What can we learn about the ratio $|G_E^p(q^2)/G_M^p(q^2)|$ by using space-like, time-like data and dispersion relations

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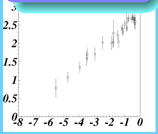
Outline

Three model-independent applications of the dispersion relations to the nucleon form factors

1. Dispersive description of the ratio G_E/G_M
   - Introduction
   - Dispersive approach
   - Results and conclusions
2. The "inverse problem": extracting unphysical form factors from space-like and time-like data
   - Motivations
   - The opportune dispersive technique
   - Results and conclusions
3. Two photon contribution to $e^+e^- \to p\bar{p}$
   - Work in progress

Motivations

$G_E(q^2)/G_M(q^2)$



2000 measurements of the space-like ratio G_E/G_M give an astonishing result: this ratio decreases dramatically as $-q^2$ increases and a linear extrapolation is in agreement with a space-like zero at $(-q^2) \sim 8\ GeV^2$

Many models have been constructed in order to fit these space-like data

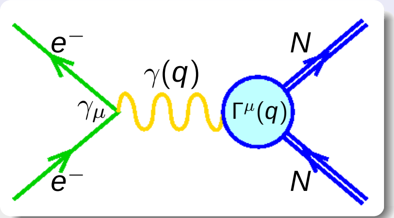
In the q^2 data region $[-5.5\ GeV^2; -0.4\ GeV^2]$ all these models agree

Problems !

Extending the space-like fits outside the data region to verify the presence of a zero

Performing a rigorous continuation of these fits in the time-like region

Nucleon Form Factors

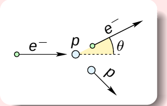


Nucleon current operator (Dirac & Pauli)

$$\Gamma^\mu(q) = \gamma^\mu F_1(q^2) + \frac{i}{2M_N}\sigma^{\mu\nu} q_\nu F_2(q^2)$$

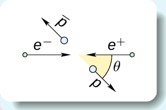
Electric and Magnetic Form Factors

$$G_E(q^2) = F_1(q^2) + \tau F_2(q^2) \qquad \tau = \frac{q^2}{4M_N^2}$$
$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$



Elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_e' \cos^2\frac{\theta}{2}}{4E_e^3 \sin^4\frac{\theta}{2}}\left[G_E^2 + \tau\left(1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right)G_M^2\right]\frac{1}{1+\tau}$$



Annihilation

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\sqrt{1-1/\tau}}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta|G_E|^2\right]$$

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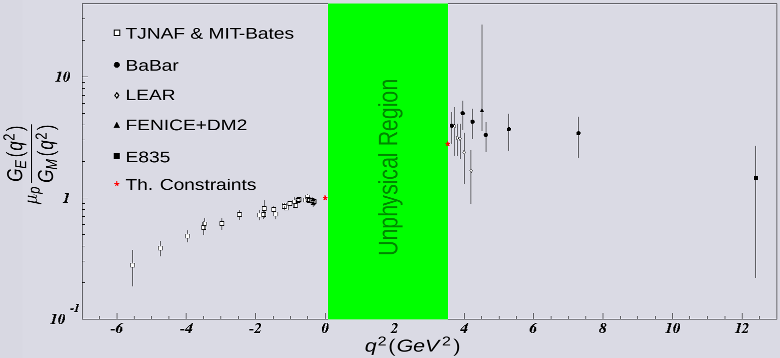
Space-like and time-like data



The ratio

$$R(q^2) = \mu_p \frac{G_E(q^2)}{G_M(q^2)}$$

Theoretical constraints

$$R(0) = 1 \qquad R(4M_N^2) = \mu_p$$

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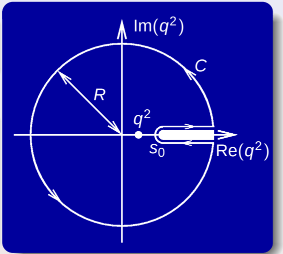
Dispersion Relations

A form factor $f(q^2)$ is an analytic function on the q^2 complex plane with the cut $[s_0 = 4m_\pi^2, \infty)$.



Dispersion relation for the imaginary part

$$f(q^2) = \lim_{R\to\infty} \frac{1}{2\pi i} \oint_C \frac{f(z)dz}{z-q^2} = \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\mathrm{Im} f(s)ds}{s-q^2}$$

DR can be used for the ratio only if G_M has no zeros

Subtraction at $q^2 = 0$ to account a non-vanishing asymptotic limit of the ratio

For $q^2 \le s_0$ R is real

$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{s_0}^{\infty} \frac{\mathrm{Im} R(s)ds}{s(s-q^2)}$$

For $s > s_0$ R is complex

$$\mathrm{Re} R(s) = R(0) + \frac{s}{\pi} \mathrm{Pr} \int_{s_0}^{\infty} \frac{\mathrm{Im} R(s')ds'}{s'(s'-s)}$$

Parameterization and constraints

The imaginary part of R is parameterized by two series of orthogonal polynomials $T_i(x)$

$$\mathrm{Im}R(q^2) \equiv I(q^2) = \begin{cases} \sum_i C_i T_i(x) & x = \frac{2q^2 - s_1 - s_0}{s_1 - s - 0} & s_0 \le q^2 \le s_1 \\ \sum_j D_j T_j(x') & x' = \frac{2s_1}{q^2} - 1 & q^2 > s_1 \end{cases}$$

Thresholds

$$s_0 = 4m_\pi^2$$

$$s_1 = 4M_N^2$$

Theoretical conditions on $ImR(q^2)$

- $R(4m_\pi^2)$ is real $\Longrightarrow I(4m_\pi^2) = 0$
- $R(4M_N^2)$ is real $\Longrightarrow I(4M_N^2) = 0$
- $R(\infty)$ is real $\Longrightarrow I(\infty) = 0$

Theoretical conditions on $R(q^2)$

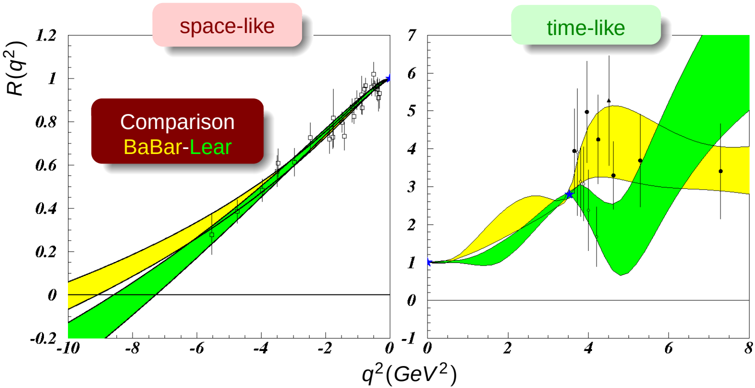
- Continuity at $q^2 = 4m_\pi^2$
- $R(4M_N^2)$ is real and $\mathrm{Re}R(4M_N^2) = \mu_p$

Experimental conditions on $R(q^2)$ and $|R(q^2)|$

- Space-like region ($q^2 < 0$) data for R from TJNAF and MIT-Bates
- Time-like region ($q^2 \ge 4M_N^2$) data for $|R|$ from FENICE+DM2, BaBar, Lear and E835

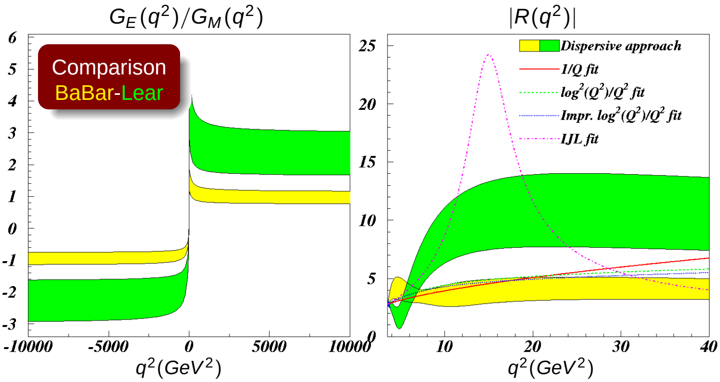
$R(q^2)$

Reconstructed R in space-like and time-like region



Asymptotic $G_E(q^2)/G_M(q^2)$

Asymptotic behaviour of R and comparison with some existing models



Asymptotic value and space-like zero

Real asymptotic values for R

$$R_{\text{BaBar}}(\infty) = 2.65 \pm 0.55 = (0.95 \pm 0.20)\mu_p$$
$$R_{\text{Lear}}(\infty) = 6.4 \pm 1.9 = (2.3 \pm 0.7)\mu_p$$

BaBar in agreement with the scaling law $|G_E| \simeq |G_M|$ but with opposite sign

Asimptotic behaviour of F_2/F_1

$$\lim_{s\to\infty} \frac{s}{4M_N^2}\frac{F_2}{F_1} = \frac{R(\infty)}{\mu_p} - 1 = \begin{cases} -0.05 \pm 0.20 & \text{BaBar} \\ 1.3 \pm 0.7 & \text{Lear} \end{cases}$$

F_2/F_1 decreases like $(1/s)$ (Lear) or faster (BaBar) as s diverges

Space-like zero

The analysis foresees, in a model-independent way, the presence of a space-like zero t_0 for R

$$t_0^{\text{BaBar}} = (-10 \pm 1)\ GeV^2$$
$$t_0^{\text{Lear}} = (-7.9 \pm 0.7)\ GeV^2$$

BaBar only!

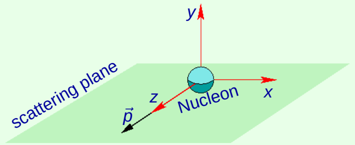
In spite of this the asymptotic scaling law seems preserved

Polarization Formulae

The ratio $R(q^2)$ is complex for $q^2 \geq 4m_\pi^2$

$$R(q^2) = |R(q^2)|e^{i\rho(q^2)}$$

The polarization depends on the phase ρ



Polarization components

$$\mathcal{P}_y = -\frac{\sin(2\theta)|R|\sin(\rho)}{D\sqrt{\tau}}$$ — Does not depend on P_e

$$\mathcal{P}_x = -P_e\frac{2\sin(2\theta)|R|\cos(\rho)}{D\sqrt{\tau}}$$

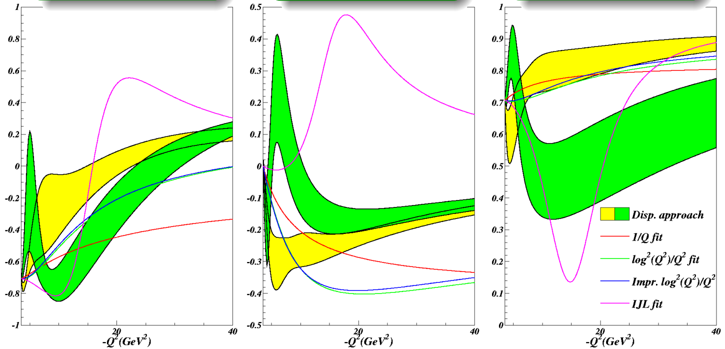
$$\mathcal{P}_z = P_e\frac{2\cos(\theta)}{D}$$ — Does not depend on ρ

$$D = \frac{1+\cos^2\theta + \frac{1}{\tau}|R|^2\sin^2\theta}{\mu_p} \qquad \tau = \frac{q^2}{4M_N^2} \qquad P_e = \text{electron polarization}$$

Single Polarization

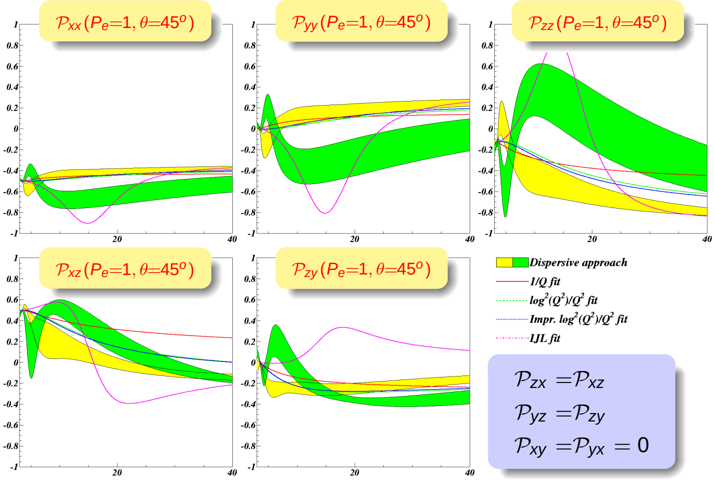
Results for the polarization vector $\vec{\mathcal{P}}$ in comparison with some existing models

$\mathcal{P}_x(P_e{=}1, \theta{=}45^o)$ $\mathcal{P}_y(\theta{=}45^o)$ $\mathcal{P}_z(P_e{=}1, \theta{=}45^o)$



Comparison BaBar-Lear

Double Polarization



$$\mathcal{P}_{zx} = \mathcal{P}_{xz}$$
$$\mathcal{P}_{yz} = \mathcal{P}_{zy}$$
$$\mathcal{P}_{xy} = \mathcal{P}_{yx} = 0$$

BaBar
Lear

G_E/G_M: conclusions

- A dispersive approach, based on the space-like and time-like data for $R(q^2)$ and on the analyticity of the ff's, has been used to obtain a complex expression of the ratio $R(q^2)$ holding for all values of q^2.
- A space-like zero for R has been found in a model independent way at $q^2 \sim -10\ GeV^2$, with BaBar data and at $q^2 \sim -8\ GeV^2$, with Lear data. New space-like data, mainly below $q^2 \sim -10\ GeV^2$, are needed to verify the presence of the zero. (TJNAF plans to attain $q^2 \sim -9\ GeV^2$ in the near future).

- Only by considering the BaBar time-like data the scaling law seems restored, even if asyntotically and with opposite sign.

- The polarization appears the most powerful tool to disentangle among various models.
- New data of polarized annihilation cross section $e^+e^- \to N\bar{N}$ or $p\bar{p} \to e^+e^-$ are expected from GSI, VEPP-2000 and other experiments, they could not only disentangle among the models, but mainly give a complete measurement of the nucleon ff's.

Motivations

Target

To reconstruct the nucleon ff's in the unphysical region, which is the portion of the time-like region where these functions are not experimentally accessible.

Space-like
$e^-N \to e^-N$

Only $q^2 < 0$

Time-like
$e^+e^- \to N\overline{N}$

Only $q^2 \geq 4M_N^2$

Means

- Analyticity
- Space-like and time-like data
- Dispersion relations

Dispersion relation for the logarithm

$$\ln G(q^2) = \frac{\sqrt{s_0 - q^2}}{\pi} \int_{s_0}^{\infty} \frac{\ln|G(s)|ds}{\sqrt{s - s_0}(s - q^2)}$$

$q^2 \leq 0 \qquad s_0 = 4m_\pi^2$

Why?

- To verify the presence of the structure near by the threshold
- To uncover the resonance region to study the vector mesons couplings
- The pleasure of knowing in a model-independent way the ff's where there are no data, but they are still defined

The integral equation

The dispersion relation subtracted at $t = 0$

$$\ln G(t) = \frac{t\sqrt{s_0 - t}}{\pi}\int_{s_0}^{\infty}\frac{\ln|G(s)|ds}{s\sqrt{s-s_0}(s-t)}$$

- It is less dependent on the asymptotic behaviour of the ff.
- No further terms have to be added thanks to the normalization $\ln G(0) = 0$.

Splitting the time-like integral $\int_{s_0}^{\infty}$ into $\int_{s_0}^{s_1} + \int_{s_1}^{\infty}$ we obtain an integral equation of the first kind

$$\ln G(t) - I(t) = \frac{t\sqrt{s_0 - t}}{\pi}\int_{s_0}^{s_1}\frac{\ln|G(s)|ds}{s\sqrt{s-s_0}(s-t)}$$

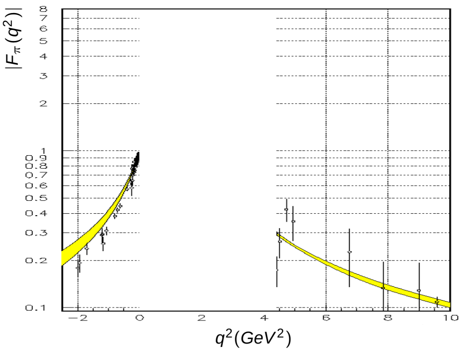
Known from data and theory → $\ln G(t) - I(t)$; Unknown → $G(s)$

- To avoid instabilities at threshold $s_1 = 4M_N^2$, the upper boundary has been shifted to $s_2 = s_1 + \Delta$, with $\Delta \simeq 0.5\ GeV^2$.
- Continuity through the upper boundary s_2 is imposed, at the lower boundary s_0 only very mild continuity is demanded.
- A regularization, depending on a free parameter τ, is introduced by requiring the total curvature of the ff in the unphysical region to be limited.

Testing the method

The test

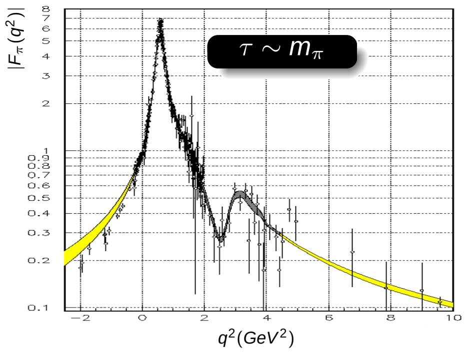
To fix the regularization parameter τ, the space-like (DR) and the time-like (experimental data) pion ff (yellow area) have been used as input in the integral equation to retrieve the time-like ff in the unphysical region.



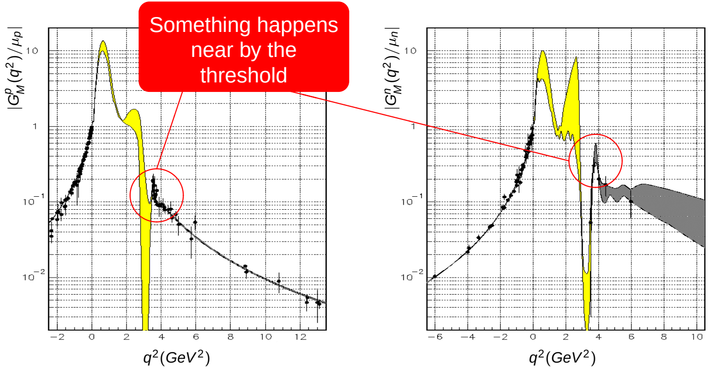
Testing the method

The test

To fix the regularization parameter τ, the space-like (DR) and the time-like (experimental data) pion ff (yellow area) have been used as input in the integral equation to retrieve the time-like ff in the unphysical region (gray area).



Nucleon magnetic form factors



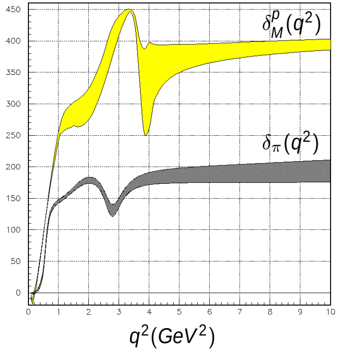
Two resonances

$M_1 \sim 770\ MeV$	$M_2 \sim 1600\ MeV$
$\Gamma_1 \sim 350\ MeV$	$\Gamma_2 \sim 350\ MeV$

The phase and the zero hypothesis

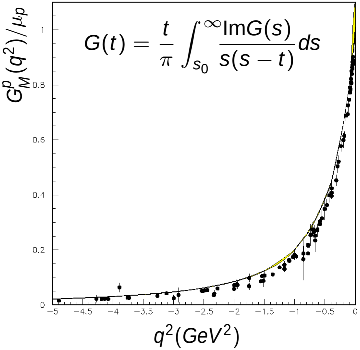
The phase

$$\delta(q^2) = -\frac{q^2\sqrt{q^2 - s_0}}{\pi} \int_{s_0}^{\infty} \frac{\ln|G(s)|ds}{s\sqrt{s-s_0}(s-q^2)}$$



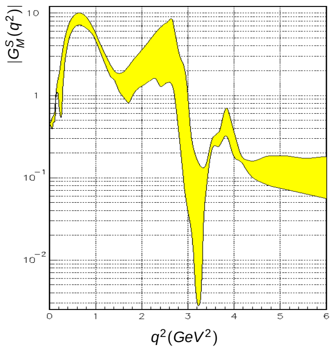
Hypothesis of no zeros in the physical sheet

This hypothesis can be validated by comparing the space-like data and the ff obtained from its time-like imaginary part.

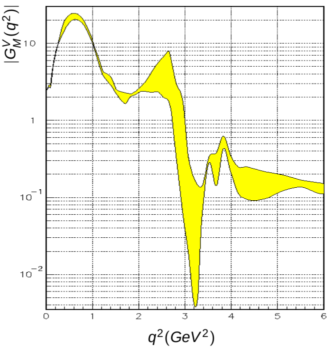


Isospin components

$$G_M^S = \frac{1}{2}(G_M^p + G_M^n)$$

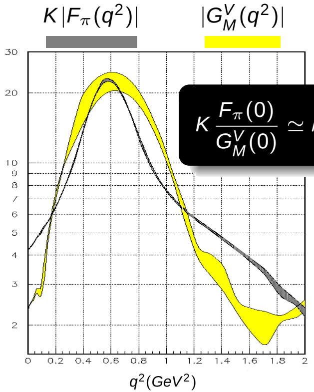


$$G_M^V = \frac{1}{2}(G_M^p - G_M^n)$$



Isospin=1 comparison

Comparison between the pion ff (gray band) and the isovector component of the nucleon magnetic ff (yellow band) which contains only the ρ-family resonances.



$$K\frac{F_\pi(0)}{G_M^V(0)} \simeq \kappa_p$$

The "inverse problem": conclusions

- The unphysical magnetic nucleon ff's have been reconstructed in a model independent way by using data, analyticity and a dispersion relation for $\log|G|$.
- Resonances and peak values have been found consistent with $\rho(770)$ and $\rho'(1600)$ mesons, however very large widths are obtained.
- Phases for the nucleons are consistent with the expectations: $\delta_N \to 360^o$.
- There is an interference pattern near the nucleon threshold ($M \sim 1.88\ GeV$).

- No sizeable Φ contribution (strangeness content) has been found.
- The problem of the very large widths seems underlying "some physics" rather than a technical lack. The regularization procedure works well in finding the width of the ρ in the pion ff.

- The analysis have to be updated by including new data on proton magnetic and electric ff's.

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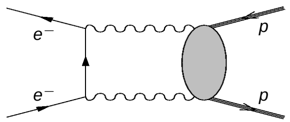
Target

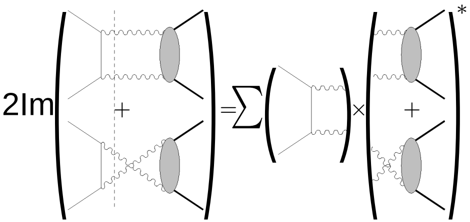
To give an estimate of the two-photon contribution to the time-like cross section for the annihilation

$$e^+e^- \to p\bar{p}$$



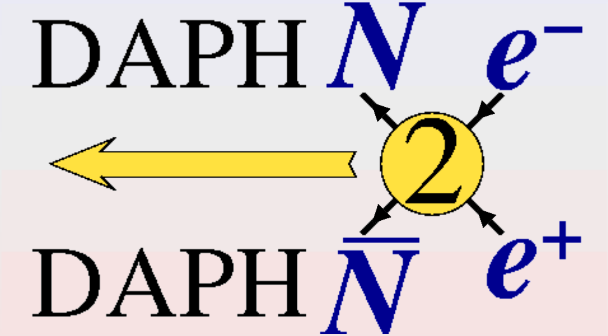
Means

- Data for $\gamma\gamma \to p\bar{p}$
- Cutkosky rule
- Dispersion relations



Why?

To verify how much this contribution affects the extraction of the time-like ff's from the annihilation cross section and if this quantity is measurable.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e^-
2
DAPH$\bar{N}$
e^+