Axial form factors of the nucleon and low Q² pion electroproduction

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Physics Motivation

Axial form factors Multipoles

Experiment

LT separation Kinematics Particle identification

Results

Comparison with photoproduction Short-Orbit Spectrometer Summary



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- How we "see" the nucleon depends on the probe
- Vector current γ_{μ} : G_E , G_M Sachs form factors \rightarrow elastic electron scattering
- Axial current $\gamma_{\mu}\gamma_{5}$: G_{A} , G_{P} (isovector) axial form factors \rightarrow neutrino scattering
 - \rightarrow pion electroproduction close to threshold
- Matrix element of axial current between two nucleons:

$$\left\langle N(P_f)|J_{5\mu}^{\alpha}|N(P_i)\right\rangle = \bar{u}_{P_f} \left[G_{\mathcal{A}}(t)\gamma_{\mu}\gamma_5 + \frac{(P_f - P_i)_{\mu}}{2m}G_P(t)\gamma_5\right] \frac{\tau^{\alpha}}{2}u_{P_i}$$

where $t=(P_f-P_i)^2=(q-k_\pi)^2$

Physics Motivation Axial form factor

Form factors G_A and G_P of the nucleon are not well known.

Axial form factor G_A

Coupling constant $G_A(Q^2=0)=g_A=1.2601\pm 0.0025$ known from neutron β decay.

Form factor usually parametrized by dipole formula

$$G_A(Q^2) = \frac{G_A(0)}{(1+Q^2/M_A^2)^2}$$

"Axial mass" parameter M_A determined from:

Neutrino-nucleon scattering:

 $M_A = 1.017 \pm 0.023 \, \text{GeV}/c^2$ (world average)

Pion electroproduction:

 $M_A = 1.068 \pm 0.017 \,\text{GeV/c}^2$ (world average)

The 2 σ discrepancy can be understood within ChPTh.



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The 2 σ discrepancy can be understood within ChPTh. [Bernard, Kaiser, Meißner: PRL 69 (1992) 1877, PRL 72 (1994) 2810]



Axial mass M_A

from neutrino scattering experiments



Axial mass M_A

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Physics Motivation Induced pseudoscalar form factor

- Induced pseudoscalar form factor G_P Coupling constant G_P(Q² = 0.88 m²_µ) = g_p determined from:
 - Muon capture $\mu^- p \rightarrow \nu_{\mu} n$: $g_p = 8.7 \pm 1.9$. This is in agreement with theory (PCAC): $q_p = 8.21 \pm 0.09$.
 - However: Radiative muon capture (TRIUMF)

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\mu^- p \rightarrow \nu_\mu n \gamma gives g_p = 12.3 \pm 0.9
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[Jonkmans et al.: PRL 77 (1996) 4512]

New result: $g_p = 10.6 \pm 1.1$ by reanalysing TRIUMF data with new Λ_{op} value [Clark et al.: nucl-ex/0509025]

Form factor $G_P(Q^2)$:

Only one pion electroproduction experiment so far: SACLAY 1993 (However, no LT separation)

[Choi et al.: PRL 71 (1993) 3927]

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Experimental Method Kinematics and cross section

Kinematics of the ${}^{1}H(e, e'\pi^{+})n$ reaction:



$$\frac{d\sigma}{d\Omega dE' d\Omega_{\pi}^{*}} = \Gamma \frac{d\sigma_{\nu}}{d\Omega_{\pi}^{*}}$$

$$\frac{\mathrm{d}\sigma_{\nu}}{\mathrm{d}\Omega_{\pi}^{*}} = \frac{\mathrm{d}\sigma_{\mathrm{T}}}{\mathrm{d}\Omega_{\pi}^{*}} + \varepsilon_{\mathrm{L}}\frac{\mathrm{d}\sigma_{\mathrm{L}}}{\mathrm{d}\Omega_{\pi}^{*}} + \sqrt{2\varepsilon_{\mathrm{L}}(1+\varepsilon)}\frac{\mathrm{d}\sigma_{\mathrm{LT}}}{\mathrm{d}\Omega_{\pi}^{*}}\cos\varphi_{\pi}^{*} + \varepsilon\frac{\mathrm{d}\sigma_{\mathrm{TT}}}{\mathrm{d}\Omega_{\pi}^{*}}\cos(2\varphi_{\pi}^{*}) \quad \text{and} \quad \varepsilon_{\mathrm{LT}}^{*} = \frac{\mathrm{d}\sigma_{\mathrm{TT}}}{\mathrm{d}\Omega_{\pi}^{*}}\cos\varphi_{\pi}^{*} + \varepsilon_{\mathrm{LT}}^{*}\frac{\mathrm{d}\sigma_{\mathrm{TT}}}{\mathrm{d}\Omega_{\pi}^{*}}\cos(2\varphi_{\pi}^{*})$$

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Experimental Method Kinematics and cross section

Kinematics of the ${}^{1}H(e, e'\pi^{+})n$ reaction:



Differential cross section (unpolarized):

$$\frac{d\sigma}{d\Omega dE' d\Omega_{\pi}^{*}} = \Gamma \frac{d\sigma_{\nu}}{d\Omega_{\pi}^{*}}$$

where

$$\frac{d\sigma_{\nu}}{d\Omega_{\pi}^{*}} = \frac{d\sigma_{T}}{d\Omega_{\pi}^{*}} + \epsilon_{L} \frac{d\sigma_{L}}{d\Omega_{\pi}^{*}} + \sqrt{2\epsilon_{L}(1+\epsilon)} \frac{d\sigma_{LT}}{d\Omega_{\pi}^{*}} \cos \phi_{\pi}^{*} + \epsilon \frac{d\sigma_{TT}}{d\Omega_{\pi}^{*}} \cos(2\phi_{\pi}^{*}) \quad \text{Algorithms}$$

Parallel kinematics ($\theta_{\pi}^* = 0$): Interference terms σ_{LT} and σ_{TT} vanish. Rosenbluth separation of σ_L and σ_T by variation of virtual photon polarization ϵ_L . (σ_{LT} accessible by additional measurement in non-parallel kinematics, $\theta_{\pi}^* \neq 0$.)

Threshold limit: s-wave multipoles E_{0+} and L_{0+} :

$$\frac{d\sigma_T}{d\Omega_\pi^*} = \frac{|\mathbf{q}|}{k_\gamma^*} |E_{0+}|^2 \quad \text{and} \quad \frac{d\sigma_L}{d\Omega_\pi^*} = \frac{|\mathbf{q}|}{k_\gamma^*} |L_{0+}|^2$$



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Differential cross section (unpolarized):

Multipoles

 E_{0+} and L_{0+} in terms of electromagnetic and axial form factors

$$\begin{split} E_{0+}\left(k^{2}\right) &= \sqrt{1-\frac{k^{2}}{4m_{N}^{2}}} \, \frac{1}{\sqrt{2}f_{\pi}} \left\{ \frac{G_{A}\left(k^{2}\right) + \frac{k^{2}}{4m_{N}^{2}} \, G_{A}\left(0\right) G_{M}^{V}\left(k^{2}\right) + \delta(k^{2}) \right\} \\ L_{0+}\left(k^{2}\right) &= \sqrt{1-\frac{k^{2}}{4m_{N}^{2}}} \left[D(t) - 2m_{N} \, G_{A}\left(0\right) \right] \frac{\omega F_{\pi}(k_{\pi}^{2})}{\sqrt{2}m_{\pi} f_{\pi}(2m_{N}+m_{\pi})} \\ &+ \frac{\omega}{m_{\pi}} E_{0+}\left(m_{\pi}^{2}\right) \end{split}$$

with

$$D(t) = 2m_N G_A(t) + tG_P(t) = \frac{2f_\pi g_{\pi N} m_\pi^2}{m_\pi^2 - t} + 2(m_N G_A(0) - f_\pi g_{\pi N}) \frac{\lambda^2}{\lambda^2 - t}$$

D(t) contains free parameter λ to be determined by experiment, providing the connection between L_{0+} and $G_P(t).$

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Experiment

The method of LT separation was applied in our previous measurement: Liesenfeld, Richter, Širca et al., PLB **468** (1999) 20

which aimed at a precise extraction of the axial mass parameter M_A from the transverse cross section.

 $G_A(Q^2) = \frac{G_A(0)}{(1+Q^2/M_A^2)^2}$



LT separation

Separated transverse and longitudinal cross sections for $H(e, e'\pi^+)n$ from that measurement:



Solid line: "Theoretical" fit.

 $M_A=1.077\pm0.039\,\text{GeV}$

To pin down the low- Q^2 end of the transverse cross section, a rather imprecise value of $d\sigma_T$ at $Q^2 = 0$ had to be adopted from (scarce) low-energy photo-production data [SAID, MAID].

Dotted line: Unconstrained fit.

 $M_A=1.089\pm0.106\,\text{GeV}$

 \rightarrow Proposal A1/1-99 for an additional measurement at low Q²:

"From the viewpoint of the axial mass parameter, an additional data point in the vicinity of the photon point would significantly improve the reliability of the extraction since the theoretical fit is most sensitive to data points at lower values of Q^2 ."



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Measurements at W = 1125 MeV

- Aim: Low four-momentum transfer $Q^2 = 0.058 (GeV/c)^2$
- Reaction ${}^{1}H(e, e'\pi^{+})n$
- Measurement with the A1 standard setup
- Electron beam from MAMI accelerator, up to 20 μA beam current
- Liquid Hydrogen target, 5 cm long
- For LT separation, the lever arm in ϵ should be as large as possible.
- Data taking in 2000 and 2002

Dagmar Baumann, doctoral thesis (2005), to be published



Kinematic parameters



Control of systematic errors:

- Measurement with spectrometers exchanged
- Elastic reaction ${}^{1}H(e, e')$
- Spek. C as luminosity monitor

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Kinematic parameters



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A1 Three-Spectrometer Setup





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Particle Identification

Protons ($E_{kin} = 15$ MeV) in trigger:



Trigger is "proton blind"

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Particle Identification

Positrons and muons



Muons

Cuts in coincidence time spectrum:



Pion decay losses and muon background are determined by simulation.





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Corrections and systematic errors

Target density

• Pressure and temperature \rightarrow equation of state

Pion decay

- Determine decay factor for each accepted pion
- Global correction by mean value of decay factor

Cut in missing mass

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- Correction by simulation
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Results Differential cross sections from 2002 data taking



 open symbols: results from: Blomqvist et al., ZPA 353 (1996) 415 Liesenfeld et al., PLB 468 (1999) 20

solid symbols: new results



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Results Separated transverse and longitudinal cross sections



- Cross sections at low Q² lower than expected from previous experiments
- ▶ At higher Q² consistent with Liesenfeld et al.
- Curves show "old" fit from previously published results



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Results Comparison with photoproduction



Why another spectrometer?

- Experiments completed at $\Delta W = 46 \text{ MeV}$
- Contributions from p-wave multipoles
- ightarrow Measurement closer to threshold, $\Delta W =$ 5 MeV
 - Pion momentum decreases with CMS energy W
 - Charged pion: lifetime 26 ns
 - Path length in A1 spectrometers \approx 10 m
 - At $\Delta W = 5$ MeV about 90 % of pions decay
 - Systematic errors due to muon contamination

ightarrow Additional "short-orbit" spectrometer (SOS) for low-energy charged pions



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Schematic view

- Dipole magnet of Browne-Buechner type
- Short flight path of only 1.6 m minimizes pion decays
- Two focal planes (only one is equipped with detectors, for the time being)
- Momentum acceptance up to 147 MeV/c (at 1.4 T)
- Solid angle acceptance between 2 and 4 msr
- Minimal forward angle 15.4°



Goal: Measure pions with kinetic energies of only 16–32 MeV with an angular resolution of a few mrad.

- Trigger and particle identification
 - Range telescope of five consecutive layers of scintillator (3, 10, 20, 20 and 10 mm thickness).
 - Online particle ID by stopping / energy thresholds.
 - Positron veto by last scintillator layer.
 - Offline particle ID by $\Delta E_i / \Delta E_j$ cuts.
- Drift chamber
 - "Volume type" chamber.
 - Optimized for small effective thickness.
 - Cathode wires: 80 μm aluminium, silver coated.
 - Operated with helium-hydrocarbon mixture.
- Shielding house of lead and borated polyethylene



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Detector System





 Range telescope: Trigger and particle identification

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Drift chamber



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Test measurement

- Calibration with quasi-elastic process ${}^{12}C(e, e'p){}^{11}B$
- Electron detected in SOS
- Cut on ground state of residual ¹¹B nucleus
- ► Mass resolution 1.5 MeV/c² FWHM with first order transfer matrix



Matthias Ding, doctoral thesis

Experiments with 3-Spectrometer Setup

- Charged pion electroproduction ${}^{1}H(e, e'\pi^{+})n$ at W = 1125 MeV
- Three different Q² values (3 to 5 kinematical settings each) measured in one experiment
- Results indicate that the cross section at low Q² is considerably lower than expected from previous experiments
- Future experiments with Short-Orbit Spectrometer
 - Measure ${}^{1}H(e, e'\pi^{+})n$ at $\Delta W = 5$ MeV
 - Access to $D(e, e'\pi^{\pm})$ also possible



M. Ases Antelo, P. Bartsch, D. Baumann, R. Böhm, M. Ding, M. O. Distler, D. Elsner, J. Friedrich, S. Grözinger, S. Hedicke, P. Jennewein, F. Klein, K. W. Krygier, H. Merkel, P. Merle, U. Müller, R. Neuhausen, R. Pérez Benito, J. Pochodzalla, Th. Pospischil, M. Seimetz, A. Süle, Th. Walcher, M. Weis *Inst. für Kernphysik, Univ. Mainz, Germany*

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Multipoles

Data for $(L_{0+}/\omega)_{thr}$ at threshold from older π^+ electroproduction experiments (Frascati, DESY).

Theoretical fits for different values of λ are shown.





Kinematic parameters

Invariant mass W = 1125 MeV for all measurements

q ²	e	E	Ε′	θ _{e'}	\mathfrak{p}_{π^+}	θ_{π^+}	π^+
$(GeV/c)^2$		MeV	MeV	0	MeV/c	0	
0.058	0.900	855	618.6	19.1°	170.5	36.8°	A
0.058	0.705	525	288.6	36.2°	170.5	30.2°	A,B
0.058	0.484	405	168.6	55.1°	170.5	24.1°	Α
0.058	0.286	345	108.6	77.3°	170.5	18.3°	В
0.117	0.834	855	587.4	27.9°	188.8	39.3°	Α
0.117	0.529	525	257.3	55.5°	188.8	29.2°	В
0.117	0.219	405	137.3	93.0°	188.8	18.4°	В
0.195	0.742	855	545.8	37.7°	209.0	38.3°	A,B
0.195	0.489	615	305.8	61.2°	209.0	29.8°	В
0.195	0.229	495	185.8	93.5°	209.0	20.1°	В
0.273	0.648	855	504.6	46.8°	228.0	35.8°	В



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Particle Identification Positrons

Identify positrons by energy loss?
No, both are minimum ionising particles:



Elastic measurements H(e, e'p)

Cross-checks by elastic H(e, e'p) measurements:

particles	E	θ_A	p _A	$\theta_{\rm B}$	р _В	
	MeV		MeV/c		MeV/c	
<mark>А: е'</mark> , В: р	345	100°	241, 231	31.5°	453, 435	
<mark>А: е'</mark> , В: р	525	55.5°	439, 422	50.6°	468, 450	
A: p, <mark>B: e'</mark>	525	50.6°	468, 450, 432	55.5°	439, 422, 405	
A: p, <mark>B: e'</mark>	855	56.9°	545, 524, 503	37.7°	747, 719, 690	
<mark>А: е'</mark> , В: р	495	93.5°	323, 311	31.6°	615, 591	
<mark>А: е'</mark> , В: р	615	61.2°	468, 450	45.6°	573, 551	
<mark>А: е'</mark> , В: р	855	67.1°	546	38.3°	810	
<mark>А: е'</mark> , В: р	855	46.8 °	665	50.4°	628	



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Results Differential cross sections from 2000 data taking





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Results Differential cross sections from 2002 data taking



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Results Comparison with photoproduction



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Results Comparison with photoproduction



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Kinematics

Kinematic parameters for W = 1084 MeV. Cross sections are calculated by the MAID program.

Q ²	e	E	Ε'	θ _{e'}	θ_{π}	pπ	$exp(-\Delta t/\gamma \tau)$	dσ/dE′dΩdΩ
GeV ²		MeV	MeV	0	0	MeV/c		pb/MeV sr ²
0.156	0.820	855	614.7	31.6	44.2		0.75	4.57
	0.535	540	299.7	58.8	33.7	99.9		1.21
	0.255	420	179.7	91.9	22.9	1		0.510
0.078	0.902	855	656.2	21.5	44.5	82.7	0.71	27.3
	0.614	450	251.2	49.0	33.6			4.46
	0.289	330	131.2	84.2	22.4	1		1.47
0.035	0.904	660	484.0	19.1	38.0	70.5	0.67	70.8
	0.666	375	199.0	40.0	29.9			13.5
	0.404	285	109.0	64.1	22.5]		4.87

