Recent and future measurements of the proton electric form factor

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Workshop on Nucleon Form Factors
Elastic Electron-Nucleon Scattering

Nucleon vertex:
\[ \Gamma_\mu(p', p) = F_1(Q^2) \gamma_\mu + \frac{i\kappa_p}{2M_p} F_2(Q^2) \sigma_{\mu\nu} q^\nu \]

\[ \begin{align*} 
G_E(Q^2) &= F_1(Q^2) - \kappa_N \tau F_2(Q^2) \\
G_M(Q^2) &= F_1(Q^2) + \kappa_N F_2(Q^2), \quad \tau = \frac{Q^2}{4M_N} 
\end{align*} \]

At \( Q^2 = 0 \):
\[ \begin{align*} 
G_{Mp} &= 2.79 & G_{Mn} &= -1.91 \\
G_{Ep} &= 1 & G_{En} &= 0 
\end{align*} \]
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At \( Q^2 = 0 \)
- \( G_{Mp} = 2.79 \)
- \( G_{Mn} = -1.91 \)
- \( G_{Ep} = 1 \)
- \( G_{En} = 0 \)

Extract \( G_E^2 \) and \( G_M^2 \) from:
- Cross-section measurements \( N(e, e') \)

Extract \( G_E/G_M \) from:
- Beam-target Asymmetries \( \bar{N}(\bar{e}, e')N \)
- Recoil polarization \( N(\bar{e}, e')\bar{N} \)

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Rosenbluth separation technique

$ep$ elastic cross-section:

$$\sigma_r \propto \frac{\epsilon}{\tau} \left( \frac{G_E}{G_D} \right)^2 + \left( \frac{G_M}{G_D} \right)^2$$

$$G_D = (1 + Q^2/.71)^{-2} \quad \tau = Q^2/4M^2$$

At fixed $Q^2$: Vary $\epsilon$

$G_E$ is slope
$G_M$ is intercept
Rosenbluth separation technique

$ep$ elastic cross-section:

$$\sigma_T \propto \frac{\epsilon}{\tau} \left( \frac{G_E}{G_D} \right)^2 + \left( \frac{G_M}{G_D} \right)^2$$

$G_D = (1 + Q^2/0.71)^{-2}$ $\tau = Q^2/4M^2$

At fixed $Q^2$: Vary $\epsilon$

- $G_E$ is slope
- $G_M$ is intercept

For $Q^2 > 1$ measuring $G_E$ becomes more difficult

- $G_E$ becomes a smaller fraction of $\sigma$
- At $Q^2 = 5$, $G_E$ maximum 8% contribution to $\sigma$

(assuming $\mu G_E/G_M = 1$)
Proton Form Factors: $G_M$ and $G_E$

At all $Q^2$

$G_M$ well measured

At $Q^2 > 1$

Error on $G_E$ large
Proton Form Factors: \(G_M\) and \(G_E\)

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Recent global fit to world cross section data

Proton Form Factors: \( G_M \) and \( G_E \)

At all \( Q^2 \)
- \( G_M \) well measured

At \( Q^2 > 1 \)
- Error on \( G_E \) large

Recent global fit to world cross section data

Recent Hall C data
M. E. Christy, PRC 70, 015206 (2004)
JLab Recoil Polarization Experiments

1st JLab experiment

Electrons detected HRS-L
Protons detected HRS-R

Covered $Q^2$ from 0.5 to 3.5 GeV$^2$. 
2nd JLab experiment  Protons detected HRS-L

Covered $Q^2$ from 3.5 to 5.6 GeV$^2$. 
JLab Recoil Polarization Experiments

2nd JLab experiment
Protons detected HRS-L
Electrons detected calorimeter

Covered $Q^2$ from 3.5 to 5.6 GeV$^2$. 
Spin Transfer Reaction $^1\text{H}(\vec{e}, e'\vec{p})$

Helicity dependent: $P_N, P_T, P_L$

$P_N = 0$ for elastic

In $1\gamma$ approx.

Helicity independent $P_N = 0$

\[
\frac{G_E}{G_M} = -\frac{P_T}{P_L} \frac{E_e + E_{e'}}{2M} \tan\left(\frac{\theta_e}{2}\right)
\]

- Experiments detect the electron and proton in coincidence
- Proton spin measured by second scattering in polarimeter
- Experiments done at MIT-Bates, Mainz and JLab
Spin precession

FPP measures $P_{N}^{fp}, P_{T}^{fp}$

Simple dipole magnet

$$P_{T}^{fp} = P_{T}^{tgt}$$
$$P_{N}^{fp} = P_{L}^{tgt} \sin \chi$$

In general

$$P_{T}^{fp} = S_{tt} P_{T}^{tgt} + S_{tl} P_{L}^{tgt}$$
$$P_{N}^{fp} = S_{nt} P_{T}^{tgt} + S_{nl} P_{L}^{tgt}$$

$S_{tt}, S_{tl}, S_{nt}, S_{nl}$ calculated from model of spectrometer

Precession angle

$$\chi = \gamma \kappa_{p} \theta_{b}$$

For Hall A HRS $\theta_{b} = 45^\circ$

For $Q^{2} = 2.2$, $\chi = 180^\circ$
Focal Plane Polarimeter

Measure $N^\pm(\theta, \phi)$, $\phi$ distribution of protons scattered in an analyzer ($A_c$) for each beam helicity ($h$)

$$N_D = \frac{1}{2} \left[ \frac{N^+(\theta, \phi)}{N_o^+(\theta)} - \frac{N^-(\theta, \phi)}{N_o^-(\theta)} \right] = h A_c \left[ P_N^{fp} \cos \phi + P_T^{fp} \sin \phi \right]$$
FPP $\phi$ Distributions

$Q^2 = 4.0 \text{ GeV}^2$

Simple dipole magnet

$$\frac{P_T^{tgt}}{P_L^{tgt}} = \frac{P_T^{fp}}{P_N^{fp}} \sin \chi$$

$hA_c$ cancels!

$$N_D = hA_c \left[ P_N^{fp} \cos \phi + P_T^{fp} \sin \phi \right]$$
$G_E/G_M$ from recoil polarization

Global Fit to xsec data

MIT-Bates experiment
\( \frac{G_E}{G_M} \) from recoil polarization

- **Global Fit to xsec data**
- **MIT-Bates experiment**
- **1st JLab experiment**
  - Used two HRS

\[
\mu \frac{G_E}{G_M}
\]

\( Q^2 \) (GeV^2)

- **Recoil Pol. Systematic**
- **MIT-Bates (1998)**
- **JLab (2000)**
\( \frac{G_E}{G_M} \) from recoil polarization

- MIT-Bates experiment
- 1st JLab experiment
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- 2nd JLab experiment
  - Used HRS+Calo

\( Q^2 (\text{GeV}^2) \)

Recoil Pol. Systematic

Global Fit to xsec data

JLab (2000)
JLab (2002)
$G_E/G_M$ from recoil polarization

- Global Fit to xsec data
- MIT-Bates experiment
- 1st JLab experiment
  - Used two HRS
- 2nd JLab experiment
  - Used HRS+Calo
- Mainz experiment

Recoil Pol. Systematic

Nucleon05 – p.10/26
\( G_E/G_M \) from recoil polarization

- MIT-Bates experiment
- 1st JLab experiment
  Used two HRS
- 2nd JLab experiment
  Used HRS+Calo
- Mainz experiment
- Secondary part of other JLab experiments.
At JLab in Hall A did Rosenbluth separation

Single arm proton detection

Advantages:

- Proton momentum fixed at each $\epsilon$
- Rate is nearly constant with $\epsilon$
- Reduces size of $\epsilon$-dependent radiative corrections
- Reduces systematic error from beam energy and scattering angle

I. Qattan et al. PRL 94, 142301 (2005)
Plot $\sigma_r$ versus $\epsilon$

- Solid black line is fit to data
- Dashed blue line is from a fit to previous cross section data.
- Dotted red line is using $G_E/G_M$ from recoil polarization

Systematic error on slope is 0.55%
Comparison of $G_E/G_M$

- Hall A Rosenbluth $G_E/G_M$ agrees with previous cross section measurements.
- Discrepancy between $G_E/G_M$ from recoil polarization and from cross section measurement persistent.
Comparison of $G_E/G_M$

Coulomb correction (soft multi-$\gamma$) is small correction to xsec data

Rosenbluth method

Coulomb corrected

Global fit

Recoil polarization

Recoil Pol. Systematic

$Q^2$ (GeV$^2$)

$\mu G_E/G_M$
Comparison of $G_E/G_M$

- Coulomb correction (soft multi-$\gamma$) is small correction to xsec data
- Missing physics from $2\gamma$ exchange?

$$\sigma_r \propto \frac{\epsilon}{\tau} G_E^2 + G_M^2 + \epsilon \sigma_{2\gamma}(\epsilon, Q^2)$$

Explain discrepancy with

$$\sigma_{2\gamma}(\epsilon, Q^2) \sim 6\%$$

with small $\epsilon, Q^2$ dependence
2\(\gamma\) exchange contribution

\[ p_1 \rightarrow k \downarrow q-k \rightarrow p_3 \]
\[ p_2 \rightarrow \rightarrow p_4 \]

- Nucleon elastic intermediate state \textit{P.G. Blunden, W. Melnitchouk, J.A. Tjon}
- GPD calculation \textit{A. V. Afanasev, S. J. Brodsky, C. E. Carlson, Y. Chen, M. Vanderhaeghen}
- Various proposals to measure 2\(\gamma\) exchange contribution
  - \(\epsilon\) dependence of \(\sigma_{e^+p}/\sigma_{e^-p}\)
  - Precision \(\epsilon\) dependence of \(\sigma_{e^-}\)
  - \(\epsilon\) dependence of \(P_T/P_L\).
- Talks in the afternoon 2\(\gamma\) session
Future $G_E/G_M$ at JLab in Hall C

- At JLab to reach higher $Q^2$ need to detect protons in the Hall C HMS
- As in Hall A, to maximize rate need to use large calorimeter to detect electron
- At Dubna measured $A = 0.05$ for $CH_2$ at proton momentum 5.3 GeV/c and test various thickness of $CH_2$
- From these tests decided on using a double polarimeter in Hall C.
Future $G_E/G_M$ at JLab in Hall C

- FPP has been built at Dubna and will be installed in the HMS.
- A large calorimeter has been assembled with help of IHEP and Yerevan.
- Scheduled to run in 2007.
Future $G_E/G_M$ at JLab in Hall C

- FPP has been built at Dubna and will be installed in the HMS.
- A large calorimeter has been assembled with help of IHEP and Yerevan.
- Scheduled to run in 2007.
- With the 12 GeV upgrade at JLab can reach $Q^2 = 14$. 

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![Graph showing $G_E/G_M$ versus $Q^2$](#)

- **MIT-Bates (1998)**
- **Mainz (2001)**
- **JLab (2001)**
- **JLab (2003)**
- **JLab (2000)**
- **JLab (2002)**
- **JLab Rosenbluth (2004)**
- **Global Fit**

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*Upcoming Hall C exp*
Summary

Proton $G_E/G_M$ has been measured to $Q^2 = 5.6$
- In Hall C at JLab 2007
Measure $G_E/G_M$ at $Q^2 = 7.5$ and $9 \text{ GeV}^2$

Precision data on $G_E/G_M$ from recoil polarization and cross section measurements disagree
- Need to include $2\gamma$ contributions when extracting $G_E/G_M$ from cross section measurements.
- Need other experiments to study $2\gamma$

Combined with precision data on the neutron has stimulated study of nucleon structure
- Relativistic effects
- Angular momentum
- Constituent quark models
Polarized target experiments

\[ \frac{\mu_E}{G_M} \]

\( Q^2 \) (GeV\(^2\))

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIT-Bates</td>
<td>1998</td>
</tr>
<tr>
<td>Mainz</td>
<td>2001</td>
</tr>
<tr>
<td>JLab (2003)</td>
<td></td>
</tr>
<tr>
<td>JLab (2000)</td>
<td></td>
</tr>
<tr>
<td>JLab (2002)</td>
<td></td>
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<tr>
<td>RSS Hall C</td>
<td></td>
</tr>
<tr>
<td>JLab Rosenbluth</td>
<td>2004</td>
</tr>
<tr>
<td>Global Fit</td>
<td></td>
</tr>
<tr>
<td>SLAC</td>
<td></td>
</tr>
</tbody>
</table>

Hall C RSS

SLAC
**$F_2/F_1$ and pQCD**

- $F_1 \sim 1/Q^4$, $F_2$ is helicity-flip amplitude and expected to have extra $1/Q^2$ dependence.

- Brodsky, Hiller and Hwang propose a pQCD-motivated fit with form: 
  \[ \frac{F_2(Q^2)}{F_1(Q^2)} \sim \frac{\kappa_p}{1+(Q^2/c) \ln^b(1+Q^2/a)} \]
  with $a, b, c$ fit to the data. The extra logarithmic factor is motivated by expected higher twist.

- Belitsky, Ji and Yuan is a pQCD calc which has quark angular momentum and gluon polarization as the dominant mechanism for helicity-flip.

  \[ \frac{F_2(Q^2)}{F_1(Q^2)} \sim \frac{\ln(Q^2/\Lambda^2)}{Q^2} \]

  $\Lambda$ is the soft scale related to the nucleon's size.

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[Chart showing data points and fits for $F_2/F_1$ vs. $Q^2$]
Systematic error

TABLE VII: Dominant contributions to the systematic uncertainty of $\mu_pG_{Ep}/G_{Mp}$, which results from the variations in the quantities listed in the table. The total systematic uncertainties $\Delta_{sys}$ given in Table VI are obtained from the values below in quadrature.

<table>
<thead>
<tr>
<th>$Q^2$ GeV$^2$</th>
<th>$\theta - \theta^{fp}$</th>
<th>$\phi - \phi^{fp}$</th>
<th>$\phi^d, \Delta \phi^d$</th>
<th>$\phi^{fp}$</th>
<th>$\Delta_{sys}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.49</td>
<td>-0.004</td>
<td>0.001</td>
<td>0.001</td>
<td>-0.004</td>
<td>0.006</td>
</tr>
<tr>
<td>0.79</td>
<td>-0.009</td>
<td>0.001</td>
<td>0.003</td>
<td>-0.004</td>
<td>0.010</td>
</tr>
<tr>
<td>1.18</td>
<td>-0.017</td>
<td>0.002</td>
<td>0.004</td>
<td>-0.003</td>
<td>0.018</td>
</tr>
<tr>
<td>1.48</td>
<td>-0.025</td>
<td>0.002</td>
<td>0.004</td>
<td>-0.005</td>
<td>0.026</td>
</tr>
<tr>
<td>1.77</td>
<td>-0.034</td>
<td>0.002</td>
<td>0.006</td>
<td>-0.002</td>
<td>0.035</td>
</tr>
<tr>
<td>1.88</td>
<td>-0.032</td>
<td>0.002</td>
<td>0.006</td>
<td>-0.005</td>
<td>0.033</td>
</tr>
<tr>
<td>2.13</td>
<td>-0.032</td>
<td>0.003</td>
<td>0.009</td>
<td>-0.006</td>
<td>0.034</td>
</tr>
<tr>
<td>2.47</td>
<td>0.030</td>
<td>0.003</td>
<td>0.010</td>
<td>-0.009</td>
<td>0.033</td>
</tr>
<tr>
<td>2.97</td>
<td>0.018</td>
<td>0.004</td>
<td>0.009</td>
<td>0.003</td>
<td>0.021</td>
</tr>
<tr>
<td>3.47</td>
<td>0.009</td>
<td>0.005</td>
<td>0.010</td>
<td>-0.001</td>
<td>0.014</td>
</tr>
</tbody>
</table>

- $\theta - \theta^{fp}$ is bend angle in dispersive plane
- $\phi - \phi^{fp}$ is bend angle in nondispersive plane
- $\phi^d$ is mean angle in dipole
- $\Delta \phi^d$ is half of the bending angle in nondispersive plane
- $\phi^{fp}$ is the azimuthal angle of the scattering proton in the analyzer

Used geometrical model of HRS to estimate error in magnetic model.
Focal plane $P_N$ at $Q^2 = 2.2$

For $Q^2 = 2.2$ expect $P_{fp}^N$ to be zero at $\chi = 180^\circ$
$P_T / P_L$ versus target quantities

- **Solid circle:** Final $P_T / P_L$ from full COSY model of HRS
- **Open circle:** $P_T / P_L$ assuming the HRS is a simple dipole
Transverse polarization

- Open triangles: $P_T$ calculated assuming $\mu G_E/G_M = 1$.
- Open circles: $P_T$ extracted from data assuming that magnet is a simple dipole.
- Solid circles: Final result from $P_T$ extracted from data.
Analyzing power

- Solid Triangles: Data from Hall A experiment
- Solid Lines: Parametrizations of analyzing power data from hadron experiments.
- Open squares: Analyzing power data from hadron experiments.
Estimate of $2\gamma$ exchange contribution

$$\Gamma_\mu(p', p) = \tilde{G}_M \gamma_\mu - \tilde{F}_2 \frac{P^u}{M} + \tilde{F}_3 \frac{\gamma \cdot K P^u}{M^2}$$

$$\tilde{G}_M = G_M + \delta \tilde{G}_M, \quad \tilde{F}_2 = F_2 + \delta \tilde{F}_2, \quad \tilde{F}_3 \text{ purely from } 2\gamma$$

$$\sigma_R \sim \frac{\tilde{G}_M^2}{\tau} \left\{ \tau + \epsilon \frac{\tilde{G}_E^2}{G_M^2} + 2\epsilon (\tau + \frac{\tilde{G}_E}{G_M}) \mathcal{R}\left(\frac{\nu \tilde{F}_3}{M^2 G_M}\right) \right\}$$

$$\frac{P_T}{P_L} \sim - \sqrt{\frac{2\epsilon}{\tau(1+\epsilon)}} \left\{ \frac{\tilde{G}_E}{G_M} + (1 - \frac{2\epsilon}{1+\epsilon}) \frac{\tilde{G}_E}{G_M} \mathcal{R}\left(\frac{\nu \tilde{F}_3}{M^2 G_M}\right) \right\}$$

To explain discrepancy need $\mathcal{R}(\frac{\nu \tilde{F}_3}{M^2 G_M}) \sim 3\%$ with small $Q^2$ and $\epsilon$ dependence. P.A.M. Guichon and M. Vanderhaegen, PRL (2003)
Calculation $2\gamma$ exchange contribution

Nucleon elastic intermediate state

\[ \frac{\sigma_R}{G_D^2} \] vs $\varepsilon$

P.G. Blunden, W. Melnitchouk, J.A. Tjon, nucl-th/0506039
Calculation 2$\gamma$ exchange contribution

Nucleon elastic intermediate state

P.G. Blunden, W. Melnitchouk, J.A. Tjon, nucl-th/0506039

GPD calculation

Cross section for ep elastic scattering