Novel dispersion-theoretical analysis of the nucleon electromagnetic form factors

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- **Why dispersion theory?**
- **Theoretical framework**
- **Discussion of the spectral functions**
- **Results for space- and time-like form factors**
- **Summary and outlook**

with: Maxim A. Belushkin, Hans-Werner Hammer

Introduction

WHY DISPERSION RELATIONS ?

- Model-independent approach $\rightarrow$ important non-perturbative tool to analyze data
- Dispersion relations are based on fundamental principles: unitarity & analyticity
- Connect data from small to large momentum transfer
  as well as time- and space-like data
- Allow for a **simultaneous analysis** of all four em form factors
- Spectral functions encode perturbative and non-perturbative physics
  e.g. vector meson couplings, multi-meson continua, pion cloud, ...
- Spectral functions also encode information on the strangeness vector current
  $\rightarrow$ sea-quark dynamics, strange matrix elements
- Allow to extract nucleon electric and magnetic radii
- Can be matched to chiral perturbation theory

Theoretical framework

BASIC DEFINITIONS

- Nucleon matrix elements of the em vector current J_μ^I

$$\langle N(p')|J_\mu^I|N(p)\rangle = \bar{u}(p') \left[F_1^I(t)\,\gamma_\mu + i\,\frac{F_2^I(t)}{2m}\,\sigma_{\mu\nu}q^\nu \right] u(p)$$

⋆ isospin $I = S, V$ (isoScalar, isoVector)

⋆ four-momentum transfer $t \equiv q^2 = (p' - p)^2$

⋆ F_1 = Dirac form factor, F_2 = Pauli form factor

⋆ Normalizations: $F_1^V(0) = F_1^S(0) = 1/2$, $F_2^{S,V}(0) = (\kappa_p \pm \kappa_n)/2$

⋆ Sachs form factors: $G_E = F_1 + \frac{t}{4m^2} F_2\ ,\ G_M = F_1 + F_2$

⋆ Nucleon radii: $F(t) = F(0)\ \left[1 + t\langle r^2\rangle/6 + \ldots\right]$ [expect for the neutron charge ff]

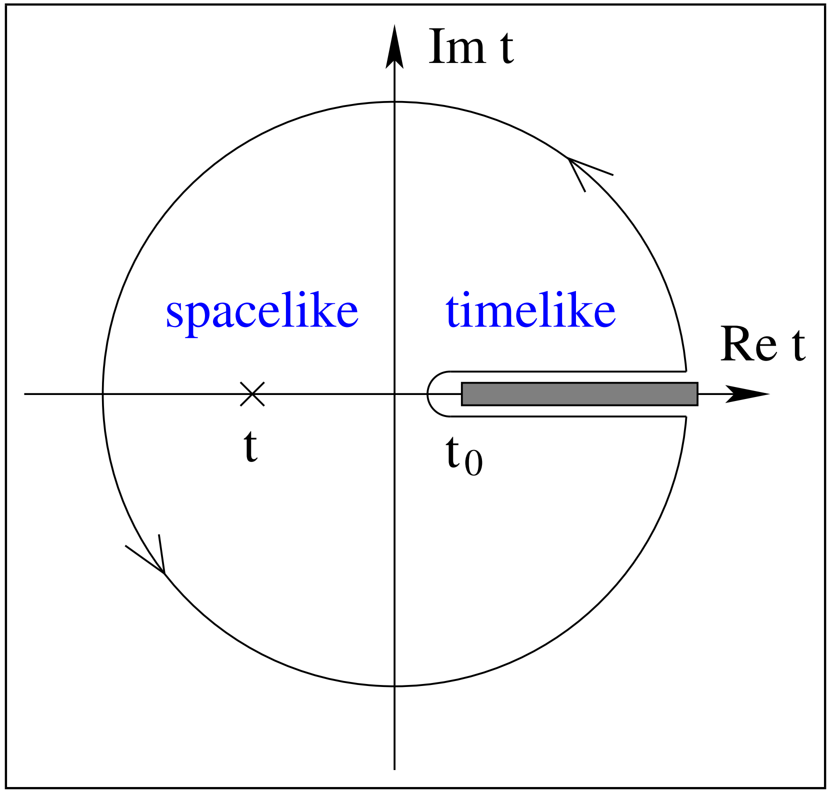
DISPERSION RELATIONS

Federbush, Goldberger, Treiman, Drell, Zachariasen, Frazer, Fulco, Höhler, . . .

- The form factors have cuts in the interval $[t_n, \infty[$ $(n = 0, 1, 2, \ldots)$ and also poles

$\Rightarrow$ Dispersion relations for $\boldsymbol{F_i(t)}$ $\boldsymbol{(i = 1, 2)}$:

$$\boldsymbol{F_i(t) = \frac{1}{\pi} \int_{t_0}^{\infty} dt' \frac{\text{Im } F_i(t')}{t' - t}}$$



- no subtractions
  [only proven in perturbation theory]
- suppression of higher mass states
- central objects: spectral functions

Im $\boldsymbol{F_i(t)}$

– cuts = multi-meson continua
– poles = vector mesons

SPECTRAL FUNCTIONS – GENERALITIES

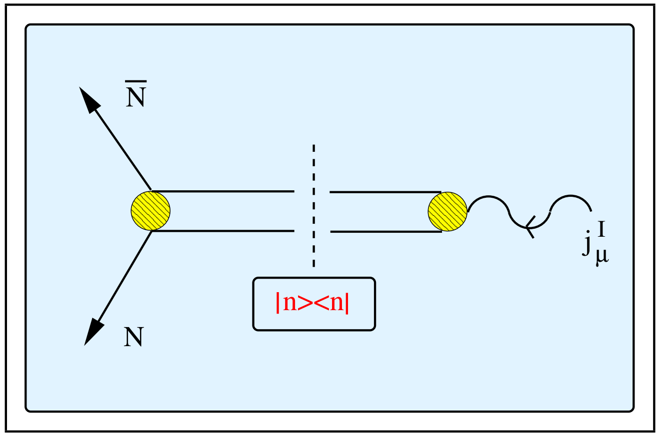
- Spectral decomposition:

$$\mathrm{Im}\,\langle \bar{N}(p')N(p)|J^I_\mu|0\rangle \sim \sum_n \langle \bar{N}(p')N(p)|n\rangle\langle n|J^I_\mu|0\rangle \Rightarrow \mathrm{Im}\,F$$

⋆ on-shell intermediate states

⋆ generates imaginary part

⋆ accessible physical states



- *Isoscalar* intermediate states: $3\pi, 5\pi, \ldots, K\bar{K}, K\bar{K}\pi, \pi\rho, \ldots +$ poles $\longrightarrow t_0 = 9M_\pi^2$
- *Isovector* intermediate states: $2\pi, 4\pi, \ldots +$ poles $\longrightarrow t_0 = 4M_\pi^2$
- Note that some poles are *generated* from the appropriate continua

ISOVECTOR SPECTRAL FUNCTIONS

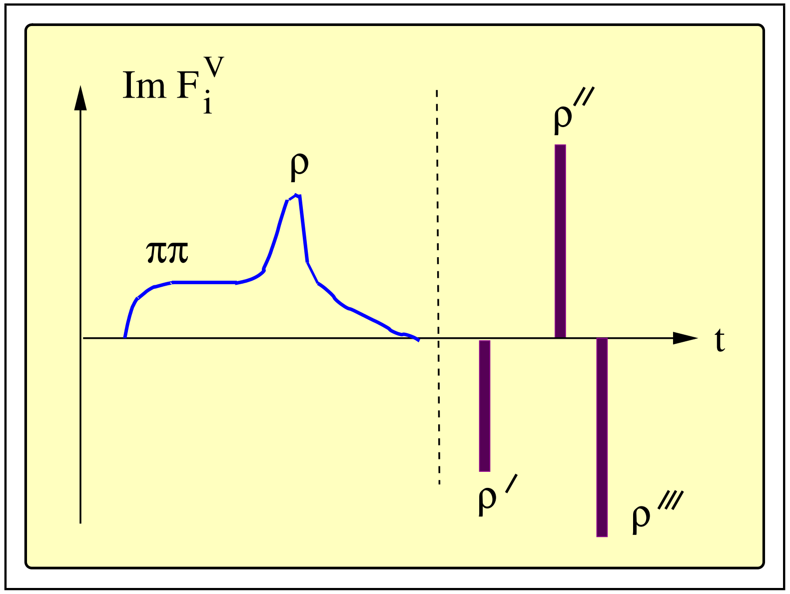
Frazer, Fulco, Höhler, Pietarinen, . . .

- exact 2π continuum is known from threshold $t_0 = 4M_\pi^2$ to $t \simeq 40\,M_\pi^2$

$$\mathrm{Im}\,F_i^V(t) = \frac{q_t^3}{\sqrt{t}}|F_\pi(t)|^2 J_i(t)$$

  $\star$ $F_\pi(t)$ = pion vector form factor

  $\star$ $J_i \sim$ P-wave pion-nucleon partial waves in the t-channel



- Spectral functions inherit singularity on the second Riemann sheet in $\pi N \to \pi N$

$$t_c = 4M_\pi^2 - M_\pi^4/m^2 \simeq 3.98\,M_\pi^2$$ $\to$ strong shoulder $\to$ isovector radii

- This singularity can also be analyzed in CHPT — Bernard, Kaiser, M, Nucl. Phys. A **611** (1996) 429

- Higher mass states represented by poles (with a finite width)

  $\to$ not necessarily physical masses

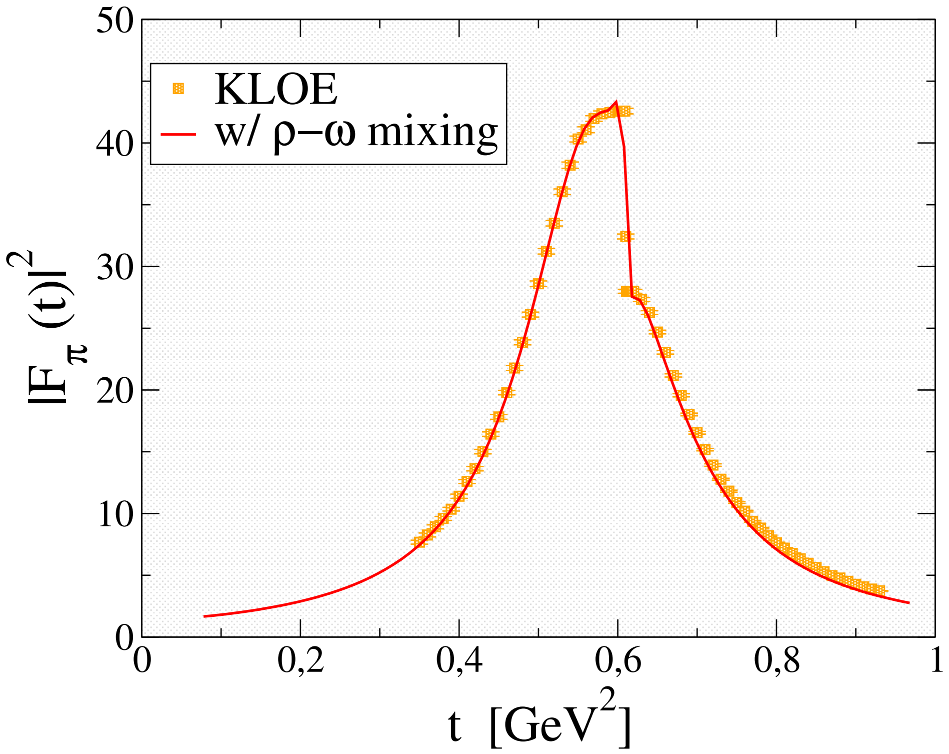
For related work, see Dubnicka et al., J.Phys. G **29** (2003) 405

NEW DETERMINATION OF THE 2π CONTINUUM

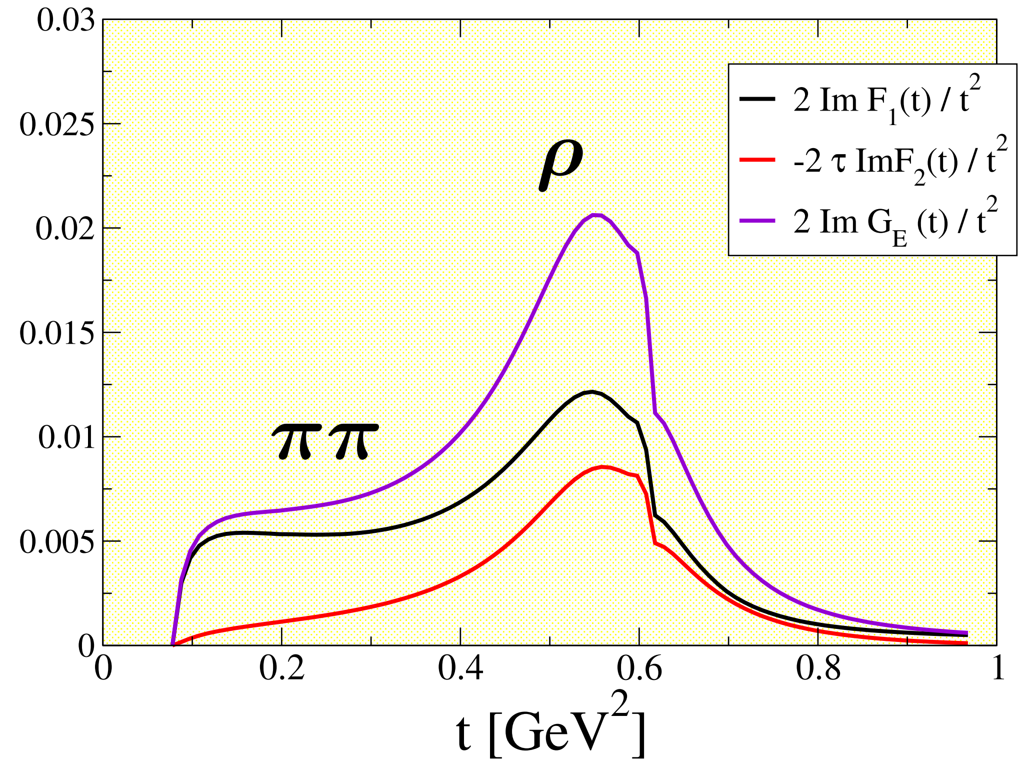
Belushkin, Hammer, M., in preparation

- Pion FF from KLOE

KLOE Coll., Phys. Lett. B **606** (2005) 12



- Nucleon isovector spectral functions



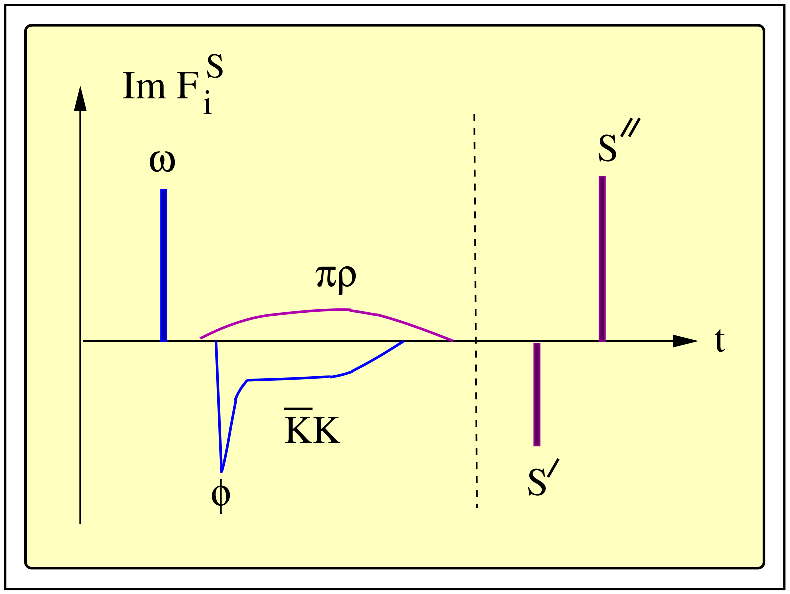
⋆ pronounced $\rho - \omega$ mixing

⋆ pronounced ρ peak

⋆ strong shoulder on the left wing

⇒ isovector radii

ISOSCALAR SPECTRAL FUNCTIONS



- $K\bar{K}$ continuum can be extracted from analytically cont. KN scattering amplitudes

  $\longrightarrow$ analytic continuation must be stabilized

  $\longrightarrow$ generates most of the ϕ contribution

  Hammer, Ramsey-Musolf, Phys. Rev. C **60** (1999) 045204, 045205

- Further strength in the ϕ-region generated by correlated $\pi\rho$ exchange

  $\longrightarrow$ strong cancellations ($K\bar{K}, K^*K, \pi\rho$)

  $\longrightarrow$ takes away sizeable strength from the ϕ

  M., Mull, Speth, van Orden, Phys. Lett. B **408** (1997) 381

- Spectral functions exhibit anomalous threshold (analyzed in 2-loop CHPT)

$$t_c = M_\pi^2 \left(\sqrt{4 - M_\pi^2/m^2} + \sqrt{1 - M_\pi^2/m^2}\right)^2 \simeq 8.9\, M_\pi^2$$

$\longrightarrow$ effectively masked

Bernard, Kaiser, M, Nucl. Phys. A **611** (1996) 429

- Higher mass states represented by poles (with a finite width)

CONSTRAINTS ON THE SPECTRAL FUNCTIONS

- Normalizations: electric charges, magnetic moments

- Neutron charge radius imposed from low-energy neutron-atom scattering
  [NB: other radii can also be imposed, not generally done]

- Superconvergence relations ≅ leading pQCD behaviour

$$\boxed{F_1(t) \sim 1/t^2 \;, F_2(t) \sim 1/t^3 \;\; (\mathrm{helicity-flip})}$$

Brodsky et al.

$$\Rightarrow \boxed{\int_{t_0}^{\infty} \mathrm{Im}\, F_1(t)\, dt = 0 \;, \quad \int_{t_0}^{\infty} \mathrm{Im}\, F_2(t)\, dt = \int_{t_0}^{\infty} \mathrm{Im}\, F_2(t)\, t\, dt = 0}$$

- Leading QCD logs or other α_S corrections can be included

see .e.g. Gari, Krümpelmann, Z. Phys. A **322** (1985) 689 , Mergell, M., Drechsel, Nucl. Phys. A **596** (1996) 367

⇒ severely restricts the number of fit parameters

SUMMARY: SPECTRAL & FIT FUNCTIONS

- Representation of the pole contributions: vector mesons
  [NB: can be extended for finite width]

$$\mathrm{Im}\, F_i^V(t) = \sum_v \pi a_i^v\, \delta(t - M_v^2)\ , \quad a_i^v = \frac{M_v^2}{f_V} g_{vNN} \ \Rightarrow\ F_i(t) = \sum_v \frac{a_i^v}{M_v^2 - t}$$

- *Isovector* spectral functions:

$$\mathrm{Im}\, F_i^V(t) = \mathrm{Im}\, F_i^{(2\pi)}(t) + \sum_{v=\rho',\rho'',\ldots} a_i^v \delta(t - M_v^2)\ , \quad (i = 1, 2)$$

- *Isoscalar* spectral functions:

$$\mathrm{Im}\, F_i^S(t) = \pi\, a_i^\omega\, \delta(t - M_\omega^2) + \mathrm{Im}\, F_i^{(K\bar{K})}(t) + \mathrm{Im}\, F_i^{(\pi\rho)}(t) + \sum_{v=S',S'',\ldots} a_i^v \delta(t - M_v^2)$$

- Parameters: 2 for the ω, 3 (4) for each other V-mesons minus # of constraints
  [NB: also fits with adjusting $K\bar{K}$ and $\pi\rho$ strengths]

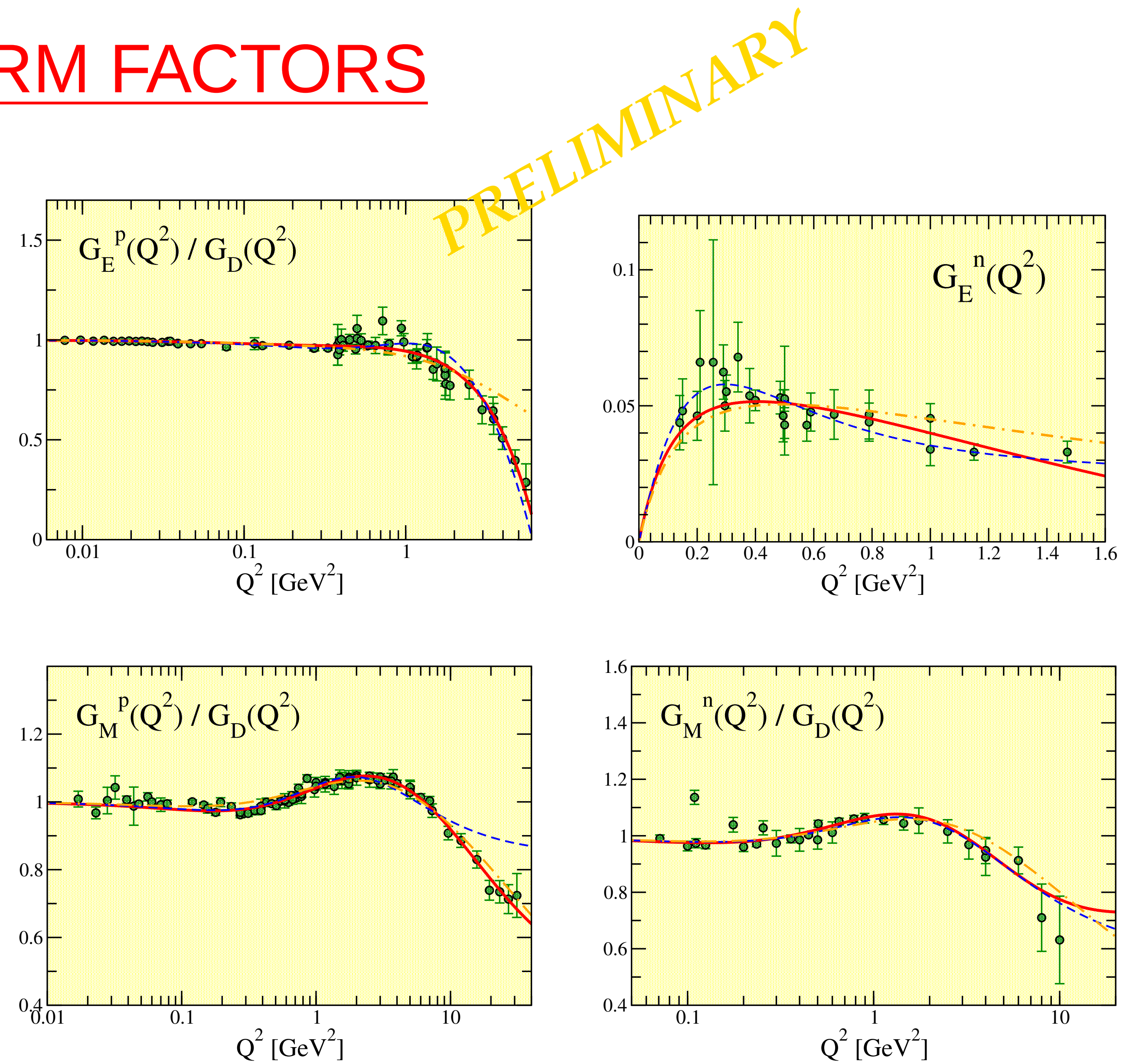
- Ill-posed problem $\rightarrow$ extra constraint: minimal # of poles to describe the data

Results

- space-like preliminary
- time-like VERY preliminary

SPACE-LIKE FORM FACTORS

- present best fit
- 4 effective IS poles
- 3 effective IV poles
- χ^2/dof = 0.84



Improved description

⋆ JLab data described

⋆ higher mass poles
not at physical values

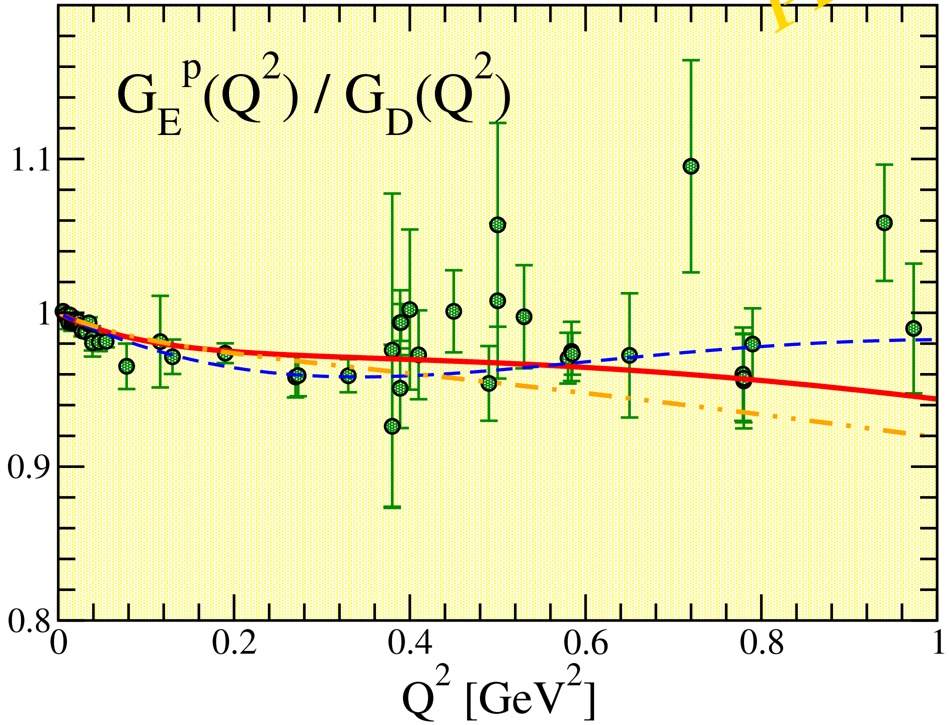
MMD 96, HMD 96, HM 04 (— · —)

$$G_D(Q^2) = \left(1 + \frac{Q^2}{0.71\ \mathrm{GeV}^2}\right)^{-2}$$

PROTON FORM FACTORS FOR $Q^2 \leq 1$ GeV^2

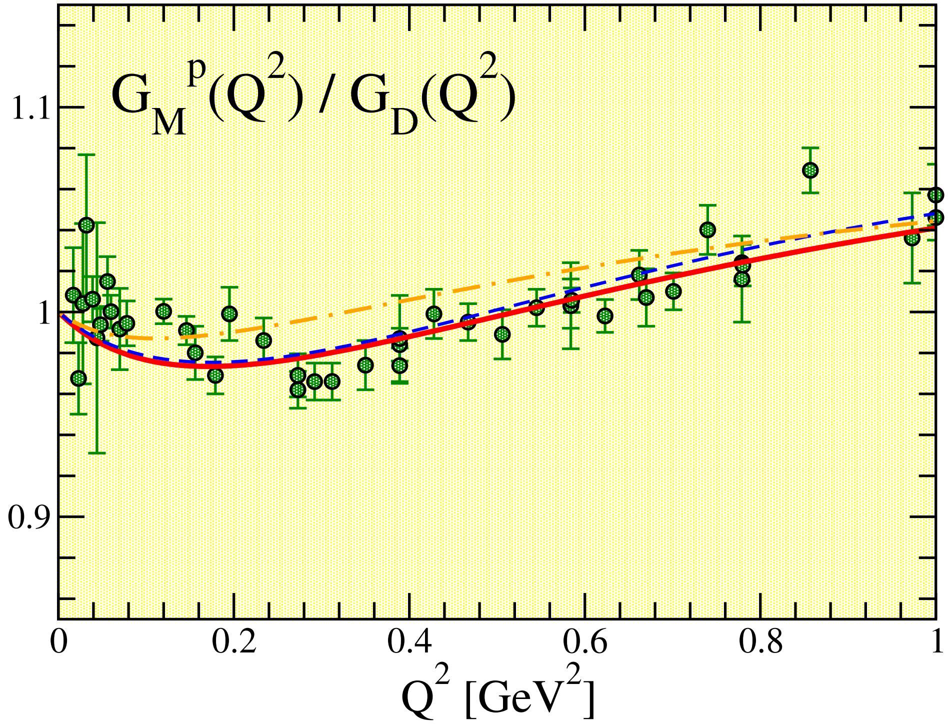
PRELIMINARY

- electric/dipole



$$\langle r_E^2 \rangle^{1/2} = 0.84 \text{ fm}$$

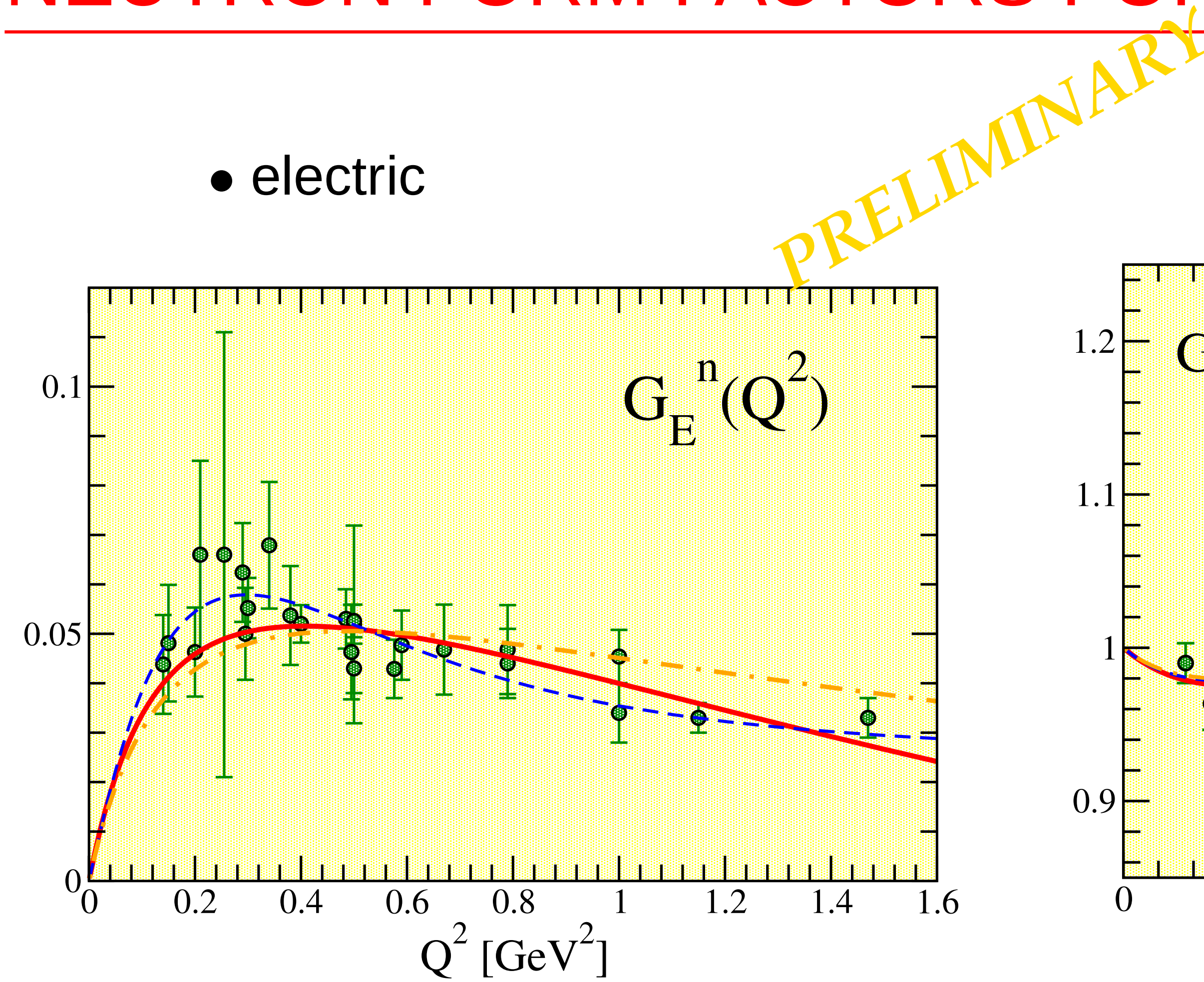
- magnetic/dipole



$$\langle r_M^2 \rangle^{1/2} = 0.86 \text{ fm}$$

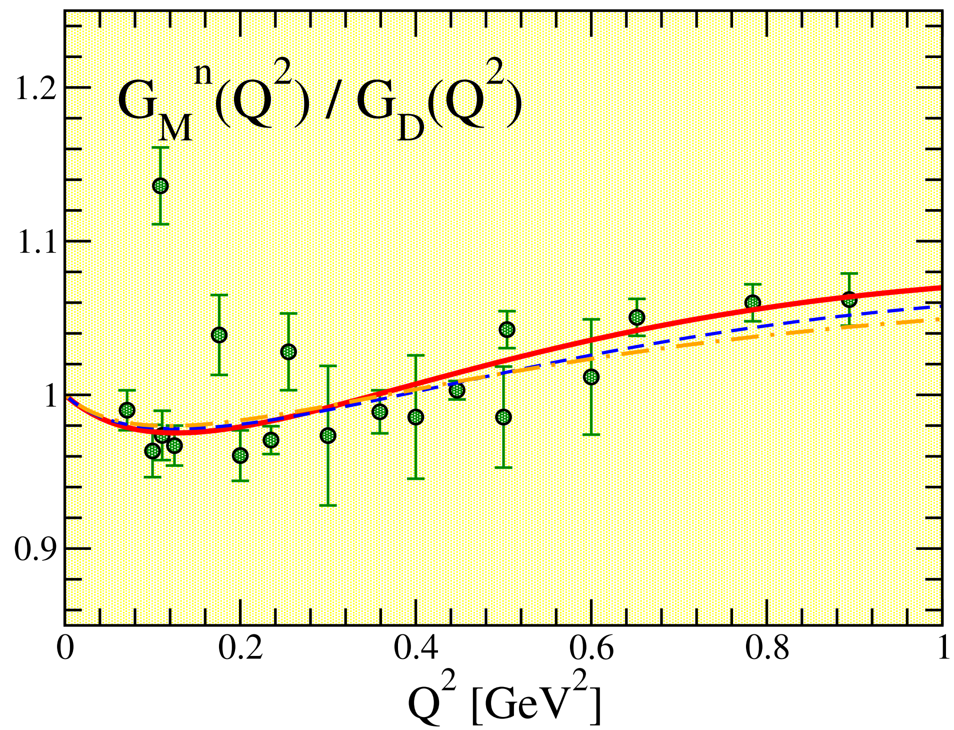
NEUTRON FORM FACTORS FOR $Q^2 \leq 1$ GeV^2

- electric



$$\langle r_E^2 \rangle = -0.12\,\mathrm{fm}^2$$

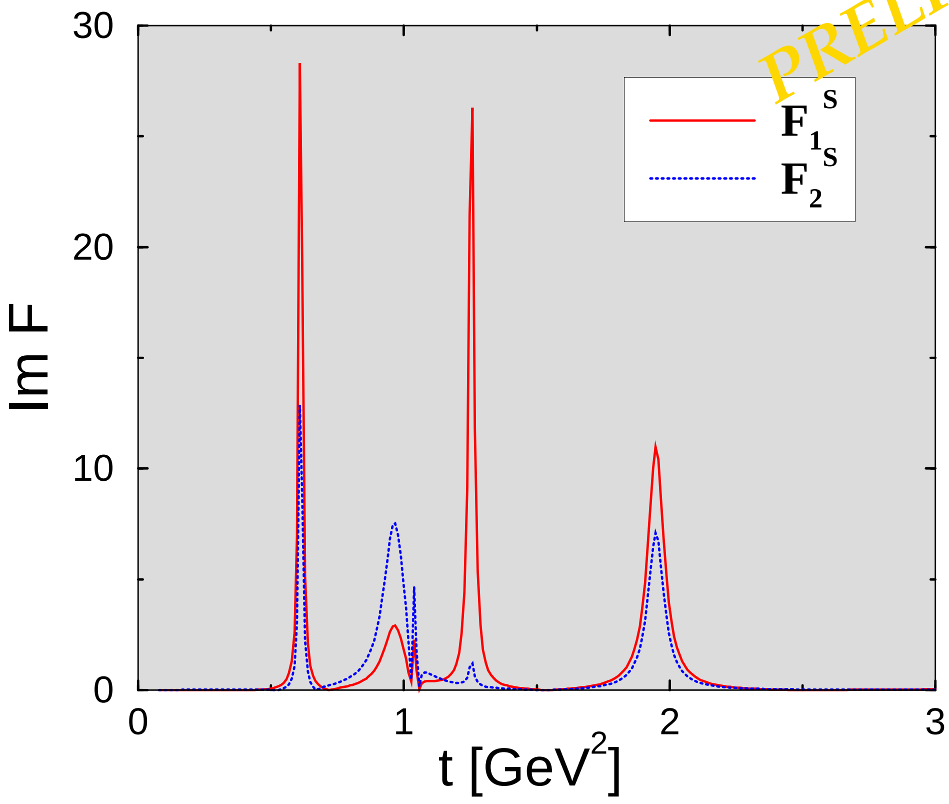
- magnetic/dipole



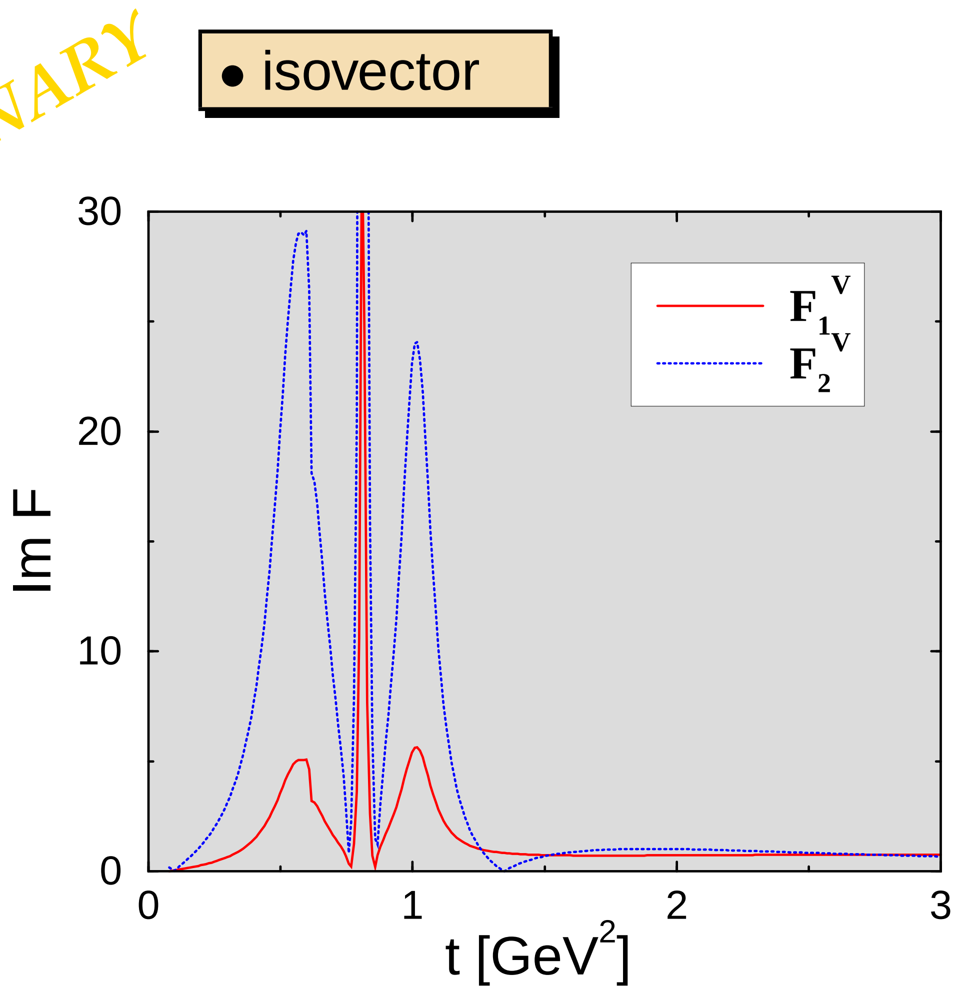
$$\langle r_M^2 \rangle^{1/2} = 0.87\,\mathrm{fm}$$

SPECTRAL FUNCTIONS

• isoscalar



• isovector



Note: Isoscalar spectral functions more similar than isovector ones

GENERAL COMMENTS ON THE FITS

- large MC sampling for initial values, successive improvement by pole reduction, new MCs, . . .

  $\Rightarrow$ must still work out theoretical uncertainty

- extraction of radii imposing soft constraints for the electric ones

	this work	HM 04	recent determ.
r_E^p [fm]	0.84. . .0.857	0.848	0.880(15) [1,2,3]
r_M^p [fm]	0.85. . .0.875	0.857	0.855(35) [4]
r_E^n [fm]	−0.12. . .−0.10	−0.12	−0.115(4) [5]
r_M^n [fm]	0.86. . .0.88	0.879	0.873(11) [6]

[1] Rosenfelder, Phys. Lett. B **479** (2000) 381
[2] Sick, private communication
[3] Melnikov, van Ritbergen, Phys. Rev. Lett. **84** (2000) 1673
[4] Sick, Phys. Lett. B **576** (2003) 62
[5] Kopecky et al., Phys. Rev. C **56** (1997) 2229
[6] Kubon et al., Phys. Lett. B **524** (2002) 26

$\star$ Magnetic radii in good agreement with recent determinations

$\star$ Proton electric radius comes out $\lesssim$ 0.87 fm

TIME-LIKE FORM FACTORS

Hammer, M., Drechsel, Phys. Lett. B **385** (1996) 343

- fitting also time-like data more complicated
- shown update from 2001

  Hammer, eConf **C010430** (2001) W08

- experimental extraction ambiguous

– E/M separation

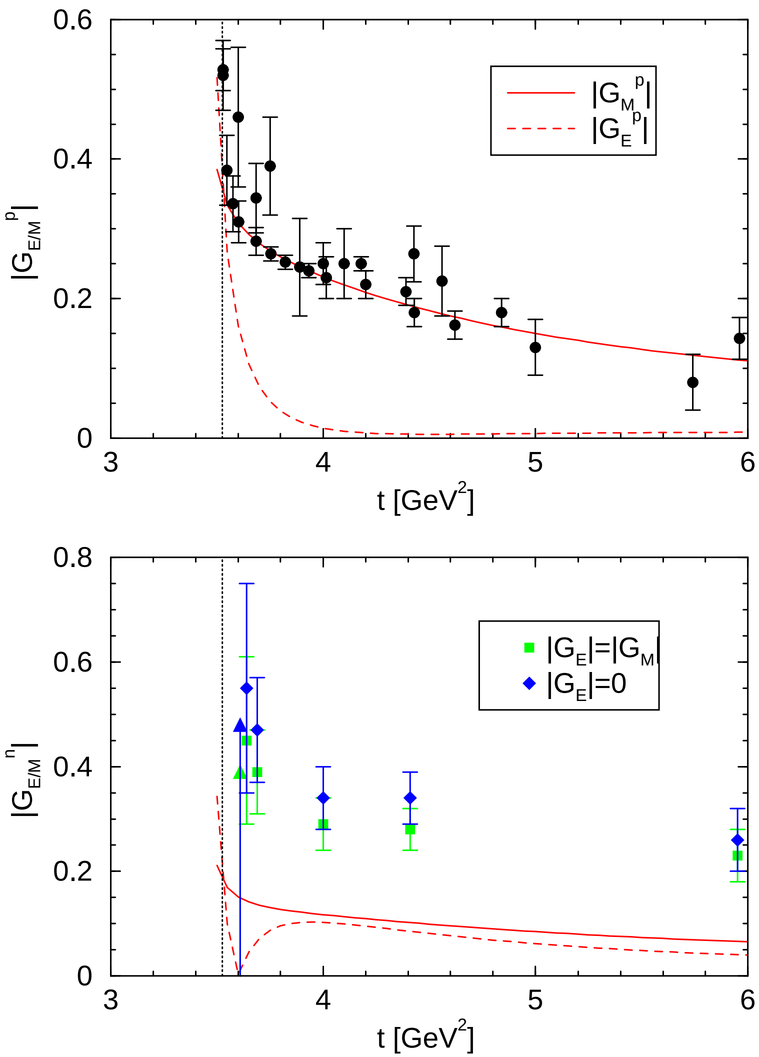
– $\bar{N}N$ final-state interactions?

– subthreshold resonance ? (or FSI ?)

  Antonelli et al., Nucl. Phys. B **517** (1998) 3

  [similar to $J/\psi \to \bar{p}p\gamma$ from BES]

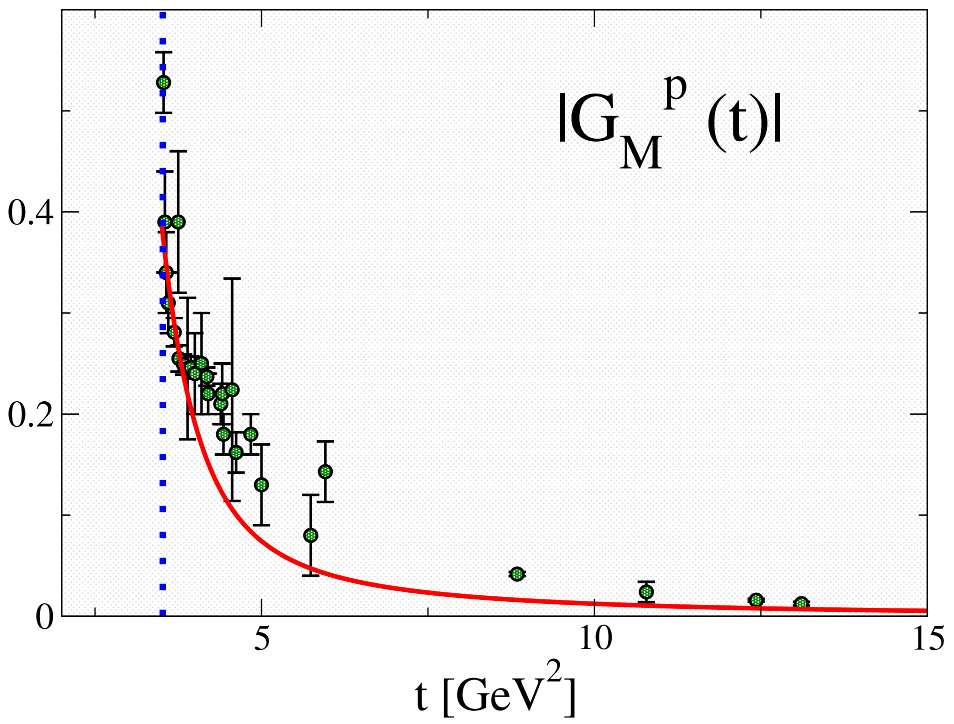
  Sibirtsev et al., Phys. Rev. D **71** (2005) 054010



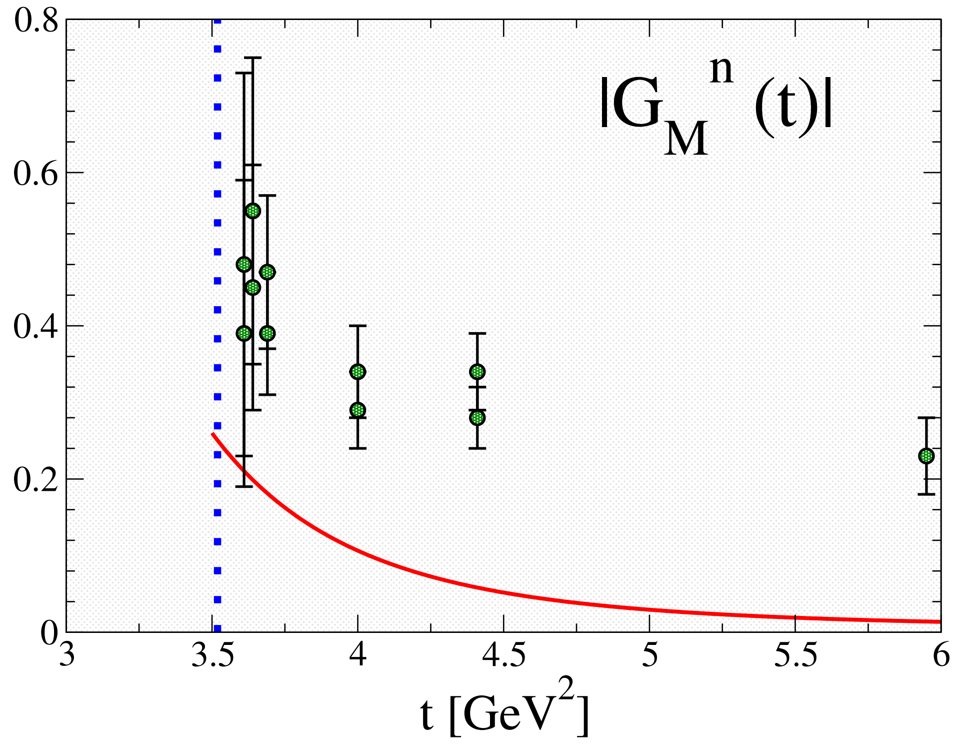
TIME-LIKE FORM FACTOR FITS: PRESENT STATUS

- Present best fit including space- and time-like data VERY PRELIMINARY

  - proton magnetic



  - neutron magnetic



[NB: data at $t > 10$ GeV^2 now included]

ON THE PION CLOUD OF THE NUCLEON

Hammer, M., Drechsel, Phys. Lett. B **586** (2004) 291

- FW find a very long-ranged contribution of the pion could, $r \simeq 2$ fm

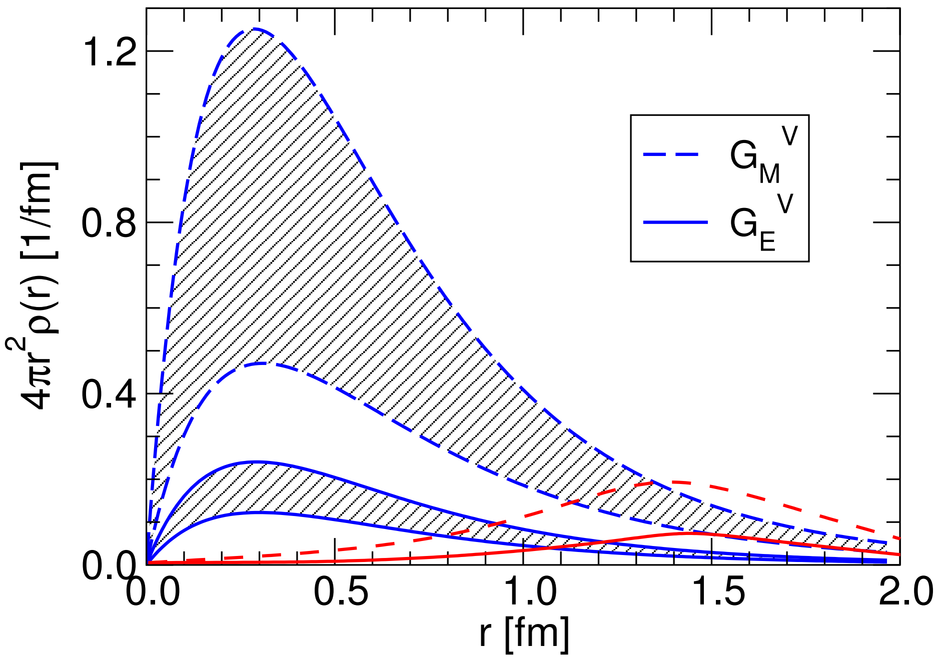
  Friedrich, Walcher, EPJ A **17** (2003) 607

- longest range component can be extracted from the isovector spectral function

  $\longrightarrow$ separation of the ρ-contribution

  $\longrightarrow$ three methods applied to do this

  $\longrightarrow$ theoretical band

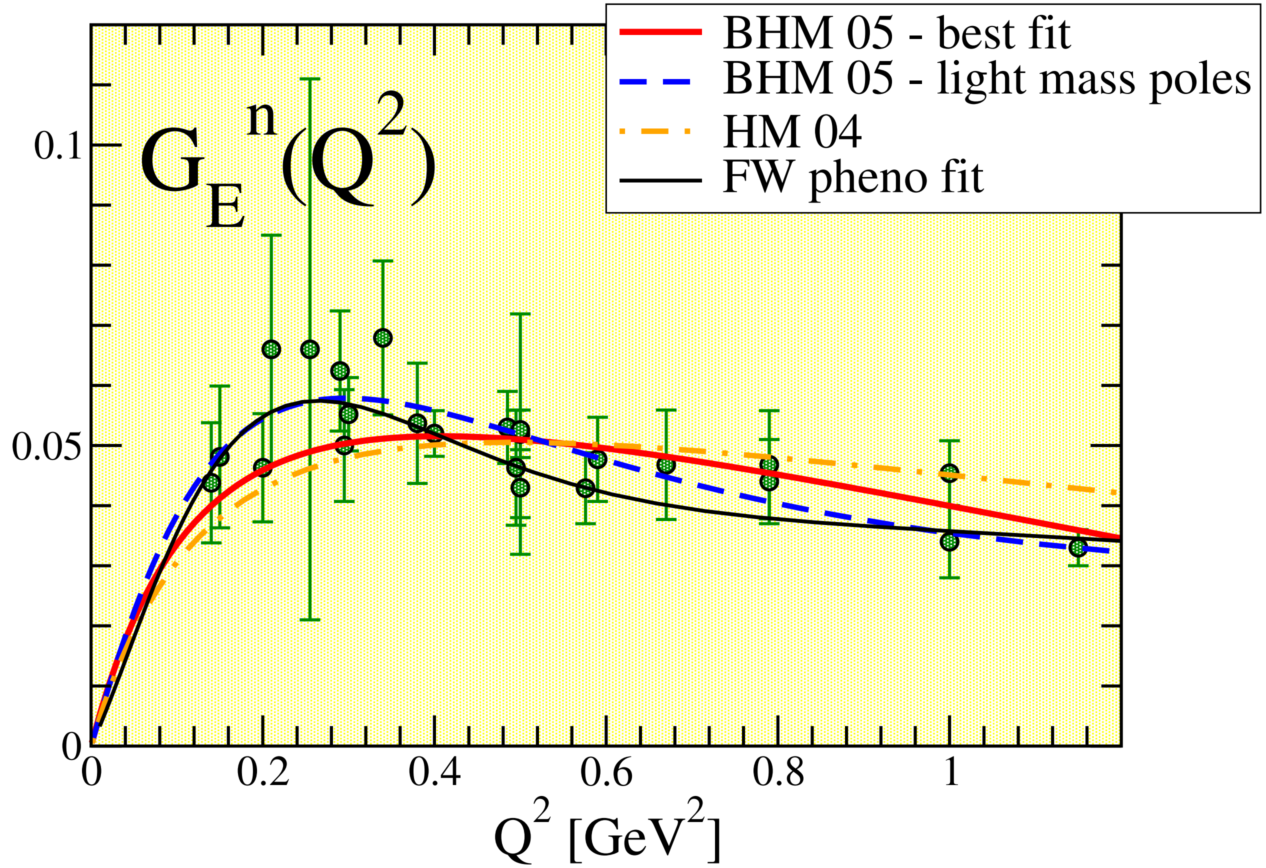


$$\rho_i^V(r) = \frac{1}{4\pi^2}\int_{4M_\pi^2}^{40M_\pi^2} dt\,\mathrm{Im}\,G_i^V(t)\,\frac{e^{-r\sqrt{t}}}{r} \qquad (i = E, M)$$

- maxima at $r_{\rm max} \simeq 0.3$ fm $\leftrightarrow$ FW find maxima at $r_{\rm max} \simeq 1.5$ fm
- much smaller pion cloud contribution for $r \geq 1$ fm compared to FW
- results independent of the contributions from $t > 40M_\pi^2$

$G_E^n(Q^2)$ w/ a BUMP-DIP STRUCTURE

- can one generate a bump-dip structure in the dispersive approach?



$\Rightarrow$ yes, but need **low-mass** poles: $M_S = 358$ MeV & $M_V = 558$ MeV

what shall these be? – not consistent w/ spec ftcs!

SUMMARY & OUTLOOK

- New dispersive analysis of the nucleon em form factors
- Improved spectral functions $\Rightarrow$ many results
- Still to be done
  - better fits w/ time-like form factors
  - theoretical/systematic uncertainty $\to$ bands
  - including pQCD corrections at large-t beyond SCR
  - two-photon effects? $\to$ fit to X sections
  - consequences for the strangeness vector form factors
  - and much more ...