Two-photon exchange in elastic $e$ scattering

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Outline

- Two-photon exchange and nucleon structure

- Extraction of proton $G_E/G_M$ ratio
  - Rosenbluth separation and polarization transfer

- Excited state contributions
  - $\Delta, N^*(1/2^+), N^*(1/2^-)$ contributions

- Effect on neutron form factors

- Summary
Proton $G_E / G_M$ Ratio

Rosenbluth (Longitudinal-Transverse) Separation

Polarization Transfer

\[ \sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2) \]

\[ \tau = \frac{Q^2}{4M^2} \]

\[ \varepsilon = \left[ 1 + 2(1 + \tau) \tan^2 \theta/2 \right]^{-1} \]

$G_E / G_M$ from slope in $\varepsilon$ plot

\[ \frac{G_E}{G_M} = -\sqrt{\frac{\tau(1 + \varepsilon)}{2\varepsilon}} \frac{P_T}{P_L} \]

$P_{T,L}$ polarization of recoil proton
Two-photon exchange & nucleon structure
QED Radiative Corrections

cross section modified by $1\gamma$ loop effects

d\sigma = d\sigma_0 (1 + \delta)

$\delta$ contains additional $\varepsilon$ dependence

mostly from box (and crossed box) diagram

$\rightarrow$ can modify $\varepsilon$ dependence in $d\sigma_0$
Box diagram

\[ \mathcal{M}_{\gamma\gamma} = e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N(k)}{D(k)} \]

where

\[ N(k) = \bar{u}(p_3) \gamma_\mu (p_1 - k + m_e) \gamma_\nu u(p_1) \]
\[ \times \bar{u}(p_4) \Gamma^\mu (q - k) (p_2 + k + M) \Gamma^\nu (k) u(p_2) \]

and

\[ D(k) = (k^2 - \lambda^2) ((k - q)^2 - \lambda^2) \]
\[ \times ((p_1 - k)^2 - m^2) ((p_2 + k)^2 - M^2) \]

with \( \lambda \) an IR regulator, and e.m. current is

\[ \Gamma^\mu (q) = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(q^2) \]
Various approximations to $\mathcal{M}_{\gamma\gamma}$ used

- **Mo-Tsai**: soft $\gamma$ approximation
  - integrand most singular when $k = 0$ and $k = q$
  - replace $\gamma$ propagator which is not at pole by $1/q^2$
  - approximate numerator $N(k) \approx N(0)$
  - neglect all structure effects

- **Maximon-Tjon**: improved loop calculation
  - exact treatment of propagators
  - still evaluate $N(k)$ at $k = 0$
  - first study of form factor effects
  - additional $\varepsilon$ dependence

- **Blunden-WM-Tjon**: exact loop calculation
  - no approximation in $N(k)$ or $D(k)$
  - include form factors
Two-photon correction

\[ \delta^{(2\gamma)} \rightarrow \frac{2 \text{Re}\{M_0^\dagger M_{\gamma\gamma}\}}{|M_0|^2} \]

\[ \delta_{\text{full}}^{(2\gamma)} - \delta_{\text{Mo-Tsai}}^{(2\gamma)} \]

\[ \Delta(\varepsilon, Q^2) \]

\[ \varepsilon \]

\[ Q^2 \]

\[ 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1 \]

\[ -0.04 \quad -0.02 \quad 0 \quad 1 \]

few % magnitude

positive slope

non-linearity in \( \varepsilon \)

Blunden, WM, Tjon
PRL 91 (2003) 142304;
PRC72 (2005) 034612
Two-photon correction

\[ \Delta (\varepsilon, Q^2) \]

-0.04
-0.02
0
0.02
0 0.2 0.4 0.6 0.8 1

\( Q^2 = 1 \text{ GeV}^2 \)

\( \text{dipole} \)

\( \text{“realistic”} \)

Blunden, WM, Tjon
PRL 91 (2003) 142304;
PRC 72 (2005) 034612
Two-photon correction

\[ \Delta \left( \varepsilon, Q^2 \right) \]

\[ Q^2 = 6 \text{ GeV}^2 \]

* different form factors

\{ Mergell, Meissner, Drechsel (1996) \\
Brash et al. (2002) \\
Arrington LT \( G_E^p \) fit (2004) \\
Arrington PT \( G_E^p \) fit (2004) \}

Blunden, WM, Tjon 
PRL 91 (2003) 142304; 
PRC 72 (2005) 034612
Effect on cross section

**Super-PRL**

Qattan et al.,

$e^+ / e^-$ comparison

- $1\gamma$ exchange changes sign under $e^+ \leftrightarrow e^-$
- $2\gamma$ exchange invariant under $e^+ \leftrightarrow e^-$
- ratio of $e^+ p / e^- p$ elastic cross sections sensitive to $\Delta(\varepsilon, Q^2)$

\[ R^{e^+e^-} = \frac{d\sigma^{e^+}}{d\sigma^{e^-}} \approx 1 - 2 \Delta \]

\[ Q^2 = 1 \text{ GeV}^2 \]
\[ Q^2 = 3 \text{ GeV}^2 \]
\[ Q^2 = 6 \text{ GeV}^2 \]

simultaneous $e^- p / e^+ p$ measurement planned in Hall B (to $Q^2 \sim 1 \text{ GeV}^2$)
Generalized form factors

Generalized electromagnetic current

\[ \Gamma^\mu = \tilde{F}_1 \gamma^\mu + \tilde{F}_2 \frac{i\sigma^{\mu\nu}q_\nu}{2M} + \tilde{F}_3 \frac{\gamma \cdot K P^\mu}{M^2} \]

\[ K = \frac{(p_1 + p_3)}{2}, \quad P = \frac{(p_2 + p_4)}{2} \]

\[ \tilde{F}_i \] are complex functions of \( Q^2 \) and \( \varepsilon \)

In 1\( \gamma \) exchange limit

\[ \tilde{F}_{1,2}(Q^2, \varepsilon) \rightarrow F_{1,2}(Q^2) \]
\[ \tilde{F}_3(Q^2, \varepsilon) \rightarrow 0 \]

* Note: decomposition not unique

Goldberger et al. (1957)
Chen et al. (2004)
Generalized form factors

Generalized (complex) Sachs form factors

\[ \tilde{G}_E = G_E + \delta G_E , \quad \tilde{G}_M = G_M + \delta G_M , \quad Y_{2\gamma} = \tilde{\nu} \frac{\tilde{F}_3}{G_M} \]

\[ K \cdot P / M^2 = \sqrt{\tau(1 + \tau)(1 + \varepsilon)/(1 - \varepsilon)} \]

\[ \sigma_R = G_M^2 + \frac{\varepsilon}{\tau} G_E^2 + 2G_M^2 \text{Re} \left\{ \frac{\delta G_M}{G_M} + Y_{2\gamma} \right\} + \frac{2\varepsilon}{\tau} G_E^2 \text{Re} \left\{ \frac{\delta G_E}{G_E} + \frac{G_M}{G_E} Y_{2\gamma} \right\} \]

\[ \begin{array}{c}
\text{(a) } Q^2 = 1 \text{ GeV}^2 \\
\text{(b) } Q^2 = 3 \text{ GeV}^2 \\
\text{(c) } Q^2 = 6 \text{ GeV}^2
\end{array} \]

cannot assume all TPE effects reside in \( Y_{2\gamma} \)
Extraction of proton $G_E/G_M$ ratio
estimate effect of TPE on \( \varepsilon \) dependence

approximate correction by linear function of \( \varepsilon \)

\[
1 + \Delta \approx a + b\varepsilon
\]

reduced cross section is then

\[
\sigma_R \approx a \ G_M^2 \left[ 1 + \frac{\varepsilon}{\mu^2 \tau} \ (R^2(1 + \varepsilon \ b/a) + \mu^2 \tau \ b/a) \right]
\]

where “true” ratio is

\[
R^2 = \frac{\tilde{R}^2 - \mu^2 \tau \ b/a}{1 + \bar{\varepsilon} \ b/a}
\]

“effective” ratio contaminated by TPE

average value of \( \varepsilon \) over range fitted
$G_E^P / G_M^P$ ratio

- Rosenbluth separation
- Rosenbluth corrected for $2\gamma$ exchange
- polarization transfer

Blunden, WM, Tjon

resolves much of the form factor discrepancy
how does TPE affect polarization transfer ratio?

\[ \tilde{R} = R \left( \frac{1 + \Delta_T}{1 + \Delta_L} \right) \]

where \( \Delta_{L,T} = \delta_{L,T}^{\text{full}} - \delta_{L,T}^{\text{IR-Mo-Tsai}} \) is finite part of \( 2\gamma \)

contribution relative to IR part of Mo-Tsai

experimentally measure ratio of polarized to unpolarized cross sections

\[ \frac{P_{L,T}^{1\gamma + 2\gamma}}{P_{L,T}^{1\gamma}} = \frac{1 + \Delta_{L,T}}{1 + \Delta} \]
Longitudinal & transverse polarizations

* Note scales!

(a) $Q^2 = 6 \text{ GeV}^2$

(b) $Q^2 = 6 \text{ GeV}^2$

**small effect** on $P_L$

**large effect** on $P_T$
$G_E^p / G_M^p$ ratio

large $Q^2$ data typically at large $\varepsilon$

< 3% suppression at large $Q^2$
Excited intermediate states
Lowest mass excitation is $P_{33} \Delta$ resonance

$\rightarrow$ relativistic $\gamma^* N\Delta$ vertex

$$\Gamma_{\gamma\Delta \to N}^{\nu\alpha}(p, q) \equiv iV_{\Delta in}^{\nu\alpha}(p, q) = i \frac{e F_\Delta(q^2)}{2 M_\Delta} \left\{ g_1 \left[ g^{\nu\alpha} p \cdot q - p^\nu \gamma^\alpha q - \gamma^\nu \gamma^\alpha p \cdot q + \gamma^\nu p^\alpha q^\alpha \right] ight.$$  
$$+ g_2 \left[ p^\nu q^\alpha - g^{\nu\alpha} p \cdot q \right] + (g_3 / M_\Delta) \left[ q^\nu (p^\nu \gamma^\alpha - g^\nu \gamma^\alpha) + q^\nu (q^\alpha p^\nu - \gamma^\alpha p \cdot q) \right] \right\} \gamma_5 T_3$$  

form factor $\frac{\Lambda_\Delta^4}{(\Lambda_\Delta^2 - q^2)^2}$

$\rightarrow$ coupling constants

$g_1$ magnetic $\rightarrow$ 7

$g_2 - g_1$ electric $\rightarrow$ 9

$g_3$ Coulomb $\rightarrow$ -2 ... 0
Two-photon exchange amplitude with $\Delta$ intermediate state

\[ M^{\gamma\gamma}_\Delta = -e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N^\Delta_{box}(k)}{D^\Delta_{box}(k)} - e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N^\Delta_{x-box}(k)}{D^\Delta_{x-box}(k)} \]

\[ N^\Delta_{box}(k) = \overline{U}(p_4) V^{\mu\alpha}_{\Delta in}(p_2 + k, q - k) [\not\! p_2 + \not\! k + M_\Delta] P^{3/2}_{\alpha\beta}(p_2 + k) V^{\beta\nu}_{\Delta out}(p_2 + k, k) U(p_2) \times \overline{u}(p_3) \gamma_\mu [\not\! p_3 - \not\! k + m_e] \gamma_\nu u(p_1) \]

\[ N^\Delta_{x-box}(k) = \overline{U}(p_4) V^{\mu\alpha}_{\Delta in}(p_2 + k, q - k) [\not\! p_2 + \not\! k + M_\Delta] P^{3/2}_{\alpha\beta}(p_2 + k) V^{\beta\nu}_{\Delta out}(p_2 + k, k) U(p_2) \times \overline{u}(p_3) \gamma_\nu [\not\! p_3 + \not\! k + m_e] \gamma_\mu u(p_1) \]

**spin-3/2 projection operator**

\[ P^{3/2}_{\alpha\beta}(p) = g_{\alpha\beta} - \frac{1}{3} \gamma_\alpha \gamma_\beta - \frac{1}{3p^2} (\not\! p \gamma_\alpha p \beta + p_\alpha \gamma_\beta \not\! p) \]
$\Delta$ has **opposite** slope to $N$

cancels some of TPE correction from $N$

*Kondratyuk, Blunden, WM, Tjon
nucl-th/0506026 (PRL 2005)*
weaker $\epsilon$ dependence than with $N$ alone

better fit to JLab data!
\[ J^P = \frac{1^+}{2}, \frac{1^-}{2} \] excited \( N^* \) states

\[ Q^2 \sim 3 \text{ GeV}^2 \]

higher mass resonance contributions small

enhance nucleon elastic contribution
Effect on neutron form factors
Neutron correction

since $G_E^n$ is small, effect may be relatively large

sign opposite to proton (since $\kappa_n < 0$)
Effect on neutron LT form factors

large effect at high $Q^2$ for LT-separation method

LT method unreliable for neutron
Effect on neutron PT form factors

FIG. 14: The data points correspond to $\varepsilon = 0.3$ (filled squares) and $\varepsilon = 0.8$ (filled circles) for $2\gamma$ exchange. The small correction for PT is evident.

- Small correction for PT
- 4% (3%) suppression at $\varepsilon = 0.3$ (0.8) for $Q^2 = 3$ GeV$^2$
- 10% (5%) suppression at $\varepsilon = 0.3$ (0.8) for $Q^2 = 6$ GeV$^2$

Summary

- First explicit calculation of TPE taking into account nucleon structure

- Nucleon elastic intermediate states resolves most of LT/PT $G_E^p/G_M^p$ discrepancy

- $\Delta$ excited state opposite sign cf. nucleon, but smaller $P_{11}(1440)$ and $S_{11}(1535)$ contributions small

- Effect on neutron form factors large for LT method, small for PT method
The End
Normal polarization

\[ Q^2 = 6 \text{ GeV}^2 \]

\[ \Delta_N \]

\[ \varepsilon \]

Graph showing the ratio of the 2γ contribution to the total cross section as a function of the center-of-mass scattering angle, \( \Theta_{cm} \), for \( Q^2 = 1 \) (dotted), 3 (dashed) and 6 GeV\(^2\) (solid).
Normal polarization

![Graph showing the ratio of the 2γ contribution to the normal polarization as a function of the center of mass scattering angle, Θ_{cm}, for Q^2 = 1 (dotted), 3 (dashed) and 6 GeV^2 (solid).]

\[ Q^2 = 1 \text{ GeV}^2 \]