New Perspectives on the Structure and Interactions of the Nucleon in QCD

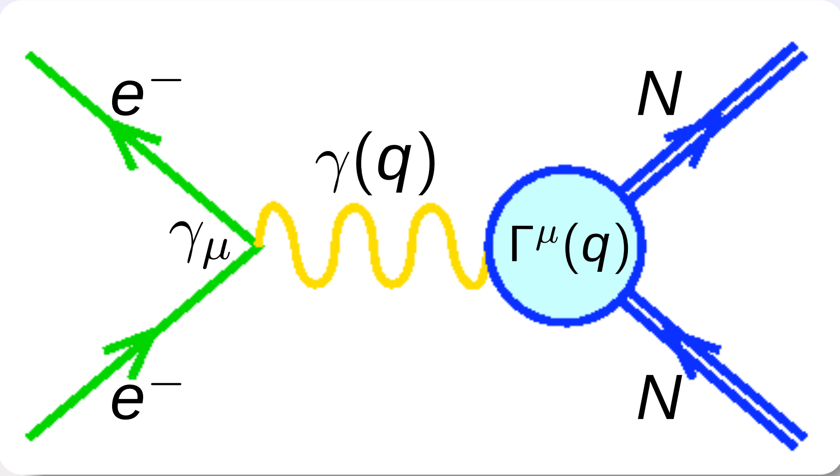
"Unique Form of Continuity in Space"
by Umberto Boccioni

N05

Workshop on Nucleon Form Factors

Frascati, 12-14 October, 2005

Nucleon Form Factors

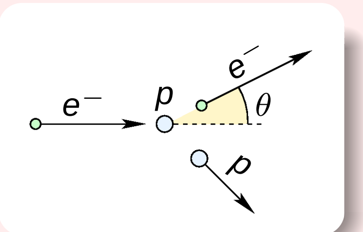


Nucleon current operator (Dirac & Pauli)

$$\Gamma^\mu(q) = \gamma^\mu F_1(q^2) + \frac{i}{2M_N}\sigma^{\mu\nu}q_\nu F_2(q^2)$$

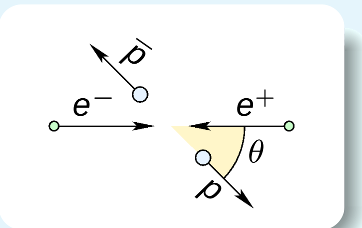
Electric and Magnetic Form Factors

$$G_E(q^2) = F_1(q^2) + \tau F_2(q^2)$$
$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$
$$\tau = \frac{q^2}{4M_N^2}$$



Elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_e' \cos^2\frac{\theta}{2}}{4E_e^3\sin^4\frac{\theta}{2}}\left[G_E^2 + \tau\left(1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right)G_M^2\right]\frac{1}{1+\tau}$$



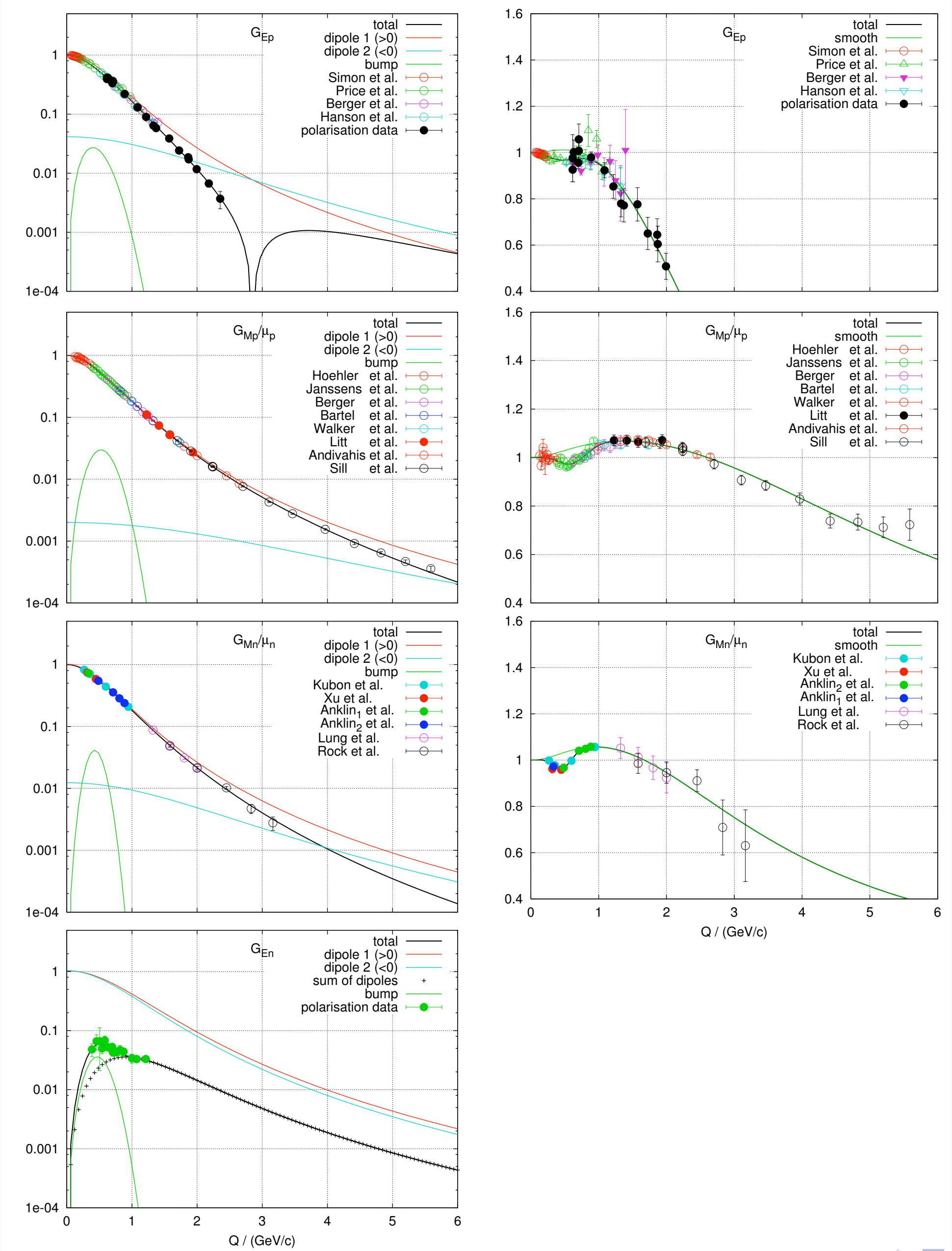
Annihilation

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\sqrt{1-1/\tau}}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta|G_E|^2\right]$$

Simone Pacetti **Ratio $|G_E^p(q^2)/G_M^p(q^2)|$ and dispersion relations**

Fit of the phenomenological ansatz to the form factors
and comparison to the standard dipole fit

form factors



form factors divided by standard dipole fit

Jörg Friedrich — A phenomenological analysis of the four standard nuclear form factors

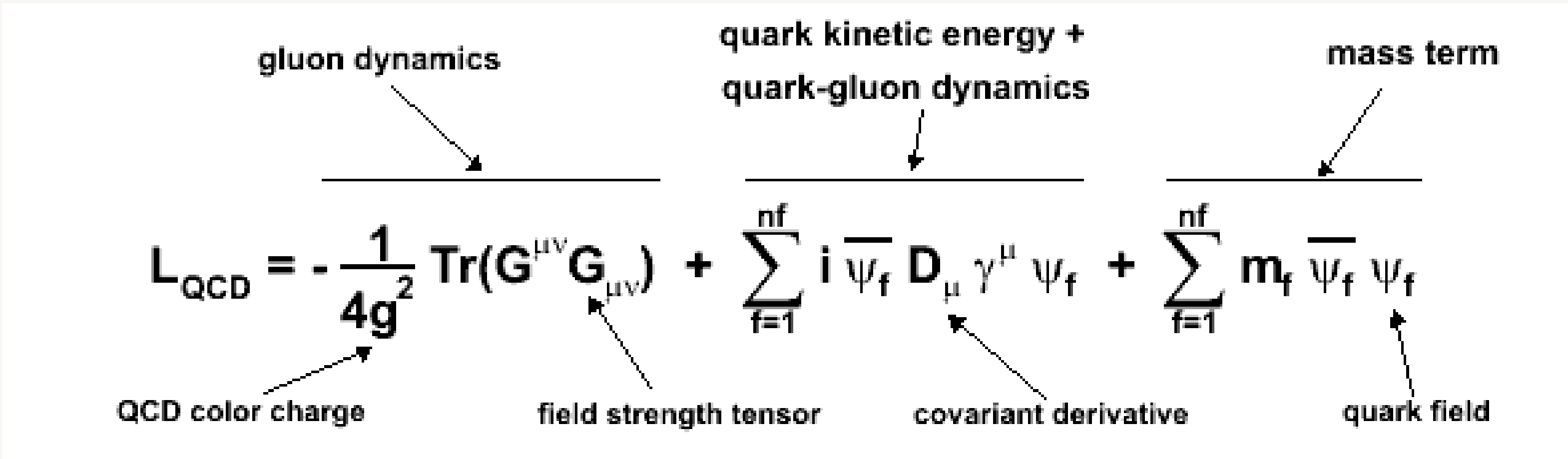
Proton Form Factors: Central objects in hadron physics

- Dynamics and structure of proton at the amplitude level: multiquark and gluon system, meson cloud
- Fundamental test of QCD: quark and gluon structure of matter, confinement
- Underlies nuclear physics
- Critical to precision atomic physics: Lamb Shift, Hfs. (See J. Hiller talk). Input to g-2

Proton Form Factors: Central objects in hadron physics

- High momentum behavior: Fundamental tests of PQCD scaling, helicity structure, asymptotic freedom, conformal scaling, AdS/CFT
- Pauli form factor: requires nonzero orbital angular momentum
- Information on shape and normalization of proton distribution amplitude -- proton decay!
- Normalization of PQCD: QCD coupling at small scales, evidence for IR fixed point

How can we compute proton form factors from first principles?



- Lattice gauge theory
- Transverse lattice
- Discretized light-cone quantization: diagonalize LF Hamiltonian -- LFWFs, spectrum
- AdS/CFT

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2 \; |\Psi_h\rangle$$

Compute LFWFS from first principles

$$H_{LC}^{QCD} = P_\mu P^\mu = \quad P^- P^+ - \vec{P}_\perp^2$$

The hadron state $|\Psi_h\rangle$ is expanded in a Fock-state complete basis of non-interacting n-particle states $|n\rangle$ with an infinite number of components

$$\left|\Psi_h(P^+, \vec{P}_\perp)\right\rangle =$$

$$\sum_{n,\lambda_i} \int \; [dx_i \; d^2\vec{k}_{\perp i}] \psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$\times \; |n : x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle$$

$$\sum_n \int [dx_i \; d^2\vec{k}_{\perp i}] \; |\psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i)|^2 = 1$$

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i >$$

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

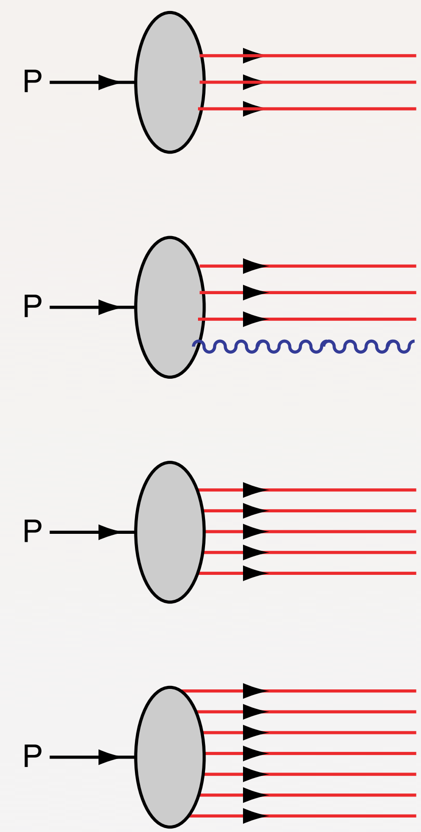
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

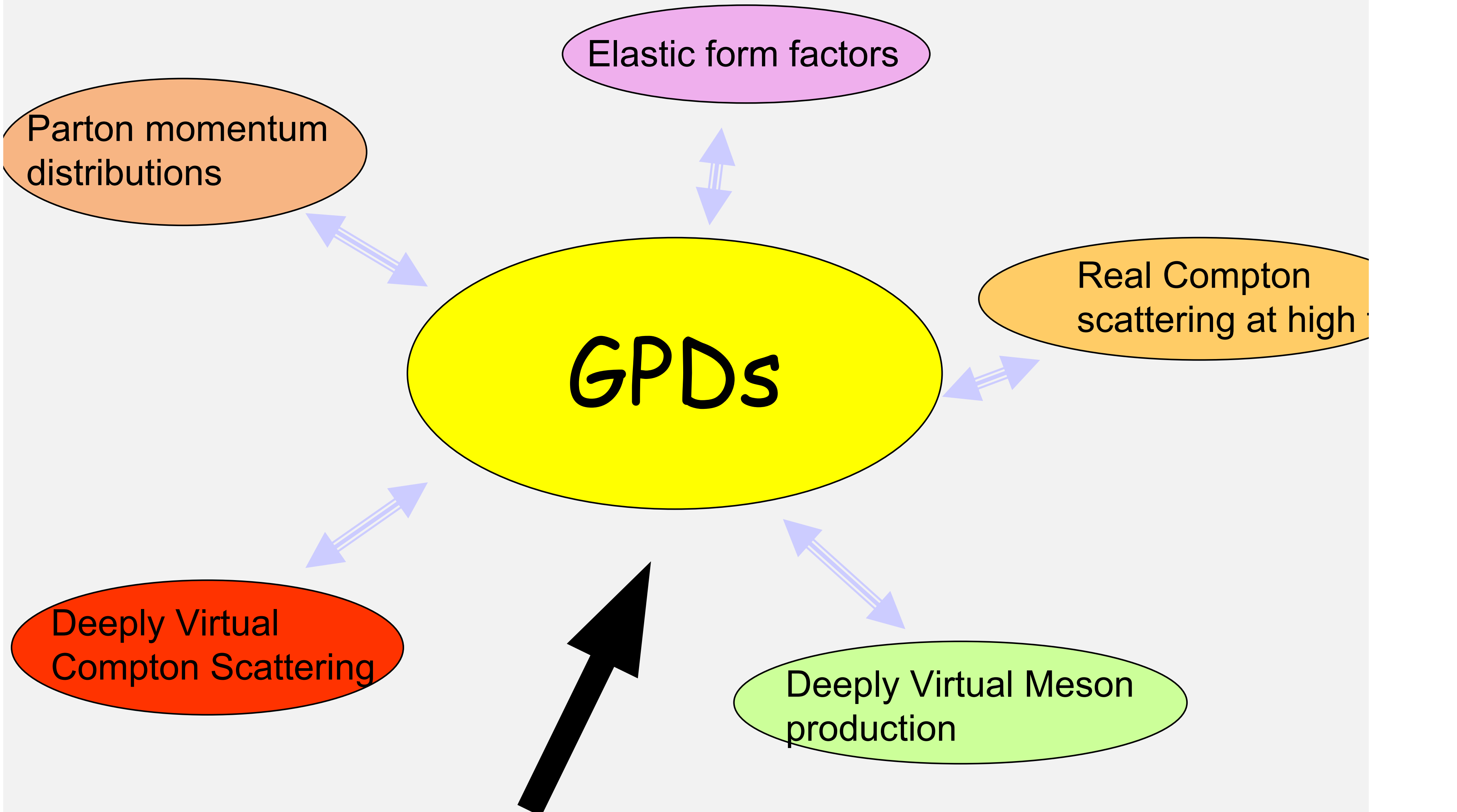
$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

Invariant under boosts. Independent of P^μ

A Unified Description of Hadron Structure



Light Front Wavefunctions

Gardner, Hwang, sjb,

Light-Cone Wavefunction Representations of Anomalous Magnetic Moment and Electric Dipole Moment

In the case of a spin-$\frac{1}{2}$ composite system, the Dirac and Pauli form factors $F_1(q^2)$ and $F_2(q^2)$, electric dipole moment form factor $F_3(q^2)$ are defined by

$$\langle P'|J^\mu(0)|P\rangle = \overline{U}(P')\left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2M}\sigma^{\mu\alpha}q_\alpha + F_3(q^2)\frac{-1}{2M}\sigma^{\mu\alpha}\gamma_5 q_\alpha\right]U(P)\,, \quad (47)$$

Dirac and Pauli form factors computed from matrix element of J^+

$$F_1(q^2) = \left\langle P+q,\uparrow \left|\frac{J^+(0)}{2P^+}\right| P,\uparrow\right\rangle = \left\langle P+q,\downarrow \left|\frac{J^+(0)}{2P^+}\right| P,\downarrow\right\rangle\,, \quad (48)$$

$$\frac{F_2(q^2)}{2M} = \frac{1}{2}\left[+\frac{1}{-q^1+\mathrm{i}q^2}\left\langle P+q,\uparrow \left|\frac{J^+(0)}{2P^+}\right| P,\downarrow\right\rangle + \frac{1}{q^1+\mathrm{i}q^2}\left\langle P+q,\downarrow \left|\frac{J^+(0)}{2P^+}\right| P,\uparrow\right\rangle\right], \quad (49)$$

$$\frac{F_3(q^2)}{2M} = \frac{i}{2}\left[+\frac{1}{-q^1+\mathrm{i}q^2}\left\langle P+q,\uparrow \left|\frac{J^+(0)}{2P^+}\right| P,\downarrow\right\rangle - \frac{1}{q^1+\mathrm{i}q^2}\left\langle P+q,\downarrow \left|\frac{J^+(0)}{2P^+}\right| P,\uparrow\right\rangle\right]. \quad (50)$$

Dirac and Pauli form factors exactly computed from convolutions of LFWFs (n'= n)

Drell Yan, West, Drell, SJB

$$\frac{F_2(q^2)}{2M} = \sum_a \int \frac{\mathrm{d}^2\vec{k}_\perp \mathrm{d}x}{16\pi^3} \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb,

$$\Big[+ \frac{1}{-q^1 + \mathrm{i}q^2} \psi_a^{\uparrow *}(x_i, \vec{k}'_{\perp i}, \lambda_i) \, \psi_a^{\downarrow}(x_i, \vec{k}_{\perp i}, \lambda_i) + \frac{1}{q^1 + \mathrm{i}q^2} \psi_a^{\downarrow *}(x_i, \vec{k}'_{\perp i}, \lambda_i) \, \psi_a^{\uparrow}(x_i, \vec{k}_{\perp i}, \lambda_i) \Big]$$

$$\frac{F_3(q^2)}{2M} = \sum_a \int \frac{\mathrm{d}^2\vec{k}_\perp \mathrm{d}x}{16\pi^3} \sum_j e_j \, \frac{i}{2} \times$$

Gardner, Hwang, sjb,

$$\Big[+ \frac{1}{-q^1 + \mathrm{i}q^2} \psi_a^{\uparrow *}(x_i, \vec{k}'_{\perp i}, \lambda_i) \, \psi_a^{\downarrow}(x_i, \vec{k}_{\perp i}, \lambda_i) - \frac{1}{q^1 + \mathrm{i}q^2} \psi_a^{\downarrow *}(x_i, \vec{k}'_{\perp i}, \lambda_i) \, \psi_a^{\uparrow}(x_i, \vec{k}_{\perp i}, \lambda_i) \Big] ,$$

Pauli form factor requires nonzero L

$\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$ struck quark $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i \vec{q}_\perp$ spectator

Relation between edm and anomalous magnetic moment

CP-violating phase

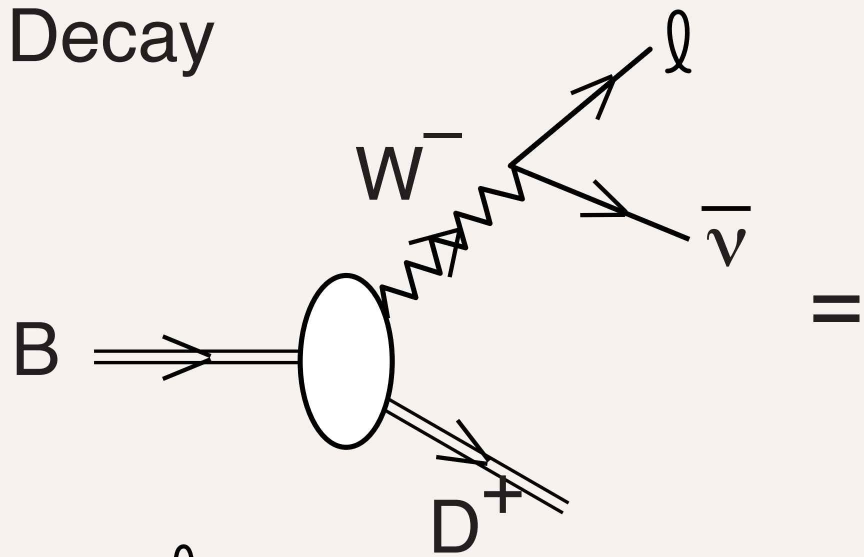
$$F_3(q^2) = F_2(q^2) \times \tan\phi$$

Fock state by Fock state

Gardner, Hwang, sjb,

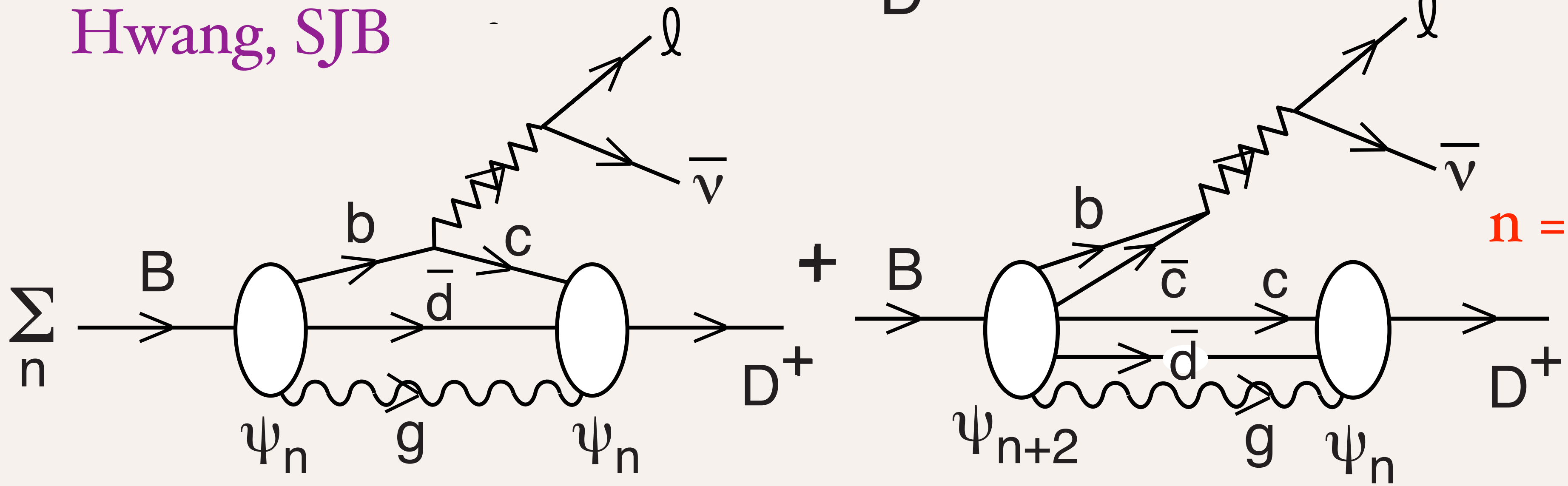
Weak Exclusive Decay

$\langle D \,|\, J^{+}(0) \,|\, B \rangle$



Exact Formula

Hwang, SJB

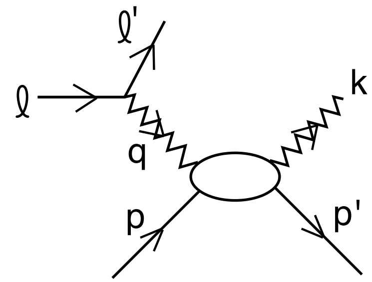


$n = n' + 2$

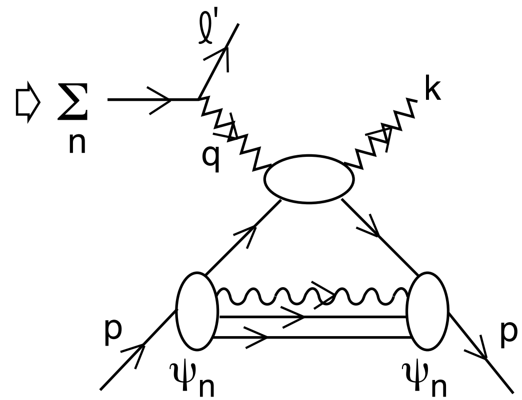
Annihilation amplitude needed for Lorentz Invariance

$\langle p' \lambda' | J^{\mu}(z) J^{\nu}(0) | p \lambda \rangle$

Large $-q^2 = Q^2$



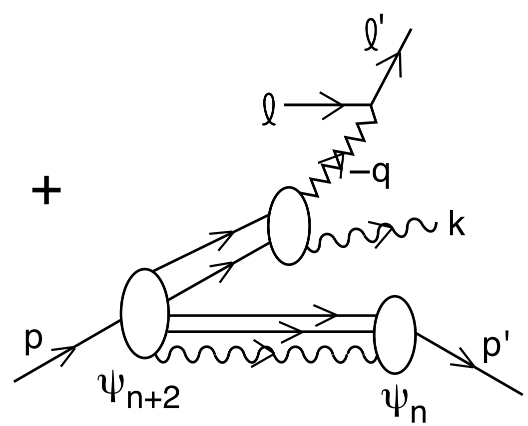
$\gamma^* p \to \gamma p'$



Given LFWFs, compute all GPDs !

Deeply Virtual Compton Scattering

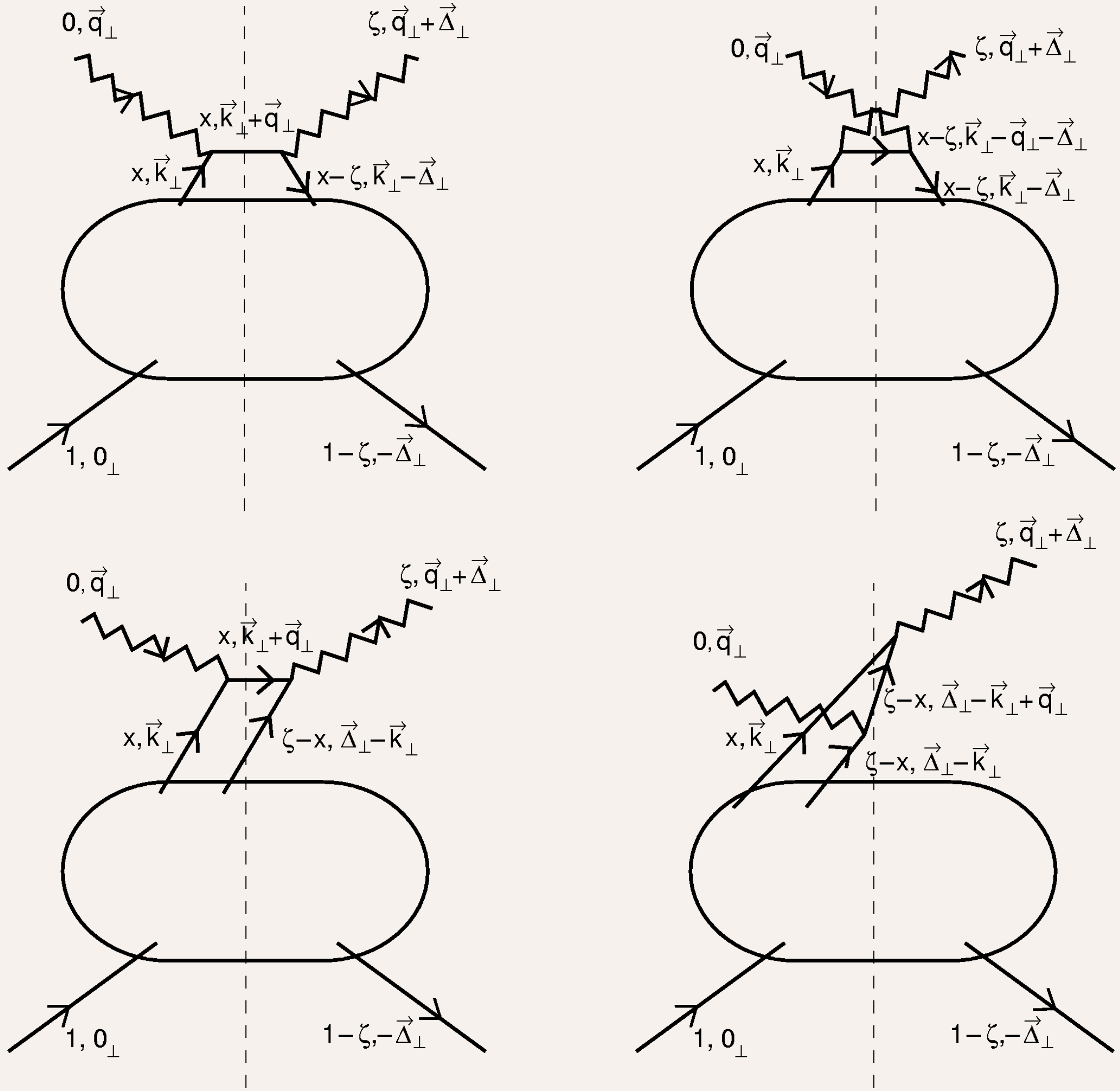
+



ERBL Evolution

n = n' + 2

Required for Lorentz Invariance



Light-cone wavefunction representation of deeply virtual Compton scattering ☆

Stanley J. Brodsky [a], Markus Diehl [a,1], Dae Sung Hwang [b]

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

Conserved
LF Fock state by Fock State

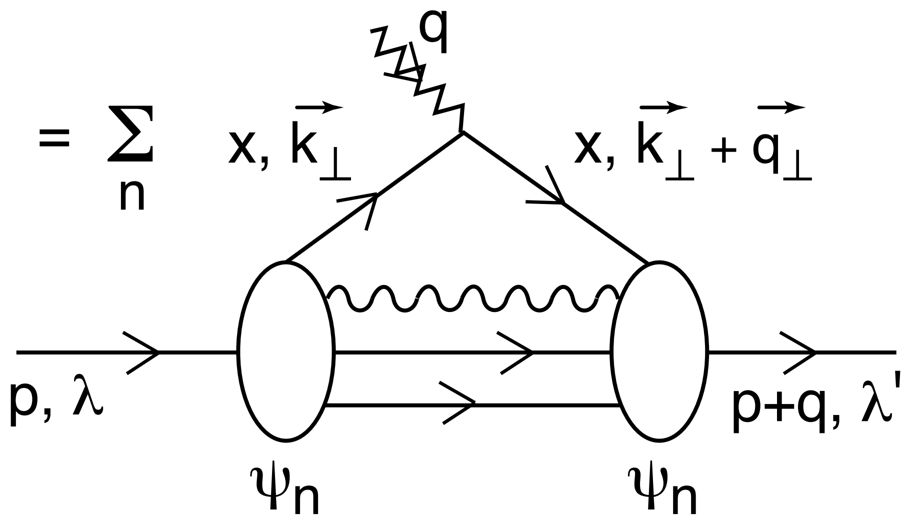
$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Pauli form factor requires nonzero L

Form Factors $\ell p \rightarrow \ell' p'$ $\langle p' \lambda' | J^+ (0) | p \lambda \rangle$

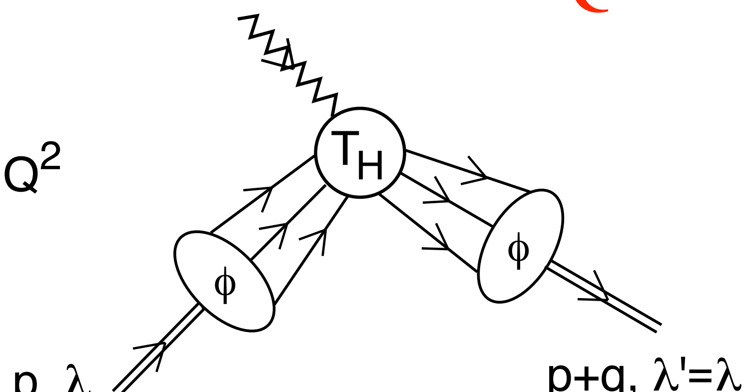
$F_{\lambda\lambda'}(Q^2) = \sum_n$



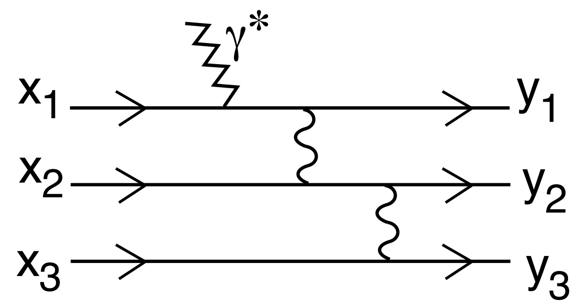
QCD Factorization

Lepage, SJB
Efremov, Radyuskin
Chernyak, Zhitnitsky

⇨ Large Q^2



$T_H = \sum$



$+ \ \ldots$

$$= \frac{\alpha_s^2}{Q^4} f(x_i, y_i,)$$

Scaling Laws from PQCD or AdS/CFT

PQCD and Exclusive Processes

Lepage, SJB
Efremov, Radyuskin
Chernyak, Zhitnitsky

$$M = \int \prod dx_i dy_i \phi_F(x, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \phi_I(y_i, Q)$$

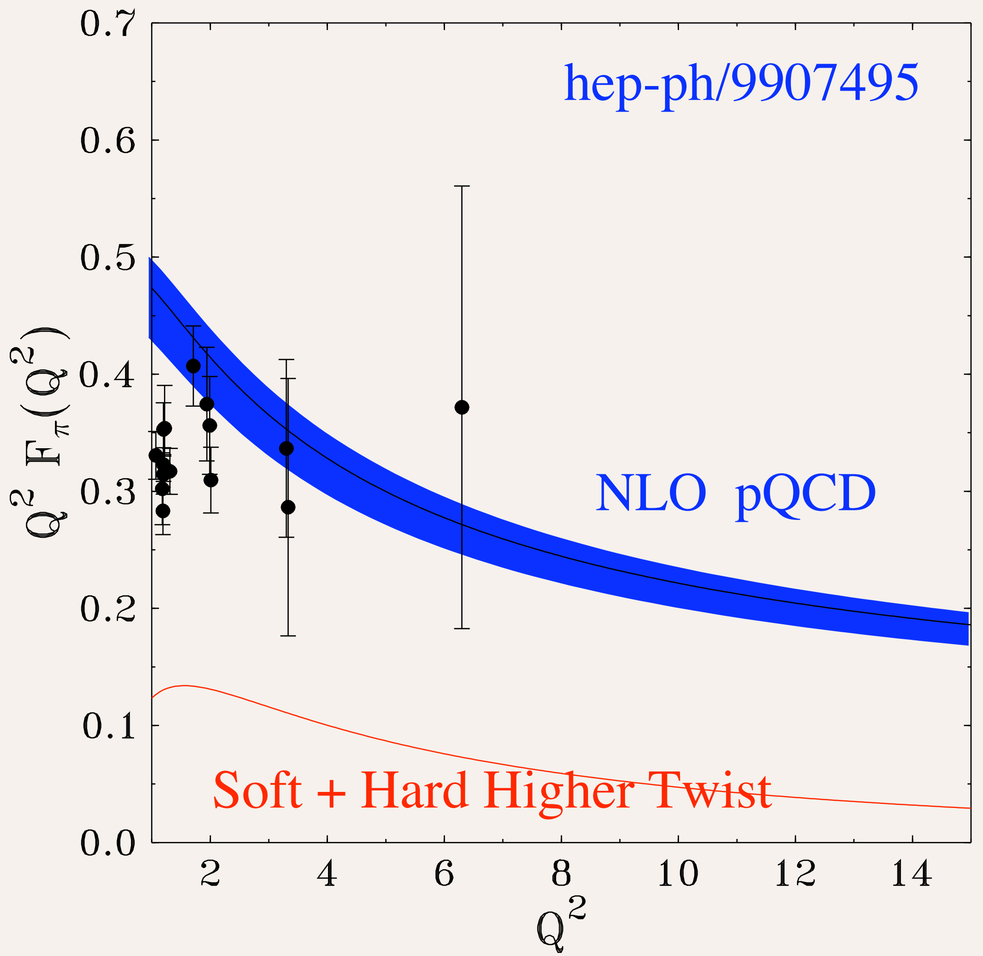
- Iterate kernel of LFWFs when at high virtuality; distribution amplitude contains all physics below factorization scale
- Rigorous Factorization Formulae: Leading twist
- Underly Exclusive B-decay analyses
- Distribution amplitude: gauge invariant, OPE, evolution equations, conformal expansions
- BLM scale setting: sum nonconformal contributions in scale of running coupling
- Derive Dimensional Counting Rules/ Conformal Scaling

Test High Momentum Transfer Domain

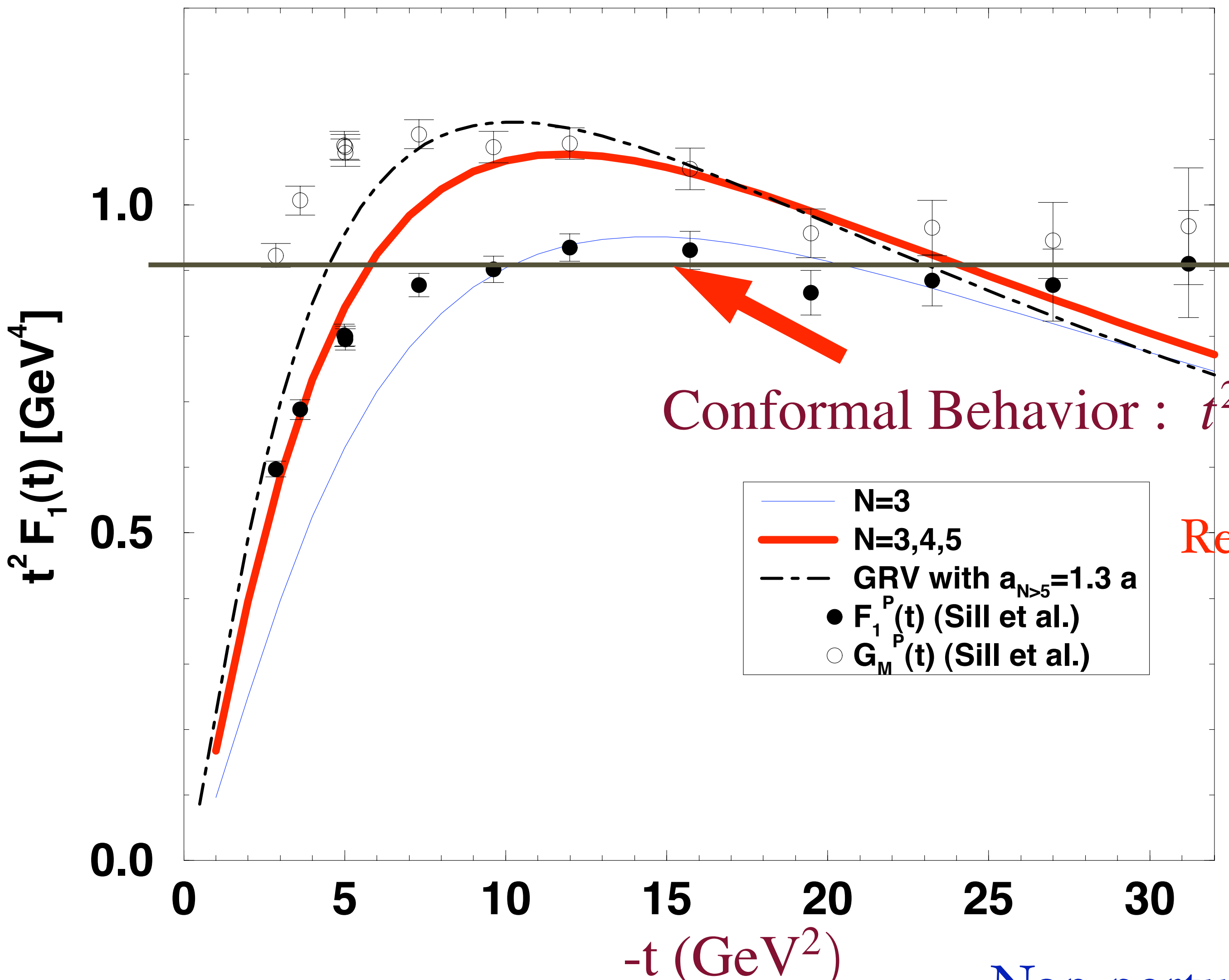
- pQCD Factorization of hard, soft domains
- Constituent Counting Rules -- reflect conformal invariance of leading twist contributions to T_H
- Hadron helicity Conservation: F_2/F_1 higher twist
- Cannot postpone validity of leading twist domain: Higher twist effects controlled by nominal QCD scales
- Running coupling evaluated at small fraction of Q^2

♡ LCSR are fully consistent with pQCD:

V.Braun



Proton Form Factor

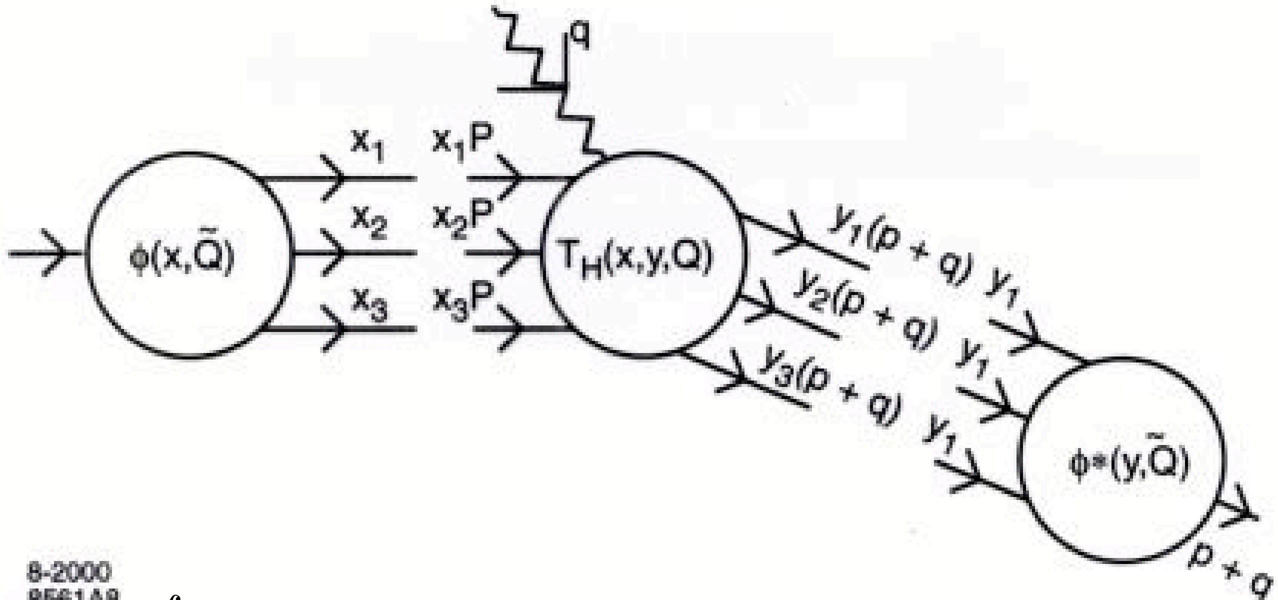


Conformal Behavior : $t^2 F_1(t) = \text{const}$

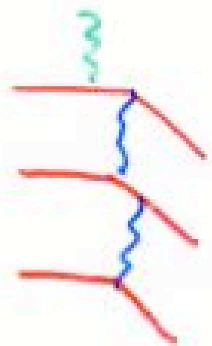
Remarkable scaling behavior

Non-perturbative model: Diehl, Kroll

Primary Test of QCD Factorization, Scaling



$$M = \int \prod dx_i dy_i \phi_F(x,\tilde{Q}) \times T_H(x_i,y_i,\tilde{Q}) \phi_I(y_i,Q)$$

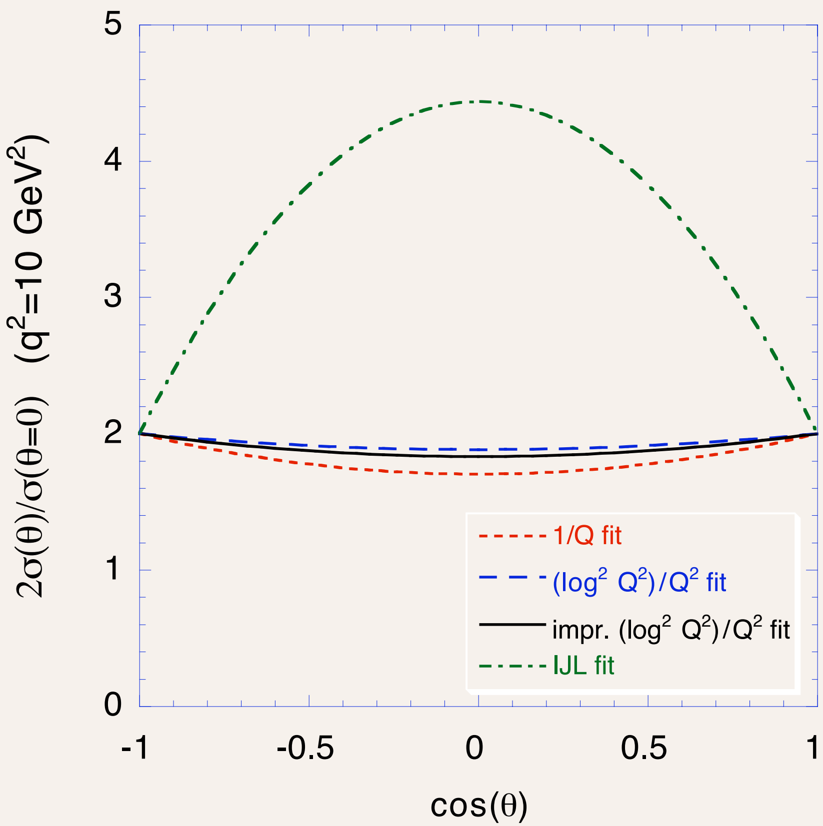


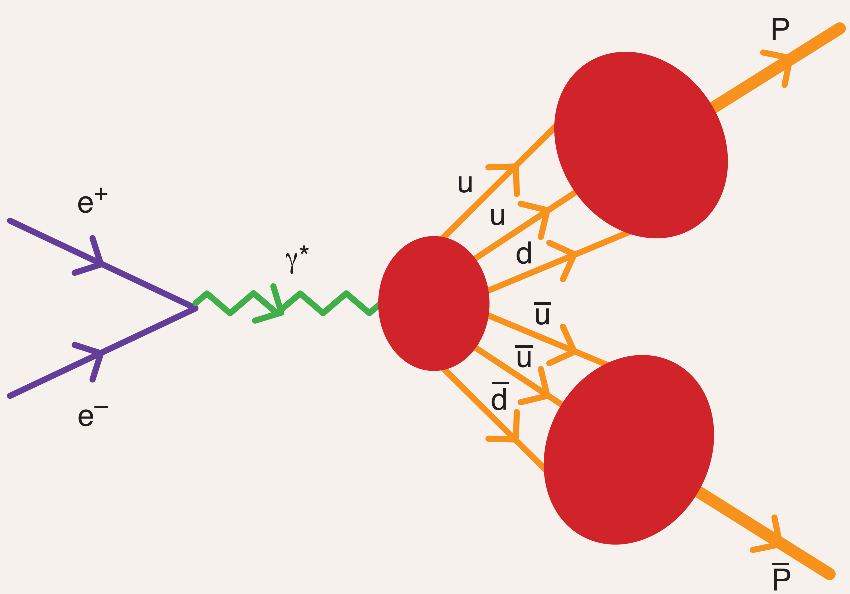
Running coupling evaluated at small scales

Resummation ?

Conformal Behavior : $t^2 F_1(t) = \text{const}$

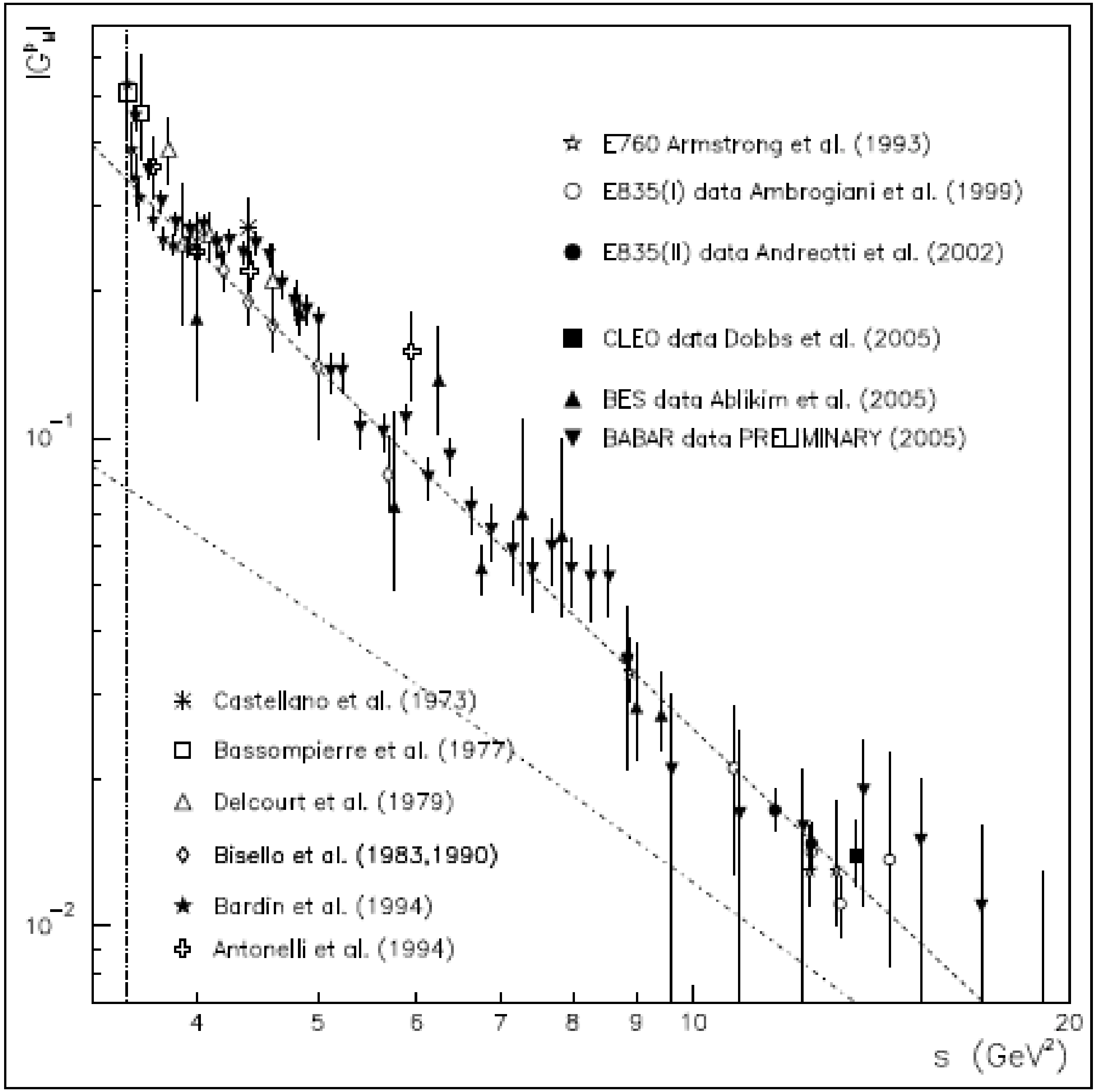
Timelike proton form factor in PQCD





$$G_M(Q^2) \rightarrow \frac{\alpha_s^2(Q^2)}{Q^4} \sum_{n,m} b_{nm} \left(\log \frac{Q^2}{\Lambda^2}\right)^{\gamma_n^B + \gamma_n^B} \times \left[1 + \mathcal{O}\left(\alpha_s(Q^2), \frac{m^2}{Q^2}\right)\right] .$$

Asymptotic Behavior



The dashed line is a fit to the PQCD prediction

$$\frac{|G_M|}{\mu_p} = \frac{C}{s^2 \ln^2\left(\frac{s}{\Lambda^2}\right)}$$

The expected Q^2 behaviour is reached quite early, however ...

... there is still a factor of 2 between timelike and spacelike.

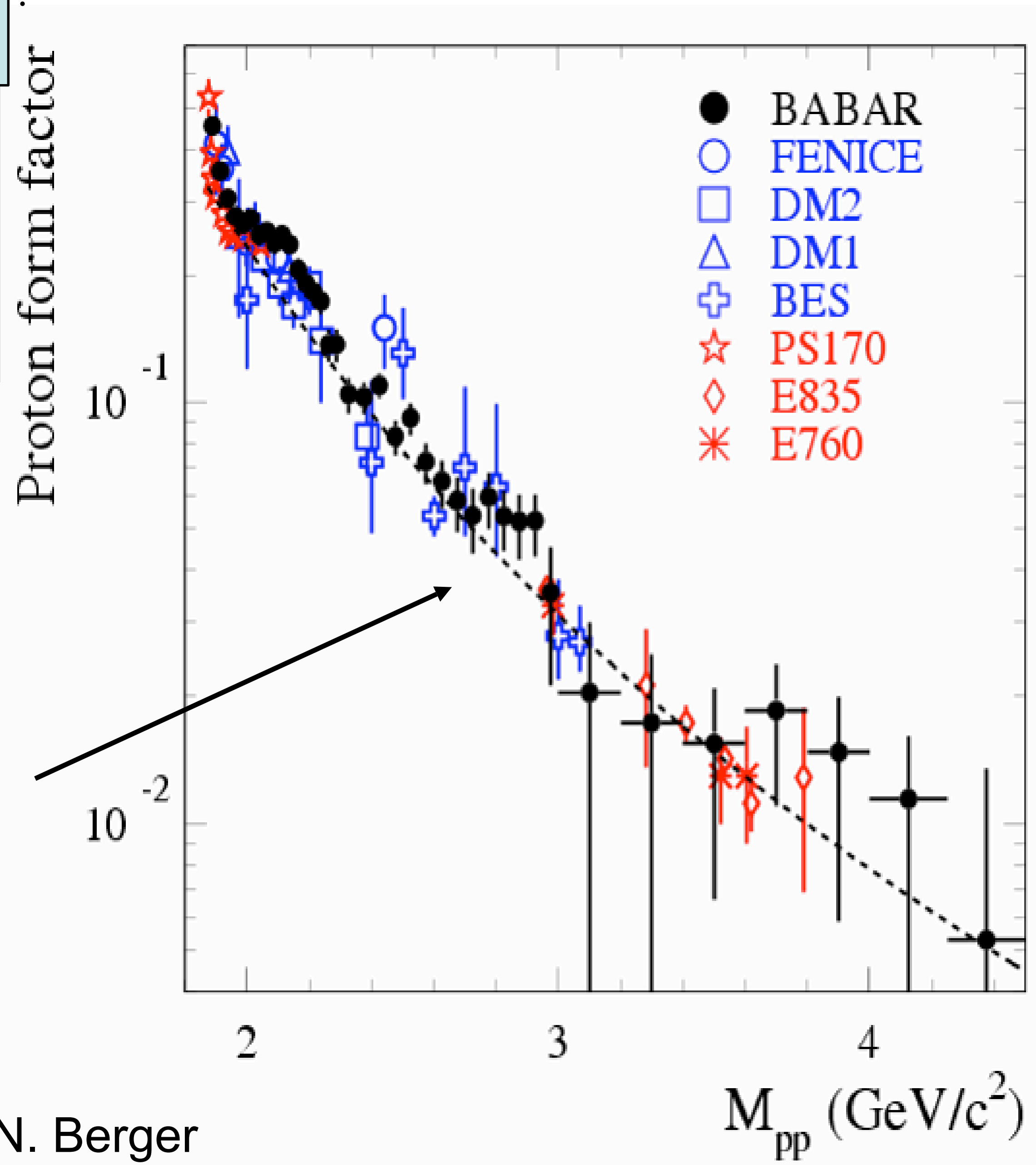
- Define "Effective" form factor by

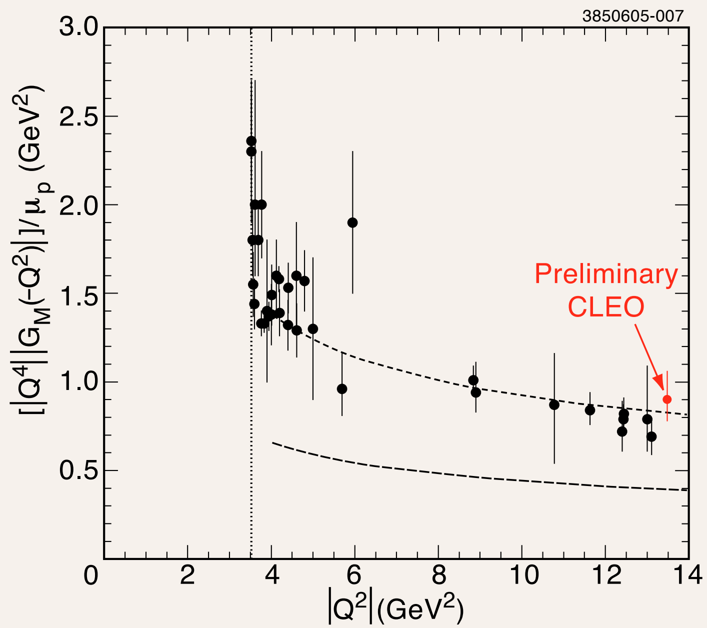
$$\sigma = \frac{4\pi\alpha^2\beta C}{3m_{p\bar{p}}^2}|F|^2,\ |F| = \sqrt{|G_M|^2 + \frac{2m_p^2}{m_{p\bar{p}}^2}|G_E|^2}.$$

- Peak at threshold, sharp dips at 2.25 GeV, 3.0 GeV.
- Good fit to pQCD prediction for high m_{pp}.

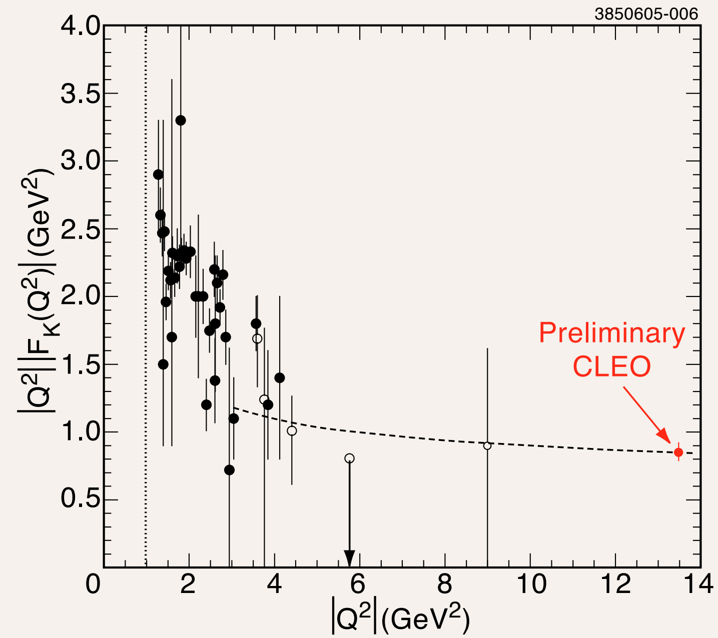
Radiative return from BaBar

$$F(s) \propto \frac{\log^{-2}\frac{s}{\Lambda^2}}{s^2}$$





Proton timelike form factor.



Kaon timelike form factor.

$$Q^2|F_K(13.48\ \mathrm{GeV}^2)| = 0.85 \pm 0.05(\mathrm{stat}) \pm 0.02(\mathrm{syst})\ \mathrm{GeV}^2$$
$$Q^4|G_M^p(13.48\ \mathrm{GeV}^2)|/\mu_p = 0.91 \pm 0.13(\mathrm{stat}) \pm 0.06(\mathrm{syst})\ \mathrm{GeV}^4$$

The proton magnetic form factor result agrees with that measured in the reverse reaction $p\bar{p} \to e^+e^-$ at Fermilab. **The kaon form factor measurement is the first ever direct measurement at $|Q^2| > 4$ GeV^2.**

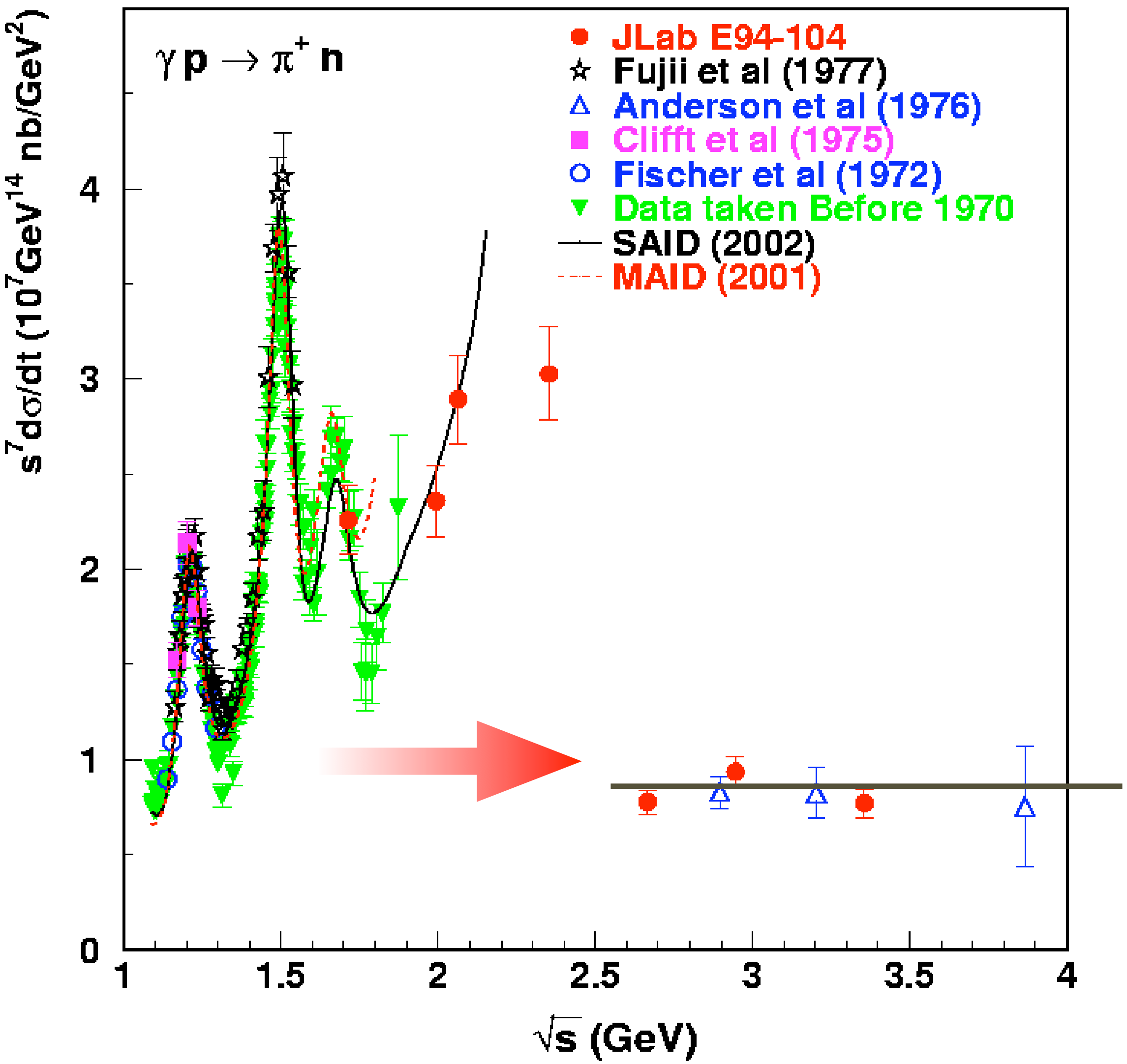
The pion form factor is being measured.

Seth

Test of PQCD Scaling

Constituent counting rules

Farrar, sjb; Muradyan, Matveev, Taveklidze



$s^7 d\sigma/dt(\gamma p \to \pi^+ n) \sim const$

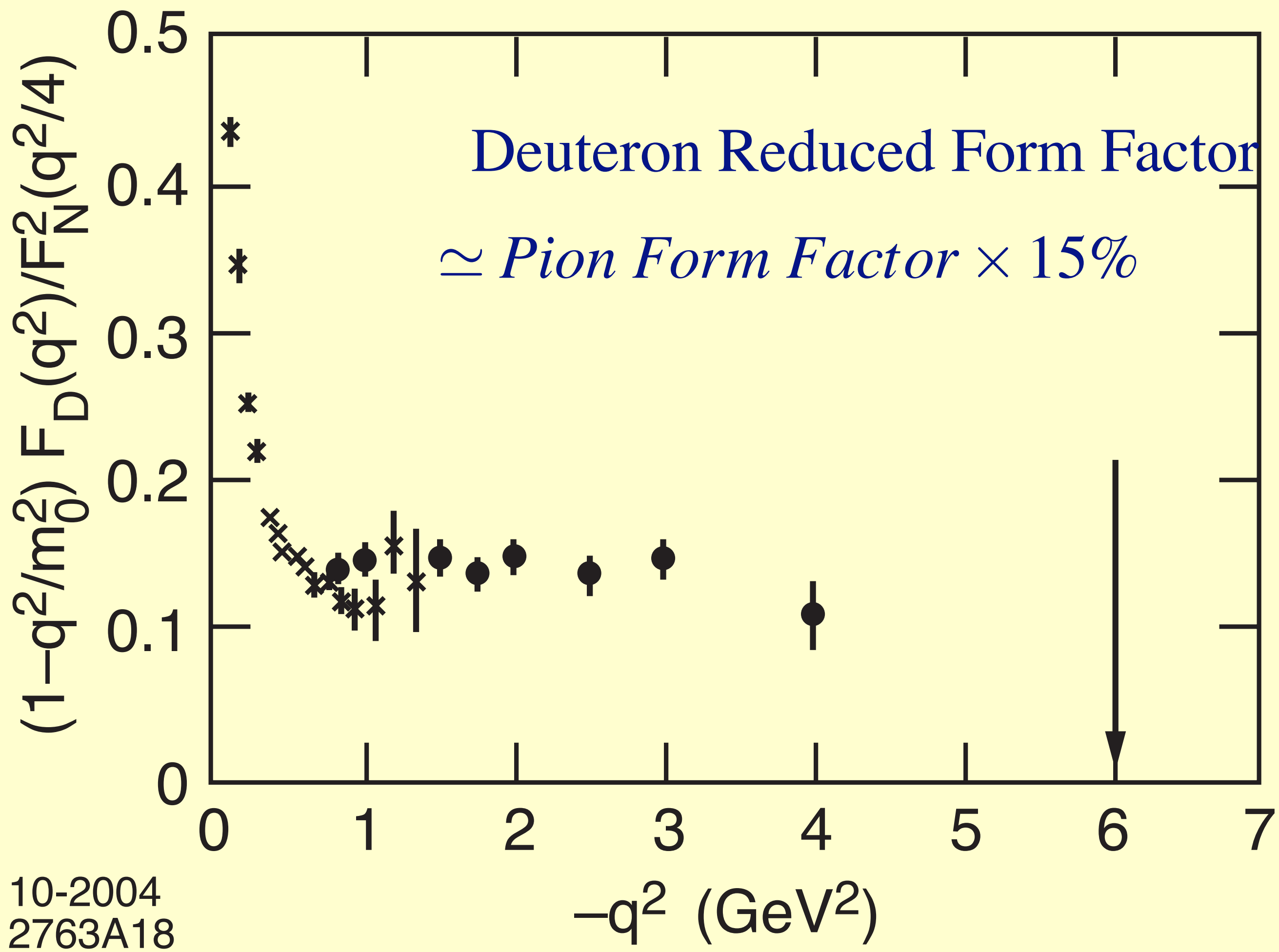
fixed θ_{CM} scaling

PQCD and AdS/CFT:

$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B \to C+D) =$
$F_{A+B\to C+D}(\theta_{CM})$

$s^7 \frac{d\sigma}{dt}(\gamma p \to \pi^+ n) = F(\theta_{CM})$
$n_{tot} = 1+3+2+3 = 9$

Conformal invariance at high momentum transfers!



- 15% Hidden Color in the Deuteron

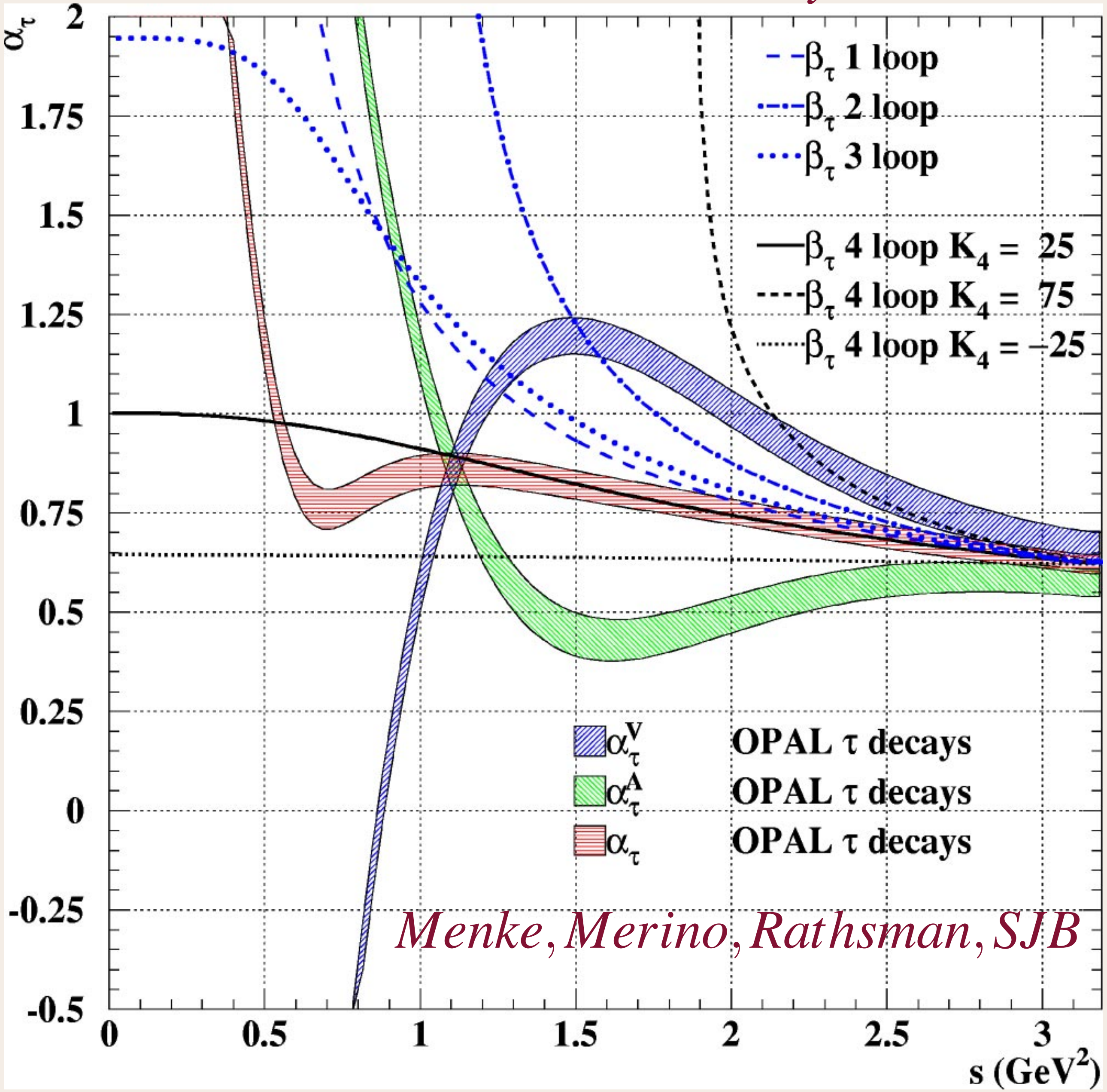
Test High Momentum Transfer Domain

- Constituent Counting Rules -- reflect conformal invariance of leading-twist contributions to T_H
- Hadron helicity conservation: F_2/F_1 higher-twist: PQCD analysis Belitsky, Tuan, Ji
- Cannot postpone validity of leading twist domain: Higher-twist effects controlled by nominal QCD scales

Braun: LC sum Rules

- Normalization: Many issues: f_N Text, shape of distribution amplitude
- Running coupling evaluated at (1/20) Q^2 Ji, Robertson, Tang, sjb
- Define effective charge from pion form factor
- IR Fixed Point for QCD Coupling?

QCD Effective Coupling from *hadronic τ decay*



New Perspectives on QCD Phenomena from AdS/CFT

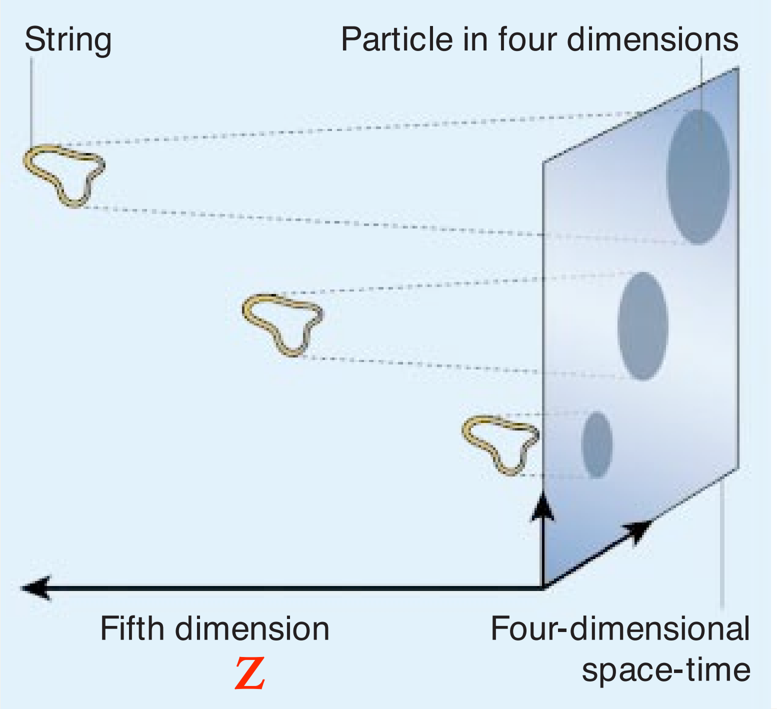
- AdS/CFT: Duality between string theory in Anti-de Sitter Space and Conformal Field Theory
- New Way to Implement Conformal Symmetry
- Holographic Model: Conformal Symmetry at Short Distances, Confinement at large distances
- Remarkable predictions for hadronic spectra, wavefunctions, interactions
- AdS/CFT provides novel insights into the quark structure of hadrons

AdS5 Metric

Holographic Model

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

J. Maldacena



Strings, particles and extra dimensions. Strings moving in the fifth dimension are represented in the everyday world by their projection onto the four-dimensional boundary of the five-dimensional space-time. The same string located at different positions along the fifth dimension corresponds to particles of different sizes in four dimensions: the further away the string, the larger the particle. The projection of a string that is very close to the boundary of the four-dimensional world can appear to be a point-like particle.

deTeramond, sjb

AdS/CFT

$z = R^2/r$

- Use mapping of SO(4,2) to AdS5
- Scale Transformations represented by wavefunction Ψ(r) in 5th dimension

$$x_\mu^2 \to \lambda^2 x_\mu^2 \quad \equiv \quad r \to \frac{r}{\lambda} \quad \equiv \quad z \to \lambda z$$

- Holographic model: Confinement at large distances and conformal symmetry at short distances

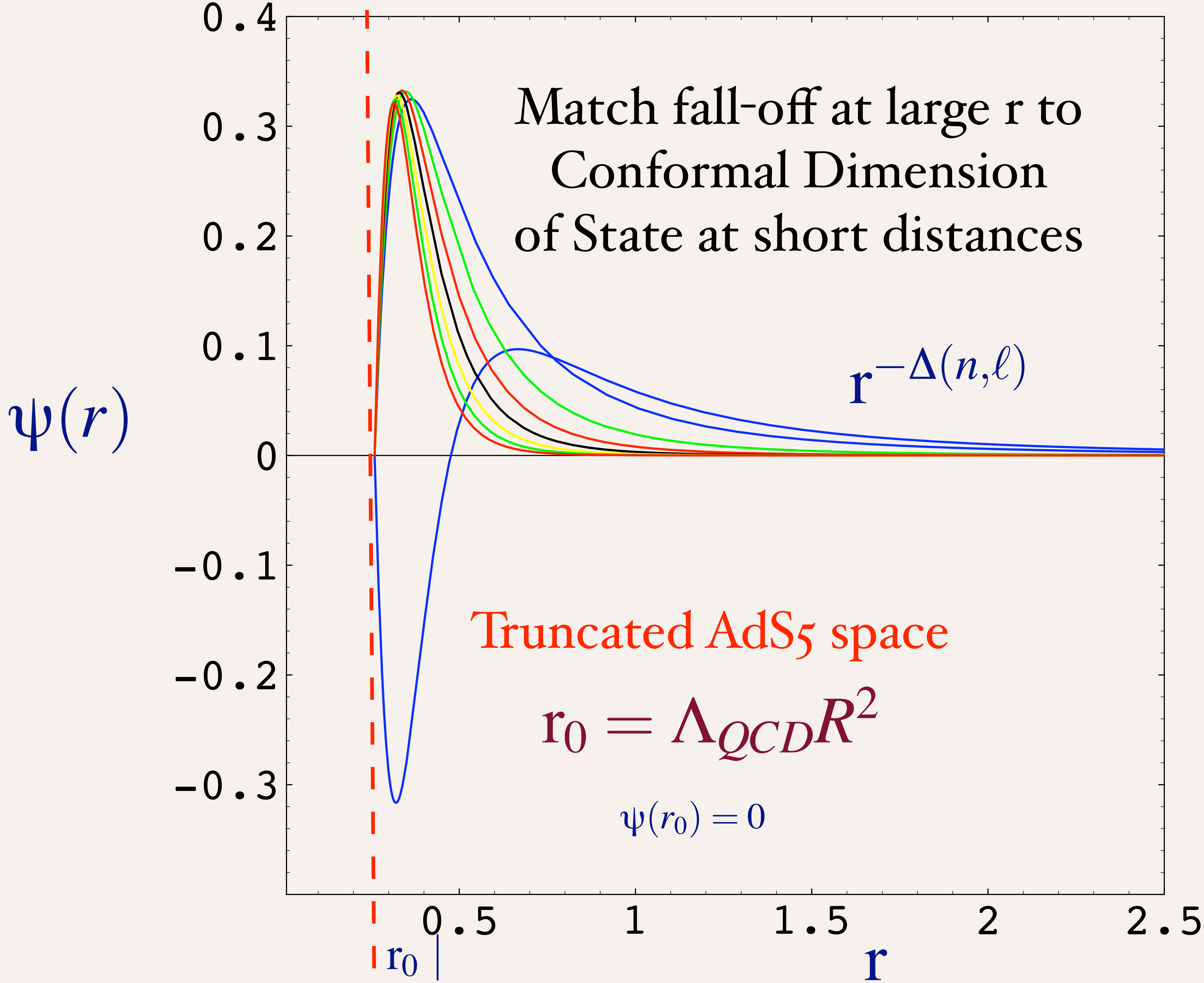
$$0 < z < z_0 = \frac{1}{\Lambda_{QCD}}, \; r > r_0 = \Lambda_{QCD} R^2$$

- Match solutions at large r to conformal dimension of hadron wavefunction at short distances

$$\psi(r) \to r^{-\Delta} \text{ at large } r, \text{ small } z$$

- Truncated space simulates "bag" boundary conditions

$$\psi(z_0) = \psi(r_0) = 0 \qquad r = \frac{R^2}{z}$$

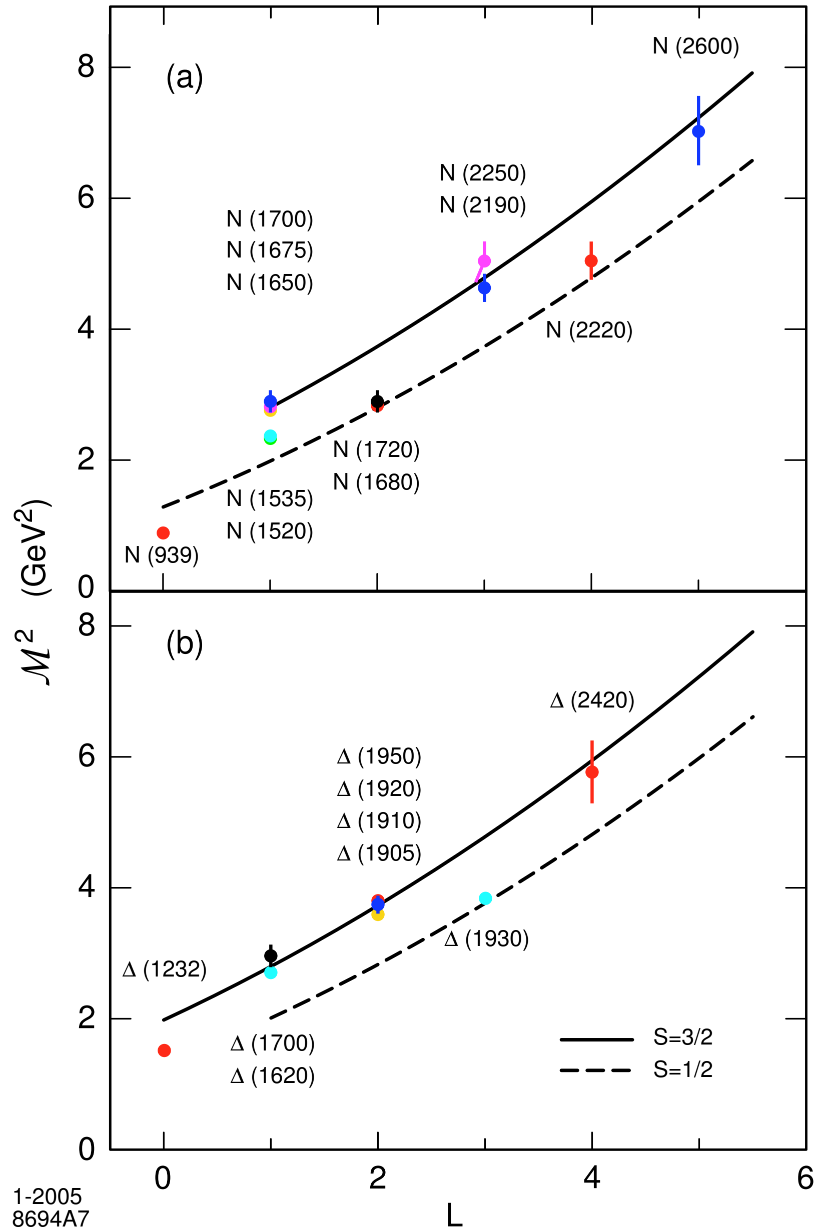


0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.5
1
1.5
2
2.5
Match fall-off at large r to
Conformal Dimension
of State at short distances
$r^{-\Delta(n,\ell)}$
$\psi(r)$
Truncated AdS5 space
$r_0 = \Lambda_{QCD} R^2$
$\psi(r_0) = 0$
r_0
r

Predictions of AdS/CFT

Only one parameter!

Entire light quark baryon spectrum



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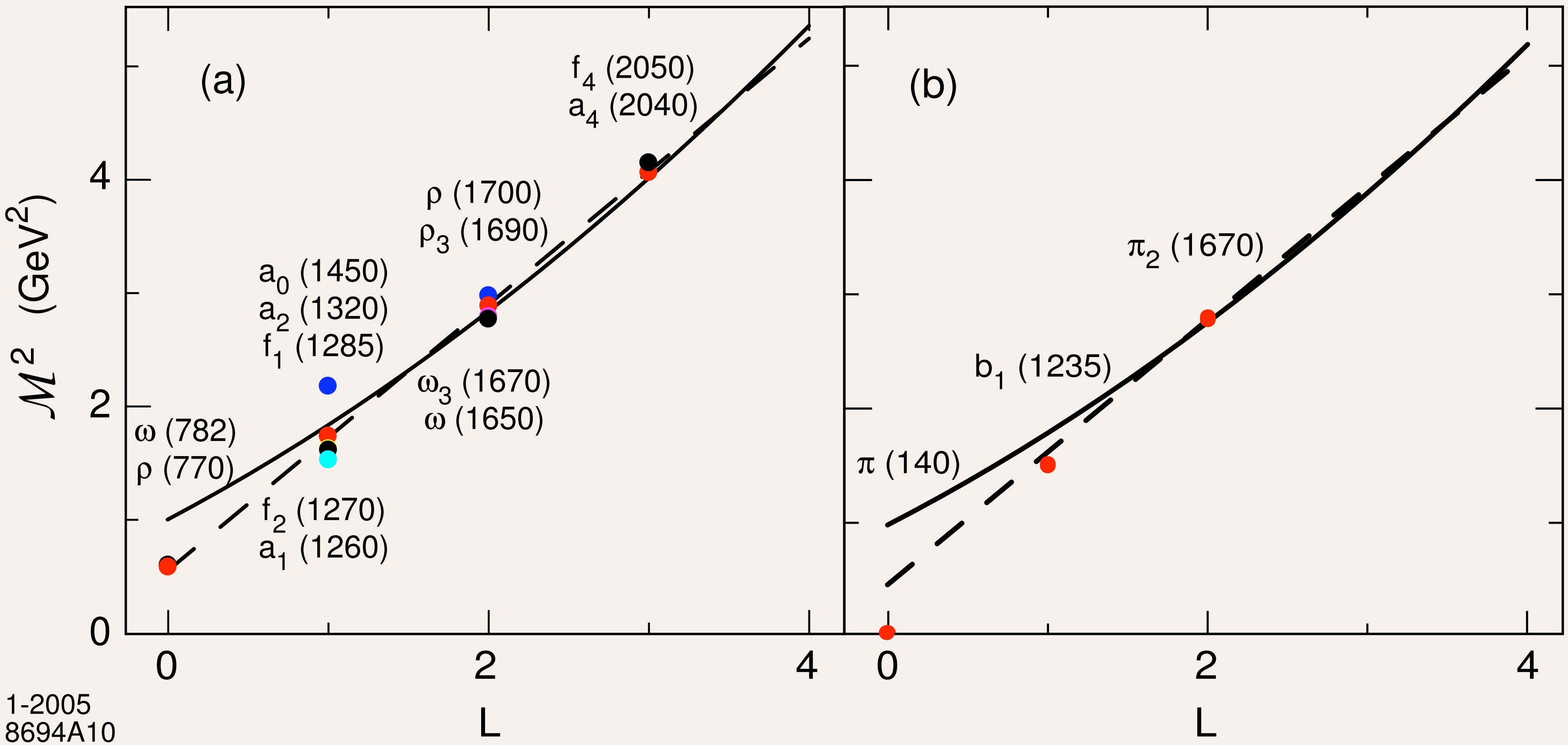
SJB

Phys.Rev.Lett.94: 201601,2005

hep-th/0501022

Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.22 GeV



: Light meson orbital spectrum: 4-dim states dual to vector fields in the bulk, $\Lambda_{QCD} = 0.26$ GeV

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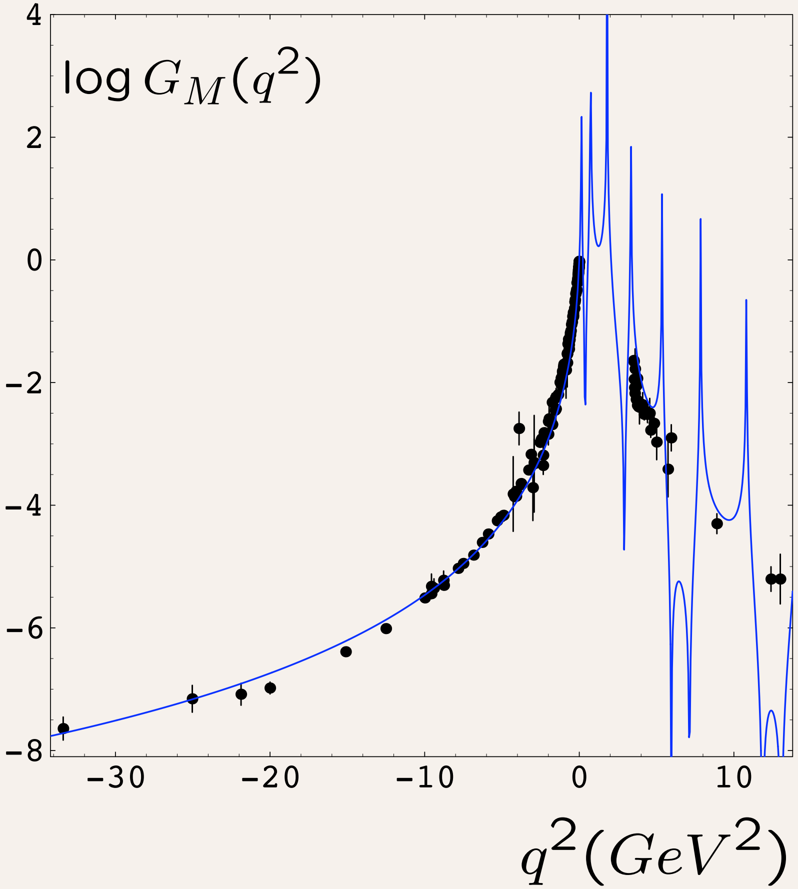
Prediction of AdS/CFT Holographic Model

One parameter: $\Lambda_{QCD} = 0.15$ GeV.



Space-like and time-like structure of the proton magnetic form factor in AdS/QCD for $\Lambda_{QCD} = 0.15$ GeV. The data are from the compilation given by Baldini et al.

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in progress

The space-like current $J_{sl}(Q, z)$ is

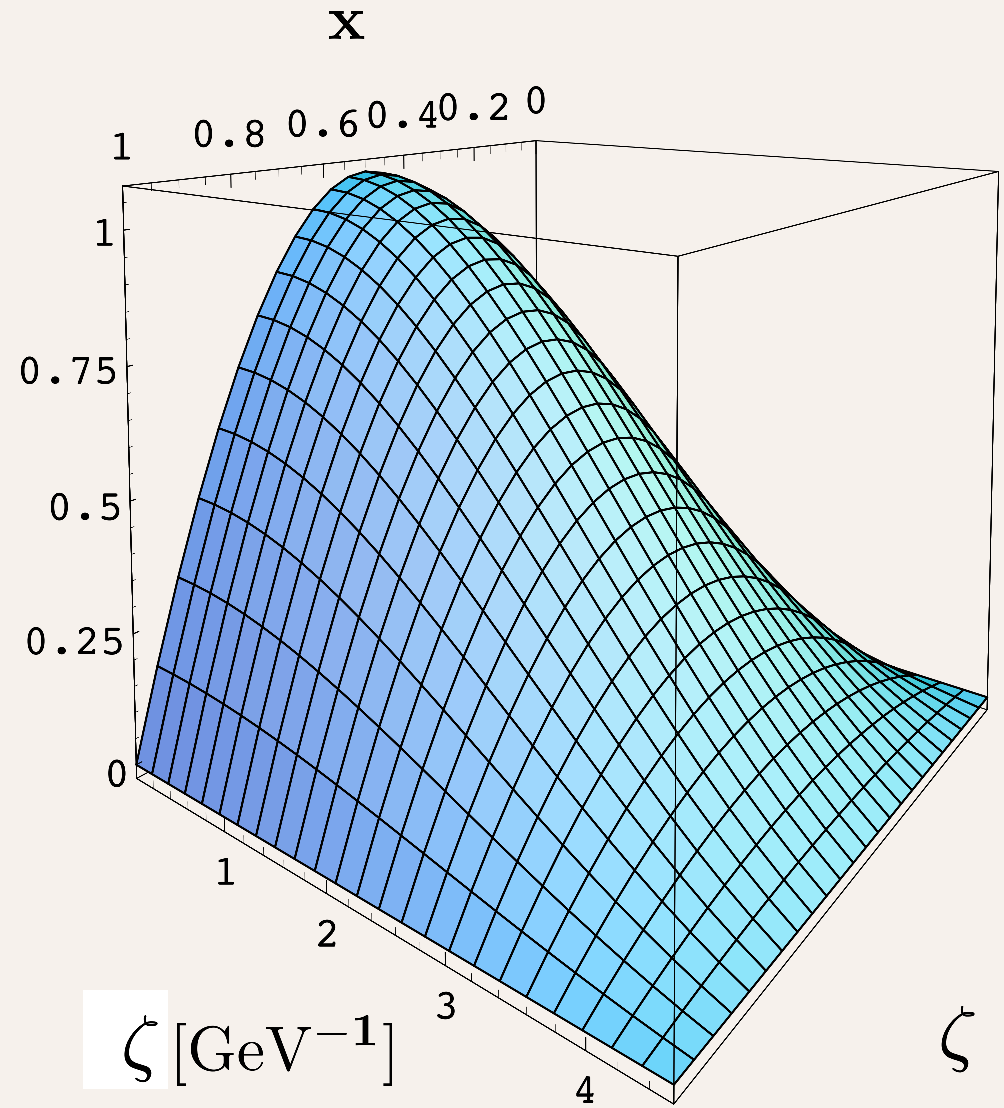
$$J_{sl}(Q,z) = zQ\left[K_1(zQ) + \frac{K_0(Q/\Lambda_{\rm QCD})}{I_0(Q/\Lambda_{\rm QCD})} I_1(zQ\right.$$

analytic continuation:

$$J_{tl}(Q,z) = -zQ\,\frac{\pi}{2}\left[Y_1(zQ) - \frac{Y_0(Q/\Lambda_{\rm QCD})}{J_0(Q/\Lambda_{\rm QCD})} J_1(zQ)\right].$$

Formalism: Polchinski and Strassler

New tool for QCD



AdS/CFT prediction for meson LFWF

Holographic Model

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SJB

$$\zeta = b\sqrt{x(1-x)}$$

Two-parton ground state LFWF in impact space $\psi(x,b)$ for a for $n=2, \ell=0, k=1$.

Holographic Model

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SJB

AdS/CFT prediction for meson LFWF

$$\widetilde{\psi}_L(x, \vec{b}_\perp)$$

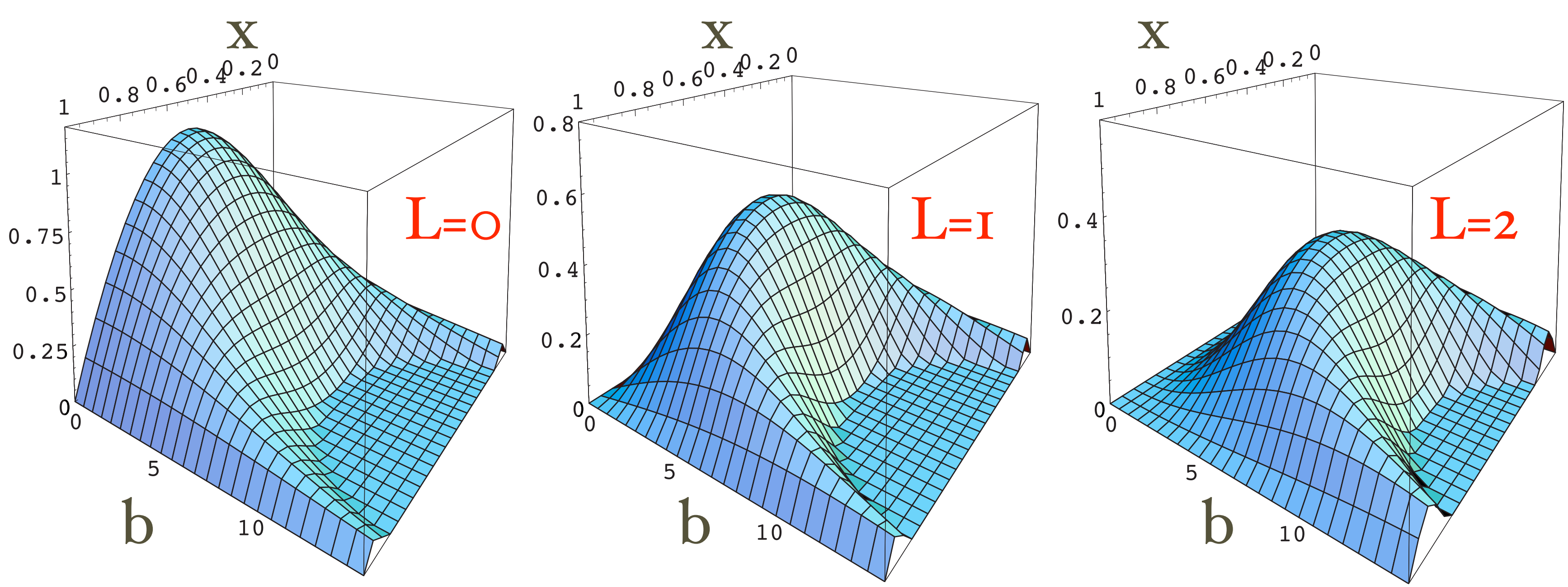

Figure 1: Two-parton bound state light-front wave function $\widetilde{\psi}_L(x, \vec{b}_\perp)$ as function of the constituents longitudinal momentum fraction x and $1-x$ and the impact space relative coordinate $\vec{b}_\perp$ in a holographic QCD model. The results for the ground state ($L = 0$) are shown in (a). The predictions for first orbital exited states ($L = 1$ and $L = 2$) are shown in (b) and (c) respectively.

Compare with Kroll et al., LGTH