A phenomenological analysis of the four standard nuclear form factors

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$\mathcal{N}^{05}$ - Nucleon Form Factors
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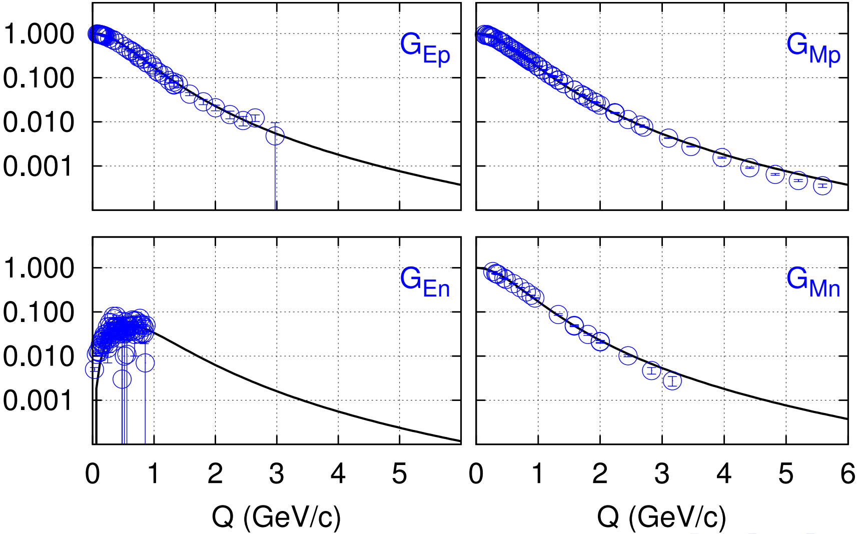
J. Friedrich and Th. Walcher: A coherent interpretation of the form factors of the nucleon in terms of a pion cloud and constituent quarks EPJ A17 (2003) 607

Standard form factors of proton and neutron

- without data from polarisation observables
- compared to standard dipole and Galster fit, respectively



Dipole form factor and charge distribution

dipole form factor:

$$\mathrm{G(Q^2)} = \frac{\mathrm{a_0}}{(1 + (\mathrm{a_1\,Q})^2)^2} \simeq \mathrm{a_0} \cdot \begin{cases} 1 - 2\,(\mathrm{a_1\,Q})^2 \\ 1 - \frac{1}{6}\,\langle \mathrm{r^2} \rangle\, Q^2 \end{cases} \rightarrow \langle \mathrm{r^2} \rangle = 12\,\mathrm{a_1^2}$$

standard dipole form factor:

$$\begin{aligned} 1\,/\,\mathrm{a}_{1,\mathrm{St}}^2 &= 0.71\ \ (\mathrm{GeV/c})^2 \ \ \hat{=}\ \ 0.843\ \mathrm{GeV/c} \\ \mathrm{a}_{1,\mathrm{St}} &= 0.234\ \mathrm{fm} \\ \rightarrow\ \langle \mathrm{r^2} \rangle_{\mathrm{St}}^{1/2} &= 0.811\ \mathrm{fm} \end{aligned}$$

Fourier transform in the Breit system → charge distribution:

$$\rho(\mathrm{r}) = \frac{1}{2\pi^2} \int_0^{\mathrm{inf}} \mathrm{G(Q)}\, \frac{\sin(\mathrm{Qr})}{\mathrm{Qr}}\, \mathrm{Q^2\, dQ}$$

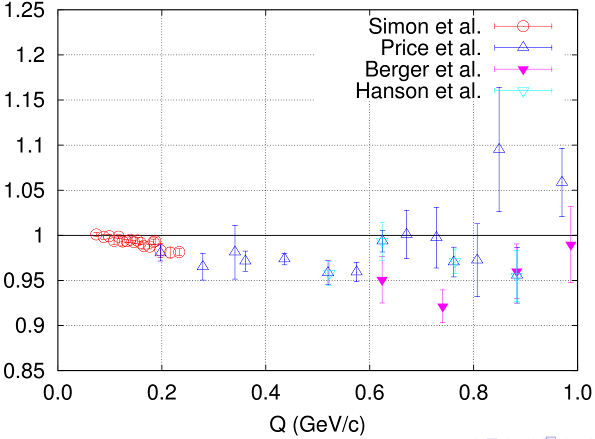
charge distribution of dipole form factor:

$$\rho(\mathrm{r}) = \rho_0\, \mathrm{e}^{-\mathrm{r}/\mathrm{r}_0}$$

$(G_{Ep}\,/$ standard dipole fit$)$ at low to medium momentum transfer

standard dipole form factor: $\langle r^2\rangle_{St}^{1/2} = 0.811$ fm

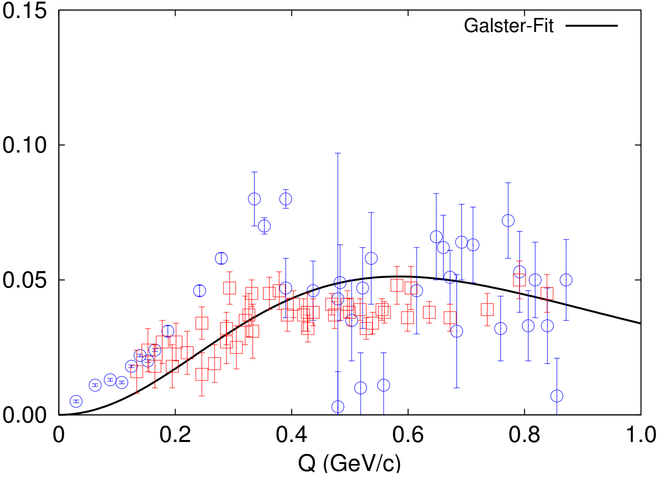
Simon et al., NP A333 (1980) 381: $\langle r^2\rangle^{1/2} = 0.862 \pm 0.012$ fm



G_{En} from D(e,e')D

Data before 1990

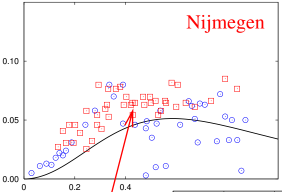
Platchkov et al., NPA 510 (1990) 740, D(e,e')D analysed with Paris Potential

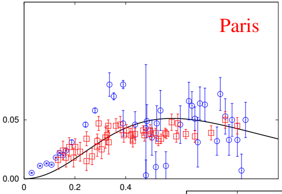


G_{En} from D(e,e')D

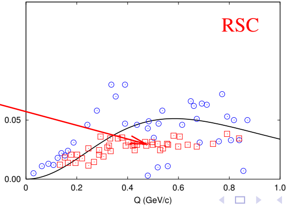
Data before 1990

Platchkov et al. NP A510 (1990)
analysed with different potentials





factor 2.2 !!!



G_{En} from polarisation measurements

Galster parametrisation:

$$G_{En}(Q^2) = \frac{a_G\,\tau}{1+b_G\,\tau}\cdot\frac{1}{(1+Q^2/m_D^2)^2}\;,\quad \tau = Q^2/(4m_n^2)\;,\quad m_D^2 = 0.71\ (\mathrm{GeV/c})^2$$

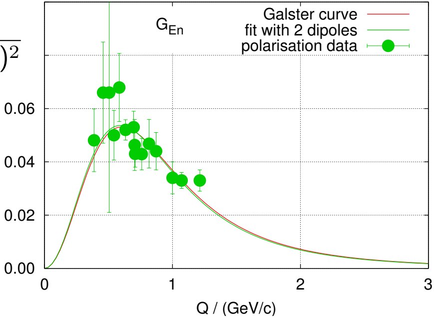
$$\langle r^2\rangle = -6\,\frac{dG_{En}^2}{dQ^2}(Q^2=0) = 6\,a_G/(4m_n^2)\;,\quad \langle r^2\rangle_{exp} = -0.115\mathrm{fm}^2 \rightarrow a_G = 1.737$$

$b_G = 4.63 \pm 0.36$ (fit to polarisation data)

(practically) **equivalent description:**

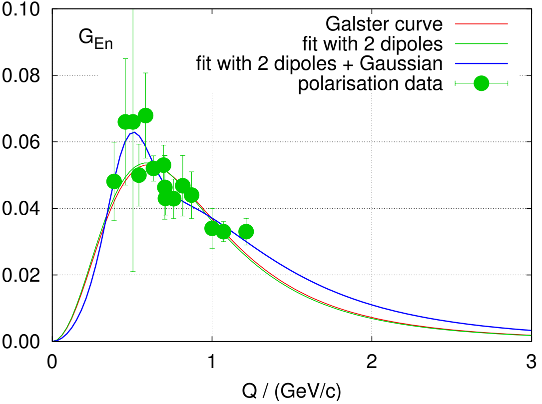
$$G_{en}(Q^2) = \frac{a_{d,0}}{(1+(a_{d,1}Q)^2)^2} - \frac{a_{d,0}}{(1+(a_{d,2}Q)^2)^2}$$

$a_{d,0} = 1.0$ (practically arbitrary)
$a_{d,1} = 0.225 \pm 0.011\,\mathrm{fm}$
$a_{d,2} = 0.246 \pm 0.012\,\mathrm{fm}$
(fit to 15 polarisation data:
χ^2/d.f. = 1.07)



The triggering conjecture ...

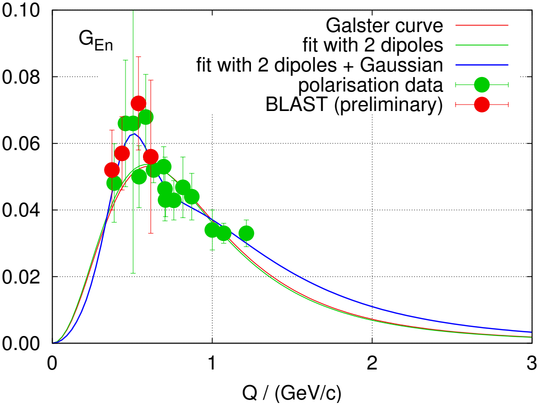
... is there a bump at low Q?



$$\Delta\chi^2 = \chi^2_{smooth} - \chi^2_{smooth+bump} = 5.4$$

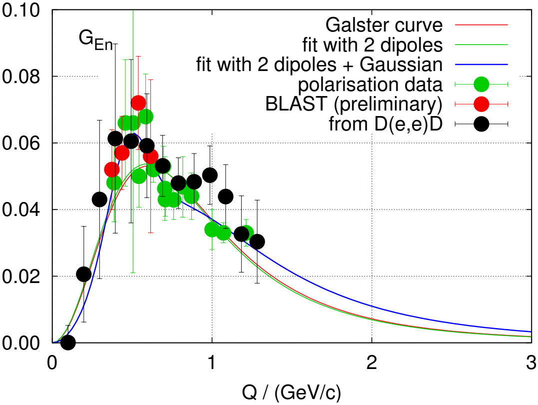
Again G_{En} ...

... now with additional (preliminary) data from BLAST:

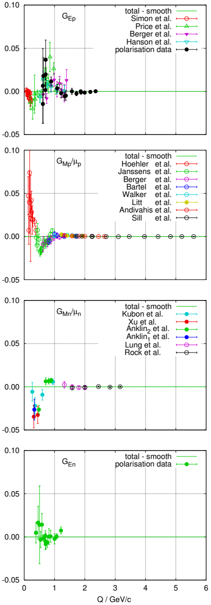


Again G_{En} from polarisation measurements ...

... now with additional data from analysis of the deuteron quadrupole form factor (R. Schiavilla and I. Sick, PR C64 (2001) 041002R)



The form factors minus sum of 2 dipoles



fit to the data:

sum of 2 dipoles

... there is something left at low momentum transfer!

Phenomenological ansatz for $G_{\rm ep}$, $G_{\rm mp}$, $G_{\rm mn}$ and $G_{\rm en}$

Smooth form: 2 dipoles

$$G_{\rm s}(Q^2) \ = \frac{a_{10}}{(1\ +\ Q^2/a_{11})^2} + \frac{a_{20}}{(1\ +\ Q^2/a_{21})^2}$$

"Bump": "Gaussian"

$$G_{\rm b}(Q^2) \ = a_{\rm b} \cdot \left[e^{-\frac{1}{2}\left(\frac{Q-Q_{\rm b}}{\sigma_{\rm b}}\right)^2} + e^{-\frac{1}{2}\left(\frac{Q+Q_{\rm b}}{\sigma_{\rm b}}\right)^2} \right] \cdot Q^2$$

Total form factor:

$$G_{\rm N}(Q^2) = G_{\rm s}(Q^2)\ +\ G_{\rm b}(Q^2)$$

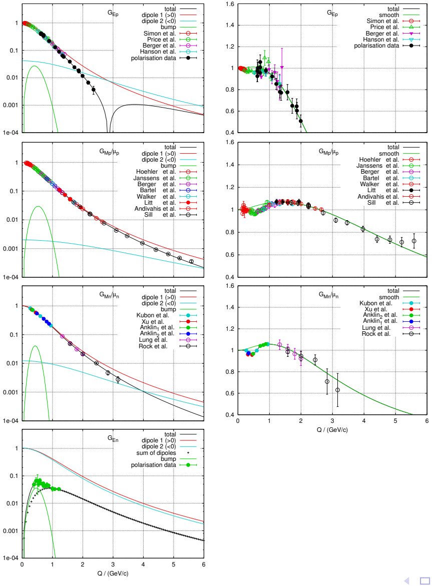
Charge in $G_{\rm S}$: $G_{\rm S}(0) = a_{10} + a_{20}$ (normalisation!)

in $G_{\rm b}$: $G_{\rm b}(0) = 0$

Fit of the phenomenological ansatz to the form factors

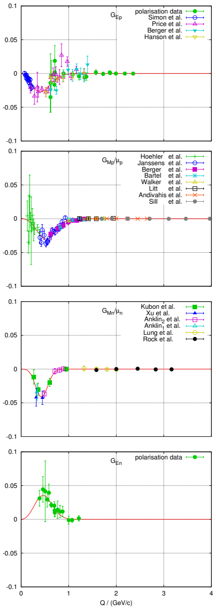
and comparison to the standard dipole fit

form factors



form factors divided by standard dipole fit

Form factors minus smooth form



“smooth form” = sum of 2 Dipoles
(fitted to the data together with a Gaussian)

... there is some “bump”-structure around 0.2 $(\mathrm{GeV/c})^2$

Model ansatz for $\mathrm{G_{ep}}$, $\mathrm{G_{mp}}$, $\mathrm{G_{mn}}$ und $\mathrm{G_{en}}$

1. The nucleons and their form factors:

$$\begin{aligned} \mathrm{p} &= \mathrm{a_p} \cdot \mathrm{p}^0 + \mathrm{b_p} \cdot (\qquad\quad \mathrm{n}^0 + \pi^+), \quad \mathrm{a_p} + \mathrm{b_p} = 1 \\ &= \qquad \mathrm{p}^0 + \mathrm{b_p} \cdot (-\mathrm{p}^0 + \mathrm{n}^0 + \pi^+) \end{aligned}$$

$$\mathrm{G_p} = \quad \mathrm{G_p^0} + \mathrm{b_p} \cdot (-\mathrm{G_p^0} + \mathrm{G_n^0} + \mathrm{G}^{\pi^+}) = \mathrm{G_p^0} + \mathrm{G_p^{pol}}$$

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&= \qquad \mathrm{p^0} + \mathrm{b_p} \cdot (\ -\mathrm{p^0} + \ \mathrm{n^0} + \pi^+ \ )
\end{aligned}$$

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$$\begin{aligned}
\mathrm{n} &= \mathrm{a_n} \cdot \mathrm{n^0} + \mathrm{b_n} \cdot (\quad \mathrm{p^0} \qquad\quad + \pi^+ \ ), \quad \mathrm{a_n} + \mathrm{b_n} = 1 \\
&= \qquad \mathrm{n^0} + \mathrm{b_n} \cdot (\quad \mathrm{p^0} - \ \mathrm{n^0} + \pi^- \ )
\end{aligned}$$

$$\mathrm{G_n} = \quad \mathrm{G_n^0} + \mathrm{b_n} \cdot (\quad \mathrm{G_p^0} - \mathrm{G_n^0} + \mathrm{G}^{\pi^-}) = \mathrm{G_n^0} + \mathrm{G_n^{pol}}$$

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$$\mathrm{G_n} = \mathrm{G_n^0} + \mathrm{b_n} \cdot (\mathrm{G_p^0} - \mathrm{G_n^0} + \mathrm{G}^{\pi^-}) = \mathrm{G_n^0} + \mathrm{G_n^{pol}}$$

2. Constituent quarks for $\mathrm{N}^0 : \mathrm{p}^0 = (\mathrm{uud})\,,\ \mathrm{n}^0 = (\mathrm{udd})$

   Ansatz for form factors of quark-distributions: $\mathrm{G^{qN}} = \dfrac{\mathrm{a_0^{qN}}}{(1 + \mathrm{Q}^2/\mathrm{a_1^{qN}})^2}$

Model ansatz for $\mathrm{G_{ep}}$, $\mathrm{G_{mp}}$, $\mathrm{G_{mn}}$ und $\mathrm{G_{en}}$

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$$\begin{aligned}
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3. Form factor of pion-distribution:

$$\mathrm{G}^{\pi^\pm} = \pm 1 \cdot (\, 1 - \frac{1}{6}(\mathrm{Q}/\mathrm{a}_1^\pi)^2 \,)\, \mathrm{e}^{-\frac{1}{4}(\mathrm{Q}/\mathrm{a}_1^\pi)^2}$$

   (p-wave in h. o. potential)

Model ansatz for G_{ep}, G_{mp}, G_{mn} und G_{en}

1. The nucleons and their form factors:

$$\begin{aligned} p &= a_p \cdot p^0 + b_p \cdot (\ n^0 + \pi^+), \quad a_p + b_p = 1 \\ &= p^0 + b_p \cdot (-p^0 + n^0 + \pi^+) \end{aligned}$$

$$G_p = G_p^0 + b_p \cdot (-G_p^0 + G_n^0 + G^{\pi^+}) = G_p^0 + G_p^{pol}$$

$$\begin{aligned} n &= a_n \cdot n^0 + b_n \cdot (p^0 + \pi^+), \quad a_n + b_n = 1 \\ &= n^0 + b_n \cdot (p^0 - n^0 + \pi^-) \end{aligned}$$

$$G_n = G_n^0 + b_n \cdot (G_p^0 - G_n^0 + G^{\pi^-}) = G_n^0 + G_n^{pol}$$

2. Constituent quarks for $N^0 : p^0 = (uud)\,,\ n^0 = (udd)$

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   (p-wave in h. o. potential)

Strict isospin invariance: $G^{u,p} \sim G^{d,n}\,,\ G^{d,p} \sim G^{u,n}\,,\ G^{\pi^+} \sim G^{\pi^-}$

Pion-cloud-model ansatz for electric form factors

proton: $$\begin{aligned} G_{\mathrm{Ep}} = {} & \left({G_{\mathrm{Ep}}}^{\mathrm{u,p}} + {G_{\mathrm{Ep}}}^{\mathrm{d,p}}\right) \\ & + b_{\mathrm{p}} \cdot \left(- \left({G_{\mathrm{Ep}}}^{\mathrm{u,p}} + {G_{\mathrm{Ep}}}^{\mathrm{d,p}}\right) + \left({G_{\mathrm{Ep}}}^{\mathrm{u,n}} + {G_{\mathrm{Ep}}}^{\mathrm{d,n}}\right) + {G_{\mathrm{Ep}}}^{\pi^{+}}\right) \end{aligned}$$

Pion-cloud-model ansatz for electric form factors

proton: $G_{\mathrm{Ep}} = \quad (G_{\mathrm{Ep}}{}^{\mathrm{u,p}} + G_{\mathrm{Ep}}{}^{\mathrm{d,p}})$

$\quad + b_{\mathrm{p}} \cdot (-\, (G_{\mathrm{Ep}}{}^{\mathrm{u,p}} + G_{\mathrm{Ep}}{}^{\mathrm{d,p}}) + (G_{\mathrm{Ep}}{}^{\mathrm{u,n}} + G_{\mathrm{Ep}}{}^{\mathrm{d,n}}) + G_{\mathrm{Ep}}{}^{\pi^+})$

neutron: $G_{\mathrm{En}} = \quad (G_{\mathrm{En}}{}^{\mathrm{u,n}} + G_{\mathrm{En}}{}^{\mathrm{d,n}})$

$\quad + b_{\mathrm{n}} \cdot (\quad (G_{\mathrm{En}}{}^{\mathrm{u,p}} + G_{\mathrm{En}}{}^{\mathrm{d,p}})$ - $(G_{\mathrm{En}}{}^{\mathrm{u,n}} + G_{\mathrm{En}}{}^{\mathrm{d,n}}) + G_{\mathrm{En}}{}^{\pi^-})$

Pion-cloud-model ansatz for electric form factors

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neutron: $$G_{En} = ({G_{En}}^{u,n} + {G_{En}}^{d,n}) + b_n \cdot (({G_{En}}^{u,p} + {G_{En}}^{d,p}) - ({G_{En}}^{u,n} + {G_{En}}^{d,n}) + {G_{En}}^{\pi^-})$$

normalisation:

proton: $$G_{Ep}(0) = G^0_{Ep}(0) + b_p[-G_{Ep}(0) + G_{En}(0) + G_{\pi^+}(0)] = 1 + 0$$

neutron: $$G_{En}(0) = G^0_{En}(0) + b_n \underbrace{[\ G_{Ep}(0) - G_{En}(0) + G_{\pi^-}(0)]}_{=0} = 0 + 0$$

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charge in “bump” $=$ pion-contribution $= +b_p, -b_n \neq 0$!!

charge in polarisation = 0

Pion-cloud-model ansatz for electric form factors

proton: $G_{Ep} = (G_{Ep}{}^{u,p} + G_{Ep}{}^{d,p})$

$+ b_p \cdot (- (G_{Ep}{}^{u,p} + G_{Ep}{}^{d,p}) + (G_{Ep}{}^{u,n} + G_{Ep}{}^{d,n}) + G_{Ep}{}^{\pi^+})$

neutron: $G_{En} = (G_{En}{}^{u,n} + G_{En}{}^{d,n})$

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amplitudes of the dipole form factors in the smooth part ....

... in the proton: $a^{u,p}_{0,E} = \frac{4}{3}, a^{d,p}_{0,E} = -\frac{1}{3}$

... in the neutron: $a^{u,n}_{0,E} = \frac{2}{3}, a^{d,n}_{0,E} = -\frac{2}{3}$

Pion-cloud-model ansatz for electric form factors

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&+ b_p \cdot (-\,(G_{Ep}{}^{u,p} + G_{Ep}{}^{d,p}) + (G_{Ep}{}^{u,n} + G_{Ep}{}^{d,n}) + G_{Ep}{}^{\pi^+}) \\
\text{neutron: } G_{En} &= \quad (G_{En}{}^{u,n} + G_{En}{}^{d,n}) \\
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charge in “bump” = pion-contribution $= +b_p, -b_n \neq 0$!!

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amplitudes of the dipole form factors in the smooth part ....

... in the proton: $a^{u,p}_{0,E} = \frac{4}{3}, a^{d,p}_{0,E} = -\frac{1}{3}$

... in the neutron: $a^{u,n}_{0,E} = \frac{2}{3}, a^{d,n}_{0,E} = -\frac{2}{3}$

→ 6 free parameters per G_{EN} : $a^{u,N}_{1,N}, a^{d,N}_{1,N}, a^{u,\bar{N}}_{1,N}, a^{d,\bar{N}}_{1,N}, b_N, a_1^{\pi^\pm}$

Parmeters of the model-ansatz fits

nº	$a_1^{u,p}$ (GeV/c)^2	$a_1^{d,p}$ (GeV/c)^2	$a_1^{u,n}$ (GeV/c)^2	$a_1^{d,n}$ (GeV/c)^2	$b_{p/n}$	a^{π^+,π^-} GeV/c	d.o.f.	$\chi^2_{d.o.f.}$
	G_{Ep}							
1	1.000(100)	2.03 (72)	57.(1200.)	78.(2500.)	0.10(4)	0.198(12)	64	0.932
2	1.008(20)	2.54 (16)	2.54(-)	1.008(-)	0.11(1)	0.203 (6)	66	0.926
3	1.051(20)	2.39 (14)	2.53(-)	2.22(-)	0.118(13)	0.204(6)	66	0.928

1: All 6 parameters free. Parameters of n^0 completely undetermined.

2: $a_1^{u,n} = a_1^{d,p}$, $a_1^{d,n} = a_1^{u,p}$ (isospin-inv.), 2 free par. less, $\Delta\chi^2 = 1.5$ (insign.)

3: $a_1^{d,n}$ from fit 3 of G_{En}

Parmeters of the model-ansatz fits

n°	$a_1^{u,p}$ (GeV/c)^2	$a_1^{d,p}$ (GeV/c)^2	$a_1^{u,n}$ (GeV/c)^2	$a_1^{d,n}$ (GeV/c)^2	$b_{p/n}$	a^{π^+,π^-} GeV/c	d.o.f.	$\chi^2_{d.o.f.}$
	G_{Ep}							
1	1.000(100)	2.03 (72)	57.(1200.)	78.(2500.)	0.10(4)	0.198(12)	64	0.932
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1: All 6 parameters free. Parameters of n^0 completely undetermined.

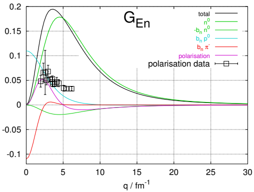
2: $a_1^{u,n} = a_1^{d,p}$, $a_1^{d,n} = a_1^{u,p}$ (isospin-inv.), 2 free par. less, $\Delta\chi^2 = 1.5$ (insign.)

3: $a_1^{d,n}$ from fit 3 of G_{En}

n°	$a_1^{u,p}$ (GeV/c)^2	$a_1^{d,p}$ (GeV/c)^2	$a_1^{u,n}$ (GeV/c)^2	$a_1^{d,n}$ (GeV/c)^2	$b_{p/n}$	a^{π^+,π^-} GeV/c	d.o.f.	$\chi^2_{d.o.f.}$
	G_{En}							
1	1.008(-)	2.54 (-)	2.54 (-)	1.008(-)	0.11(-)	0.203(-)	-	-
2	1.008(-)	2.54(-)	6.2(6.4)	5.3 (5.1)	0.086(10)	0.203(-)	12	0.807
3	1.008(-)	2.54(-)	2.54(-)	2.22(2)	0.074(5)	0.203(-)	13	0.818

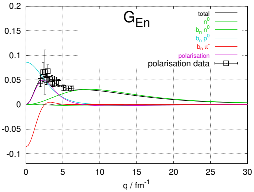
1. isospin-invariance: $a_1^{u,n} = a_1^{d,p}$, $a_1^{d,n} = a_1^{u,p}$ and $G^{\pi^-} = G^{\pi^+}$
2. reduction of G_{En} by fitting $a_1^{d,n}$ and $a_1^{u,n}$, keeping $a_0^{d,n} = -a_0^{u,n}$ = -2/3
3. as fit 2 but $a_1^{u,n} = a_1^{d,p}$, fitting only $a_1^{d,n}$

Fit of pion-cloud-model ansatz to G_{En}



Fit 1: Isospin-invariance

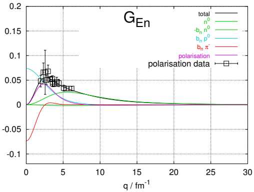
$a_1^{u,n} = a_1^{d,p}$, $a_1^{d,n} = a_1^{u,p}$ and $G^{\pi^-} = G^{\pi^+}$



Fit 2: Reduction of amplitude

by fitting $a_1^{d,n}$ and $a_1^{u,n}$ and also b_n

while keeping $a_0^{d,n} = -a_0^{u,n}$ = -2/3



Fit 3: Like fit 2 but setting $a_1^{u,n} = a_1^{d,p}$,

fit of $a_1^{d,n}$ and b_n

Parameters of the fits of the phenomenological ansatz

	a_{10}	a_{11} $(\mathrm{GeV/c})^2$	a_{20}	a_{21} $(\mathrm{GeV/c})^2$	a_b $(\mathrm{GeV/c})^{-2}$	Q_b $(\mathrm{GeV/c})$	σ_b $(\mathrm{GeV/c})$	$N_{\mathrm{d.o.f.}}$	$\chi^2_{\mathrm{d.o.f.}}$
G_{Ep}	1.0415	0.76517	-0.0415	6.1581	-0.2385	0.0739	0.2735	64	0.933
±	0.0407	0.0326	-.-	2.5248	0.1884	1.0092	0.2982		
G_{En}	1.0400	1.73010	-1.0400	1.5018	0.2057	0.3050	0.1869	11	0.753
±	-.-	-.-	-.-	0.7542	0.1274	0.1603	0.0807		
G_{Mp}	1.0024	0.75561	-0.0024	14.690	-0.1592	0.3248	0.2314	75	0.869
±	0.0004	0.00256	-.-	2.027	0.0172	0.0380	0.0156		
G_{Mn}	1.0123	0.77032	-0.0123	6.7486	-0.2816	0.3328	0.1429	14	0.579
±	0.0064	0.01028	-.-	4.57	0.0327	0.0306	0.0184		

Reference: J. Friedrich and Th. Walcher, EPJ A 17 (2003) 607, Table 2

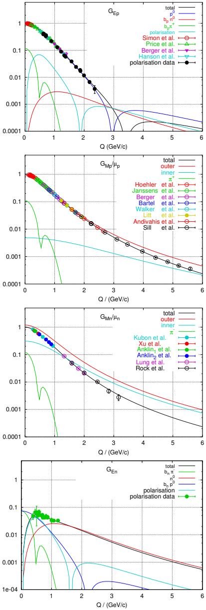
Parameters of the fits of the pion-cloud model

		G_p^0		G_n^0		$G^{\pi^\pm}$			
	fit	a_1^{up} $(\mathrm{GeV/c})^2$	a_1^{dp} $(\mathrm{GeV/c})^2$	a_1^{un} $(\mathrm{GeV/c})^2$	a_1^{dn} $(\mathrm{GeV/c})^2$	$b_{p,n}$	$a_1^{\pi^\pm}$ $(\mathrm{GeV/c})$	N_{dof}	χ^2_{dof}
G_{Ep}	3	1.0513	2.39126	2.5317	2.2341	0.1178	0.2042	66	0.928
	±	0.0249	0.10383	-.-	-.-	0.0132	0.0065		
G_{En}	3	1.0080	2.540	2.540	2.1350	0.0649	0.2042	13	0.782
	±	-.-	-.-	-.-	0.0214	0.0047	-.-		

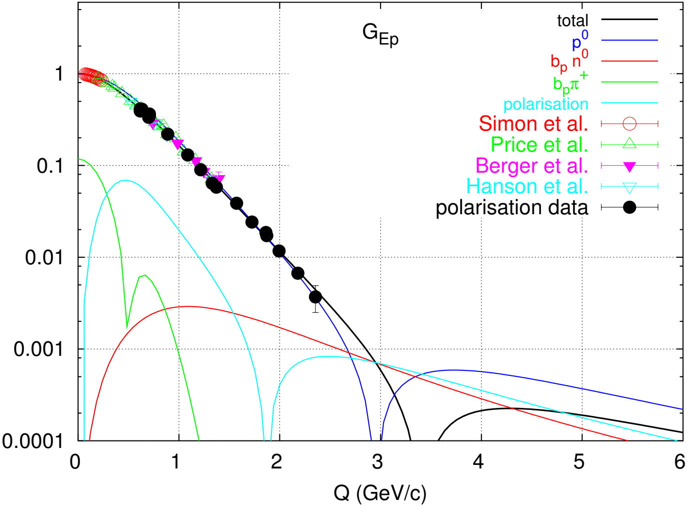
		outer distribution		inner distribution		G^π			
	fit	a_0^{out}	a_1^{out} $(\mathrm{GeV/c})^2$	a_0^{in}	a_1^{in} $(\mathrm{GeV/c})^2$	$a_0^{\pi^\pm}$	$a_1^{\pi^\pm}$ $(\mathrm{GeV/c})$		
G_{Mp}	1	0.9135	0.81848	-0.0048	9.5761	0.1098	0.2126	75	0.887
	±	0.0049	0.00769	0.0010	1.2277	0.0071	0.0065		
G_{Mn}	3	1.1899	1.06081	-0.3094	1.8519	0.1195	0.1886	15	0.0837
	±	1.347	0.466	1.383	1.8	0.0394	0.0194		

Reference: J. Friedrich and Th. Walcher, EPJ A 17 (2003) 607
Tables 3, 4

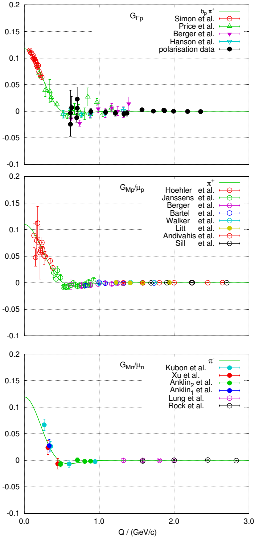
Pion-cloud model fits to G_{Ep}, G_{Mp}, G_{Mn} and G_{En}



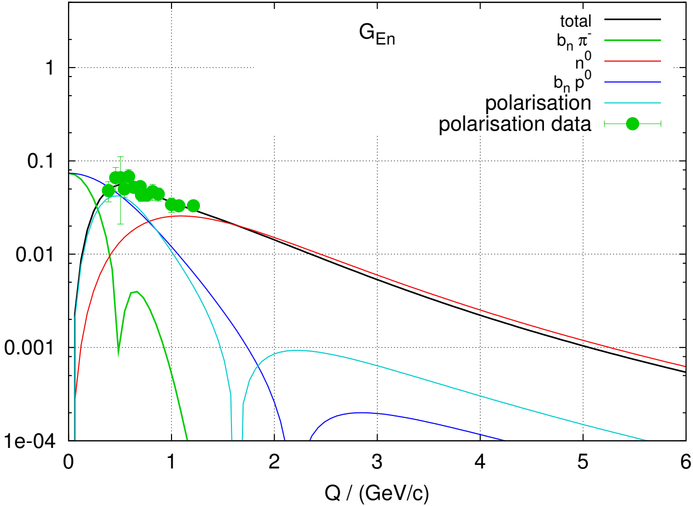
The electric form factor of the proton



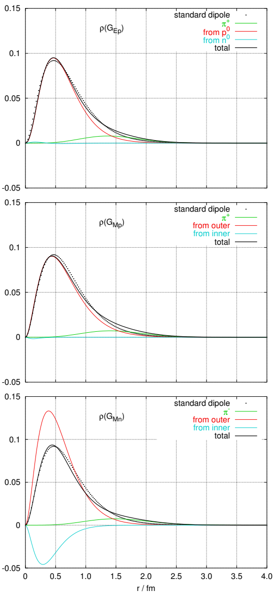
Contribution of the pion-cloud to G_{Ep}, G_{Mp} and G_{Mn}



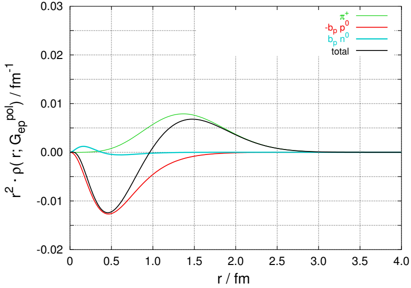
The electric form factor of the neutron



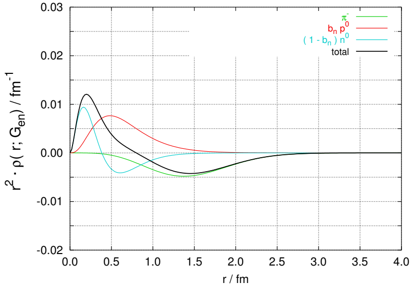
Distributions in the Breit frame (pion-cloud model)



Distributions in the Breit frame (pion-cloud model)



polarisation in the proton:
$\mathrm{r}^2 \cdot \rho(\mathrm{G}_{\mathrm{Ep}}^{\mathrm{polarisation}})$



charge in the neutron:
$\mathrm{r}^2 \cdot \rho(\mathrm{G}_{\mathrm{En}})$

rms-radii in the pion-cloud model

The proton (parameters from proton-fit 3):

$$\begin{aligned} \langle r^2\rangle_p &= (1-b_p)\cdot\langle r^2\rangle_{p^0} + b_p\cdot\left(\langle r^2\rangle_{n^0} + \langle r^2\rangle_{\pi^+} \right) \\ &= 0.88\cdot\ 0.527\ \mathrm{fm}^2 + 0.12\cdot\left(-\ 0.016\ \mathrm{fm}^2 + 2.332\ \mathrm{fm}^2 \right) \\ &= 0.464\ \mathrm{fm}^2 \qquad -\ 0.002\ \mathrm{fm}^2 + 0.280\ \mathrm{fm}^2 \\ &= 0.738\ \mathrm{fm}^2 \end{aligned}$$

$$\boxed{\langle \mathbf{r}^2\rangle_{\mathbf{p}}^{1/2} = \mathbf{0.859\ fm}}$$

The neutron (parameters from neutron-fit 3):

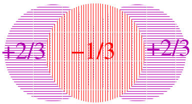
$$\begin{aligned} \langle r^2\rangle_n &= (1-b_n)\cdot\langle r^2\rangle_{n^0} + b_n\cdot\left(\langle r^2\rangle_{p^0} - \langle r^2\rangle_{\pi^-} \right) \\ &= -0.93\cdot\ 0.016\ \mathrm{fm}^2 + 0.07\cdot\left(0.527\ \mathrm{fm}^2 - 2.356\ \mathrm{fm}^2 \right) \\ &= -\ 0.015\ \mathrm{fm}^2 + \qquad 0.037\ \mathrm{fm}^2 - 0.165\ \mathrm{fm}^2 \\ &= -\ 0.150\ \mathrm{fm}^2 \end{aligned}$$

$$\boxed{-|\langle \mathbf{r}^2\rangle_{\mathbf{n}}|^{1/2} = -\ \mathbf{0.388\ fm}}$$

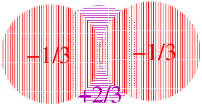
Smooth part of form factor ↔ quark distributions?

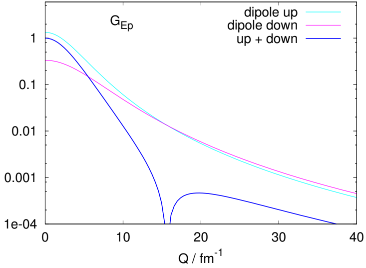
$$G_{Ep} = \frac{2\cdot 4/3}{(1+(\underbrace{0.192}_{\text{fm}}\,Q)^2)^2} - \frac{1/3}{(1+(\underbrace{0.128}_{\text{fm}}\,Q)^2)^2} \qquad G_{En} = \frac{2/3}{(1+(\underbrace{0.124}_{\text{fm}}\,Q)^2)^2} - \frac{2\cdot 1/3}{(1+(\underbrace{0.132}_{\text{fm}}\,Q)^2)^2}$$

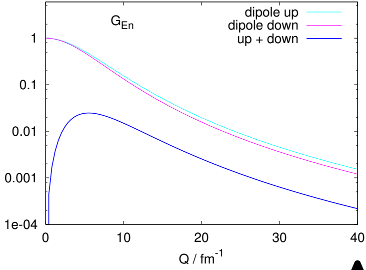
proton



neutron







Dispers.-theor. description of the nucleon form factors

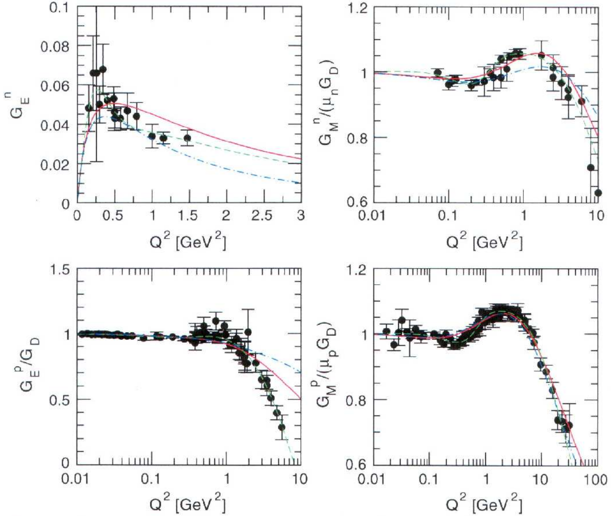


Fig. 1. The nucleon form factors for space-like momentum transfer. The solid line gives our best fit for $b_{ne} = (-1.33 \pm 0.027 \pm 0.03) \cdot 10^{-3}$ fm [12] while the dash-dotted line gives fit 1 from ref. [2]. The dashed lines give the result of the phenomenological fit of ref. [9].

H.-W. Hammer and Ulf-G. Meißner, EPJ A 20 (2004) 469

G_{Ep}: Dispersion theoretical description

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Updated dispersion-theoretical analysis of the nucleon electromagnetic form factor

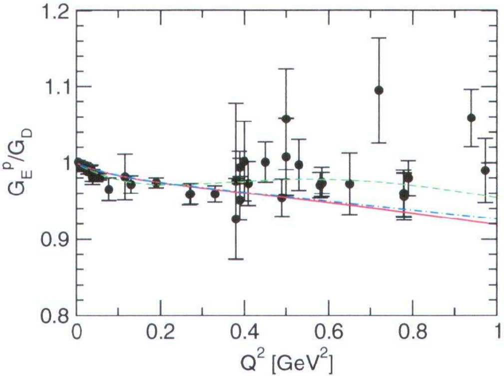

Fig. 3. The proton charge form factor G_E^p for momentum transfers $Q^2 = 0 \ldots 1$ GeV^2. The curves are as in fig. 1.

G_{En}: Dispersion theoretical description

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Updated dispersion-theoretical analysis of the nucleon electromagnetic form factor

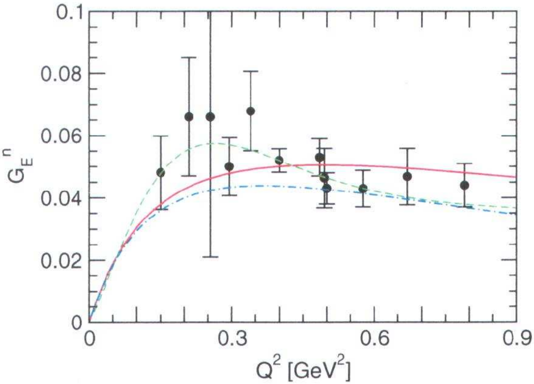

Fig. 2. The neutron charge form factor G_E^n for momentum transfers $Q^2 = 0 \ldots 0.9$ GeV^2. The curves are as in fig. 1.