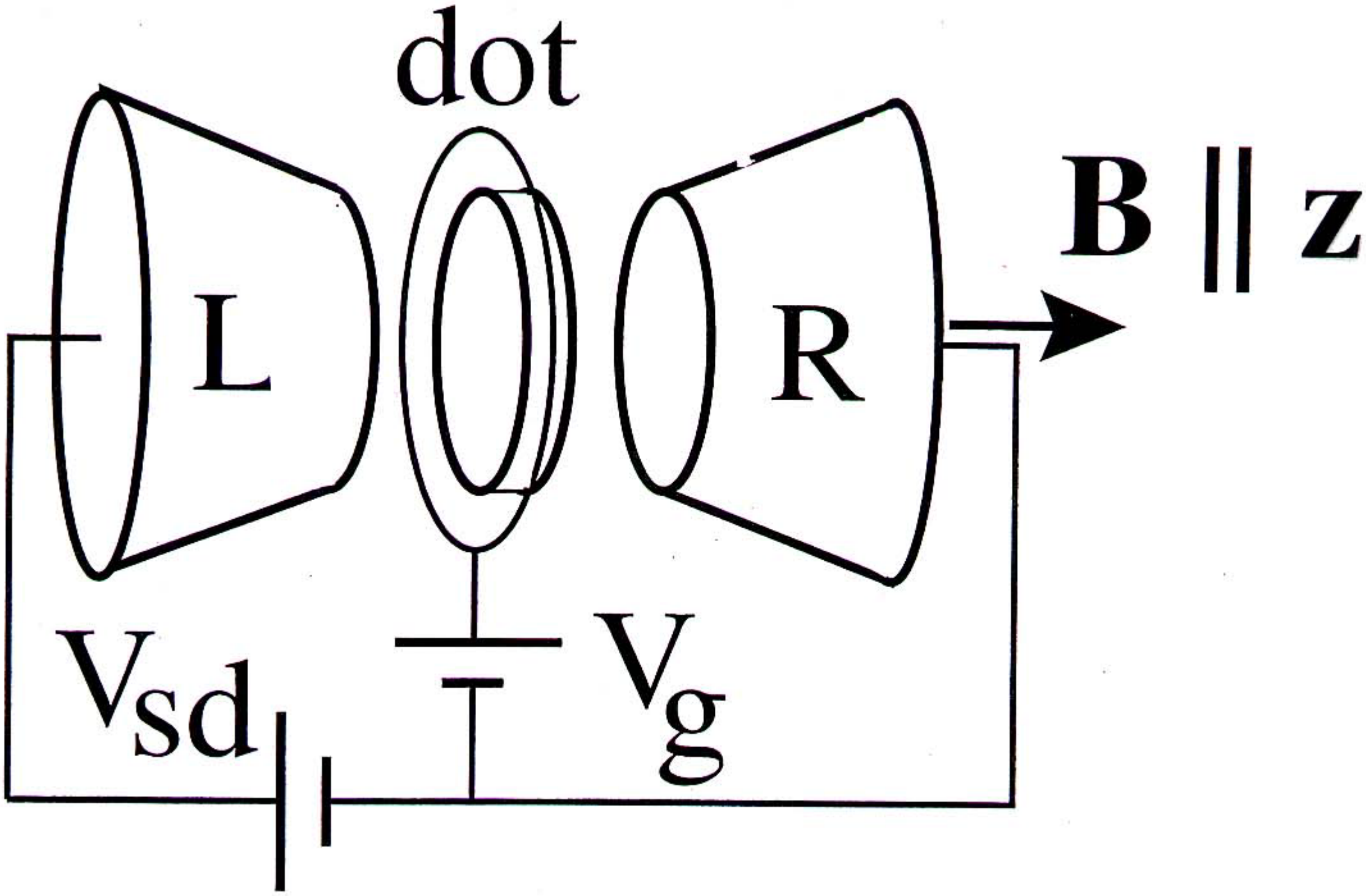
PLAN OF THE LECTURES

Lecture II: Non perturbative linear conductance: Kondo effect in Quantum dots

- Mechanism and experimental findings
- Freezing the charge degree of freedom in tunneling: Schrieffer-Wolff transformation and Kondo hamiltonian
- Poor man's scaling and Kondo screening
- Kondo conductance
- Orbital Kondo and more exotic Kondo states
- Kondo with superconducting contacts: Josephson current



B $\perp$ to the dot :

at B^* Singlet - Triplet transition

N = even

- Linear conductance
- Lowering the temperature ...

Lowering T:

Temperature dependence of the peaks in the conductance

- **halfwidth** $\Gamma \propto T$
- **height** $g_{max} \propto 1/T$

$$t(\epsilon) = -i \frac{\sqrt{\Gamma_L \Gamma_R}}{\epsilon - \epsilon_0 + i\Gamma/2}$$

$\Gamma = \Gamma_L + \Gamma_R$

$$g = \frac{dI}{dV} = \frac{1}{2\pi} \frac{e^2}{\hbar} \int d\epsilon \; |t(\epsilon)|^2 \; \frac{\partial f}{\partial \epsilon}\bigg|_{\epsilon_F}$$

$$\Rightarrow \qquad g_{max} \propto \frac{e^2}{\pi\hbar} \frac{\Gamma_L \Gamma_R}{\Gamma} \beta$$

at Coulomb Blockade : the charge degree of freedom is frozen

2 Non perturbative Kondo conductance :

SPIN *degree of freedom survives* *if it is present*

- Antiferromagnetic interaction : **Kondo screening**
- non perturbative tunneling : **resonance at the Fermi level**
- phase shift $\delta = \pi/2$:**unitary limit of the conductance**

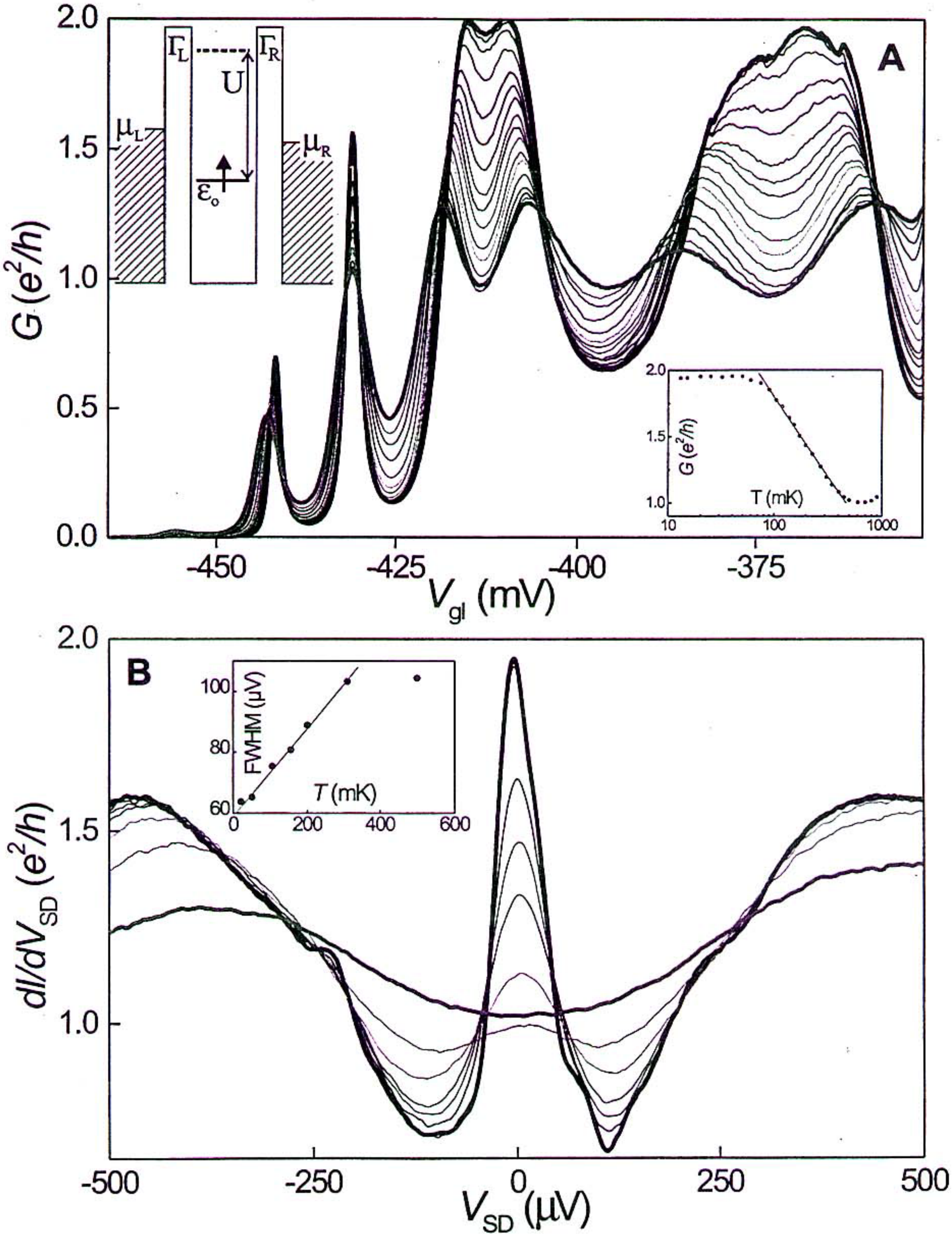


Figure 2
van der Wiel *et al.*

EXAMPLES :

Parity effect

$N = even,\ S = 0$: $\Rightarrow$ $\boxed{\begin{array}{c}\textit{no degeneracy in the GS :}\\ \textit{KONDO is ABSENT}\end{array}}$

$N = odd,\ S = \frac{1}{2}$: $\Rightarrow$ $\boxed{\textit{GS is a doublet :}\quad \sigma = \uparrow, \downarrow}$

the magnetic moment is screened: **Kondo Singlet**

$$\mathbf{H} = \sum_{\mathbf{qm}\sigma} \epsilon_{\mathbf{q,m}} \mathbf{b}^{\dagger}_{\mathbf{q,m},\sigma} \mathbf{b}_{\mathbf{q,m},\sigma} + \mathbf{J} \vec{\sigma}(\mathbf{0}) \cdot \vec{\mathbf{S}}$$

contacts electrons

J > 0

antiferromagnetic coupling

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single electron states in the leads in cylindrical coordinates

$$\psi_{\epsilon \sim \epsilon_{\mathbf{F}}, \mathbf{q,m},\sigma} \equiv \mathbf{e}^{\mathbf{i}(\mathbf{k_F}+\mathbf{q})\mathbf{z}} \varphi_{\sim \mathbf{0,m}}(\rho) \mathbf{e}^{\mathbf{im}\vartheta} \chi_\sigma$$

***D* is the cutoff in the conduction band**

Kondo effect (1960)

Magnetic impurity in non magnetic metal

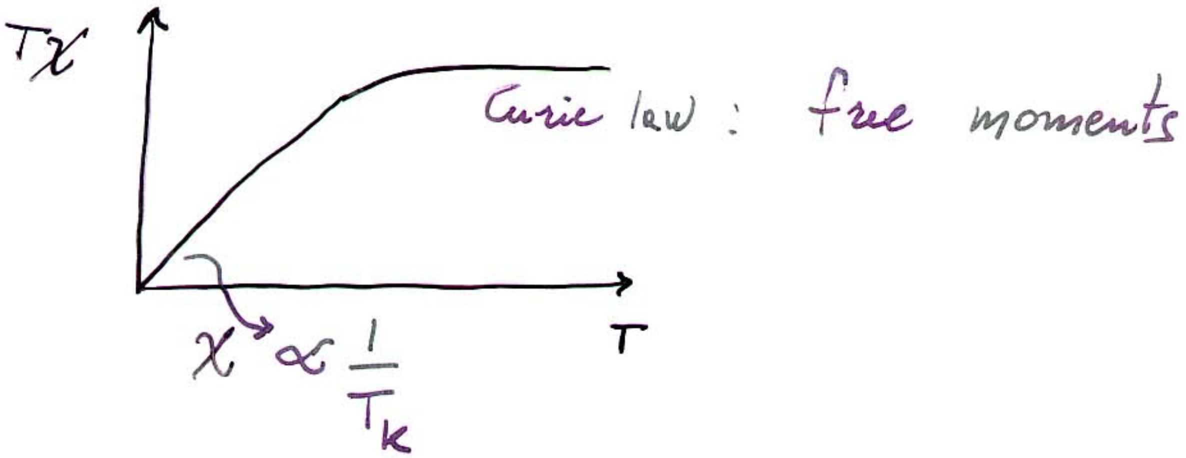
(e.g. Mn in Cu)

Diluted alloy: non interacting magnetic moments

$$\chi = \frac{C}{T} \quad \text{(Curie law)}$$

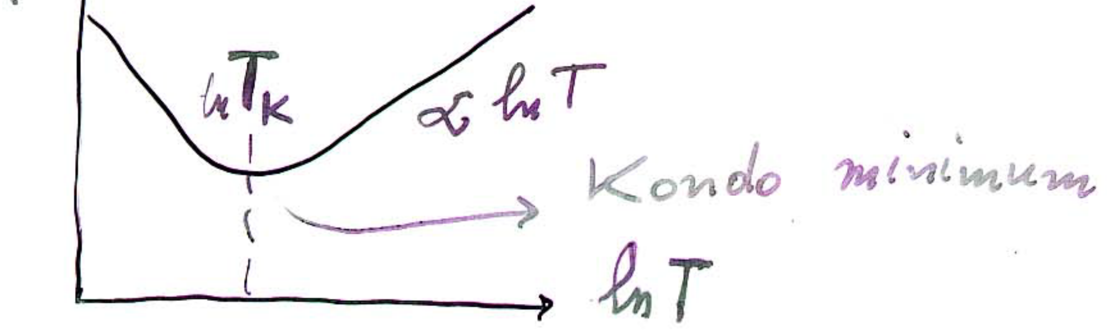
$$C = (\mu_B g)^2 \frac{J(J+1)}{3}$$

Free electrons: $\chi = \mu_B^2 \nu(0)$



Resistivity: expected $\frac{d\rho}{dT} > 0$

$$\rho \sim a T^2 + b T^5 + \dots$$



Kondo problem:

$$H = \underbrace{\sum_{k\sigma} \epsilon_{k\sigma} c^{+}_{k\sigma} c_{k\sigma}}_{\text{conduction electrons}} + \overbrace{J\,\vec{S}\cdot \vec{\sigma}(0)}^{H_K}$$

impurity spin located at $r=0$

spin density at $r=0$ of itinerant electrons

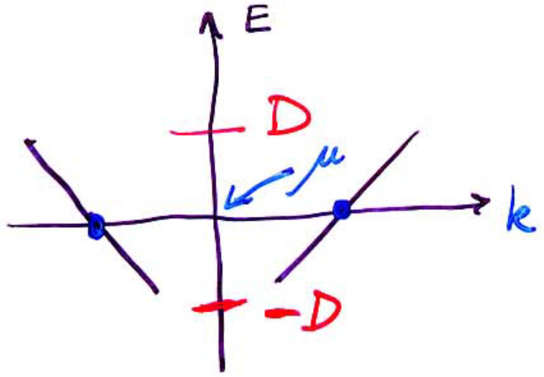
$$H_K/J = S_z \sigma_z(0) + \frac{1}{2}\left(S_+ \sigma_-(0) + S_- \sigma_+(0)\right)$$

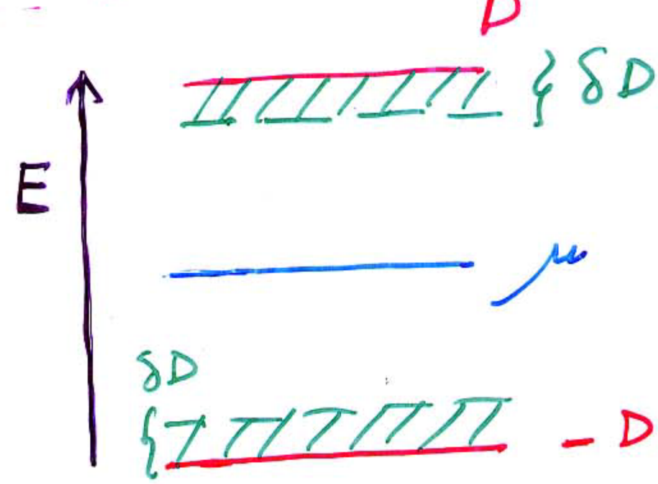
$$\sigma_-(0) \to c^{+}_{\downarrow}(0)\, c_{\uparrow}(0)$$

Note: $$c^{+}_{\sigma}(0)\, c_{\sigma'}(0) = \sum_{k k'} c^{+}_{k\sigma} c_{k'\sigma'}$$

all k, k'

$\rightarrow$ up to a cutoff



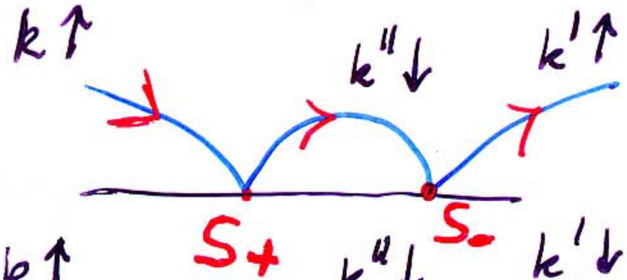


Deal with high energy k" states perturbatively

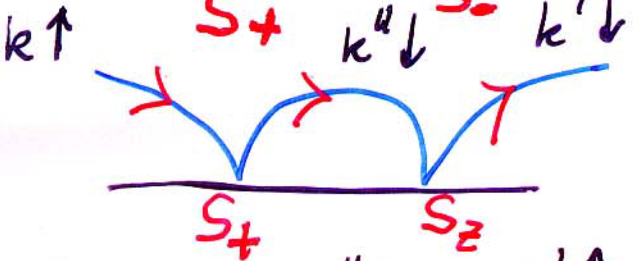
Second order correction for scattering from k to k' via intermediate state k''

$$\sum_{k''\in\delta D} \langle k'|\, H_K \,|k''\rangle\langle k''|\, \frac{1}{E-H_0} \,|k''\rangle\langle k''|\, H_K \,|k\rangle$$

3 possibilities: (particle process)



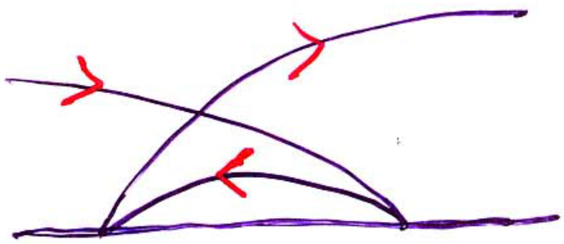
$\longrightarrow \quad c^+_{k'\uparrow}\, S_z\, c_{k\uparrow}$



$\longrightarrow \quad S_+\, \sigma_-$

$\longrightarrow \quad S_-\, \sigma_+$

hole process analogous



Correction to H_K for scattering k, k':

$$\sim -J^2\nu(0)\,\delta D\,\frac{1}{D}\,\vec{S}\cdot \underbrace{c^+_{k'\sigma'}\,\tau_{\sigma'\sigma}\,c_{k\sigma}}_{\text{the sum is } \vec{\sigma}(0)}$$

Conclusion :

when scaling $D \to D - \delta D$

$J \to J + \delta J$

$$\delta J = - J^2 \nu(0) \frac{\delta D}{D}$$

integrate from D_0 to D :

$j = \nu(0) J$

$j_0 = j(D_0)$

$$\frac{1}{j(D)} - \frac{1}{j(D_0)} = \ln \frac{D}{D_0}$$

or :
$$j(D) = \frac{j_0}{1 - j_0 \ln \frac{D_0}{D}}$$

if $j_0 > 0$ (AF coupling)

divergency at :

$$\ln \frac{D_0}{D} = \frac{1}{j_0}$$

D scales as T :

$$\ln \frac{D_0}{k_B T_K} = \frac{1}{j_0}$$

T_K : Kondo temperature : $T_K = D_0 e^{-\frac{1}{j_0}}$

at $T_K >> T \sim 0$ Strong coupling

Spin and impurity : $< \vec{S} + \vec{\sigma}(0) > \sim 0$

Kondo singlet

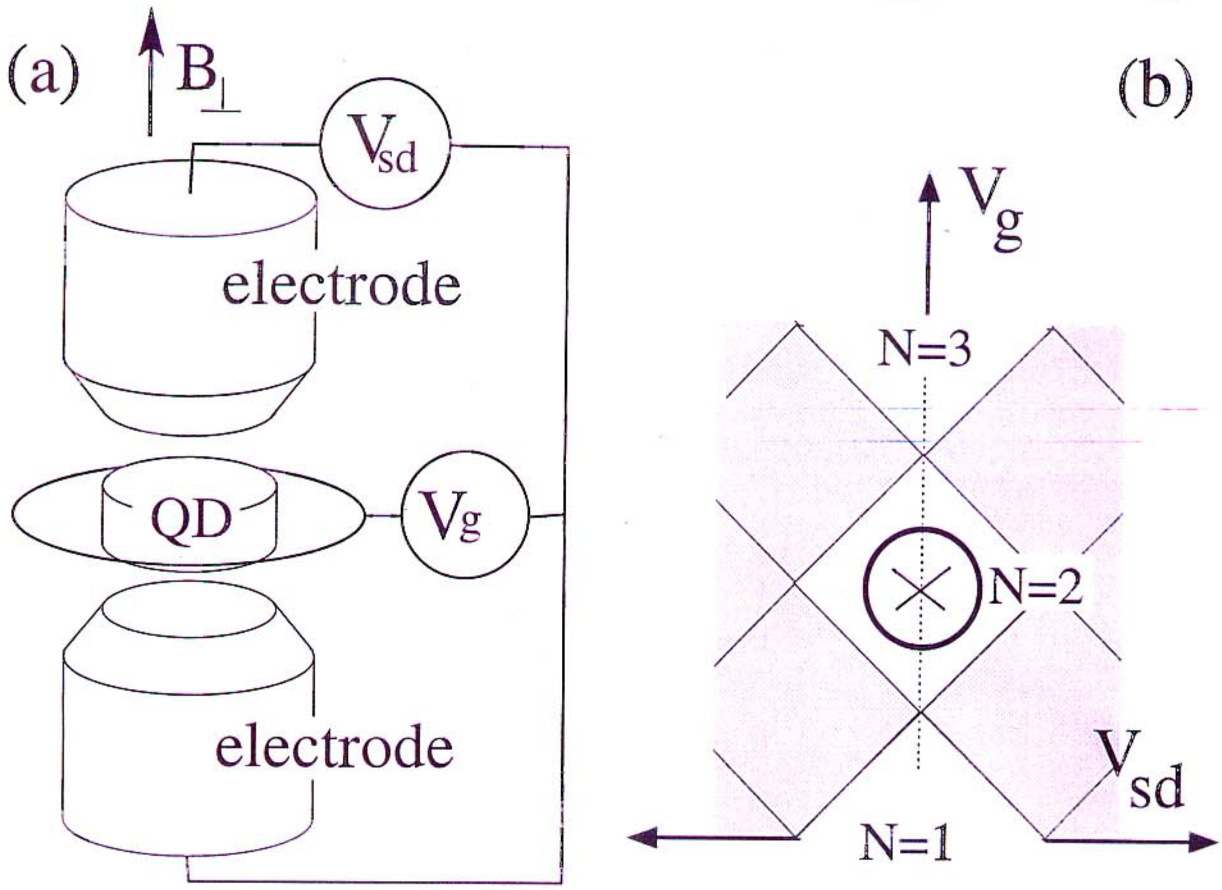


FIG. 1. Vertical Quantum Dot with contacts in a pillar configuration.

at Coulomb blockade: N, S fixed

N odd → S halfinteger

N even → S integer

Coupling to contacts makes charge and spin non conserved.

Kondo correlated state: charge and spin are again conserved but:

N odd → S integer

N even → S halfinteger

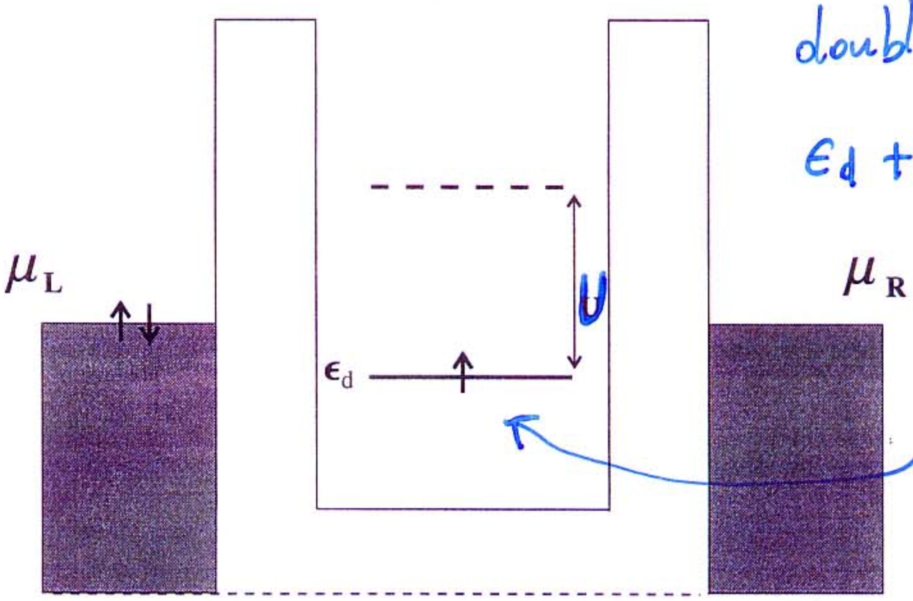
2

$\boxed{\text{Coulomb blockade}}$

Hamiltoniana di Anderson

$$H = \sum_{\sigma} \epsilon_d d^\dagger_\sigma d_\sigma + \sum_{k,\sigma} \epsilon_k c^\dagger_{k,\sigma} c_{k,\sigma} + U n_{d,\uparrow} n_{d,\downarrow}$$

$$+ \sum_{k,\sigma} \left[V_k d^\dagger_\sigma c_{k,\sigma} + V_k^* c^\dagger_{k,\sigma} d_\sigma \right]$$



double occupancy

$\epsilon_d + U$

singly occupied

e.g. $\epsilon_d = -\dfrac{U}{2}$, $\epsilon_d + U = \dfrac{U}{2}$

singly occupied doubly occupied

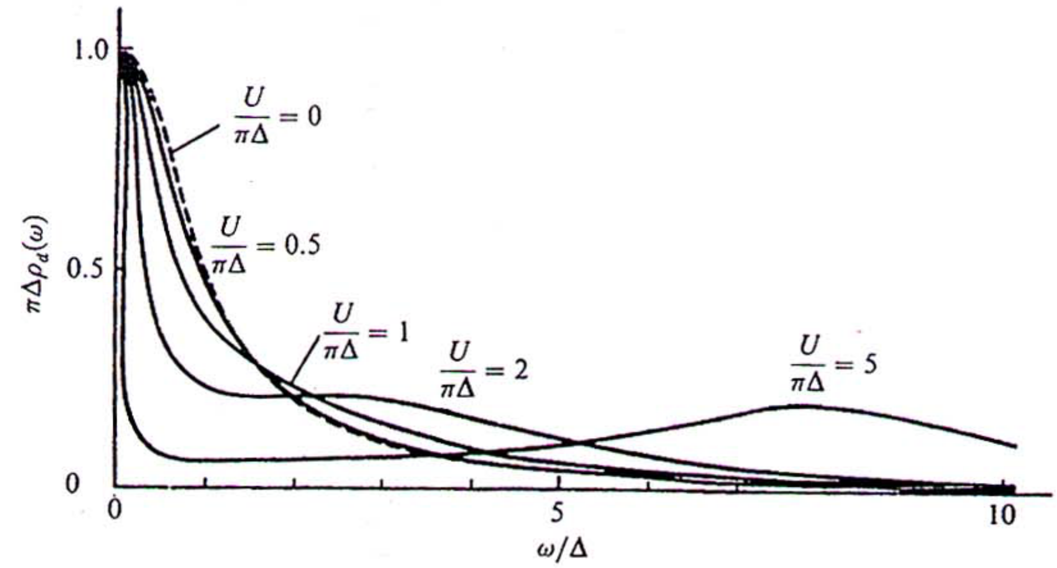


Figure 5.7 Fourth order perturbational results (Yamada, 1975) for the spectral density $\pi\Delta\rho_d(\omega)$ for the symmetric Anderson model.

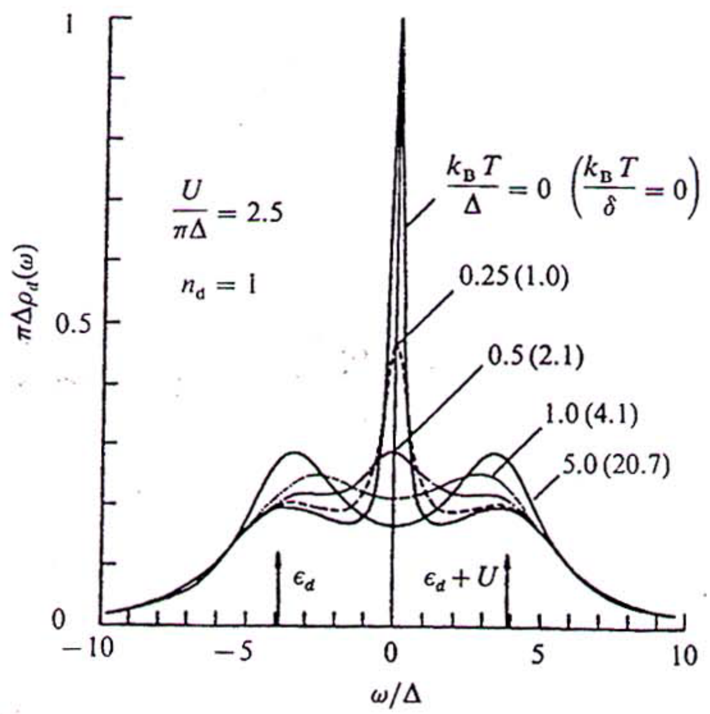


Figure 5.8 Results for the spectral density $\pi\Delta\rho_d(\omega)$ at various temperatures (Horvatić et al, 1987) for the symmetric Anderson model with $U/\pi\Delta = 2.5$ and $\langle n_d \rangle = 1$. The arrows indicate the energies of the excitations for $\Delta = 0$. The temperature is indicated in units of Δ and also in terms of the half width of the central resonance $\delta \sim T_K$.

Below $T_K = \sqrt{U\Gamma}\, e^{\frac{\pi \epsilon_0 (\epsilon_0 + U)}{2\Gamma U}}$

scattering amplitude in the

spin flip channel diverges (loghanitmically)

Symmetric Anderson model

$\epsilon_0 = -\frac{U}{2}$ singly occupied level

$\epsilon_0 + U = \frac{U}{2}$ doubly occupied level

resonance width Γ at $\mu = 0$

$T < T_K = \sqrt{U\Gamma}\, e^{-\frac{\pi U}{8\Gamma}}$

impurity is singly occupied in the average

$$\langle n_\uparrow + n_\downarrow \rangle = 1$$

magnetization on the impurity fluctuates wildly averaging to zero

$\langle n_\uparrow - n_\downarrow \rangle = 0$ Kondo singlet

COULOMB BLOCKADE at N:

Schrieffer - Wolff transformation to include

virtual hopping from the contacts:

H_0 conserves the number $N = 1$

V changes $N \rightarrow N \pm 1$

P projects onto $N = 1$ spans

$$P\left[(H_0 - E) - V^{+}(1-P)(H_0-E)^{-1}V\right]P|\psi\rangle = 0$$

$\Rightarrow$ effective Kondo hamiltonian:

$$H_{eff} = \sum_{k\sigma} \epsilon_k b^{+}_{k\sigma} b_{k\sigma} + J\,\vec{\sigma}(0)\cdot\vec{S}$$

delocalized electrons

spin 1/2

spans

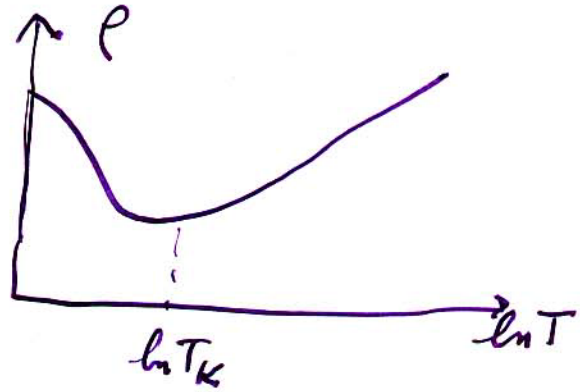
$$J = |V|^2\left[\frac{1}{\epsilon_d + U} - \frac{1}{\epsilon_d}\right] > 0$$

AF coupling $\Longrightarrow$ Kondo screening

"Paradox"

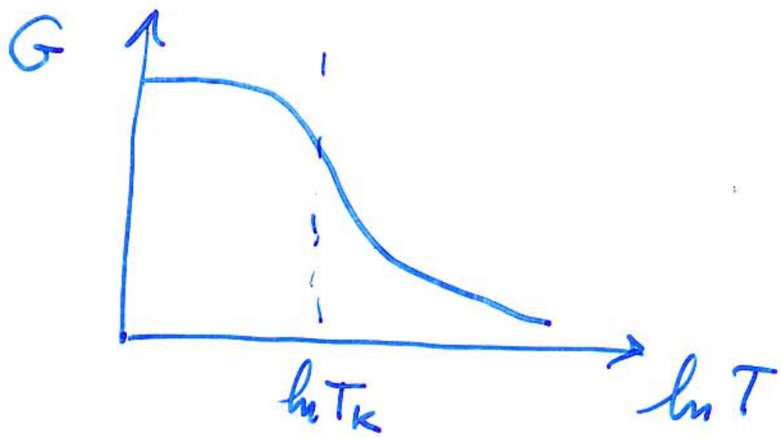
Kondo in diluted alloys:

minimum of the resistivity with temperature:

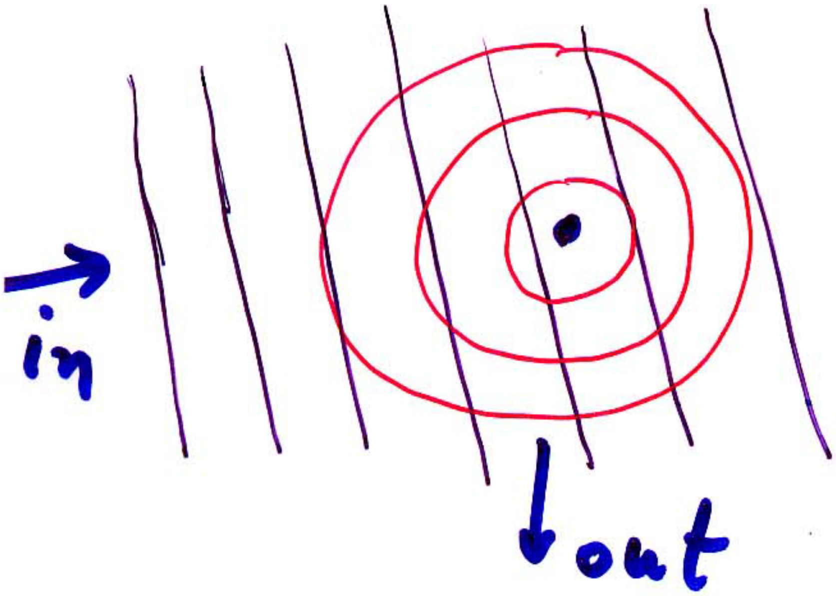


Kondo in quantum dots

Maximum of the conductance:



Scattering and transport



$$\psi = \psi_{in} + S\,\psi_{out}$$ definition of S

elastic S: $S = e^{2i\delta}$

$\delta(\epsilon)$: phase shift

Resonant scattering at $\mu = 0$: $\delta = \pi/2$

t-matrix : $-\frac{2\pi i}{k_0}\, t = S - 1$

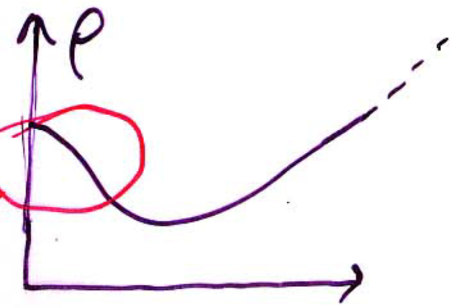
$$\Longrightarrow \quad t = -\frac{k_0}{\pi} \sin\delta \, e^{i\delta}$$

Resonance halfwidth : $\frac{\hbar}{\tau} = \operatorname{Im}\{t\} \propto T_K \sin^2\delta$

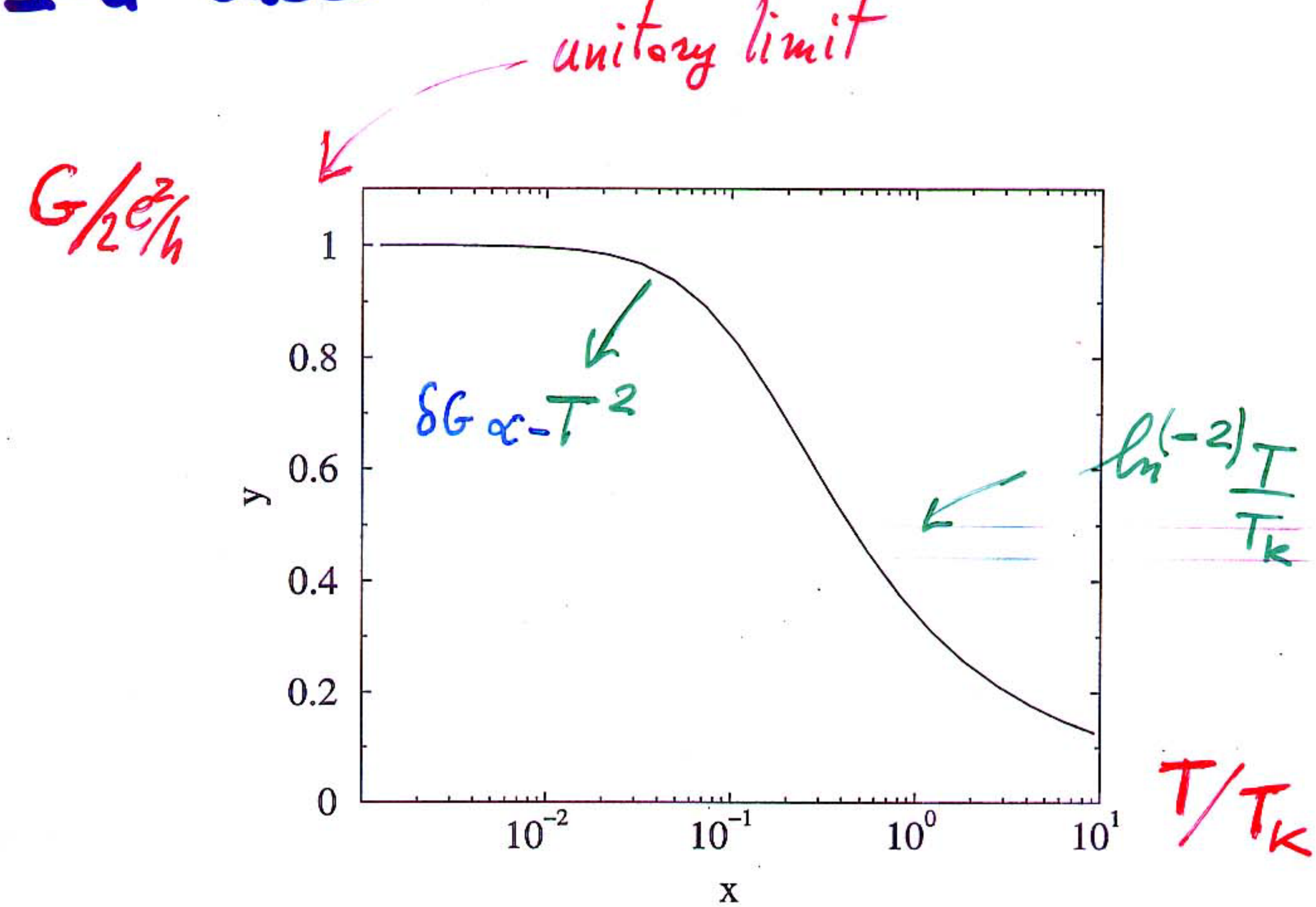
$\tau \rightarrow$ relaxation time

3d case : $\sigma = \frac{ne^2\tau}{m}$

$$\Longrightarrow \quad \rho \sim \frac{m}{ne^2\hbar} T_K \sin^2\delta$$



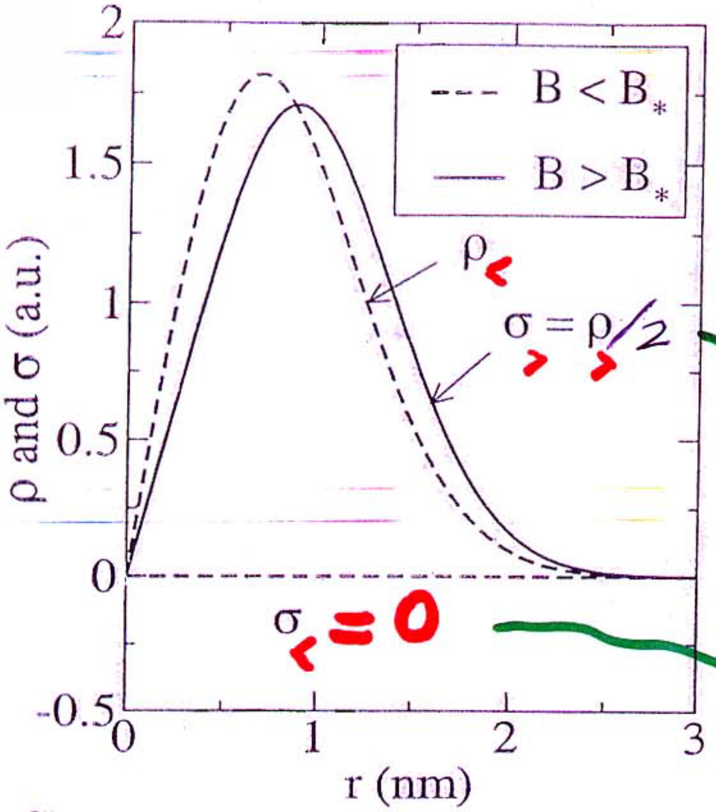
1-d case



$$G = \frac{2e^2}{h}\, T \longrightarrow \frac{2e^2}{h}\, \frac{\pi^2}{k_0^2} |t|^2$$

$$\simeq \frac{2e^2}{h} \frac{4\Gamma_L \Gamma_R}{(\Gamma_L + \Gamma_R)^2} \sin^2 \delta$$

N=2



triplet state

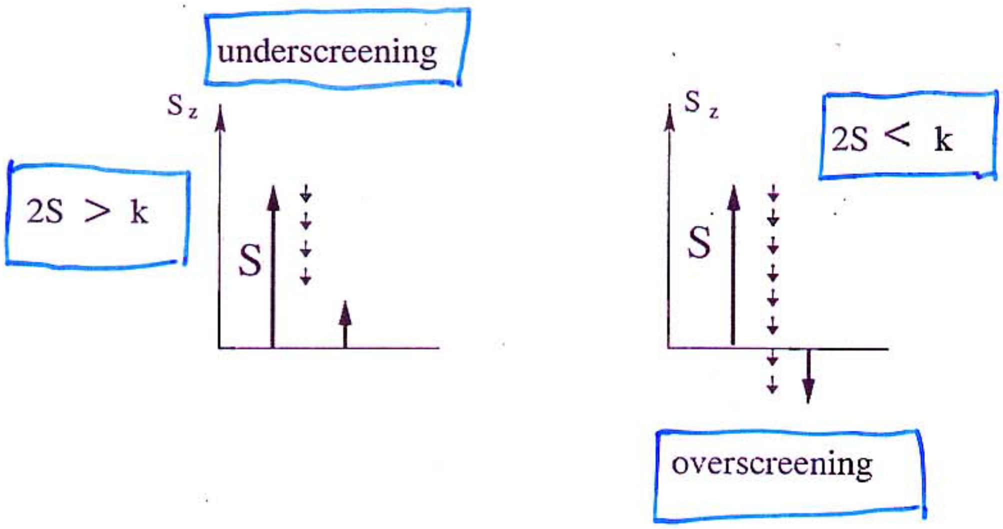
singlet state

FIG. 2. Charge density as a funtion of the radius r. $B < B_*$ ($B > B_*$) should be understood as B slightly smaller (higher) than B_*. The charge density ρ is only slighlty affected when B goes across B_*. The spin density σ is zero when $B < B_*$. It follows the profile of the charge density as $B > B_*$.

$\rho_< \sim \rho_>$ at B_*

Dynamical variable is S !

Underscreening or Overscreening of S depending on the number of channels k



Examples of underscreening *at even* N :

$N = 4, 6$
$S = 1,\ M = 0$
(Hund's rule)
and $k = 1$

$\Rightarrow$

Kondo impurity is $S = 1$ *:* *underscreening !*

$N = 2, 4$
crossing $(S, M) =$
$(0, 0) \rightarrow (1, 1)$
and $k = 1$

$\Rightarrow$

equiv. $S = 3/2$ *:* *underscreening !*

T - dependence of the conductance

$$\delta\tilde{G} = \frac{2e^2}{h}\int d\omega \frac{\partial f(\omega)}{\partial \omega} \Im m Tr\left\{\tilde{\Gamma}\delta \mathbf{G}^r(\omega)\right\} \qquad (1)$$

with

$$Tr\left\{\delta \mathbf{G}^r\right\} \approx Tr\left\{G_0 \Sigma G_0\right\}$$

Low energy correction behavior of the self energy is :

- 1CK .... $\delta\tilde{G} \propto: T^2 \Rightarrow$ $\boxed{\Im m\{\Sigma\} \propto \omega^2}$
- 2CK .... $\delta\tilde{G} \propto: \sqrt{T} \Rightarrow$ $\boxed{\Im m\{\Sigma\} \propto \sqrt{\omega}}$

the dot spin can be **screened** by lead electrons:

$$\psi_{\epsilon\sim\epsilon_F,q,m,\sigma} \equiv e^{i(k_F+q)z}\varphi_{\sim 0,m}(\rho)e^{im\vartheta}\chi_\sigma$$

$$\Rightarrow \quad \boxed{b^\dagger_{q,m,\sigma}\ ,\ b_{q,m,\sigma}}$$

Unitary limit of the conductance

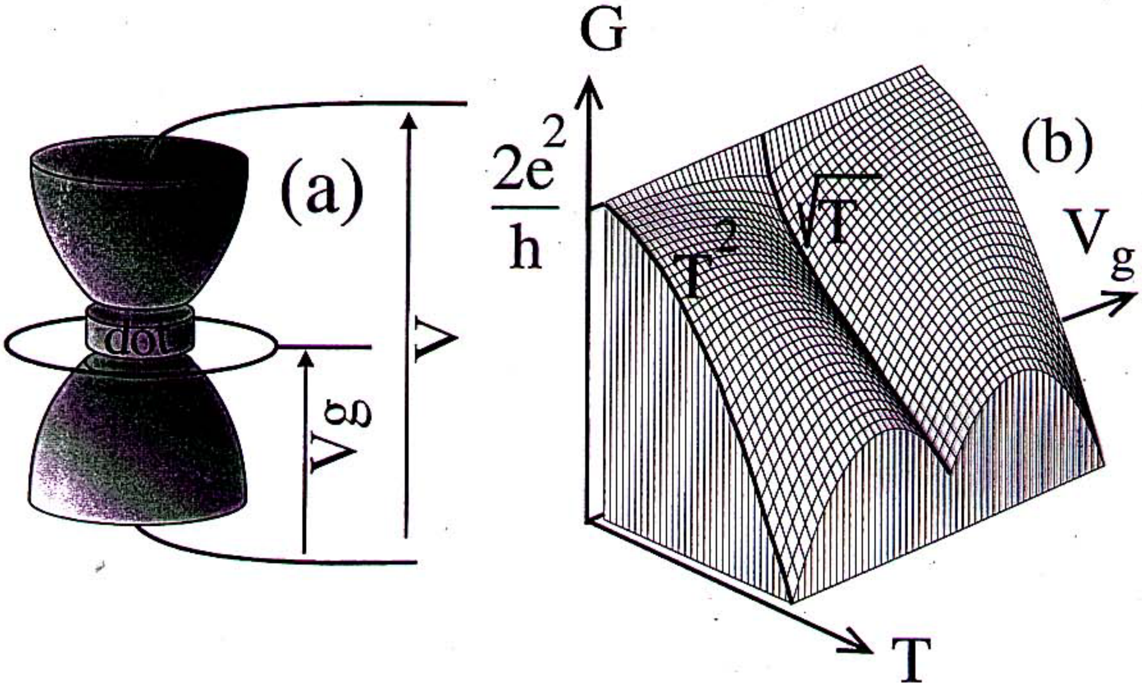


Figure 1: T- dependence of G : changes with gate voltage V_g when moving from **1CK** to **2CK**