The shape of the nucleon
Frascati Workshop on Nucleon Form Factors (N05)

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JLab and W&M

Outline:
I. Background
II. The model
II. Parameters and results
III. Discussion and implications

*This talk is based on a modified version of JLAB-THY-04-39 (under review by PRD) by FG and Peter Agbakpe.
Background

★ Beautiful new data from JLab, Mainz and other labs on the form factors

★ Lots of excitement about the decrease in the ratio $R_p = \frac{G_{Ep}}{G_{Mp}}$ with $Q^2$

★ Enter Jerry Miller, John Ralston, and the Jlab workshop of May 2002
  • Deformed protons !!!!
  • lack of helicity conservation !!!!
  • USA Today, Science News !!!
  • Great discovery !!!!

★ Whats going ON !!!?

Physicists thrown for a loop; New insight on protons changes the shape of things;
[FINAL Edition]

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Science and education

The humble proton, an atomic particle with mysteries long thought solved, turns out to have a hidden secret, scientists report.

Experimental results released this year by the Department of Energy's Jefferson Lab in Newport News, Va., have upturned the normally placid world of nuclear physics with the suggestion that protons, the positively charged particles found in the center of every atom, aren't round. Instead, they seem somewhat elliptical.

The round proton has been a staple of textbooks for 40 years, tied to the theory that protons and neutrons are built of three smaller particles called "quarks" slowly bubbling inside their interiors. What difference does it make whether protons are round or elliptical? Plenty, physicists say. Adjustments in protons and neutrons could affect scientific understanding of the magnetic "spin" of atoms. Scientists hope to use "spintronics" in future computers and tiny "nano-scale" devices. Understanding the fundamental shape of particles will affect those application's success.

At a Jefferson Lab meeting in May, about 60 nuclear physicists met to debate the "crisis," in the words of physicist John Ralston of the University of Kansas, over the odd shape of the proton. At the May meeting, a consensus emerged that Einstein's theory of special relativity, which explains how things moving near the speed of light have smooshed lengths and increased mass, seemed the likeliest explanation for the weird experiment results.

"The new thing we've figured out is that quarks are moving around inside the proton at relativistic (near speed of light) speeds," says physicist Gerald Miller of the University of Washington-Seattle. Quarks moving at those speeds simply elongate the particle's electromagnetic shape, Miller says. In a paper in the journal Physical Review C, he outlines how quarks moving at high speeds, about 90% of the speed of light, stretch out protons.

"The proton is the simplest thing around, and it is not spherical," says physicist Charles Glashausser of the Rutgers University campus in Piscataway, N.J. The neutron, the uncharged partner-particle to the proton in the nucleus of atoms, also is built of quarks, he notes.
WHAT'S GOING ON?

★ A spin 1/2 object cannot have a quadrupole deformation!

(Unless it is deformed in the sense of the transition nuclei, described by Bohr and Mottelson.* In this case there will be rotational bands and large electromagnetic transition rates, not seen.)

★ Talking about such things could make the field look foolish, or we could waste our time arguing about what we mean. It matters (confusion and ethics).

★ To remove the danger of making a mistake about shapes, we MUST be sure rotational invariance is treated exactly.

(The light-front formalism does not.)

★ Can the data be explained using standard physics?

*See: Buchmann and Henley, PRC 63,015202 (2001)
The Model
The nucleon wave function (1)

- The nucleon consists of 3 constituent quarks (CQ) with a size, mass, and form factor given by the dressing of the quark in the sea of gluons and $q\bar{q}$ pairs.
- Using the spectator theory, the nucleon is described by a 3-CQ vertex function with two of the CQ on shell

\[
\Psi_\alpha = \left( \frac{1}{m - \not{p} - i\epsilon} \right)_{\alpha\beta} \Gamma_\beta (P, p_s)
\]

- Confinement insures that this vertex function is zero when all three quarks are on shell (i.e. there is no 3q scattering)

\[
\Gamma = 0 \quad \text{Hence, model } \Psi \text{ directly}
\]
The nucleon wave function (2)

★ Spin-isospin structure of the NR nucleon wave function

- Assume a simple, fully symmetric S state spatial wave function
- Then spin-flavor wave function is fully symmetric

\[ |p \uparrow \rangle = \frac{1}{\sqrt{2}} \left\{ \Phi_0 \left( \frac{1}{2} \right) \Phi_0 \left( \frac{1}{2} \right) + \Phi_1 \left( \frac{1}{2} \right) \Phi_1 \left( \frac{1}{2} \right) \right\} \]

★ in operator form

\[ |fs \rangle = \frac{1}{\sqrt{2}} \left\{ \begin{bmatrix} \xi_0 \chi^s \end{bmatrix} \begin{bmatrix} \xi_0 \chi^f \end{bmatrix} - \frac{1}{3} \left[ \sigma \cdot \xi_1 \chi^s \right] \left[ \tau \cdot \xi_1 \chi^f \right] \right\} \]

\[ \chi^\frac{1}{2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi^{-\frac{1}{2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]

(0,0) diquark (1,1) diquark

★ Relativistically

\[ \Psi_\alpha = \frac{1}{\sqrt{2}} \left\{ u_\alpha (P,s) \begin{bmatrix} \xi_0 \chi^f \end{bmatrix} \psi_{00}(P,p_s) \right. \]

\[ + \frac{1}{3} \left( \gamma^5 \right)_{\alpha \beta} u_\beta (P,s) \left[ \tau \cdot \xi_1 \chi^f \right] \psi_{11}(P,p_s) \left\} \right. \]

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The nucleon wave function (3)

- Relativistic wave function, including spin-flavor structure
  
  $(0,0)$ diquark
  
  $\Psi_\alpha = \frac{1}{\sqrt{2}}\left\{u_\alpha(P, s)\left[\xi_0 \chi^f\right]\psi_{00}(P, p_s) + \frac{1}{3}(\gamma^5 \gamma^a)\alpha_\beta u_\beta(P, s)\left[\tau \cdot \xi_1 \chi^f\right]\psi_{11}(P, p_s)\right\}$

When $\vec{P} = 0$, this reduces to the nonrelativistic form

- choose $\Psi_{00}$ and $\Psi_{11}$ to be spherical in the rest frame (i.e. they depend only on the magnitude of $p_s$ only)

\[
\begin{align*}
\Psi_{00} &= \frac{N_0}{(\beta_1 + 2\sqrt{1 + \kappa^2} - 2)(\beta_2 + 2\sqrt{1 + \kappa^2} - 2)} \quad \text{NR} \quad \frac{N_0}{(\beta_1 + \kappa^2)(\beta_2 + \kappa^2)} \\
\Psi_{11} &= \sqrt{\frac{1}{1 + \frac{2}{3}\kappa^2}} \Psi_{00} \quad \text{NR} \quad \frac{N_0}{(\beta_1 + \kappa^2)(\beta_2 + \kappa^2)} \\
\kappa &= \frac{|\vec{p}_s|}{m_s}
\end{align*}
\]
Relativistic impulse approximation for the form factors

In the spectator theory, the photon couples to the off-shell quark, and because of the symmetry, the coupling to all three quarks is 3 times the coupling to one

\[ P_\pm = P \pm \frac{1}{2} q \quad Q^2 = -q^2 \]

approximate the two spectator quarks by a single diquark with a fixed mass

\[ J_\mu^I = \bar{u}(P_+, \lambda') \frac{3}{2} \int \frac{d^3 p_s}{(2\pi)^3 2E_s(p_s)} \left\{ j_\mu^I \psi_{00}(P_+, p_s) \psi_{00}(P_-, p_s) \right\} \]

integrate over the (on-shell) spectator three momentum

\[ - \frac{1}{9} \gamma^\nu \gamma^5 \tau_j j_\mu^I \gamma^5 \gamma^\nu' \Delta_{v\nu'} \psi_{11}(P_+, p_s) \psi_{11}(P_-, p_s) \right\} \]

quark currents with form factors

sum over the vector diquark, using

\[ \sum_\eta \eta_v \eta_{v'} \equiv \Delta_{v\nu'} = -g_{v\nu'} + \frac{p_{sv} p_{sv'}}{m_s^2} \]
Quark form factors include vector dominance and a pion cloud

The quark currents are

\[ j_i^\mu = f_1 \gamma^\mu + f_2 \frac{i \sigma^{\mu\nu} q_v}{2M}, \quad f_i = f_{i+} + \tau_3 f_{i-} \]

\[ f_{1\pm} = \frac{1}{3} f_{1u} \mp \frac{1}{6} f_{1d} \quad f_{2\pm} = \frac{1}{2} (\mu_u f_{2u} \pm \mu_d f_{2d}) \]

the quark form factors have 9 parameters:

\[
\begin{align*}
  f_{1+} &= e_+ \left( \frac{1 - \lambda}{1 + Q_0^2 \Lambda_{1+}^2} + \lambda \right) \\
  f_{2\pm} &= \mu_\pm \left( \frac{1}{1 + Q_0^2 \Lambda_{2\pm}^2} \right) \\
  f_{1-} &= e_- \left( \frac{1 - \lambda - \lambda_\pi}{1 + Q_0^2 \Lambda_{1-}^2} + \lambda_\pi \frac{\lambda_\pi}{\left(1 + Q_0^2 / \Lambda_{\pi}^2\right)^2} + \lambda \right)
\end{align*}
\]

\[ \text{VDM terms} \]

\[ \text{pion cloud} \]

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Normalization and shape

To find the shape and normalization, compute $J^0_I(q=0)$. After some algebra:

$$J^0_I(q = 0) = \frac{1}{2} (1 + \tau_3) \delta_{\lambda' \lambda} \int \frac{d^3 p_s}{(2\pi)^3 2E_s(p_s)} \psi_{00}^2 (P, p_s)$$

$$= \frac{1}{2} (1 + \tau_3) \delta_{\lambda' \lambda} \int \frac{d^3 \kappa}{(2\pi)^3 2E_s(\kappa)} \frac{N_0^2}{(\beta_1 - 2 + 2E_s(\kappa))^2 (\beta_2 - 2 + 2E_s(\kappa))^2}$$

spherical shape

correct normalization

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The Parameters and Results
The form factors data is fit with 11 parameters:

- 1 asymptotic value of the $f_1$ quark form factors, $\lambda$
- 2 nucleon wave function parameters $\beta_1$ and $\beta_2$
- 2 quark anomalous moments $\mu_u$ and $\mu_d$
- 2 pion cloud parameters, the strength $\lambda_\pi$, and its range, $\Lambda_\pi$
- 4 vector monopole dominance scales for the four quark form factors, $\Lambda_{i\pm}$
- 11 total

The model automatically normalizes $G_{Ep}(0) = 1$ and $G_{En}(0) = 0$, by adjusting the normalization constant (not a parameter) $N_0$

The fit constrains the nucleon anomalous moments and the nucleon radii to their experimental values, assuring realistic low $Q^2$ properties/

The other 7=11-4 parameters are determined my minimizing $\chi^2$. 

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The parameters \( r_p^2(\text{exp}) = 0.780(25), \quad r_n^2(\text{exp}) = -0.113(7) \)

- Fit to the data

\[
\chi^2_{\text{datum}} = 1.32, \quad \lambda = 0.156, \quad N_0^2 = 149.8, \quad r_p^2 = 0.780, \quad r_n^2 = -0.113
\]

<table>
<thead>
<tr>
<th>( \beta_1 \beta_2 )</th>
<th>( \mu_u \mu_d )</th>
<th>( \Lambda_{1+} \Lambda_{1-} )</th>
<th>( \Lambda_{2+} \Lambda_{2-} )</th>
<th>( \lambda_{\pi} \Lambda_{\pi} )</th>
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<tr>
<td>0.069</td>
<td>1.127</td>
<td>1.685</td>
<td>0.591</td>
<td>0.231</td>
</tr>
<tr>
<td>2.772</td>
<td>-0.823</td>
<td>2.217</td>
<td>1.086</td>
<td>0.796</td>
</tr>
</tbody>
</table>

\[
\phi_0 \equiv \frac{N_0}{(\beta_1 + 2\sqrt{1+\kappa^2} - 2)(\beta_2 + 2\sqrt{1+\kappa^2} - 2)} \quad \text{Nonrelativistic} \quad \frac{N_0}{(\beta_1 + \kappa^2)(\beta_2 + \kappa^2)}
\]

\[
\phi_1 = \sqrt{\frac{1}{1 + \frac{2}{3} \kappa^2}} \phi_0 \quad \text{Nonrelativistic} \quad \frac{N_0}{(\beta_1 + \kappa^2)(\beta_2 + \kappa^2)}
\]

\[
f_1^+(Q^2) = (1 - \lambda) \left( \frac{1}{1 + \frac{Q_0^2}{\Lambda_{1+}^2}} \right) + \lambda
\]

\[
f_1^-(Q^2) = (1 - \lambda - \lambda_{\pi}) \left( \frac{1}{1 + \frac{Q_0^2}{\Lambda_{1-}^2}} \right) + \frac{\lambda_{\pi}}{1 + \frac{Q_0^2}{\Lambda_{\pi}^2}} + \lambda
\]

\[
f_2^\pm(Q^2) = \frac{1}{1 + \frac{Q_0^2}{\Lambda_{2\pm}^2}}
\]

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The quark form factors

\[ f_1^+ + f_{1-} + f_1^- - f_{1-} - 

\]
The magnetic form factors

- Both magnetic form factors are similar in shape. Better data for $G_{Mn}$ a must! Pion cloud essential to explain $G_{Mn}$

![Graph of magnetic form factors](image)

- fit with pion cloud
- no pion cloud
- --- refit with NO pion cloud

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The charge form factors

- The fits to the charge form factors are both excellent. *Again the pion cloud is essential to the description of $G_{En}$*

- Prediction: $G_{Fn}$ will change sign at $Q^2 \sim 8$ $\text{GeV}^2$

![Graph showing fits with and without pion cloud](image)

- Fit with pion cloud
- ....... no pion cloud
- -- - refit with NO pion cloud

*Jefferson Lab*

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The F2/F1 puzzle for the proton

- $QF_{2p}/F_{1p}$ is indeed flat, but this is an accident; The asymptotic value will not be reached until $Q^2 \sim 100 \text{ GeV}^2$.

$$R_{\text{mag}} = \frac{Q F_{2p}(Q^2)}{\kappa_p F_{1p}(Q^2)}$$
Comparisons (light front theory)

- G. A. Miller,

- F. Cardarelli and S. Simula,
Comparisons (vector dominance)


Original model (IJL, 1973) fails, and new fit gives different prediction for $G_{Ep}$ !!

Discussion and Implications

★ The data do not require the proton to be deformed!
  • This is forbidden by quantum mechanics (unless there are rotational bands).
  • This model is a counter example to claims that the data cannot be explained by a spherical proton.

★ The data do give interesting new information about the CQ form factors, and tell us about the proton wave function.

★ The model is so simple that it can be used to study many phenomena near the quark-hadron transition.

★ Predictions
  • $G_{Ep}$ will change sign near $Q^2 \sim 8 \text{ GeV}^2$
  • $G_{Mn}$ will be larger than the older data suggests.

★ What does this problem teach us about over-selling science?
END