Workshop on Nucleon Form Factors
Frascati, October 12-14, 2005

Two-photon exchange in elastic e scattering

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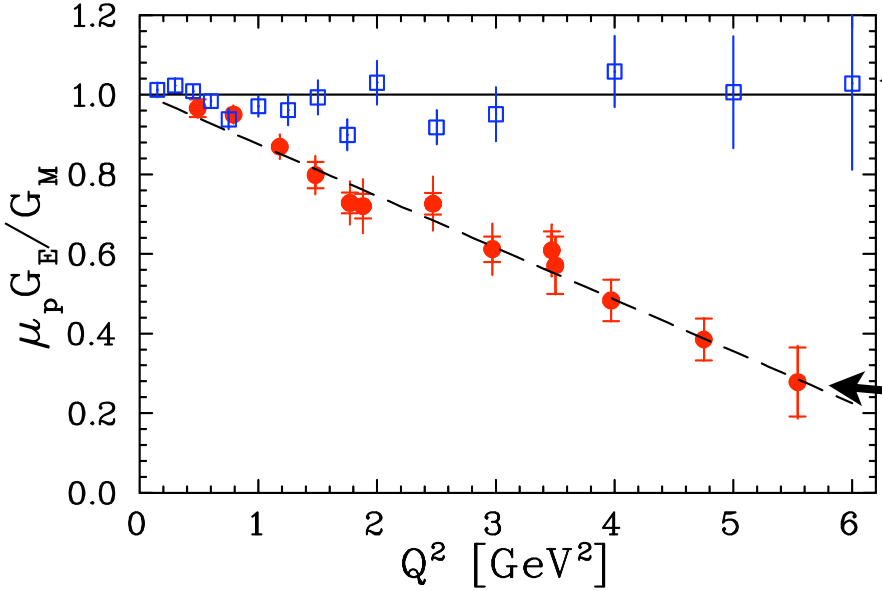
Outline

- Two-photon exchange and nucleon structure
- Extraction of proton G_E/G_M ratio
  - → Rosenbluth separation and polarization transfer
- Excited state contributions
  - → Δ , $N^*(1/2^+)$, $N^*(1/2^-)$ contributions
- Effect on *neutron* form factors
- Summary

Proton G_E/G_M Ratio



Rosenbluth (Longitudinal-Transverse) Separation

Polarization Transfer

LT

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

$$\tau = Q^2/4M^2$$
$$\varepsilon = \left[1 + 2(1+\tau)\tan^2\theta/2\right]^{-1}$$

G_E/G_M from slope in ε plot

PT

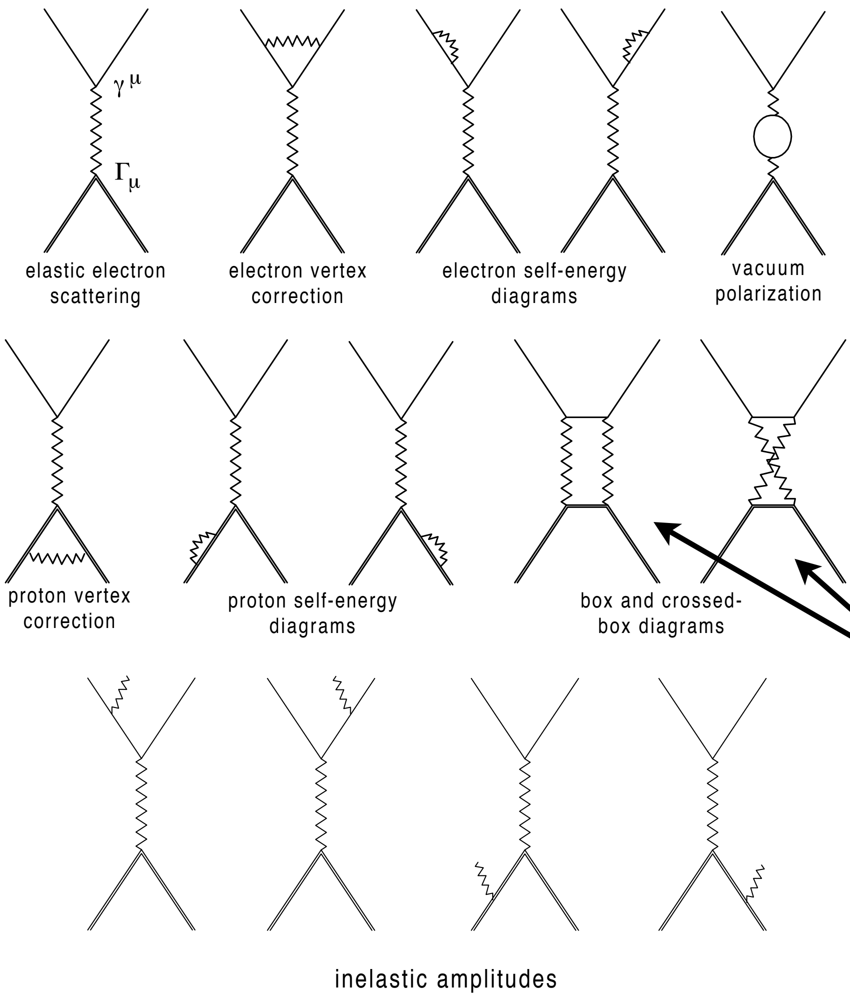
$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\frac{P_T}{P_L}$$

$P_{T,L}$ polarization of recoil proton

Two-photon exchange
& nucleon structure

QED Radiative Corrections

cross section modified by 1γ loop effects



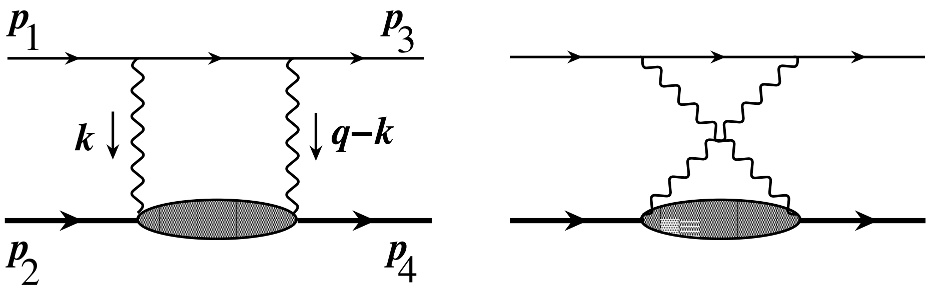
$$d\sigma = d\sigma_0\,(1+\delta)$$

→ δ contains additional ε dependence

mostly from box (and crossed box) diagram

$\longrightarrow$ can modify ε dependence in $d\sigma_0$

Box diagram



→ elastic contribution

$$\mathcal{M}_{\gamma\gamma} = e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N(k)}{D(k)}$$

where

$$\begin{aligned} N(k) &= \bar{u}(p_3)\, \gamma_\mu (\not{p}_1 - \not{k} + m_e) \gamma_\nu\, u(p_1) \\ &\times \bar{u}(p_4)\, \Gamma^\mu(q-k)\, (\not{p}_2 + \not{k} + M)\, \Gamma^\nu(k)\, u(p_2) \end{aligned}$$

and

$$\begin{aligned} D(k) &= (k^2 - \lambda^2)\, ((k-q)^2 - \lambda^2) \\ &\times ((p_1 - k)^2 - m^2)\, ((p_2 + k)^2 - M^2) \end{aligned}$$

with λ an IR regulator, and e.m. current is

$$\Gamma^\mu(q) = \gamma^\mu\, F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M}\, F_2(q^2)$$

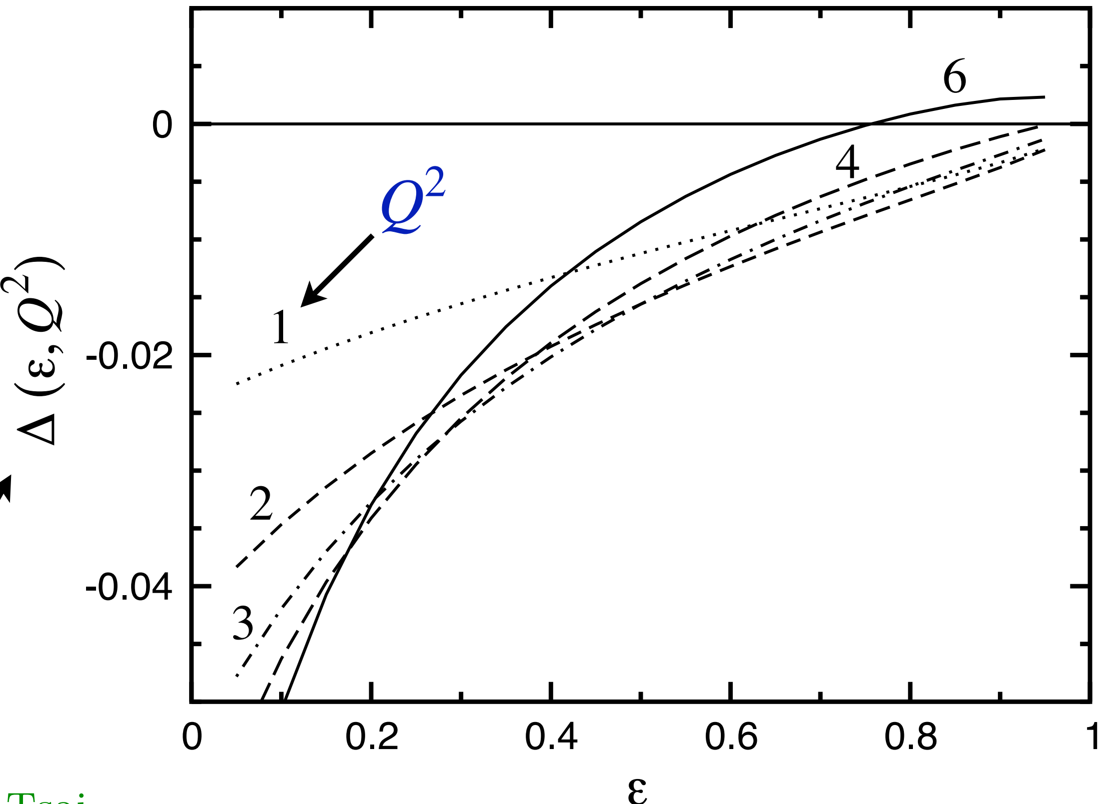
Various approximations to $\mathcal{M}_{\gamma\gamma}$ used

- Mo-Tsai: soft γ approximation
  $\longrightarrow$ integrand most singular when $k = 0$ and $k = q$
  $\longrightarrow$ replace γ propagator which is not at pole by $1/q^2$
  $\longrightarrow$ approximate numerator $N(k) \approx N(0)$
  $\longrightarrow$ neglect all structure effects

- Maximon-Tjon: improved loop calculation
  $\longrightarrow$ exact treatment of propagators
  $\longrightarrow$ still evaluate $N(k)$ at $k = 0$
  $\longrightarrow$ first study of form factor effects
  $\longrightarrow$ additional ε dependence

- Blunden-WM-Tjon: exact loop calculation
  $\longrightarrow$ no approximation in $N(k)$ or $D(k)$
  $\longrightarrow$ include form factors

Two-photon correction



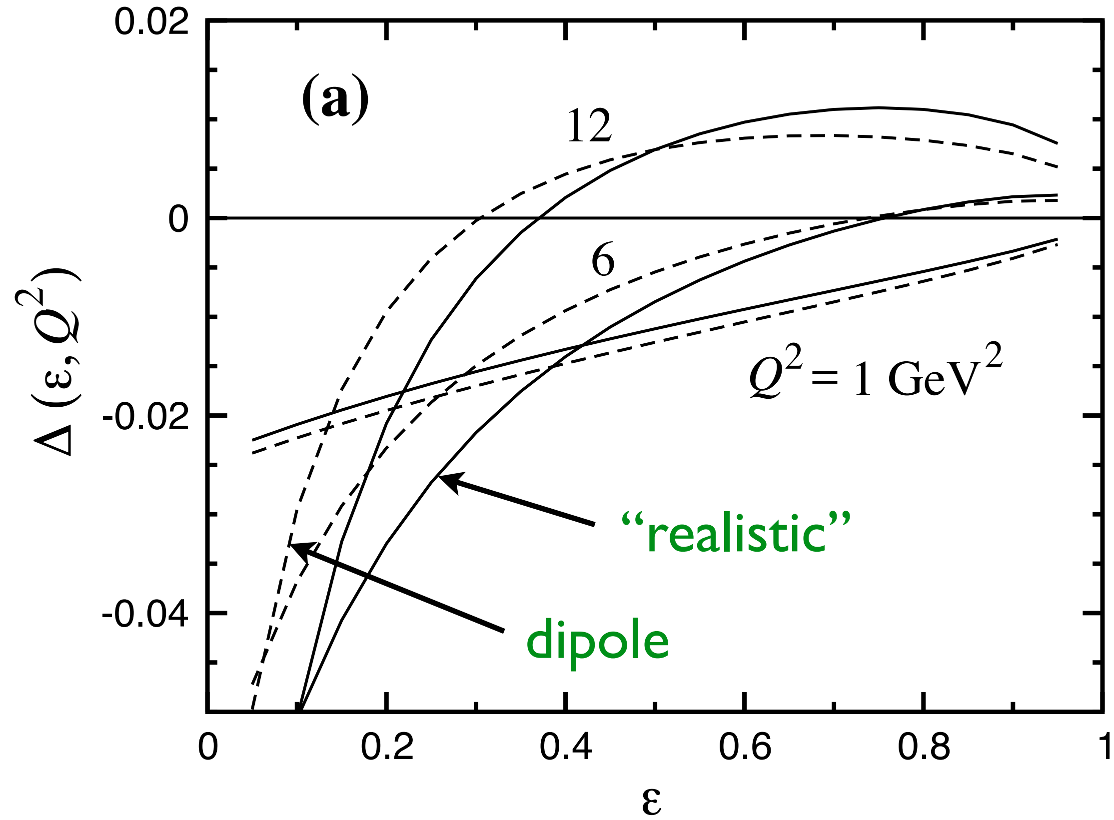
$\delta^{(2\gamma)}_{\rm full} - \delta^{(2\gamma)}_{\rm Mo-Tsai}$

$$\delta^{(2\gamma)} \rightarrow \frac{2\mathcal{R}e\{\mathcal{M}_0^\dagger \mathcal{M}_{\gamma\gamma}\}}{|\mathcal{M}_0|^2}$$

Blunden, WM, Tjon
PRL 91 (2003) 142304;
PRC72 (2005) 034612

- few % magnitude
- positive slope
- non-linearity in ε

Two-photon correction



Blunden, WM, Tjon
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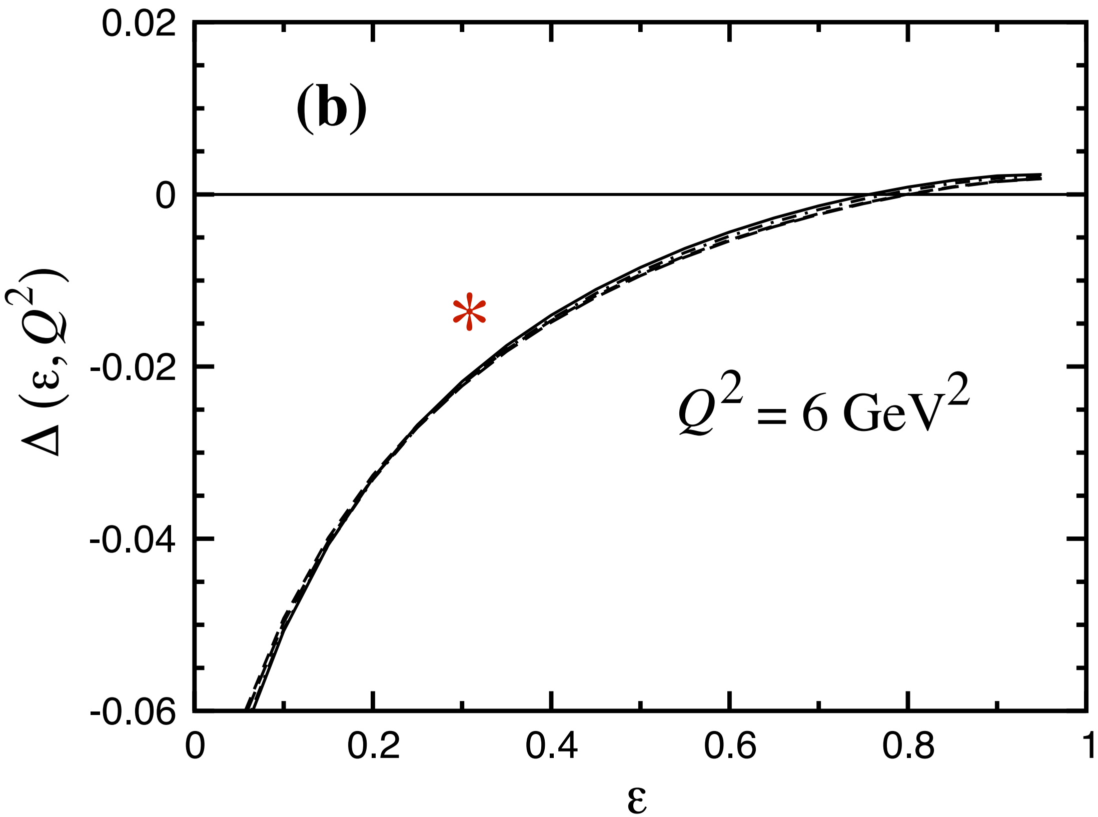
Two-photon correction

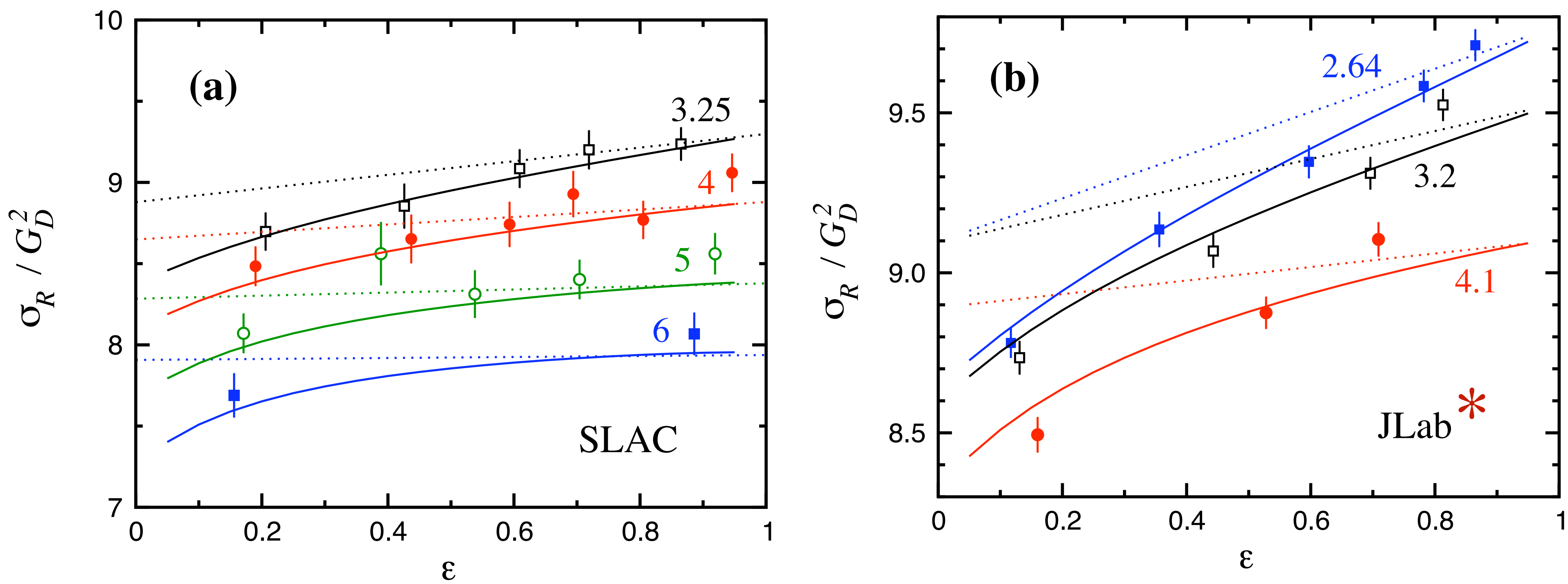


Blunden, WM, Tjon
PRL 91 (2003) 142304;
PRC72 (2005) 034612

* different form factors

- *Mergell, Meissner, Drechsel (1996)*
- *Brash et al. (2002)*
- *Arrington LT G_E^p fit (2004)*
- *Arrington PT G_E^p fit (2004)*

Effect on cross section



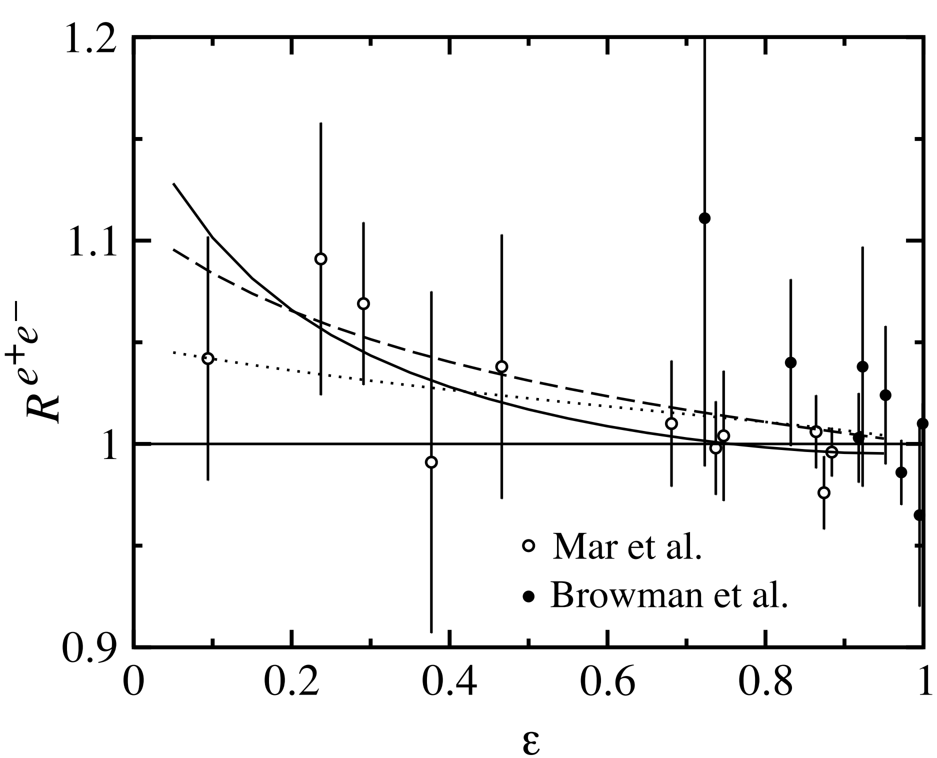
········ Born cross section with PT form factors

—— including TPE effects

* Super-Rosenbluth
Qattan et al.,
PRL 94, 142301 (2005)

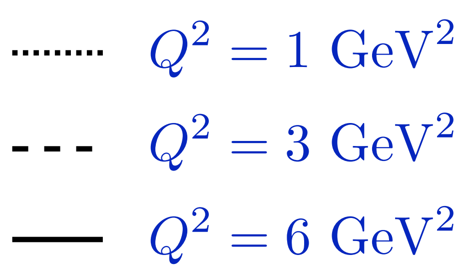
e^+ / e^- comparison

- 1γ exchange changes sign under $e^+ \leftrightarrow e^-$
- 2γ exchange invariant under $e^+ \leftrightarrow e^-$
- ratio of $e^+p \,/\, e^-p$ elastic cross sections sensitive to $\Delta(\varepsilon, Q^2)$



$$R^{e^+e^-} = \frac{d\sigma^{e^+}}{d\sigma^{e^-}}$$

$$\approx 1 - 2\,\Delta$$



⇨ simultaneous e^-p/e^+p measurement planned in Hall B (to $Q^2 \sim 1$ GeV^2)

Generalized form factors

- Generalized electromagnetic current

$$\Gamma^\mu = \tilde{F}_1\, \gamma^\mu \; + \; \tilde{F}_2\, \frac{i\sigma^{\mu\nu} q_\nu}{2M} \; + \; \tilde{F}_3\, \frac{\gamma \cdot K\, P^\mu}{M^2} \quad *$$

$$K = (p_1 + p_3)/2\,, \qquad P = (p_2 + p_4)/2$$

Goldberger et al. (1957)
Guichon, Vanderhaeghen (2003)
Chen et al. (2004)

- $\tilde{F}_i$ are complex functions of Q^2 and ε

- In 1γ exchange limit $\quad \tilde{F}_{1,2}(Q^2, \varepsilon) \rightarrow F_{1,2}(Q^2)$

$$\tilde{F}_3(Q^2, \varepsilon) \rightarrow 0$$

* Note: decomposition not unique

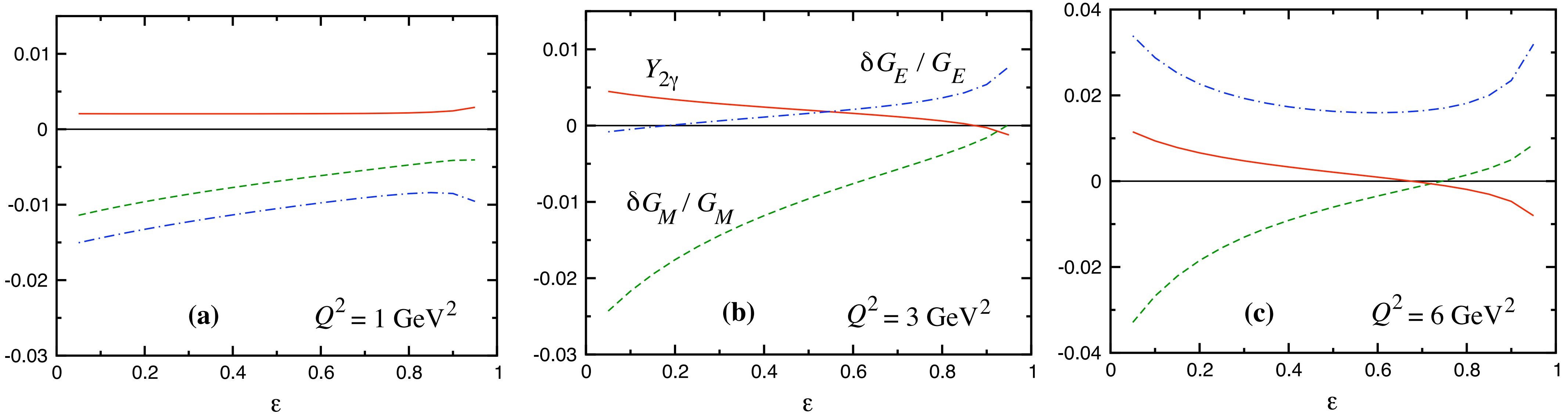
Generalized form factors

- Generalized (complex) Sachs form factors

$$\widetilde{G}_E = G_E + \delta G_E \ , \qquad \widetilde{G}_M = G_M + \delta G_M \ , \qquad Y_{2\gamma} = \tilde{\nu}\, \frac{\widetilde{F}_3}{G_M}$$

$$K \cdot P / M^2 = \sqrt{\tau(1+\tau)(1+\varepsilon)/(1-\varepsilon)}$$

$$\Rightarrow \quad \sigma_R = G_M^2 + \frac{\varepsilon}{\tau} G_E^2 + 2 G_M^2 \, Re\left\{ \frac{\delta G_M}{G_M} + Y_{2\gamma} \right\} + \frac{2\varepsilon}{\tau} G_E^2 \, Re\left\{ \frac{\delta G_E}{G_E} + \frac{G_M}{G_E} Y_{2\gamma} \right\}$$



⇒ cannot assume all TPE effects reside in $Y_{2\gamma}$

Extraction of proton G_E/G_M ratio

$G_E^p \,/\, G_M^p$ ratio

- estimate effect of TPE on ε dependence

- approximate correction by linear function of ε

$$1 + \Delta \approx a + b\varepsilon$$

→ reduced cross section is then

$$\sigma_R \approx a\, G_M^2 \left[1 + \frac{\varepsilon}{\mu^2 \tau} \left(R^2 (1 + \varepsilon\, b/a) + \mu^2\, \tau\, b/a \right) \right]$$

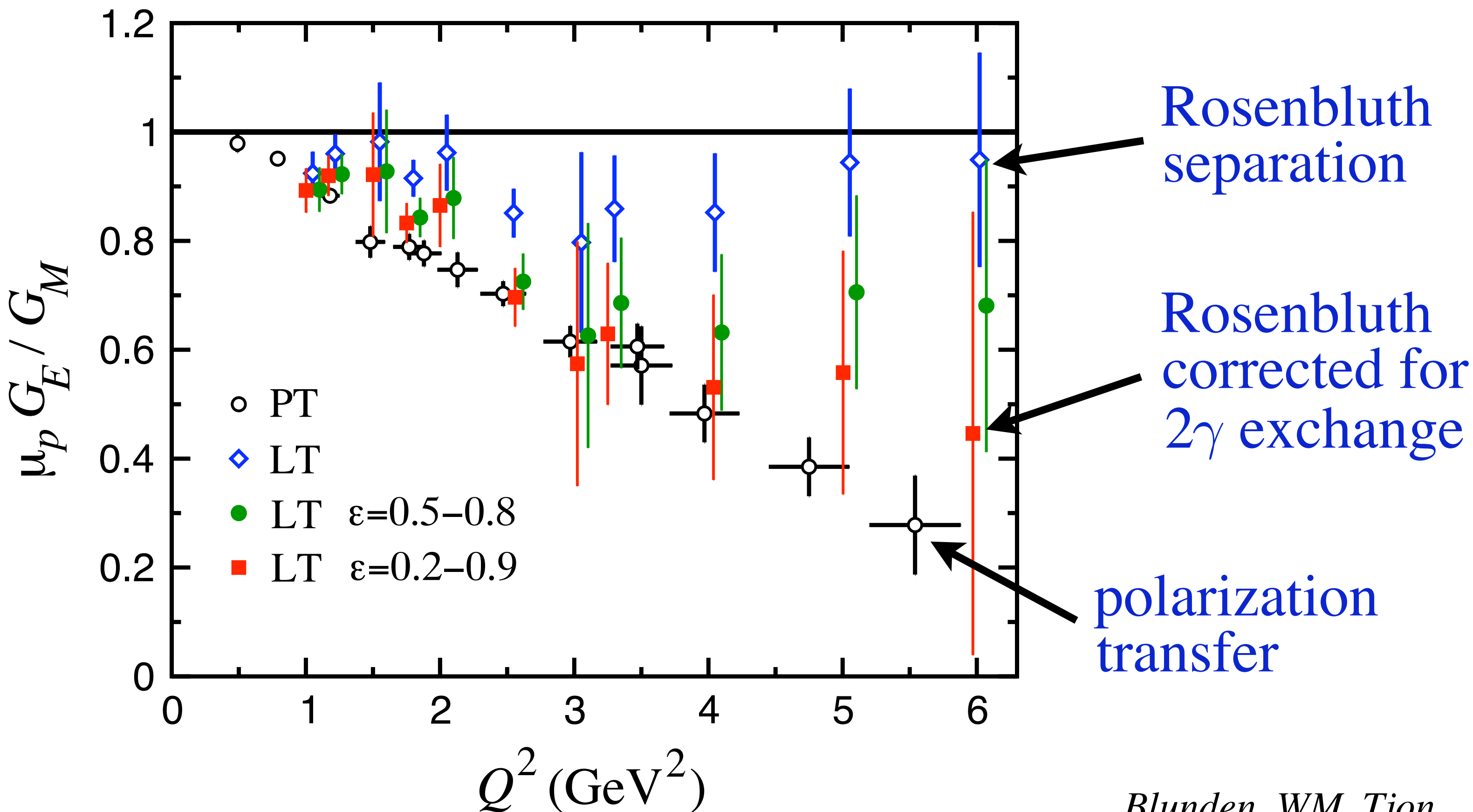
where "true" ratio is

$$R^2 = \frac{\widetilde{R}^2 - \mu^2\, \tau\, b/a}{1 + \bar{\varepsilon}\, b/a}$$

"effective" ratio contaminated by TPE

average value of ε over range fitted

$G_E^p \,/\, G_M^p$ ratio



Blunden, WM, Tjon
Phys. Rev. C72 (2005) 034612

⟹ resolves much of the form factor discrepancy

- how does TPE affect polarization transfer ratio?

$$\Rightarrow \quad \widetilde{R} = R\left(\frac{1+\Delta_T}{1+\Delta_L}\right)$$

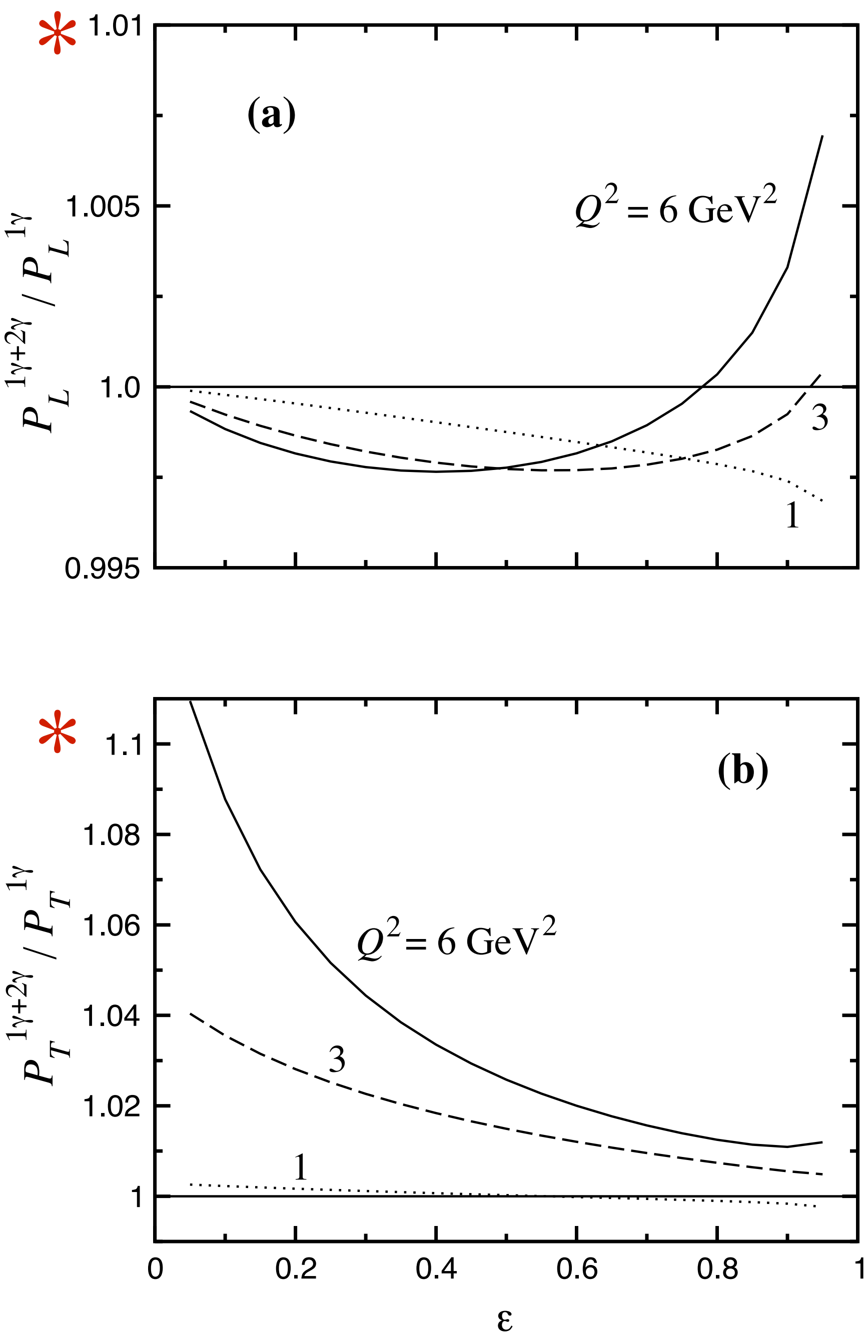
where $\Delta_{L,T} = \delta_{L,T}^{\mathrm{full}} - \delta_{\mathrm{IR}}^{\mathrm{Mo-Tsai}}$ is finite part of 2γ contribution relative to IR part of Mo-Tsai

- experimentally measure ratio of polarized to unpolarized cross sections

$$\Rightarrow \quad \frac{P_{L,T}^{1\gamma+2\gamma}}{P_{L,T}^{1\gamma}} = \frac{1+\Delta_{L,T}}{1+\Delta}$$

Longitudinal & transverse polarizations

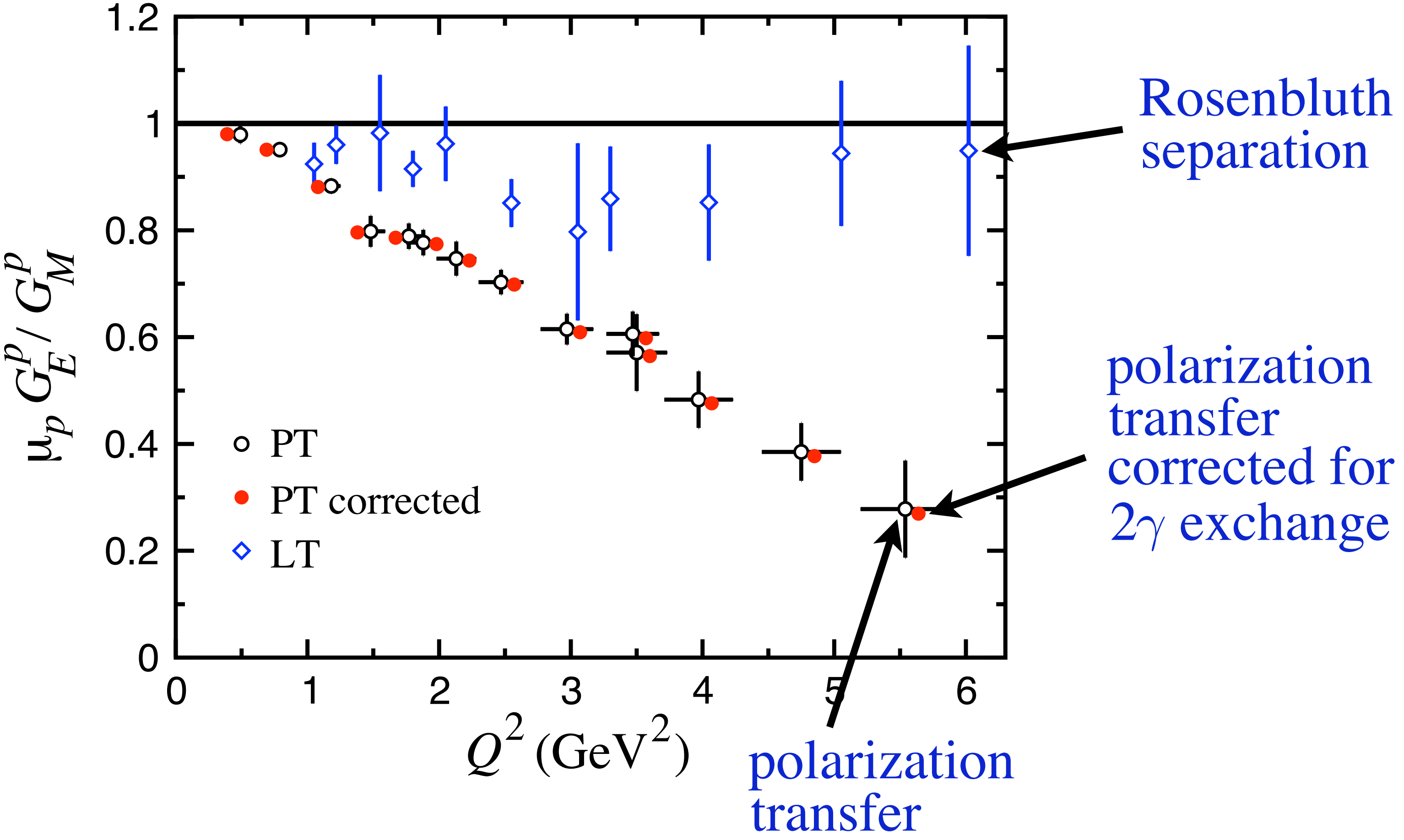
* Note scales!



small effect on P_L

large effect on P_T

$G_E^p \,/\, G_M^p$ ratio



- large Q^2 data typically at large ε
- $< 3\%$ suppression at large Q^2

Excited intermediate states

- Lowest mass excitation is P_{33} Δ resonance

  - ➡ relativistic $\gamma^* N \Delta$ vertex

$$\Gamma^{\nu\alpha}_{\gamma\Delta\to N}(p,q) \equiv iV^{\nu\alpha}_{\Delta in}(p,q) = i\frac{eF_\Delta(q^2)}{2M_\Delta^2}\Big\{g_1\left[g^{\nu\alpha}\not{p}\not{q} - p^\nu\gamma^\alpha\not{q} - \gamma^\nu\gamma^\alpha p\cdot q + \gamma^\nu\not{p}q^\alpha\right]$$

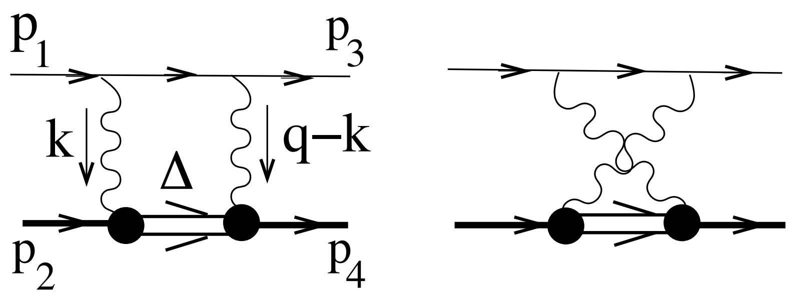
$$+g_2\left[p^\nu q^\alpha - g^{\nu\alpha}p\cdot q\right] + (g_3/M_\Delta)\left[q^2(p^\nu\gamma^\alpha - g^{\nu\alpha}\not{p}) + q^\nu(q^\alpha\not{p} - \gamma^\alpha p\cdot q)\right]\Big\}\gamma_5 T_3$$

form factor $\frac{\Lambda_\Delta^4}{(\Lambda_\Delta^2 - q^2)^2}$

  - ➡ coupling constants

g_1	magnetic	➡	7
$g_2 - g_1$	electric	➡	9
g_3	Coulomb	➡	-2 ... 0

Two-photon exchange amplitude with Δ intermediate state



$$\mathcal{M}_{\Delta}^{\gamma\gamma} = -e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N_{box}^{\Delta}(k)}{D_{box}^{\Delta}(k)} - e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N_{x-box}^{\Delta}(k)}{D_{x-box}^{\Delta}(k)}$$

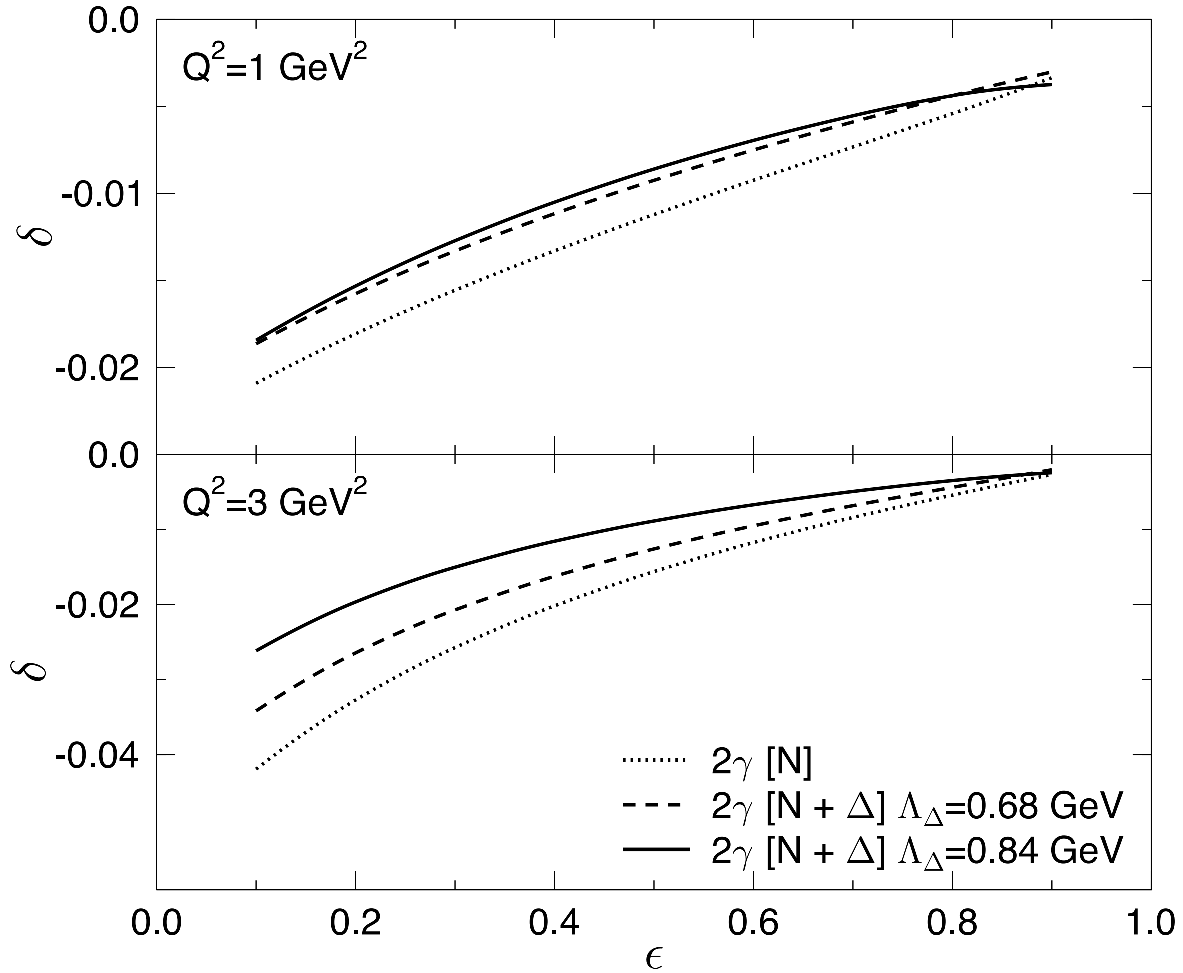
numerators

$$\begin{aligned} N_{box}^{\Delta}(k) &= \overline{U}(p_4) V_{\Delta in}^{\mu\alpha}(p_2+k, q-k) \left[\not{p}_2 + \not{k} + M_\Delta\right] \mathcal{P}_{\alpha\beta}^{3/2}(p_2+k) V_{\Delta out}^{\beta\nu}(p_2+k, k) U(p_2) \\ &\times \; \overline{u}(p_3)\gamma_\mu \left[\not{p}_1 - \not{k} + m_e\right] \gamma_\nu u(p_1) \end{aligned}$$

$$\begin{aligned} N_{x-box}^{\Delta}(k) &= \overline{U}(p_4) V_{\Delta in}^{\mu\alpha}(p_2+k, q-k) \left[\not{p}_2 + \not{k} + M_\Delta\right] \mathcal{P}_{\alpha\beta}^{3/2}(p_2+k) V_{\Delta out}^{\beta\nu}(p_2+k, k) U(p_2) \\ &\times \; \overline{u}(p_3)\gamma_\nu \left[\not{p}_3 + \not{k} + m_e\right] \gamma_\mu u(p_1) \end{aligned}$$

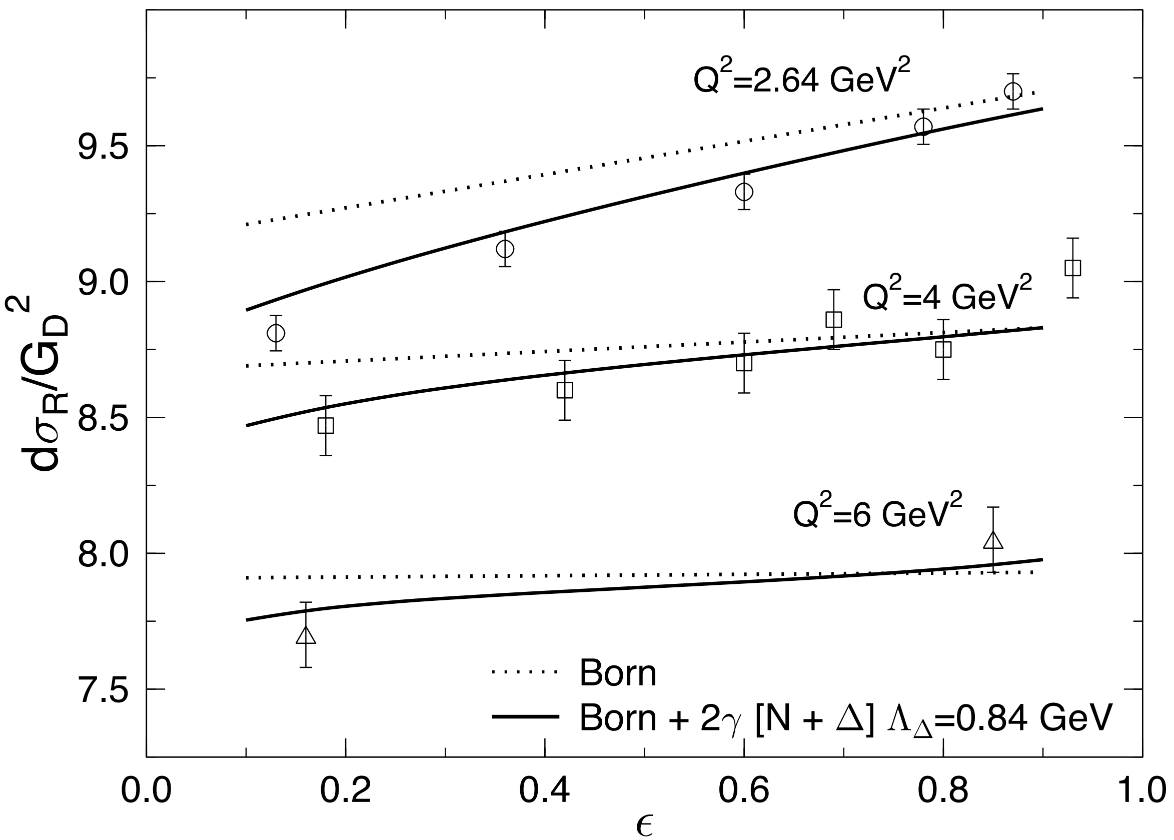
spin-3/2 projection operator

$$\mathcal{P}_{\alpha\beta}^{3/2}(p) = g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{1}{3p^2}\left(\not{p}\gamma_\alpha p_\beta + p_\alpha\gamma_\beta\not{p}\right)$$



Kondratyuk, Blunden, WM, Tjon
nucl-th/0506026 (PRL 2005)

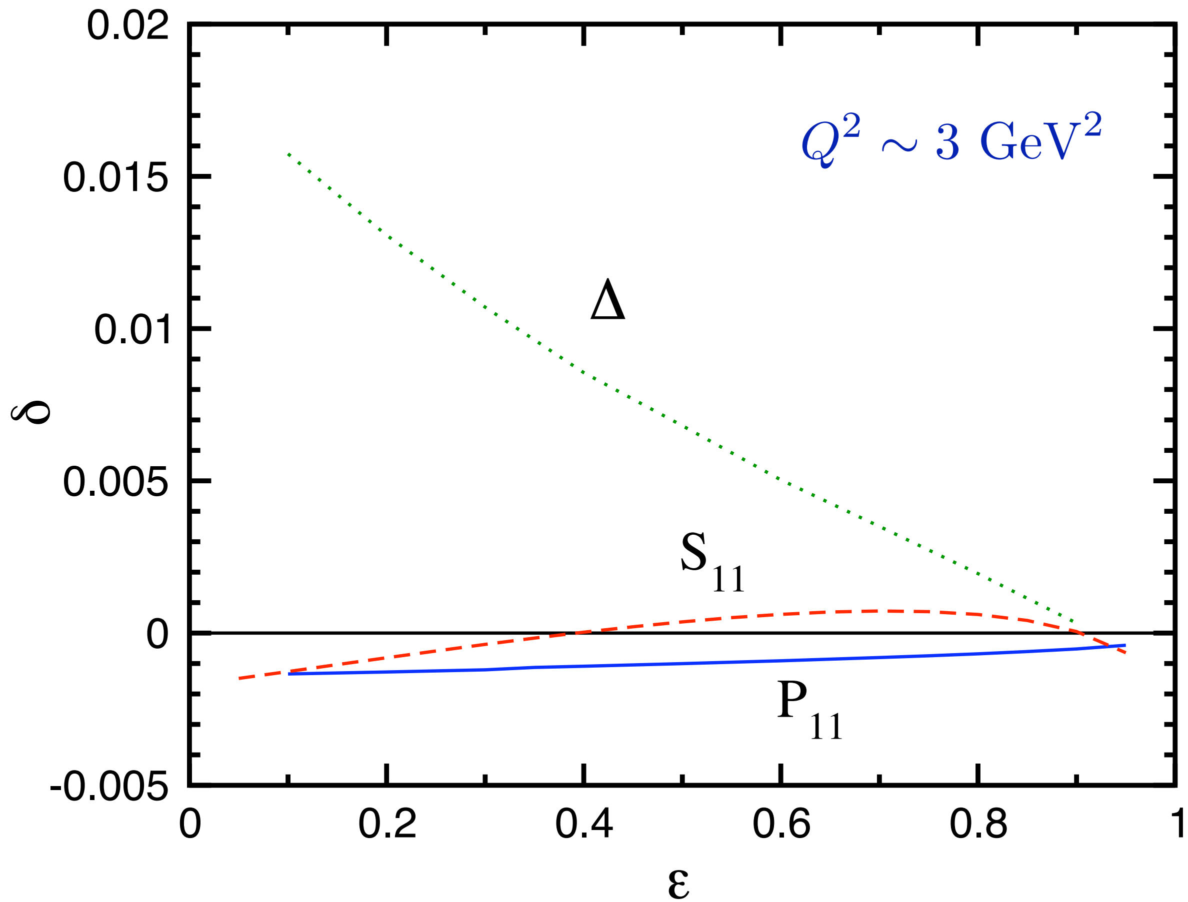
- ➡ Δ has <u>opposite</u> slope to N
- ➡ cancels some of TPE correction from N



Kondratyuk, Blunden, WM, Tjon
nucl-th/0506026 (PRL 2005)

- weaker ε dependence than with N alone
- better fit to JLab data!

$$J^P = \frac{1}{2}^+, \quad \frac{1}{2}^- \quad \text{excited } N^* \text{ states}$$

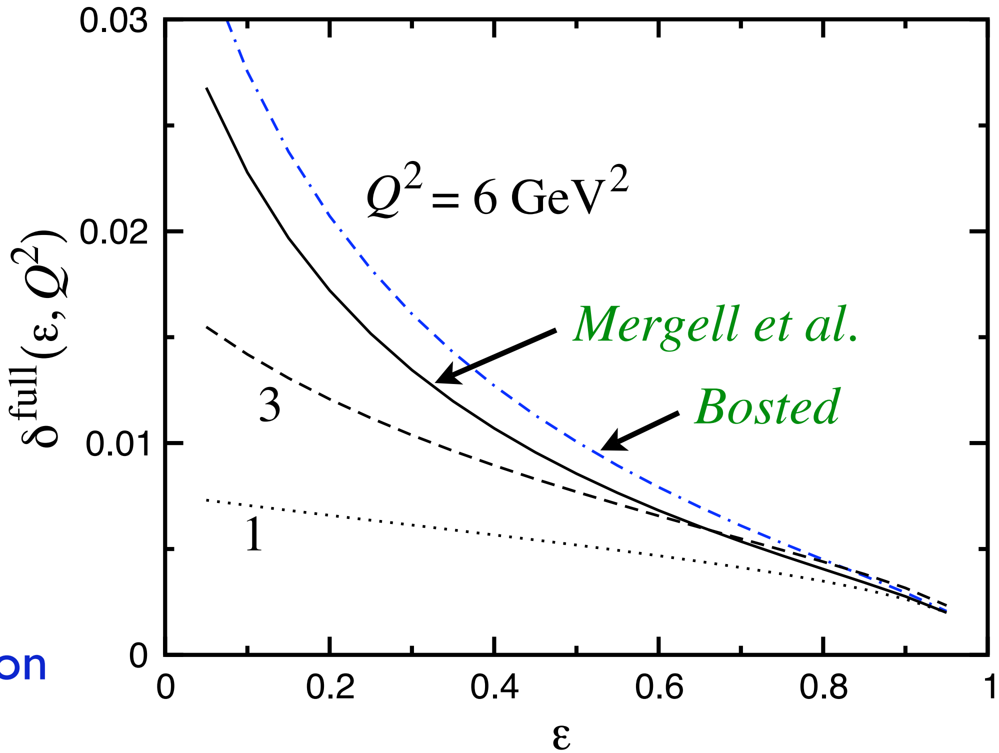


Tjon, WM, et al. (2005)

- higher mass resonance contributions small
- enhance nucleon elastic contribution

Effect on
neutron form factors

Neutron correction



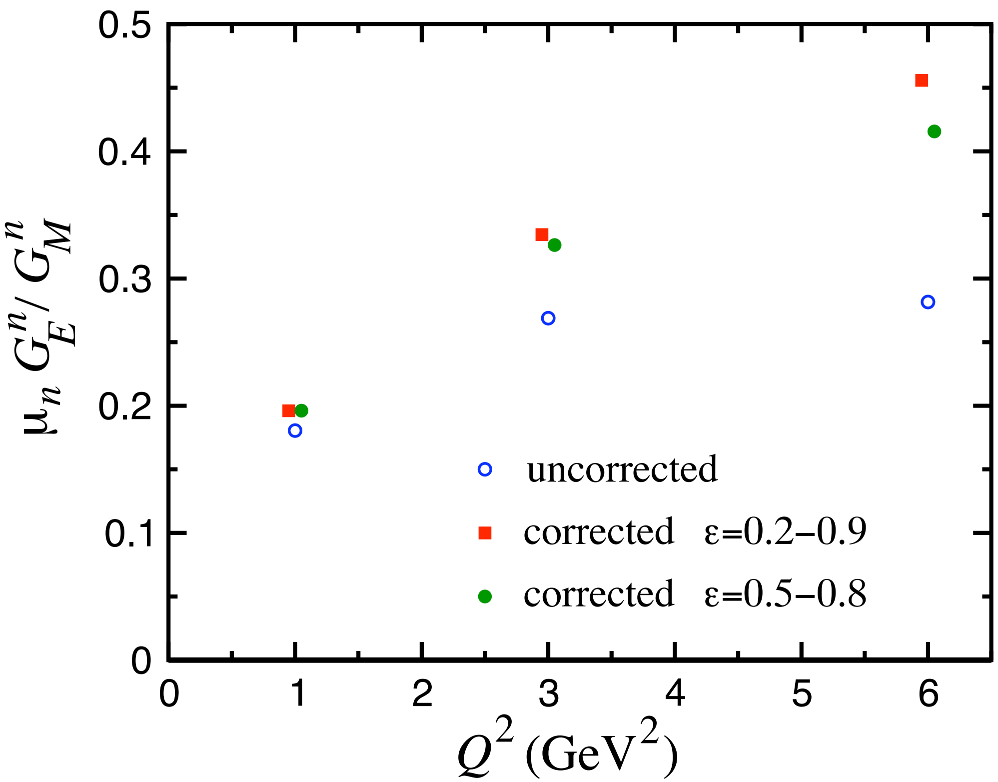
full correction
- no IR contribution

Blunden, WM, Tjon
Phys. Rev. C72 (2005) 034612

➡ since G_E^n is small, effect may be relatively large

➡ sign opposite to proton (since $\kappa_n < 0$)

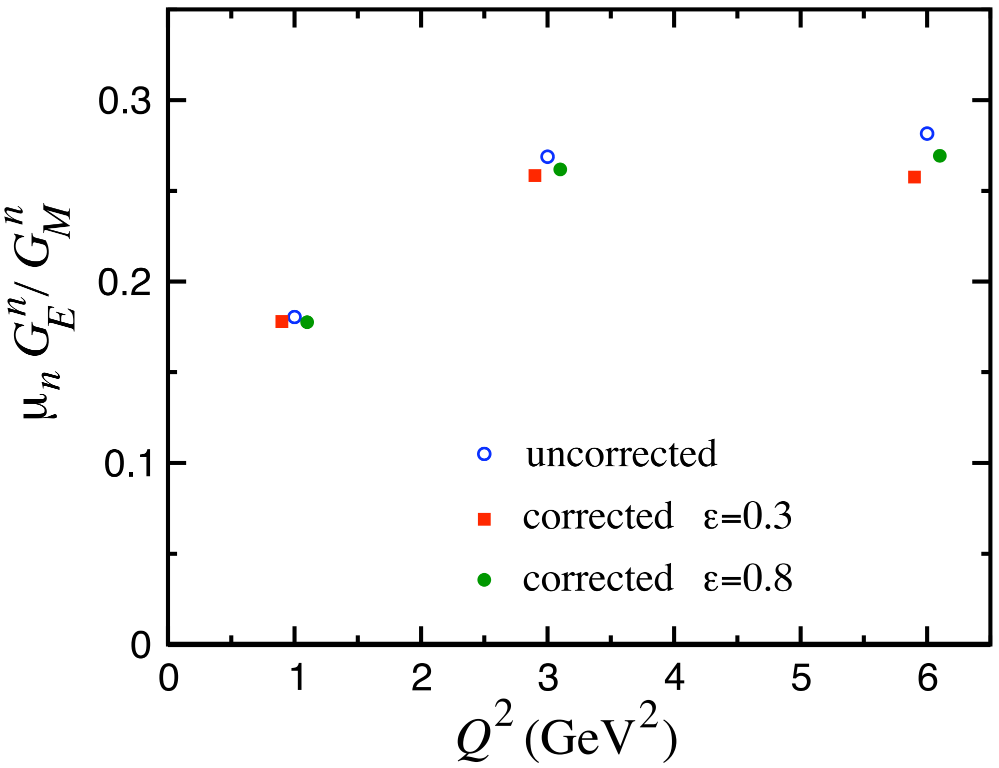
Effect on neutron LT form factors



Blunden, WM, Tjon
Phys. Rev. C72 (2005) 034612

- ⟹ large effect at high Q^2 for LT-separation method
- ⟹ LT method unreliable for neutron

Effect on neutron PT form factors



Blunden, WM, Tjon
Phys. Rev. C72 (2005) 034612

- small correction for PT
- 4% (3%) suppression at $\varepsilon = 0.3\ (0.8)$ for $Q^2 = 3\ \mathrm{GeV}^2$
  10% (5%) suppression at $\varepsilon = 0.3\ (0.8)$ for $Q^2 = 6\ \mathrm{GeV}^2$

Summary

- First explicit calculation of TPE taking into account nucleon structure

- Nucleon elastic intermediate states resolves most of LT/PT G_E^p/G_M^p discrepancy

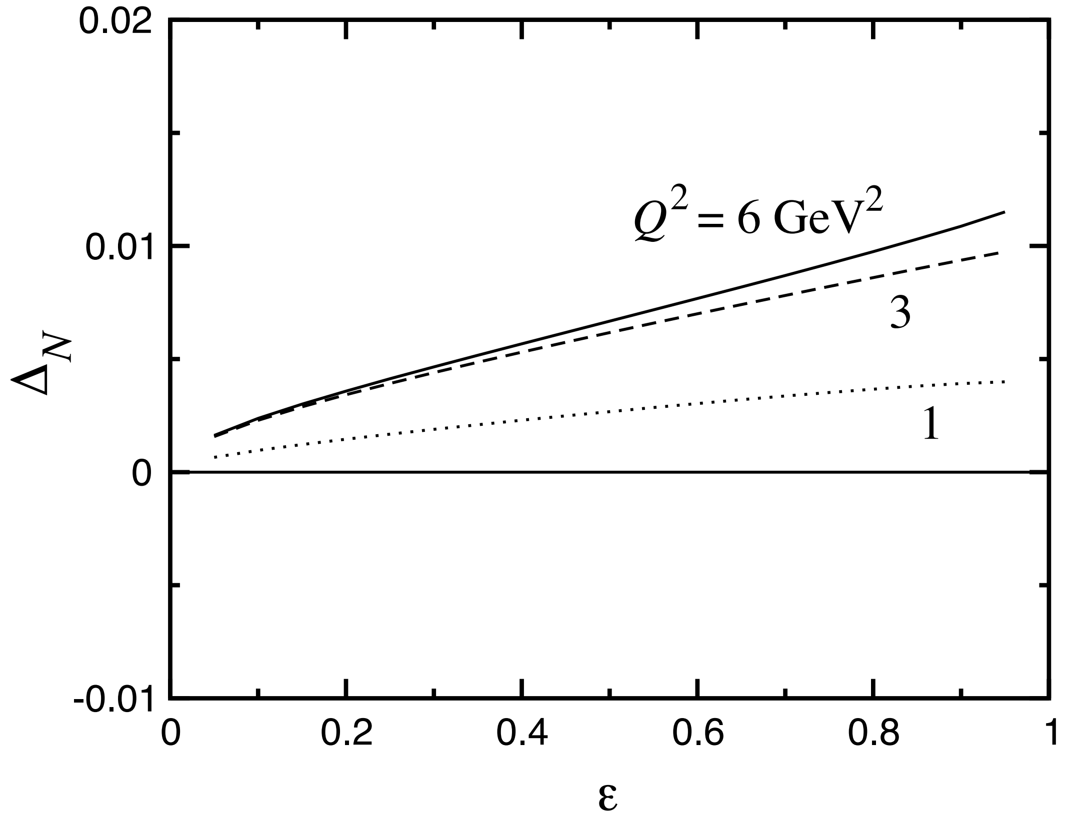
- Δ excited state opposite sign cf. nucleon, but smaller

  $P_{11}(1440)$ and $S_{11}(1535)$ contributions small

- Effect on neutron form factors large for LT method, small for PT method

The End

Normal polarization



Normal polarization