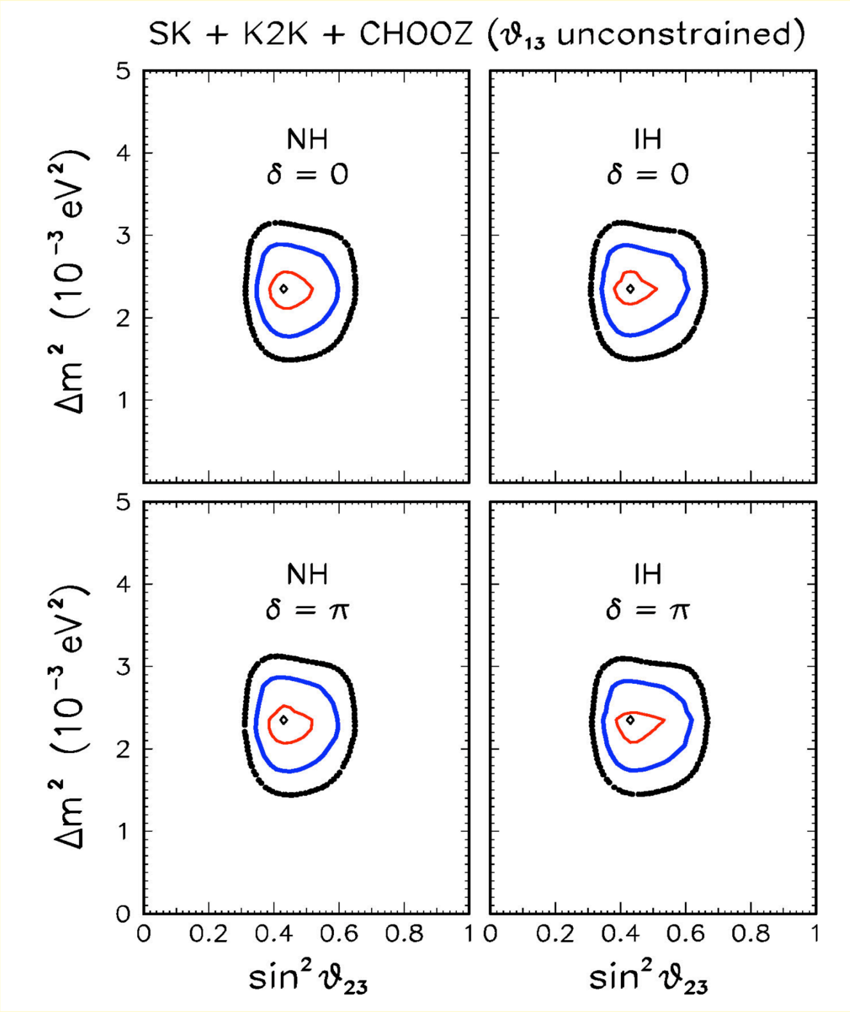


Slight preference (<1σ) for $s_{13} \neq 0$ and $\delta=\pi$ (over $\delta=0$)

Very tiny difference at $s_{13}=0$ (entirely due to $\delta m^2>0$)

Previous cases $[\text{sign}(\pm\Delta m^2) = \pm 1] \otimes [\cos\delta = \pm 1]$

in terms of the other two parameters (s_{13} marginalized)



at 1, 2, and 3 sigma*

Four cases ~equivalent phenomenologically in the parameters ($\Delta m^2, s^2_{23}$)

Weak ($<1\sigma$) but "stable" preference for less-than-maximal mixing (**$s^2_{23}<1/2$**);

preference driven by **δm^2**-induced effects; present also for s_{13}=0

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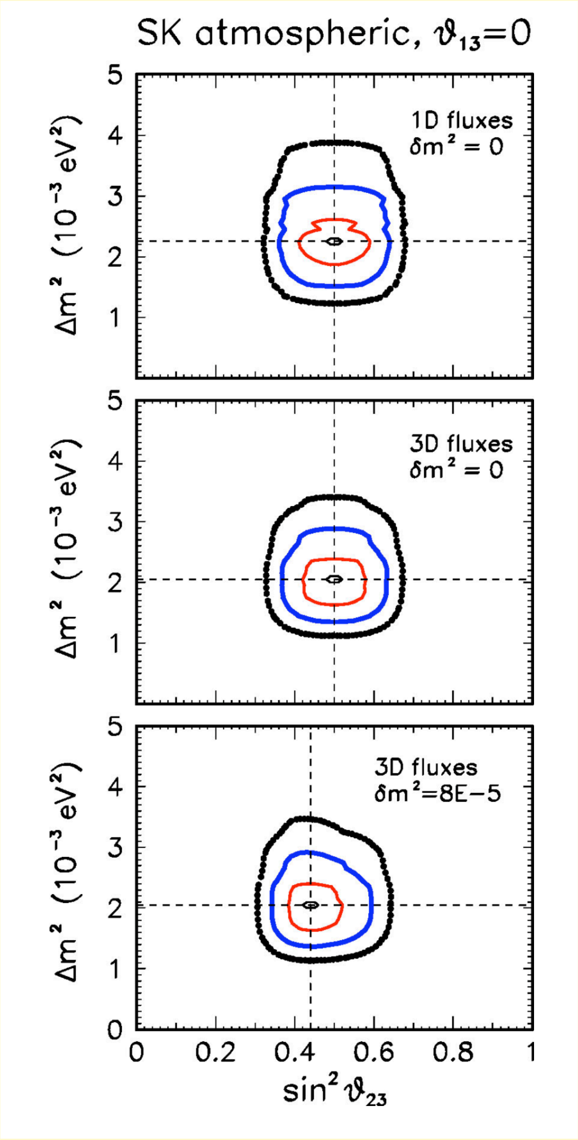
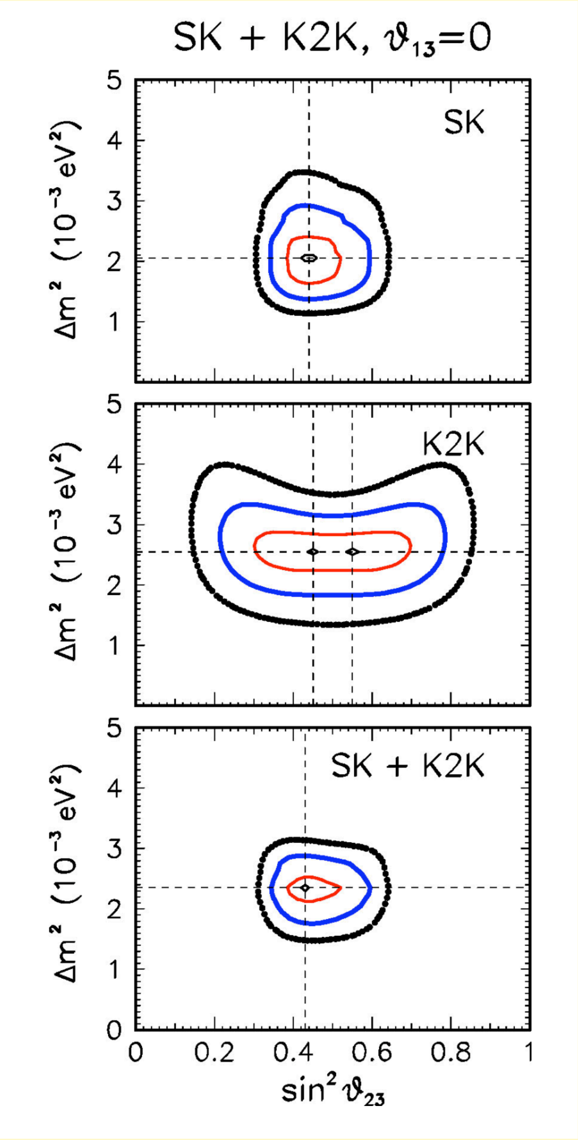
*$\Delta\chi^2=(n\sigma)^2$ hereafter. Consensus on "typical" C.L. contours also desirable

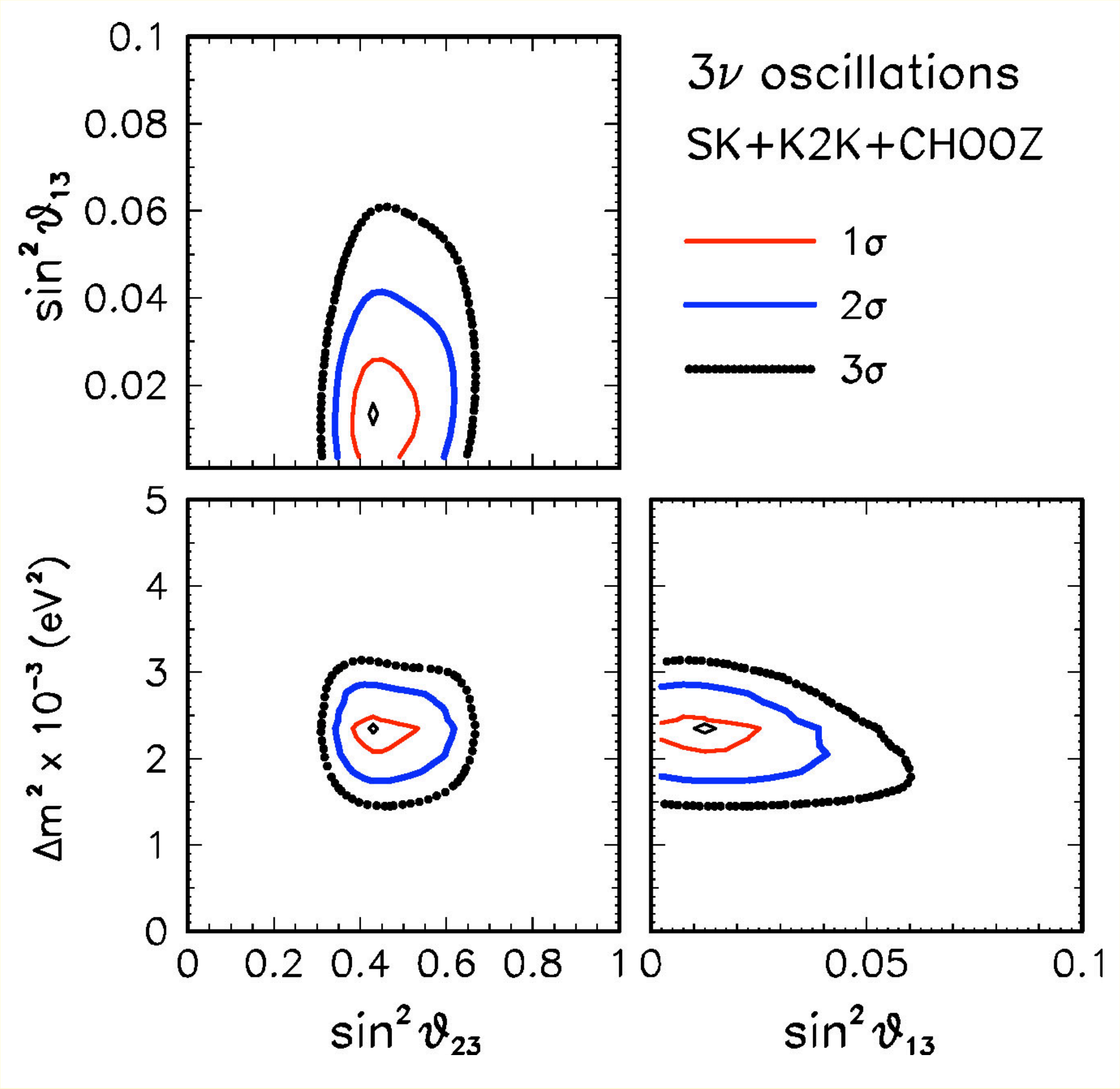
δm^2 effects are small, but not smaller than others one takes care of ….



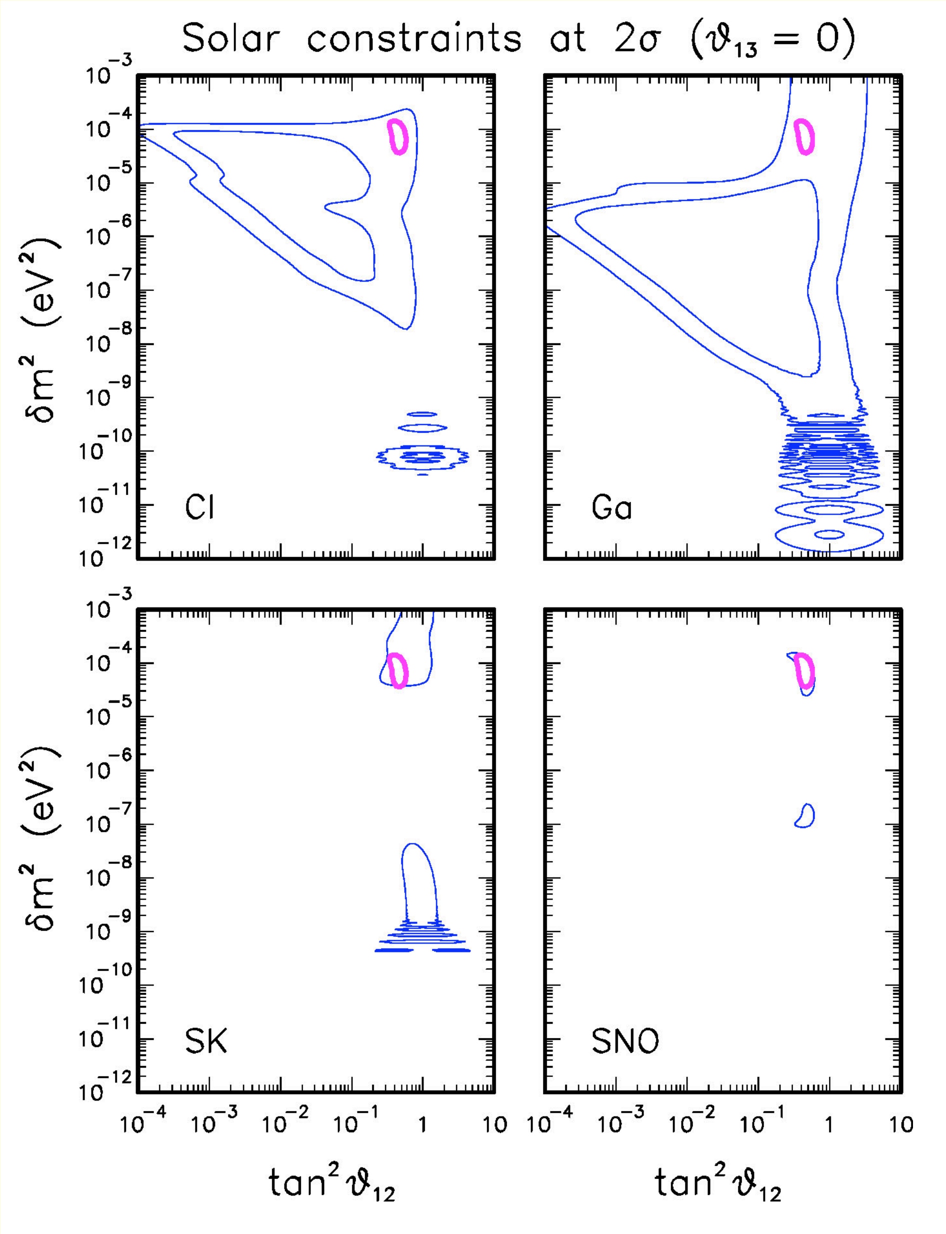


(Bounds consistent with MACRO, Soudan 2)

For free s^2_{13}, marginalizing over $[\mathrm{sign}(\pm\Delta m^2) = \pm 1] \otimes [\cos\delta = \pm 1]$, we get



Constraints on $(\delta m^2, s_{12}, s_{13})$ from Solar ν + KamLAND



Solar data alone identify a single LMA solution in the **(δm^2, $\tan^2\theta_{12}$)** plane

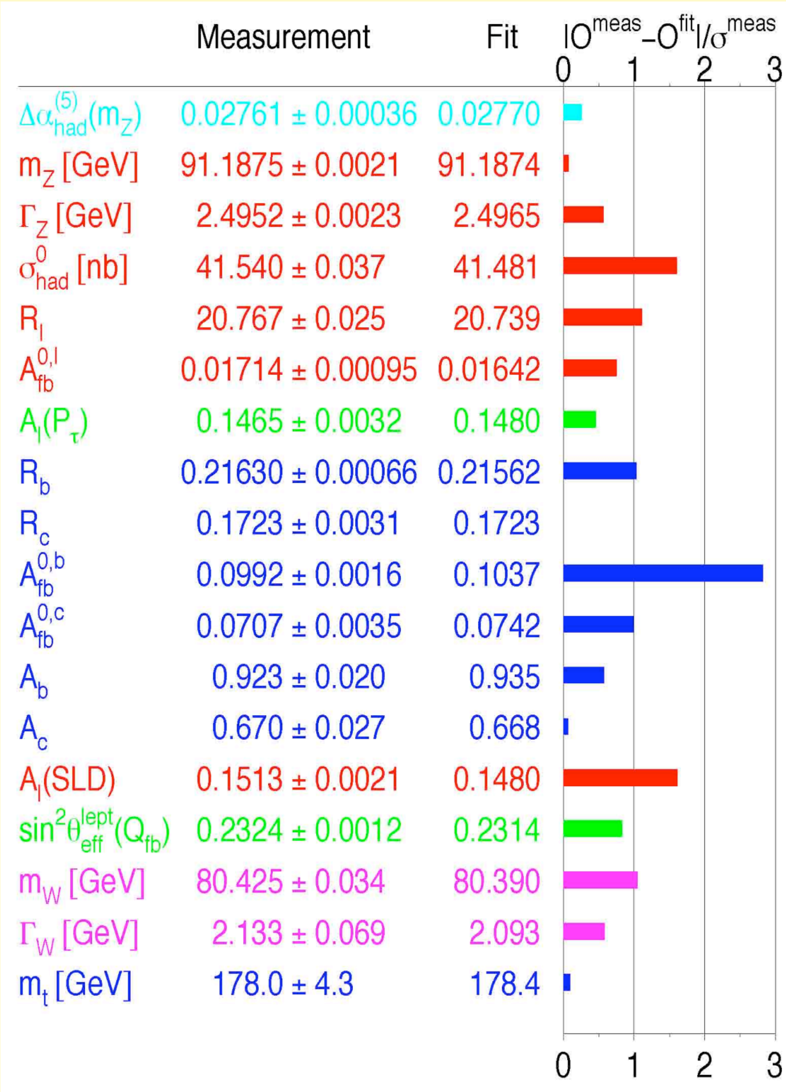
LMA parameters are dominated by SNO and SK, sensitive to the ^8B ν flux

SNO NC determination of the ^8B ν flux twice more accurate than typical SSM predictions

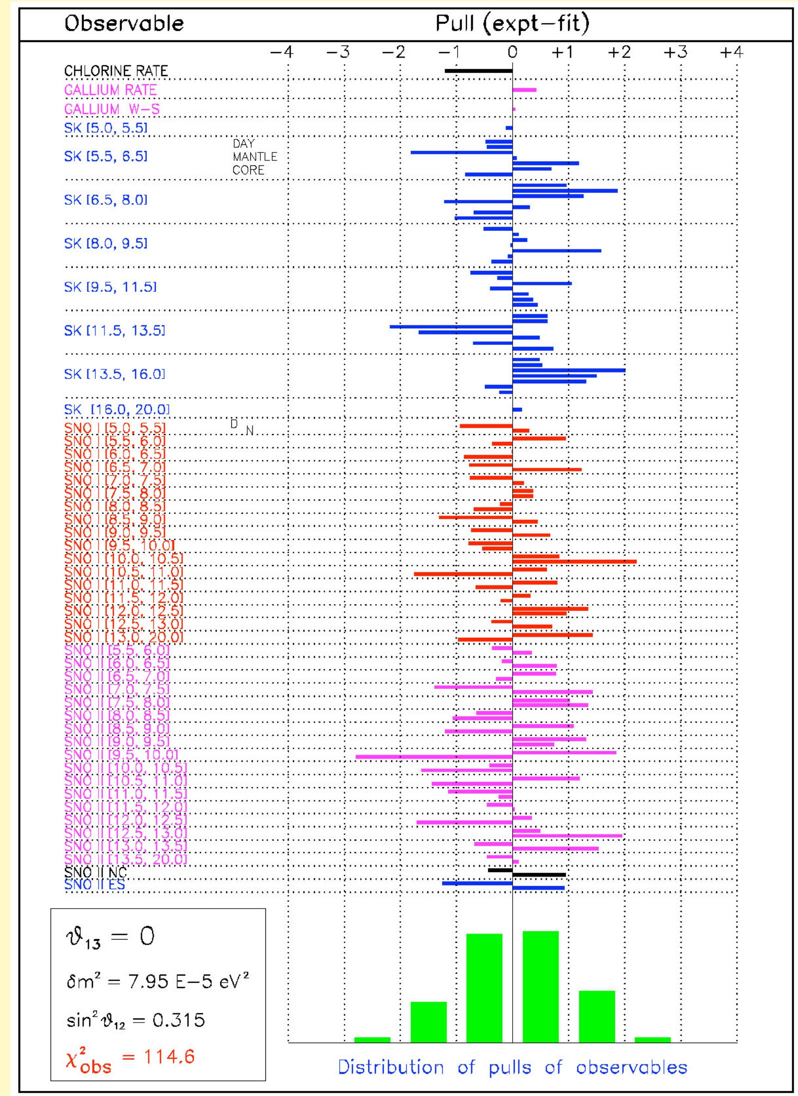
LMA param. basically SSM-independent

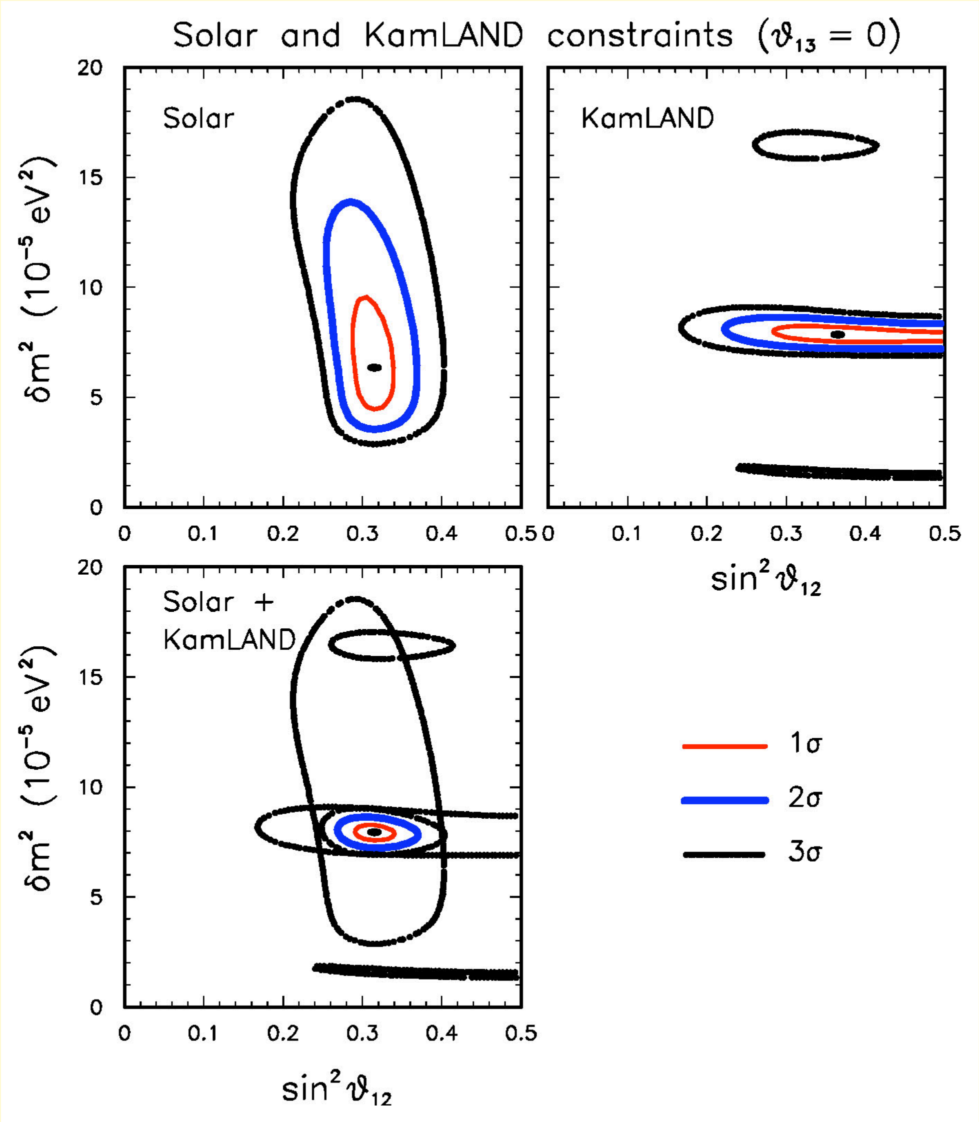
Towards precision neutrino physics ....

LEP EW Working Group, 2005



Solar neutrinos (Bari group), 2005





KamLAND dominates

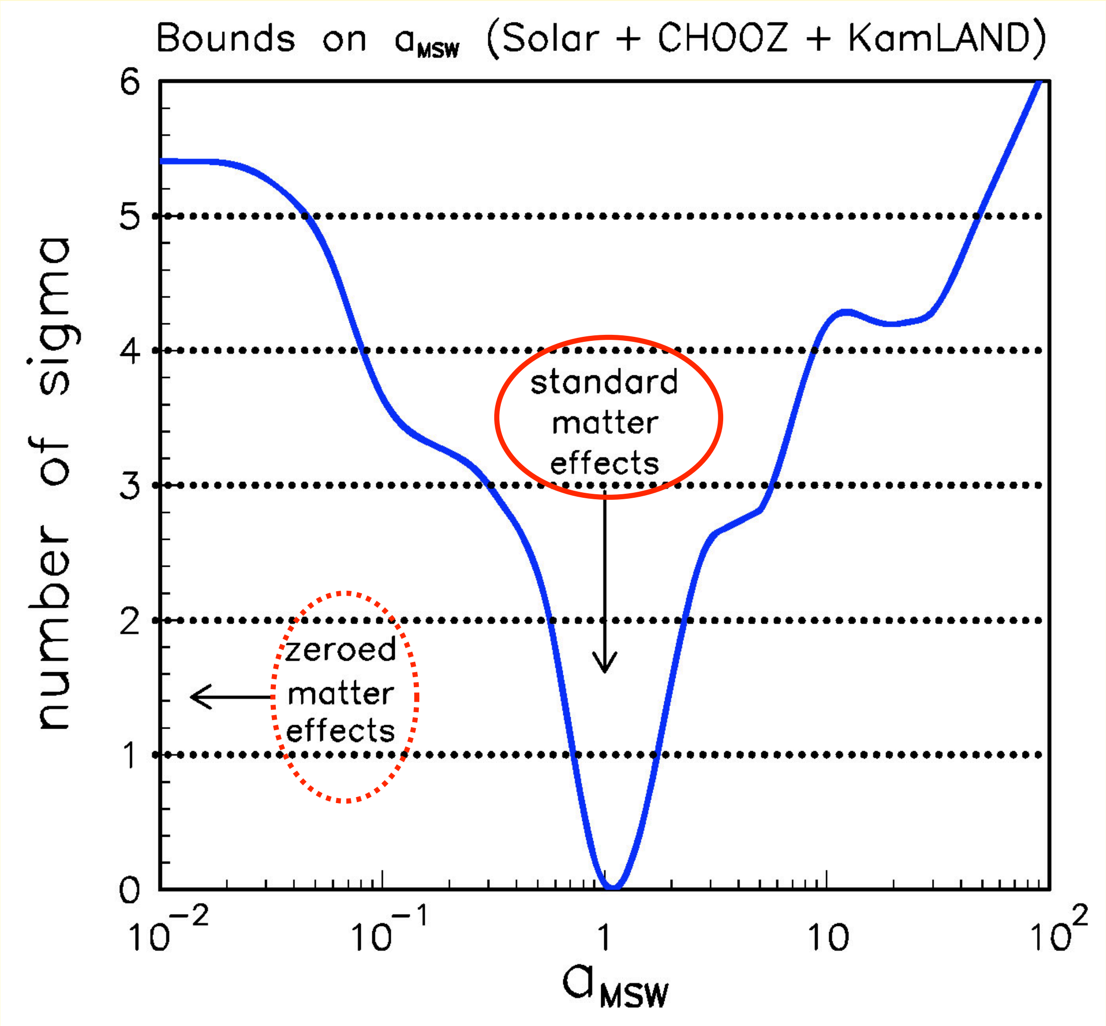
δm^2 constraints

Main impact of 2005 SNO data (at s_{13}=0):

Slight increase in solar best-fit param. (**δm^2, s^2_{12}**), and thus better agreement with the latest data from KamLAND

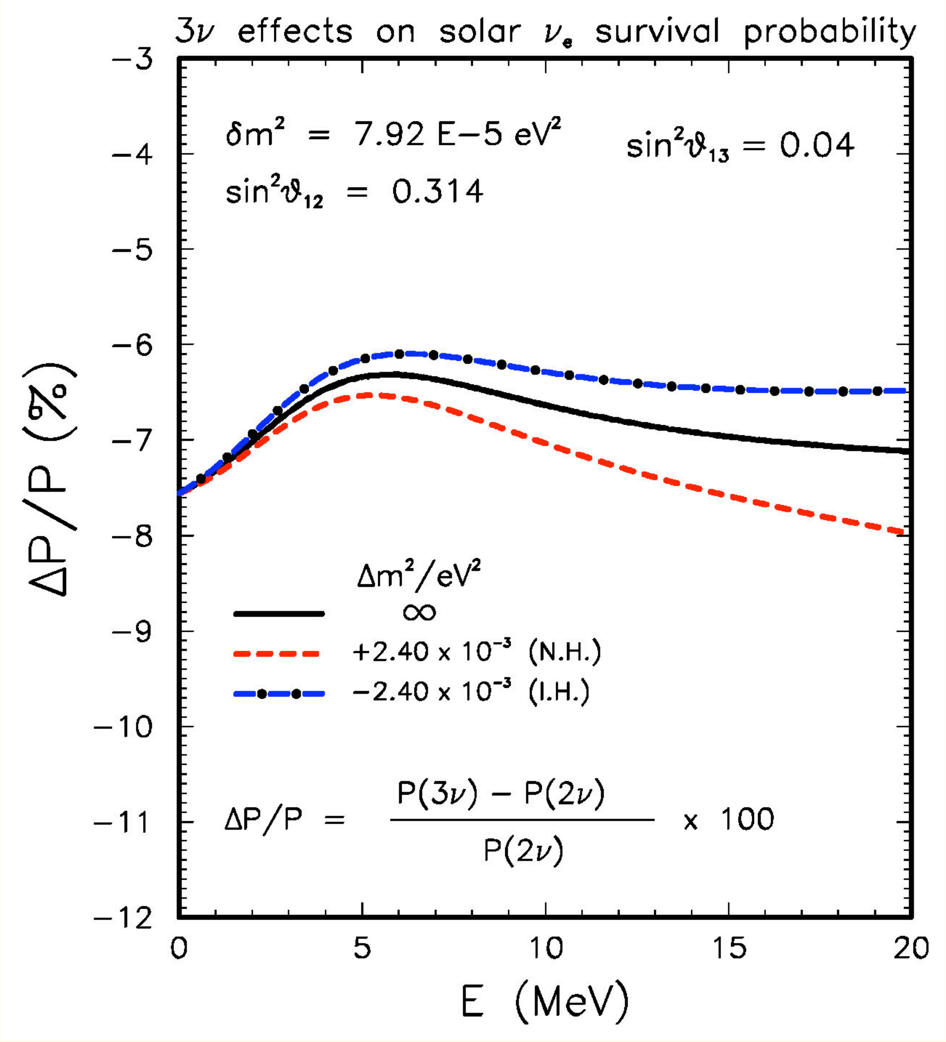
Note change of scale: log $\tan^2\theta_{12} \rightarrow$ lin $\sin^2\theta_{12}$. In general, consensus on trigonometric functions of θ_{ij} is desirable for homogeneous comparison

matter effects with standard size ($V = \sqrt{2}\, G_F\, N_e$) confirmed



$V(x) \rightarrow a_{MSW}\, V(x)$

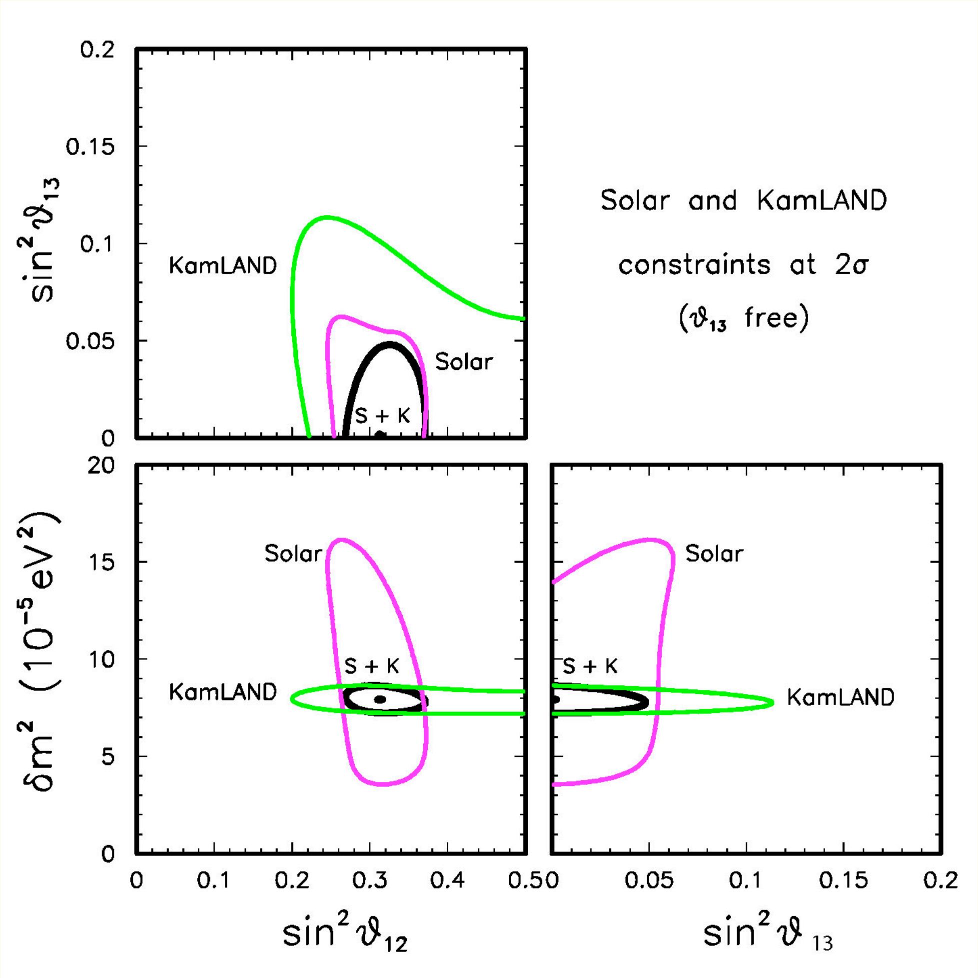
Solar ν data also sensitive to s_{13} ...



..and, in principle, to $\pm\Delta m^2$ (hierarchy)

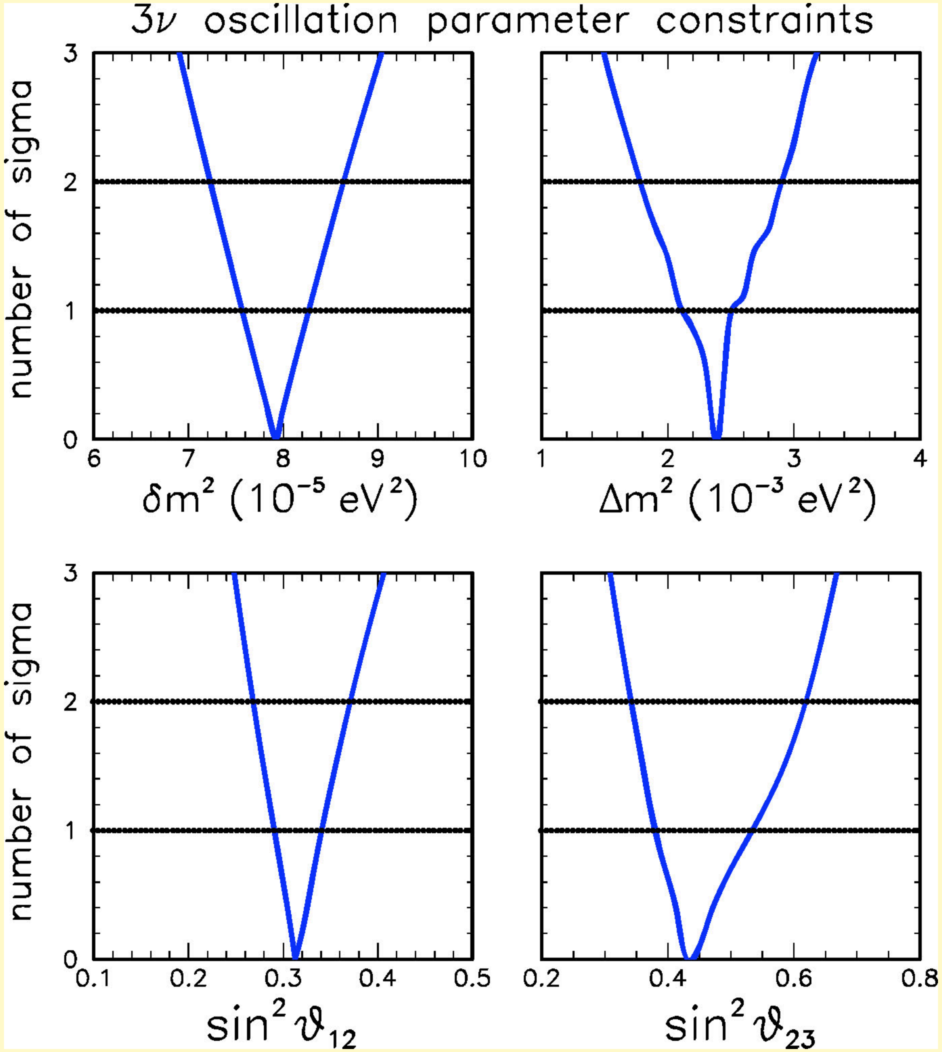
But: $\pm\Delta m^2$ **effects** are typically O(10) smaller than s_{13} **effects** (and can thus be currently neglected)

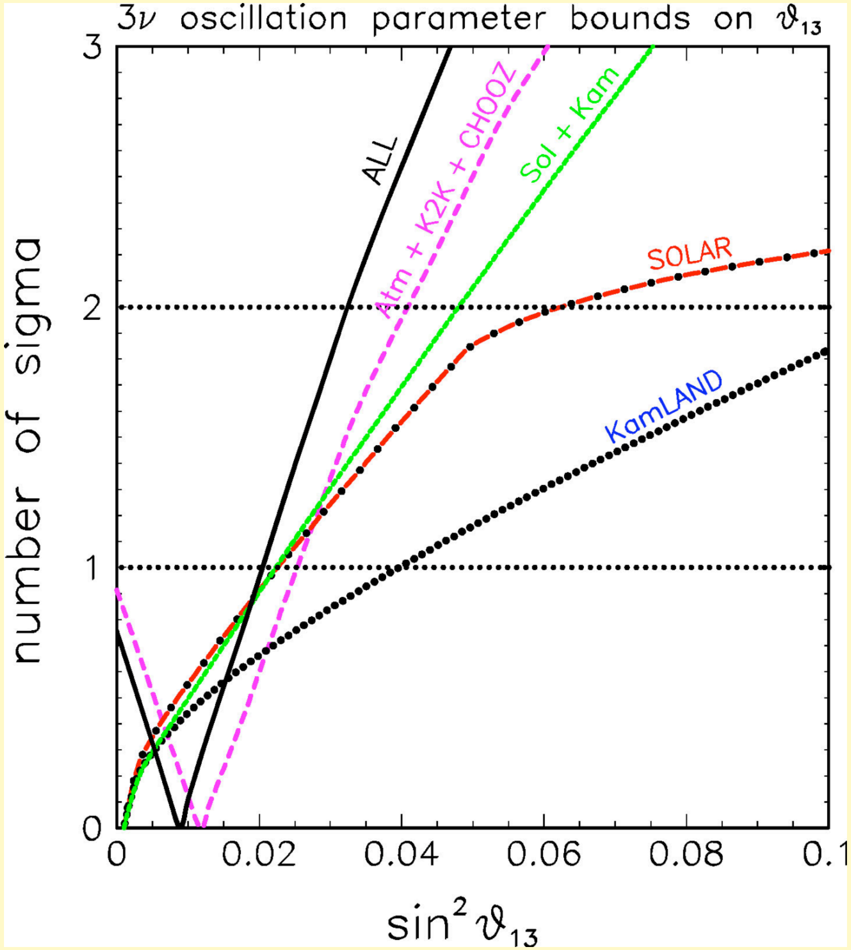
Interesting constraints on s^2_{13} from current solar+KamLAND data:



All solar data (radiochemical + Cherenkov) and KamLAND data (rate + spectrum shape) cooperate in setting limits on s^2_{13}

Finally, combining solar & terrestrial ν oscillation data (-LSND) …





2005 global 2σ bounds (95% CL), marginalized over the four cases $[\mathrm{sign}(\pm\Delta m^2)] \otimes [\cos\delta = \pm 1]$:

$$\delta m^2 = 7.92\,(1^{+0.09}_{-0.09}) \times 10^{-5}\ \mathrm{eV}^2$$

$$\Delta m^2 = 2.4\,(1^{+0.21}_{-0.26}) \times 10^{-3}\ \mathrm{eV}^2$$

$$\sin^2\theta_{12} = 0.314\,(1^{+0.18}_{-0.15})$$

$$\sin^2\theta_{23} = 0.44\,(1^{+0.41}_{-0.22})$$

$$\sin^2\theta_{13} < 3.2 \times 10^{-2}$$

with very small correlations

Needless to say, new physics beyond the standard 3ν framework (e.g., from LSND/MiniBOONE) might alter such bounds

Probing absolute ν masses through non-oscillation searches

Three main tools: (m_β, $m_{\beta\beta}$, Σ)

1) **β decay**: $m^2_i \neq 0$ can affect spectrum endpoint. Sensitive to the "effective electron neutrino mass":

$$m_\beta = \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2\right]^{\frac{1}{2}}$$

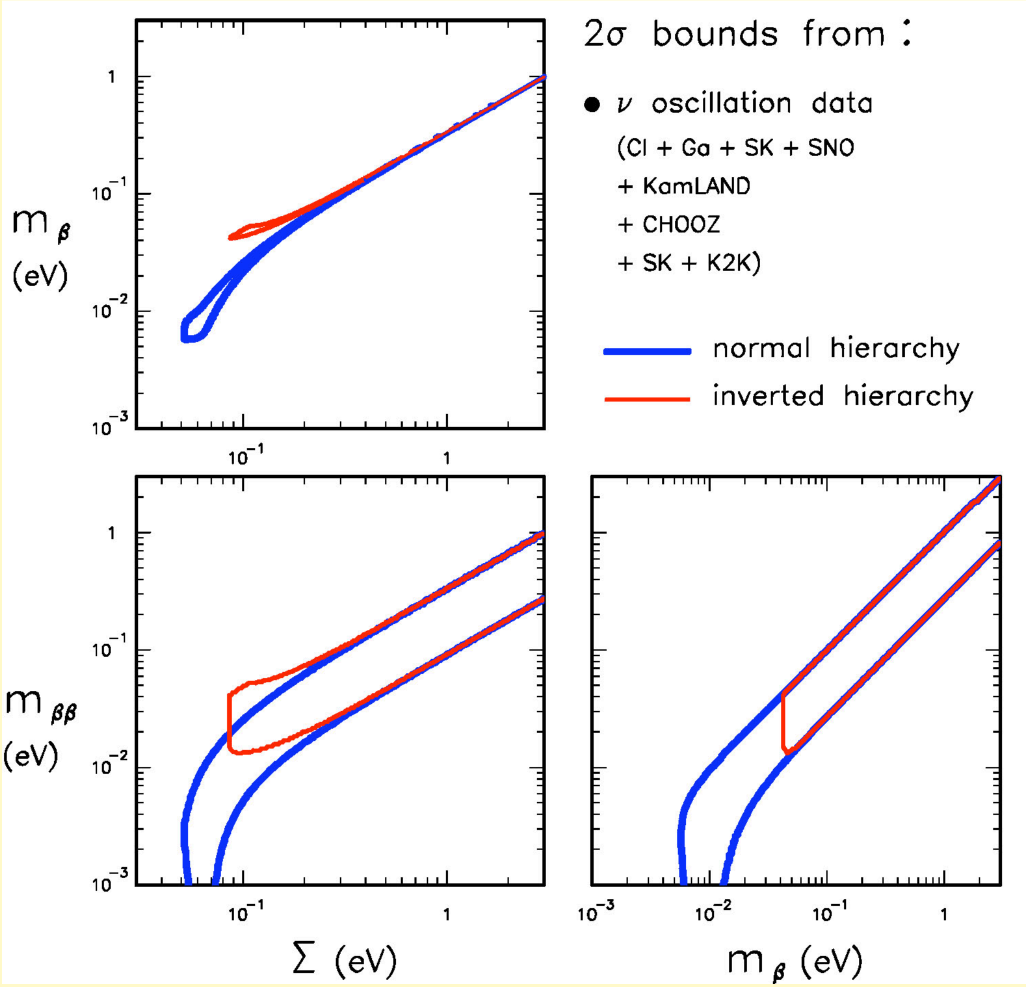
2) **0ν2β decay**: Can occur if $m^2_i \neq 0$ and $\nu=\bar{\nu}$. Sensitive to the "effective Majorana mass" (and phases):

$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right|$$

3) **Cosmology**: $m^2_i \neq 0$ can affect large scale structures in (standard) cosmology constrained by CMB+other data. Sensitive to:

$$\Sigma = m_1 + m_2 + m_3$$

Even without non-oscillation data, the (m_β, $m_{\beta\beta}$, Σ) parameter space is constrained by previous oscillation results:



Significant covariances

Partial overlap between the two hierarchies

Large $m_{\beta\beta}$ spread due to unknown Majorana phases

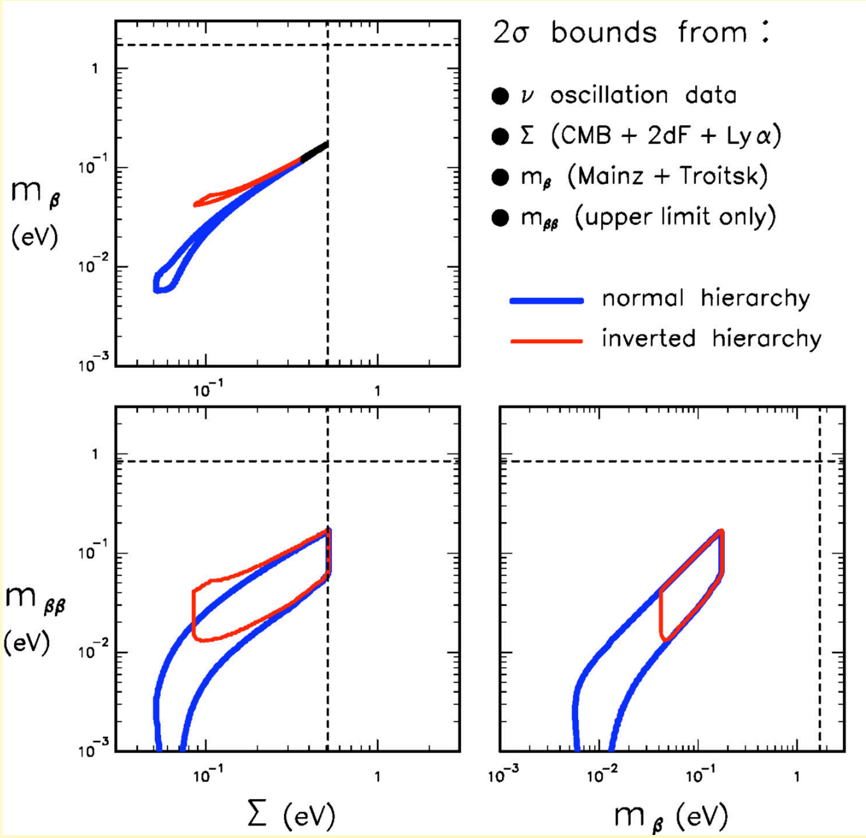
But we do have information from non-oscillation experiments:

1) β **decay**: no signal so far. Mainz & Troitsk expts: m_β < O(eV)

2) **$0\nu2\beta$ decay**, no signal in all experiments, except in the most sensitive one *(Heidelberg-Moscow)*. Rather debated claim.
Claim accepted: **$m_{\beta\beta}$ in sub-eV range** (with large uncertainties)
Claim rejected: **$m_{\beta\beta}$ < O(eV).**

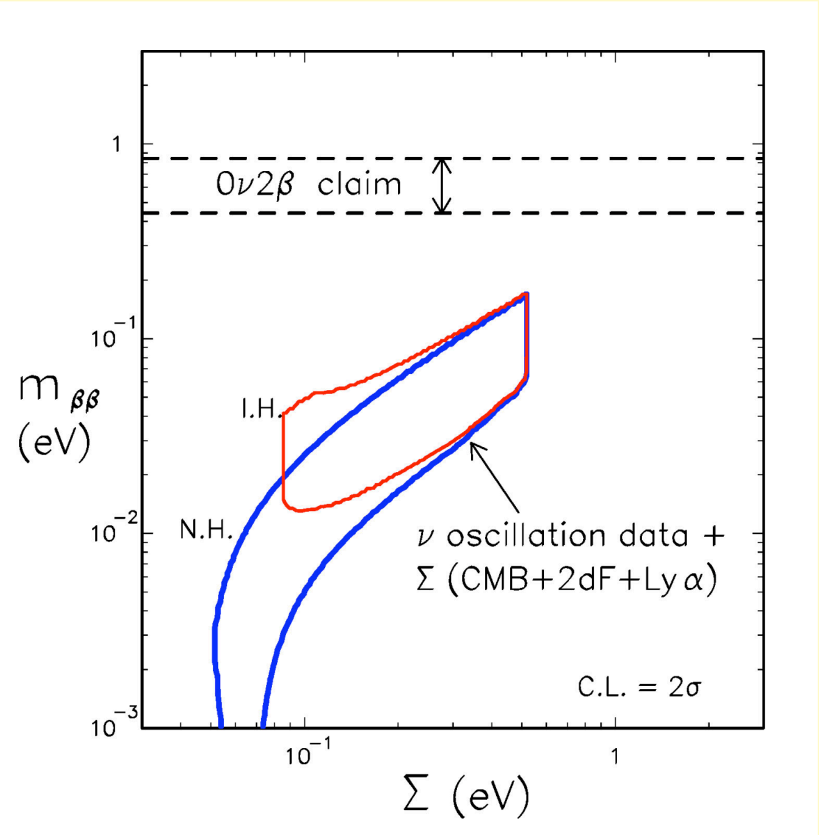
3) **Cosmology**. Upper bounds: **Σ < eV/sub-eV range**, depending on several inputs and priors. E.g., latest SDSS Lyα data crucial to reach sub-eV bounds (but: systematics?)

$0\nu2\beta$ claim rejected



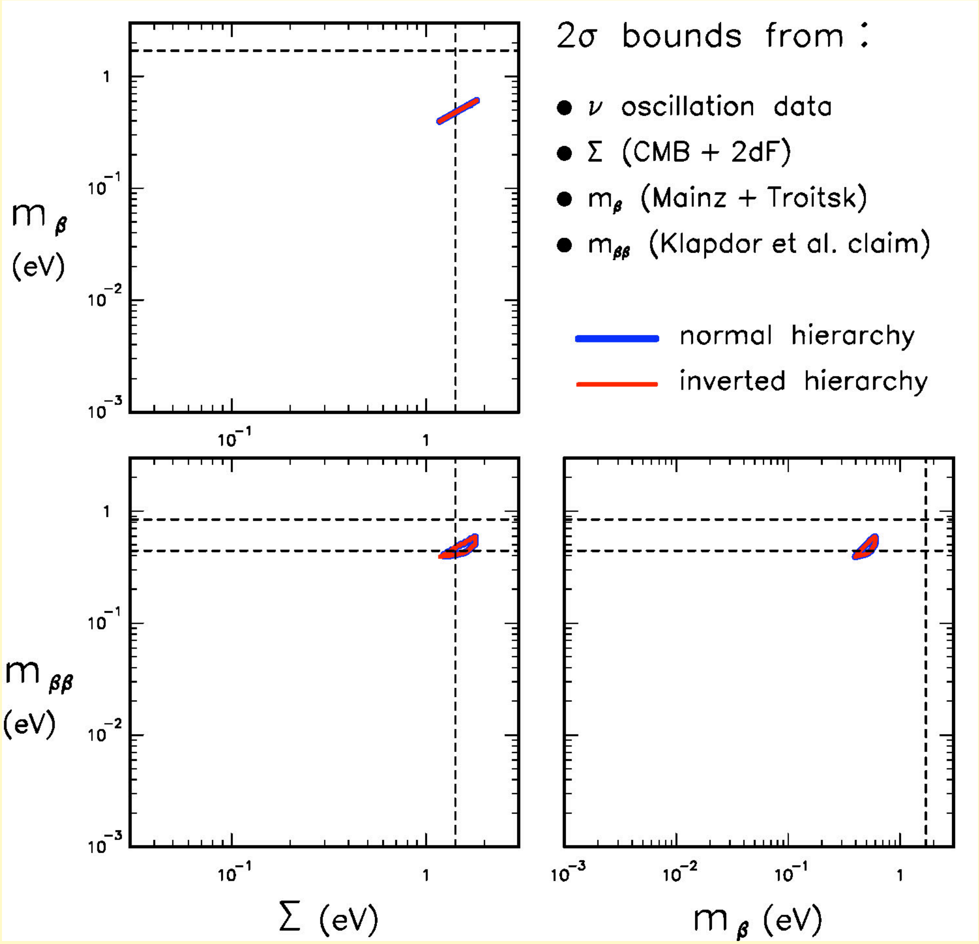
Cosmological bound dominates, but does not probe hierarchy yet

$0\nu2\beta$ claim accepted



Tension with cosmological bound
(no combination possible at face value)
But: too early to draw definite conclusions

E.g., if 0ν2β claim accepted but SDSS Lyα data discarded:



Combination of all data (osc+nonosc.) possible

Complete overlap for the two hierarchies (degenerate spectrum with "large" masses: $m_{1,2,3} \sim 0.5$ eV)

High discovery potential in future (m_β, $m_{\beta\beta}$, Σ) searches

Summary and Conclusions

- We have entered the era of precision neutrino physics. Consensus about conventions, mass-mixing parameter notation, C.L. contours, and graphical presentations is desirable for uniform comparison

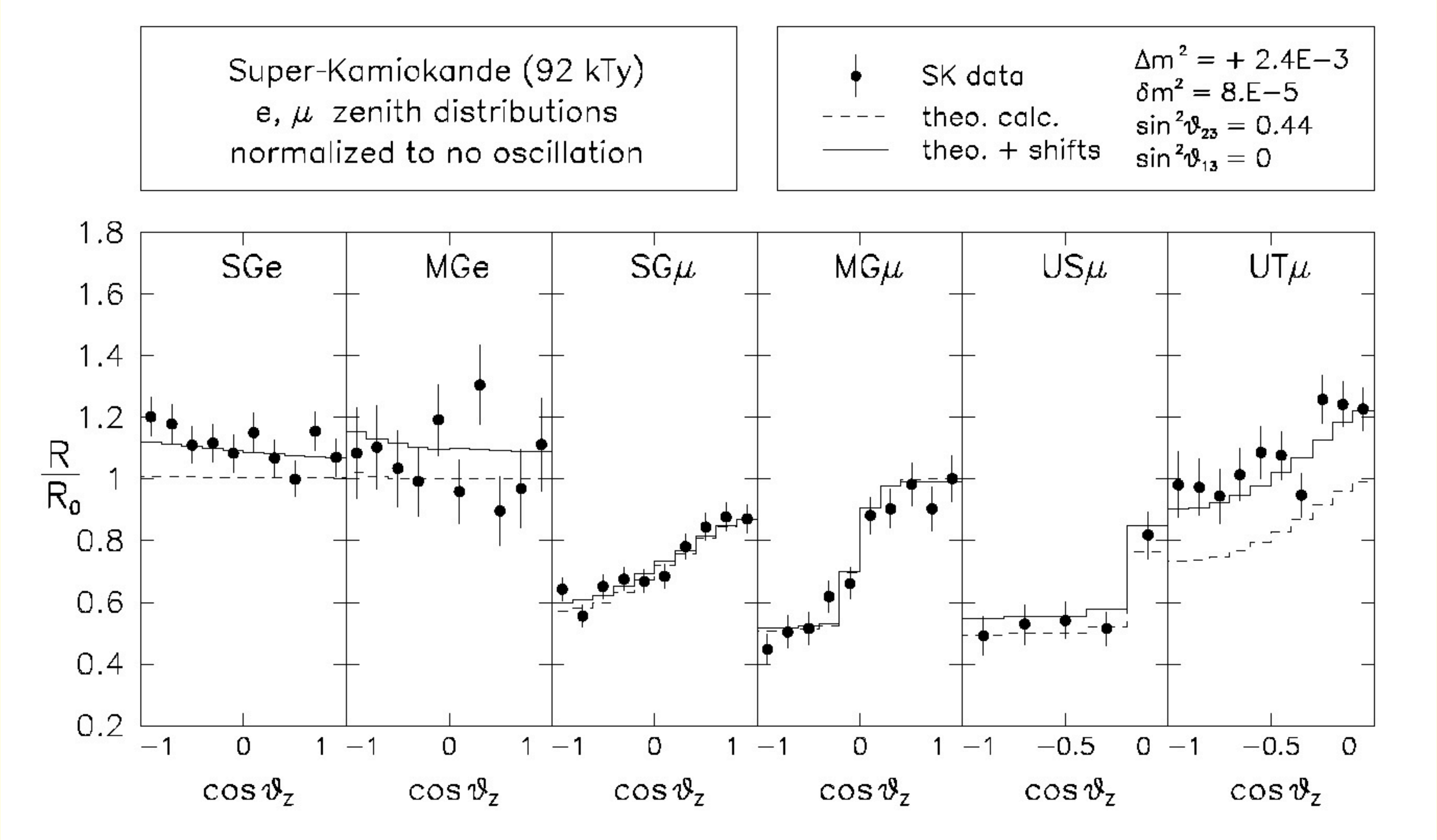
- Within the standard 3ν framework, remarkable consistency of oscillation data (except LSND) with parameters: (but sensitivity to $S_{13}\neq 0$, hierarchy, and δ_{CP} requires future searches)

$$\delta m^2 = 7.92\,(1^{+0.09}_{-0.09}) \times 10^{-5}\ \mathrm{eV}^2$$
$$\Delta m^2 = 2.4\,(1^{+0.21}_{-0.26}) \times 10^{-3}\ \mathrm{eV}^2$$
$$\sin^2\theta_{12} = 0.314\,(1^{+0.18}_{-0.15})$$
$$\sin^2\theta_{23} = 0.44\,(1^{+0.41}_{-0.22})$$
$$\sin^2\theta_{13} < 3.2 \times 10^{-2}$$

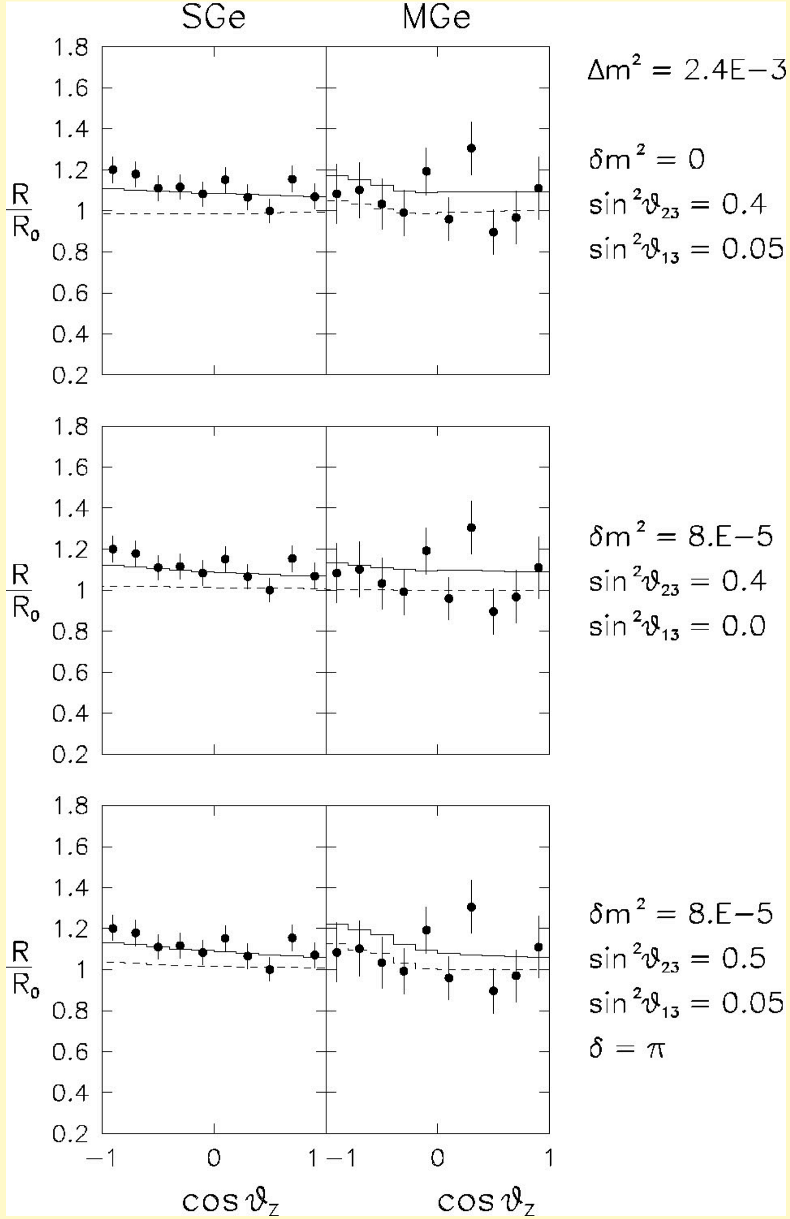
- Combination with observables sensitive to absolute ν masses (m_β, $m_{\beta\beta}$, Σ) needs further understanding and new measurements

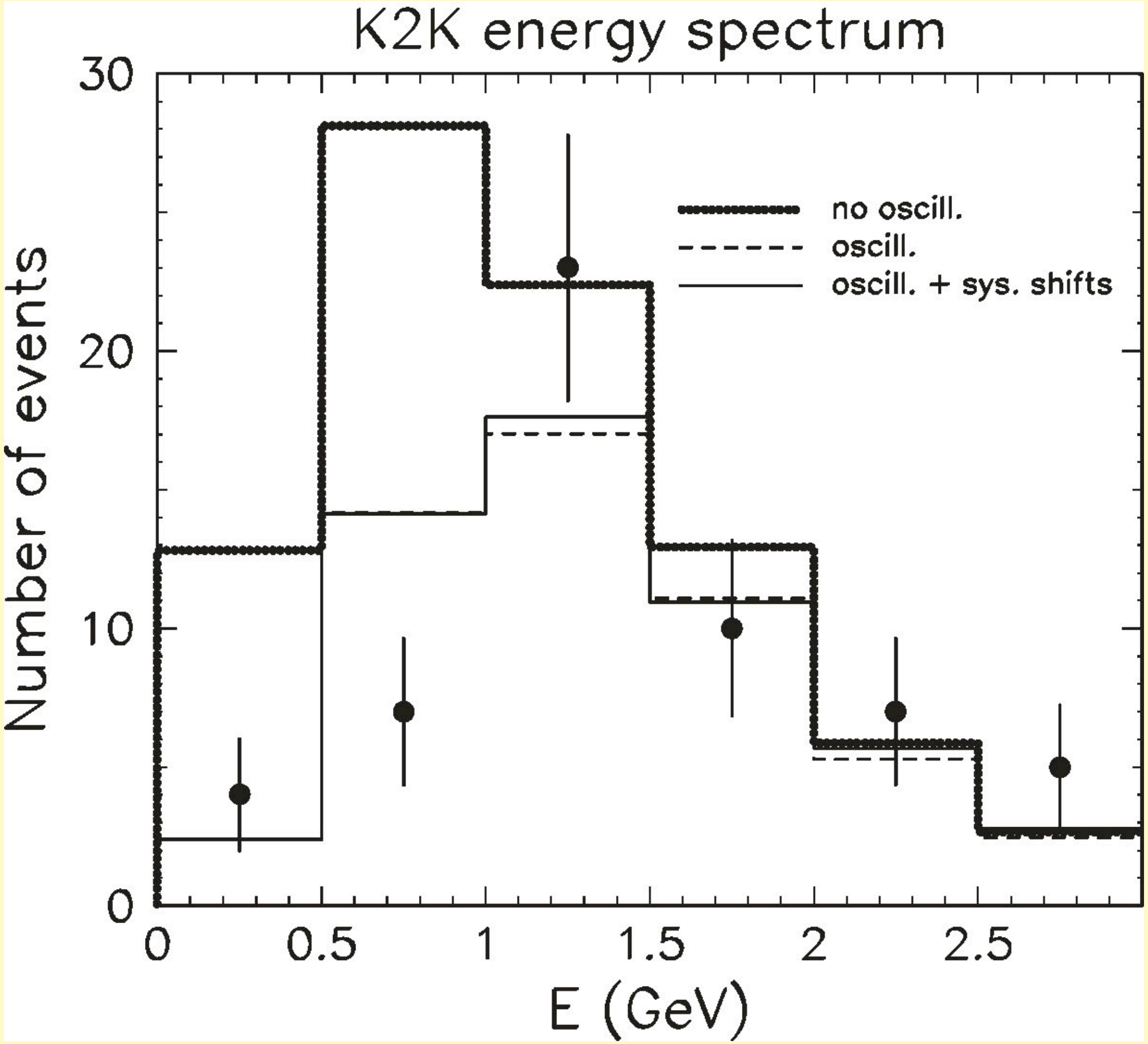
- Impressive and rapid progress in ν physics in the last few years; but exciting challenges and possible surprises are ahead of us

Backup slides

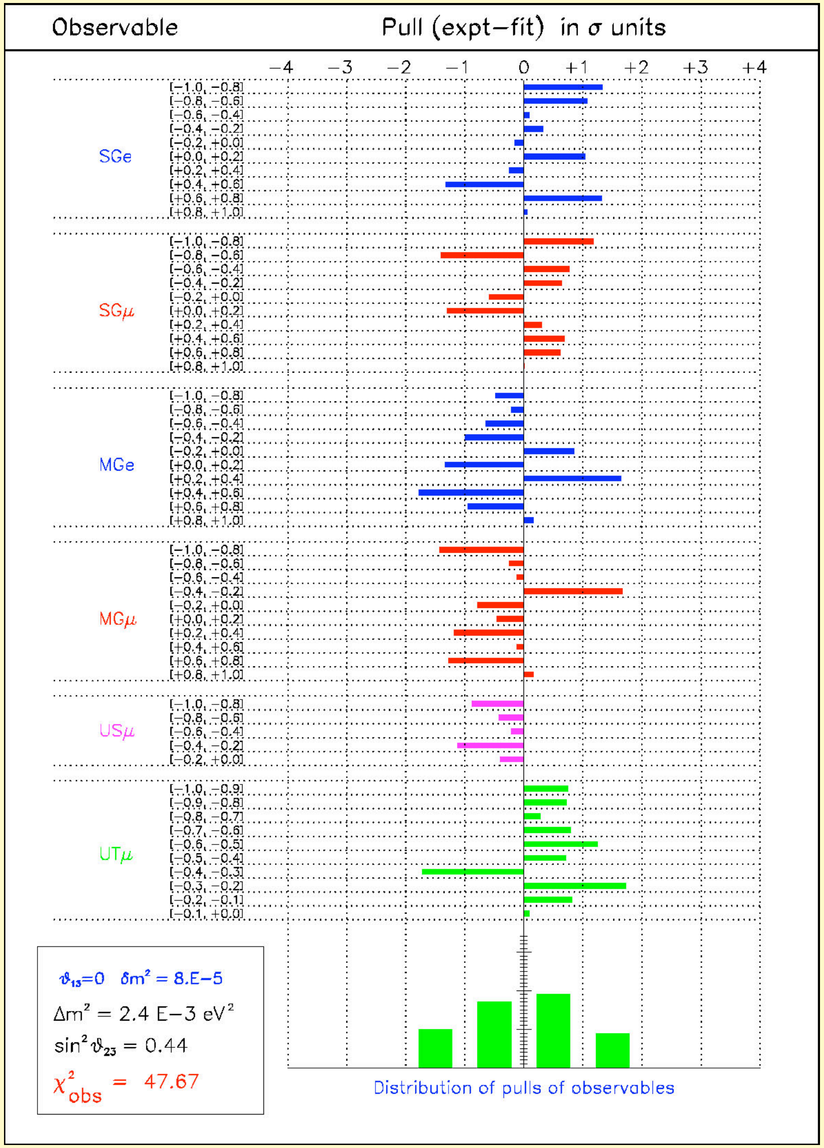


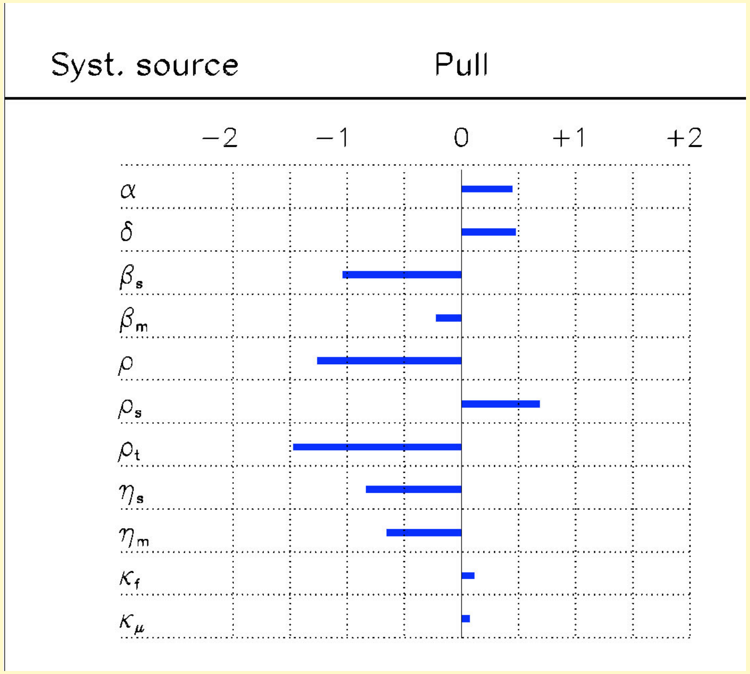
Super-Kamiokande (92 kTy)
e, μ zenith distributions
normalized to no oscillation
SK data
theo. calc.
theo. + shifts
Δm² = + 2.4E−3
δm² = 8.E−5
sin²ϑ₂₃ = 0.44
sin²ϑ₁₃ = 0
SGe
MGe
SGμ
MGμ
USμ
UTμ
R/R₀
cos ϑz

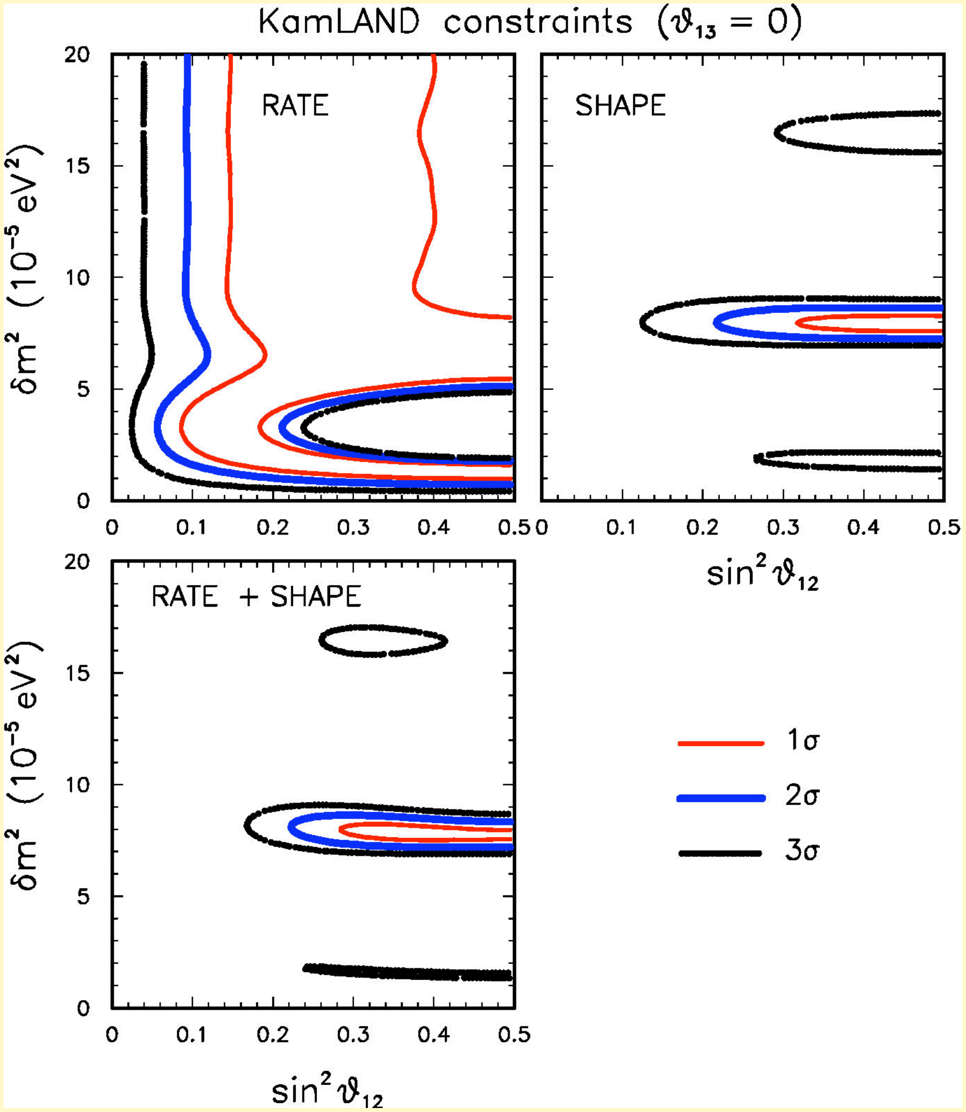

SGe
MGe
R/R0
cos ϑz
Δm² = 2.4E−3
δm² = 0
sin²ϑ23 = 0.4
sin²ϑ13 = 0.05
δm² = 8.E−5
sin²ϑ23 = 0.4
sin²ϑ13 = 0.0
δm² = 8.E−5
sin²ϑ23 = 0.5
sin²ϑ13 = 0.05
δ = π

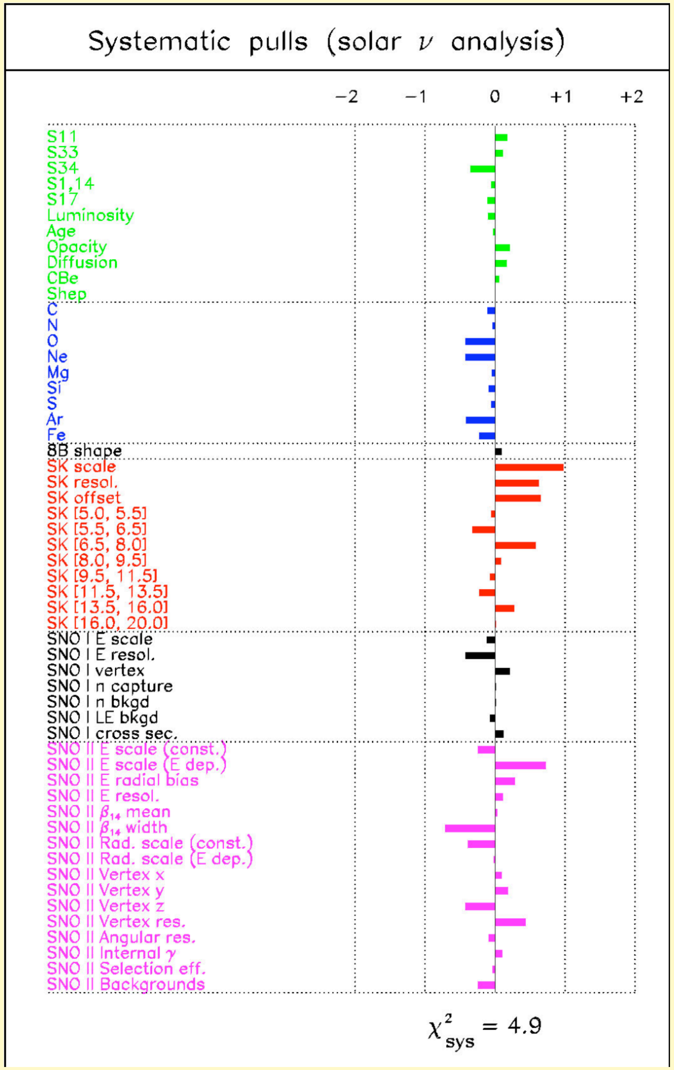


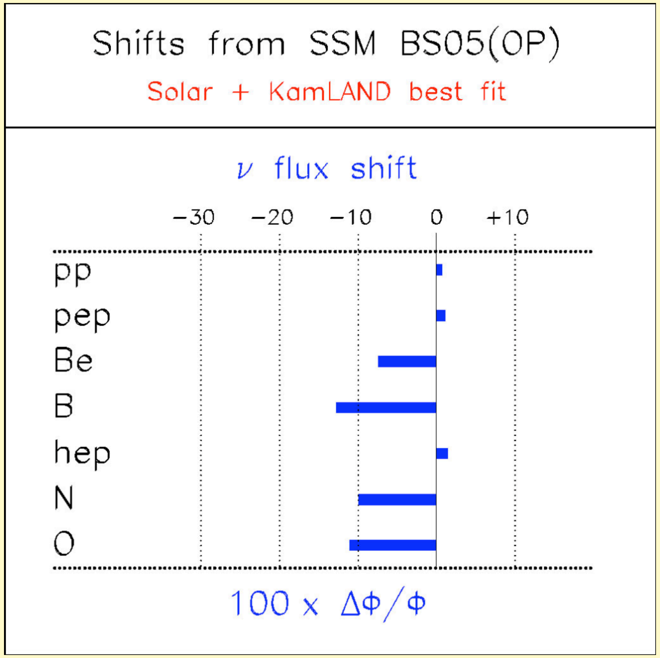
K2K energy spectrum
Number of events
E (GeV)
0
10
20
30
0
0.5
1
1.5
2
2.5
no oscill.
oscill.
oscill. + sys. shifts

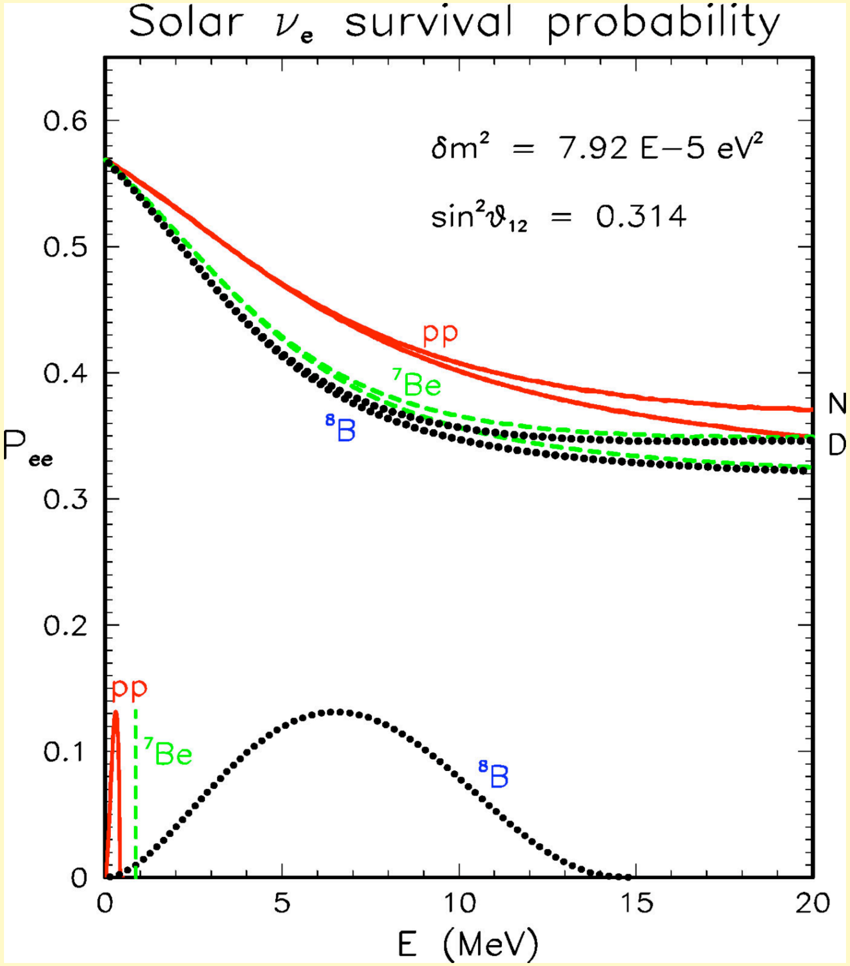

Observable
Pull (expt−fit) in σ units
−4 −3 −2 −1 0 +1 +2 +3 +4
SGe
SGμ
MGe
MGμ
USμ
UTμ
ϑ13=0 δm² = 8.E−5
Δm² = 2.4 E−3 eV²
sin²ϑ23 = 0.44
χ²obs = 47.67
Distribution of pulls of observables


Syst. source
Pull
−2 −1 0 +1 +2
α
δ
βs
βm
ρ
ρs
ρt
ηs
ηm
κf
κμ

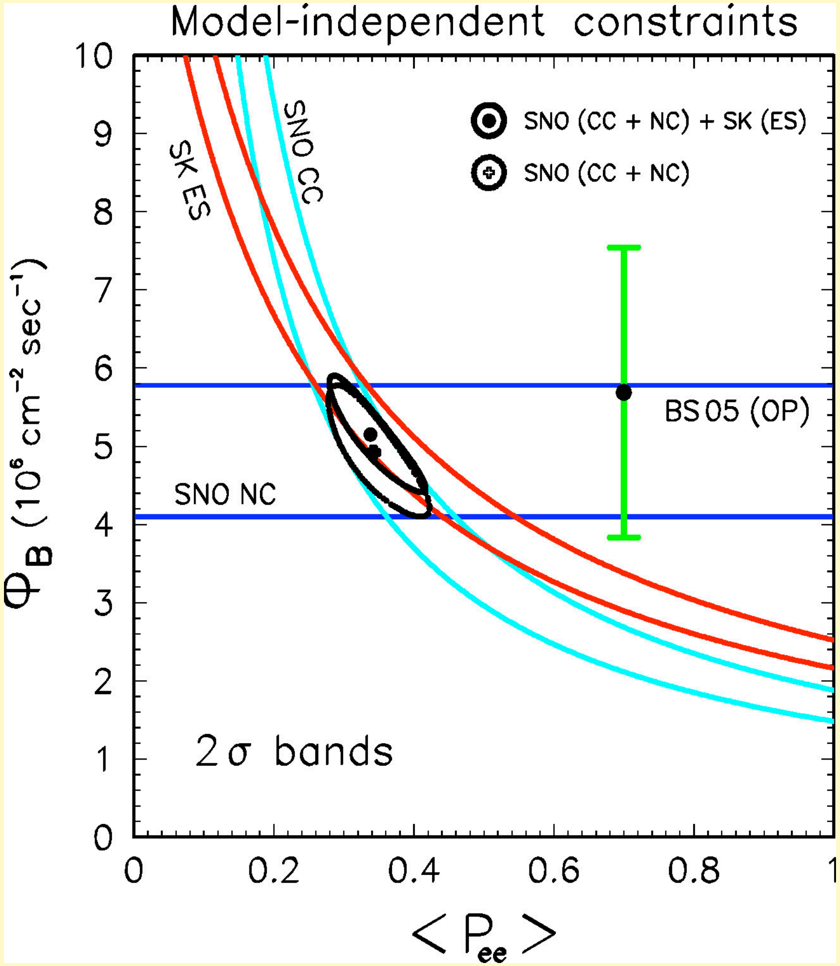

KamLAND constraints (ϑ_{13} = 0)
RATE
SHAPE
RATE + SHAPE
δm^2 (10^{-5} eV^2)
$\sin^2\vartheta_{12}$
1σ
2σ
3σ
0
5
10
15
20
0
0.1
0.2
0.3
0.4
0.5


Systematic pulls (solar ν analysis)
−2 −1 0 +1 +2
S11
S33
S34
S1,14
S17
Luminosity
Age
Opacity
Diffusion
CBe
Shep
C
N
O
Ne
Mg
Si
S
Ar
Fe
8B shape
SK scale
SK resol.
SK offset
SK [5.0, 5.5]
SK [5.5, 6.5]
SK [6.5, 8.0]
SK [8.0, 9.5]
SK [9.5, 11.5]
SK [11.5, 13.5]
SK [13.5, 16.0]
SK [16.0, 20.0]
SNO I E scale
SNO I E resol.
SNO I vertex
SNO I n capture
SNO I n bkgd
SNO I LE bkgd
SNO I cross sec.
SNO II E scale (const.)
SNO II E scale (E dep.)
SNO II E radial bias
SNO II E resol.
SNO II β14 mean
SNO II β14 width
SNO II Rad. scale (const.)
SNO II Rad. scale (E dep.)
SNO II Vertex x
SNO II Vertex y
SNO II Vertex z
SNO II Vertex res.
SNO II Angular res.
SNO II Internal γ
SNO II Selection eff.
SNO II Backgrounds
χ²sys = 4.9


Shifts from SSM BS05(OP)
Solar + KamLAND best fit
ν flux shift
−30 −20 −10 0 +10
pp
pep
Be
B
hep
N
O
100 x ΔΦ/Φ



Solar ν_e survival probability
$\delta m^2 = 7.92$ E−5 eV^2
$\sin^2\vartheta_{12} = 0.314$
pp
^{7}Be
^{8}B
N
D
P_{ee}
0.6
0.5
0.4
0.3
0.2
0.1
0
pp
^{7}Be
^{8}B
0
5
10
15
20
E (MeV)



Model-independent constraints
SNO (CC + NC) + SK (ES)
SNO (CC + NC)
SK ES
SNO CC
BS05 (OP)
SNO NC
2σ bands
Φ_B (10^6 cm^{-2} sec^{-1})
10
9
8
7
6
5
4
3
2
1
0
0
0.2
0.4
0.6
0.8
1
< P_{ee} >