Quantum transport: tunnel current in a Quantum Dot

D. Giuliano*, B. Jouault†, A. T.*, G.Santoro (SISSA), P.Onorato*

*** INFM and**
Universitá di Napoli "Federico II"
† CNRS -Université Montpellier II

Tunneling spectroscopy in a Quantum Dot

- **Shell structure and exchange effects**

Quantum Dot as a single Kondo impurity

- **At Coulomb blockade the dot has spin "S"**

PR B61, 10242 (2000)
PRL 84, 4677 (2000)
PR B63, 125318 (2001)

Local : $\quad J = \sigma E \qquad \left[\frac{e}{L^{d-1} t}\right] = [\sigma]\left[\frac{V}{L}\right]$

Global : $\quad I = G V$

$$G(L) = \sigma(L) L^{d-2}$$

Diffusion : $\quad J = \underbrace{\sigma E}_{\text{drift current}} = \underbrace{e D \nabla n}_{\text{diffusion current}}$

non equilibrium is stationary

if

gives : $\quad \sigma = e^2 D \frac{dn}{dE}$ $\qquad$ Einstein relation

Brownian motion : $\quad \langle r^2(t) \rangle = 6 D t$

thermal diffusion : $\quad L_T^2 = D \left(\frac{\hbar}{k_B T}\right)$

quantum diffusion (phase coherence)

if $L < L_T$: along a path $\sim v_F \frac{L^2}{D}$

$T < E_c = \frac{\hbar D}{L^2}$ (Thouless energy)

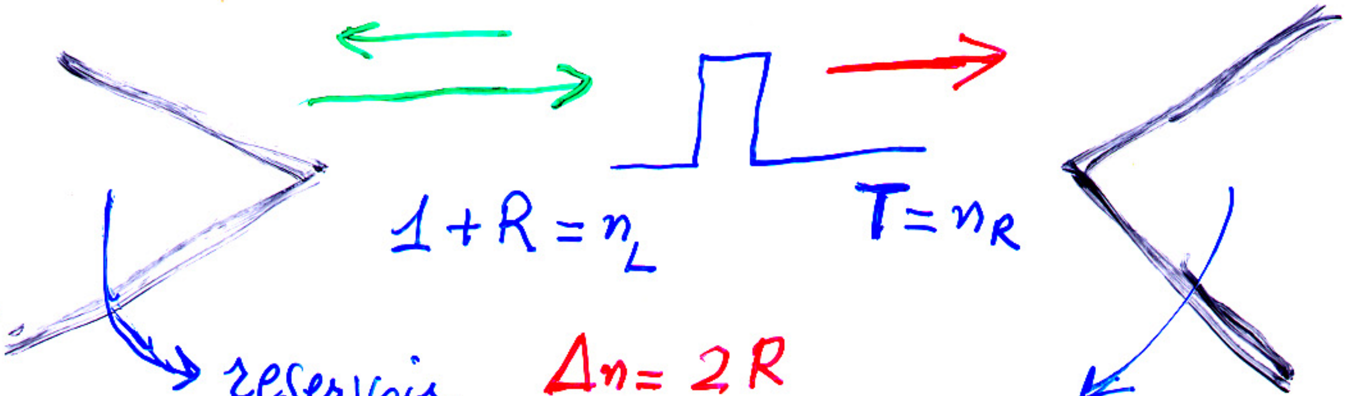
Quantum diffusion

→ Ohm's law breaks down

$$G(L) = \sigma(L)\, L^{d-2}$$

$\sigma(L)$ → no longer L independent

Easy 1-d approach : Landauer formula



$$D = \frac{\text{current density}}{\text{density gradient}} = \frac{T v_F}{2R/L}$$

$$G = \frac{\sigma}{L} = \frac{e^2}{L} D \frac{dn}{dE} = \frac{e^2 T v_F}{2R} \frac{1}{\pi \hbar v_F} = \frac{e^2}{h} \frac{T}{R}$$

v_F is canceled : all depends on coherence in the transport

PLAN OF THE LECTURES

Lecture I : Perturbative Linear conductance in vertical Quantum Dots

- Isolated dot as an artificial atom: addition energy and shell structure
- Transition to higher spin states with magnetic field: maximum density droplet
- Perturbative linear conductance: Coulomb blockade and spin blockade
- Weak coupling with the contacts: temperature corrections
- Coulomb charging: the case of a sigle level Anderson impurity

Lecture II: Non perturbative linear conductance: Kondo effect in Quantum dots

- Mechanism and experimental findings
- Freezing the charge degree of freedom in tunneling: Schrieffer-Wolff transformation and Kondo hamiltonian
- Poor man's scaling and Kondo screening
- Kondo conductance
- Orbital Kondo and more exotic Kondo states
- Kondo with superconducting contacts: Josephson current

Quantum dot :

artificial atom

N : electron number $(1 \div 100)$

size : tunable

$$\mathcal{E} = \mathcal{E}_{cin} + U_{e\text{-}e}$$

$$\mathcal{E}_{cin} \to \sim \frac{\hbar^2}{2mL^2} \qquad U_{e\text{-}e} \to \sim \frac{e^2}{L}$$

$L \sim 50 \div 150$ Å $\Longrightarrow$ U dominates

Level spacing $\Delta \sim \frac{1}{L} \to$ meV

$$\Delta \gg k_B T$$

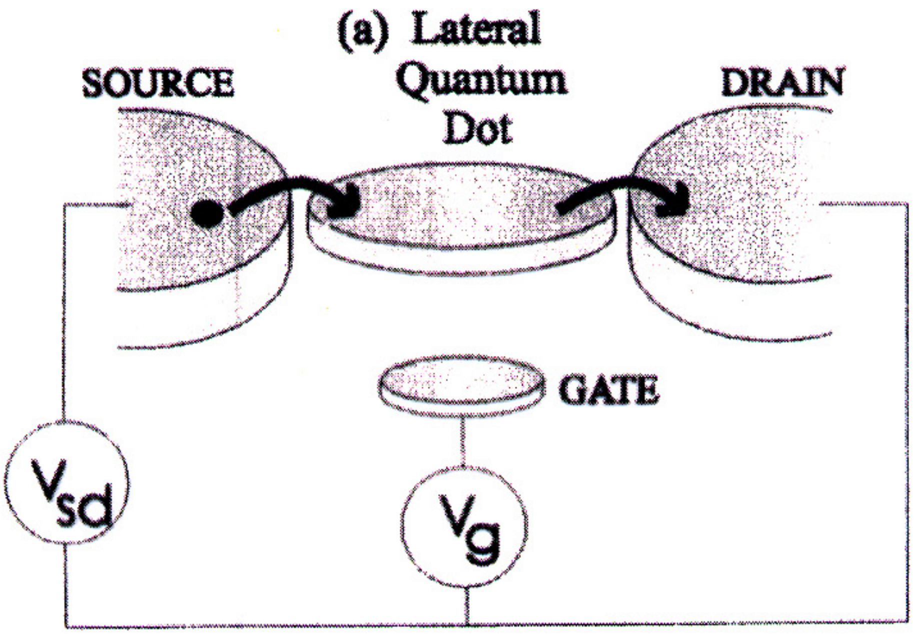
$\Longrightarrow$ Coulomb blockade &

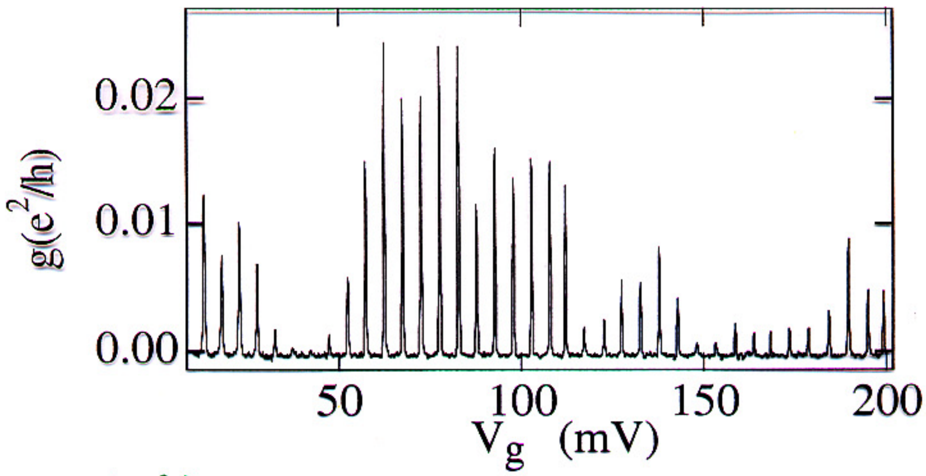
Coulomb oscillations

N particle energies $E_n^{(N)}$

Gate voltage tuning :

$$E_{V_g}^{(N)} = E_0^{(N)} - N e V_g$$



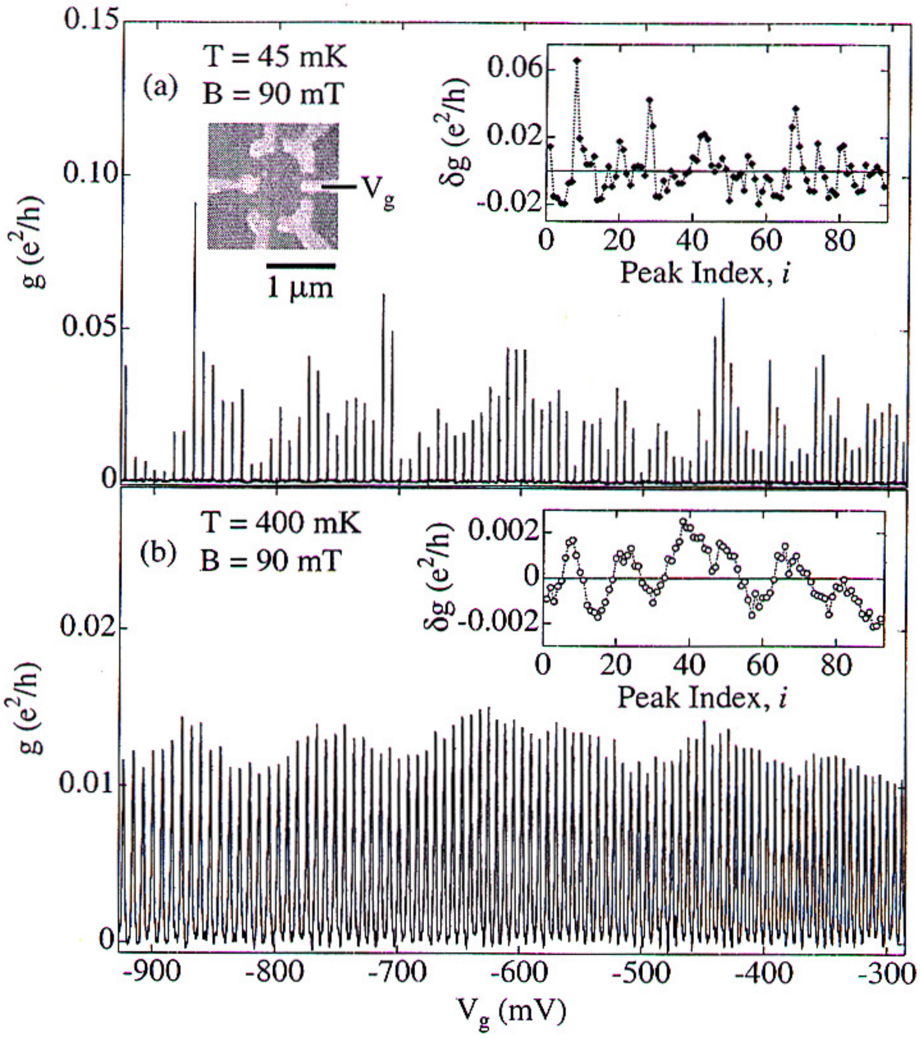


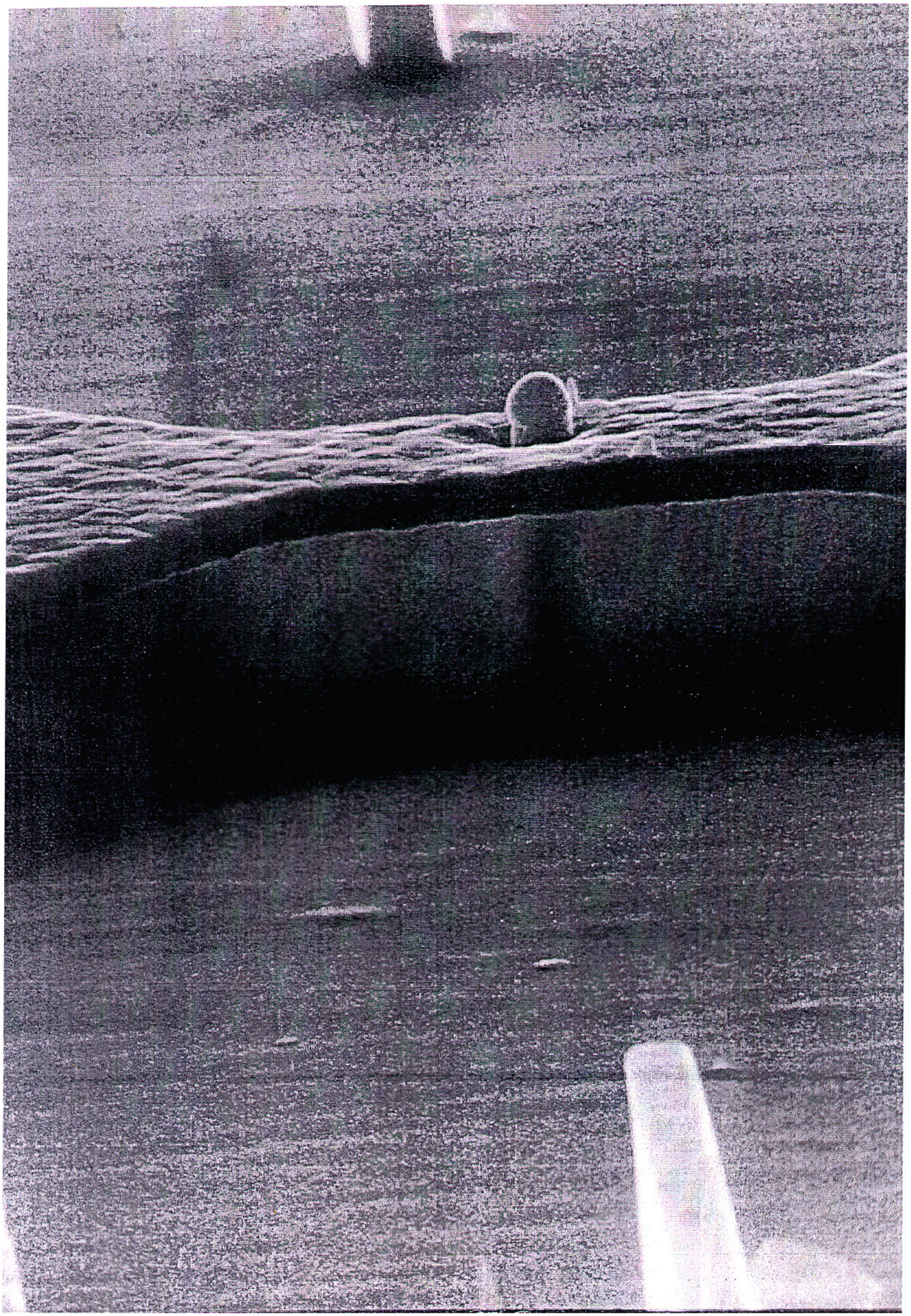
peak at

$$E_{V_g}^{(N+1)} \sim E_{V_g}^{(N)}$$

$$\Longrightarrow \quad \underbrace{E_0^{(N+1)} - E_0^{(N)}}_{\mu_N} \simeq e V_g$$

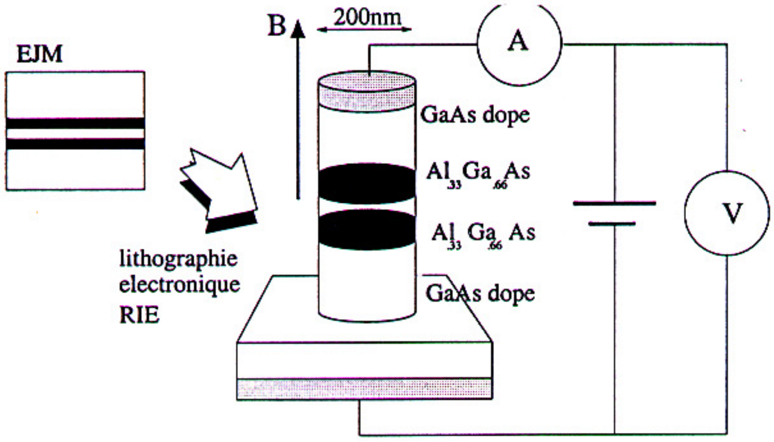
Coulomb Oscillations : dot a molti elettroni

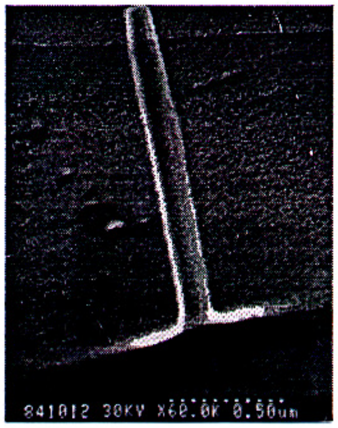




Fabrication of vertical QDs

pillars obtained by M.B.E.

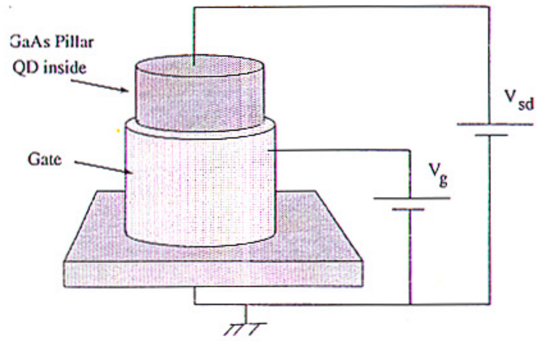


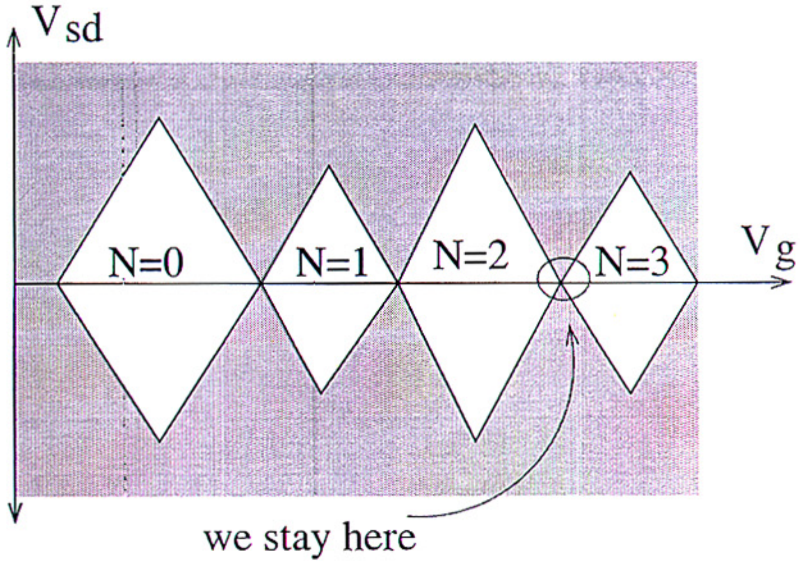


First models:

- Poisson and Thomas-Fermi to determine the conduction band.
- Near the surface the conduction band edge is at 700meV.
- the QD is a disk.
- the lateral confinement potential is of parabolic shape.

QD with a lateral gate





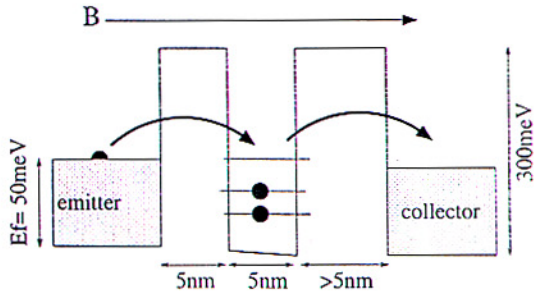
Experimental conditions:

Linear transport

Magnetic field

Low temperature (100mK)

2 or 3 electrons in the dot.



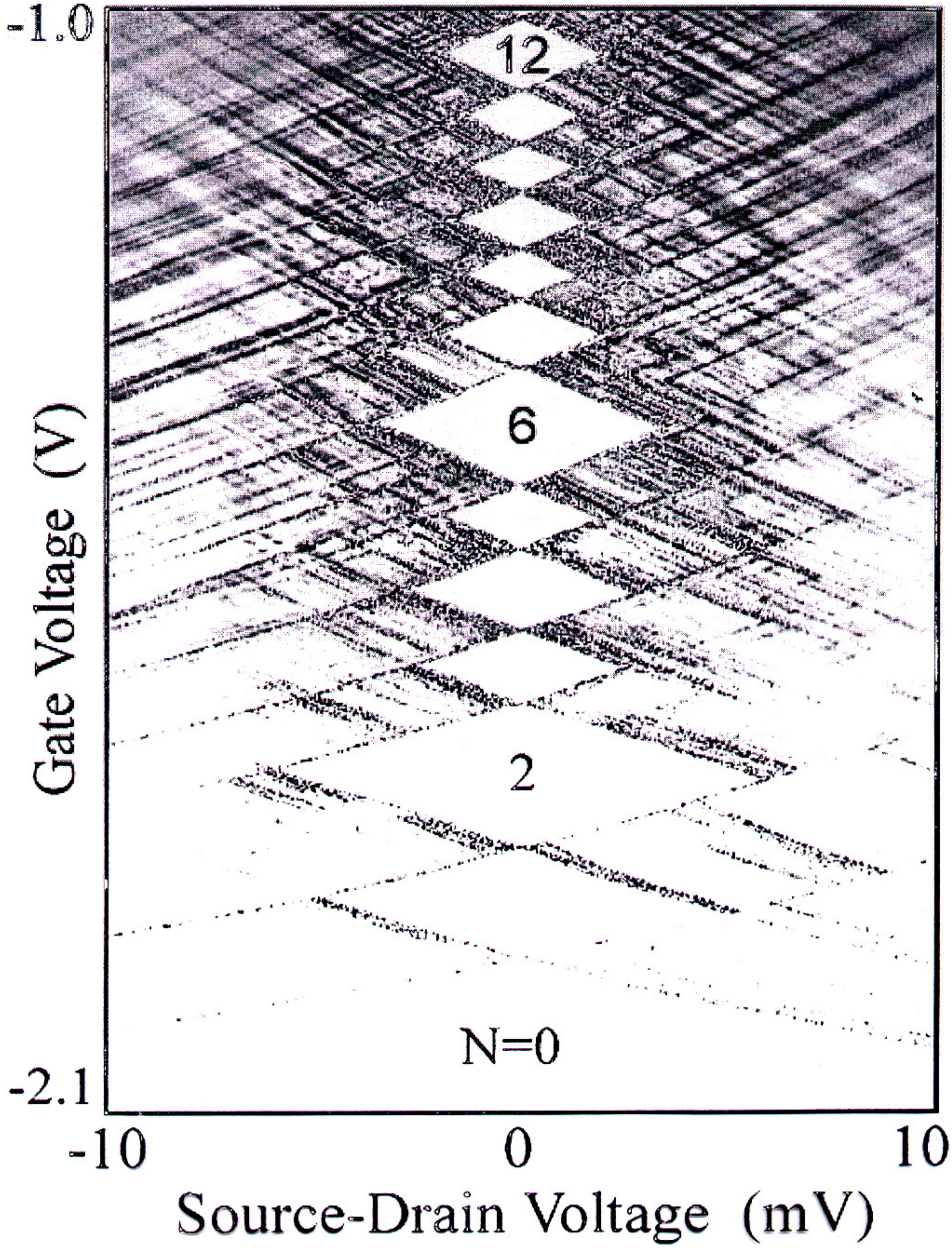


Figure 2

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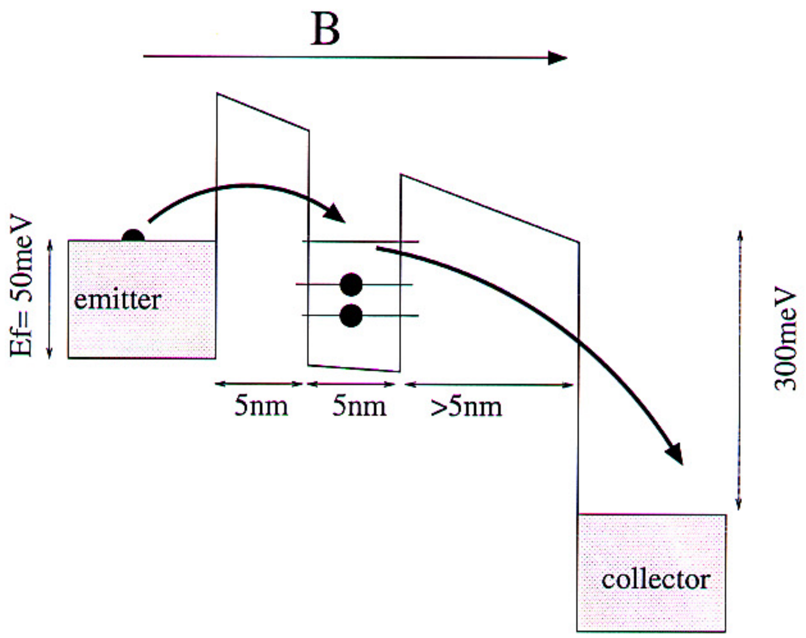
Alternative: non-linear transport

Without fabricating a lateral gate, non-linear tranport experiment can be carried out.



conductance peaks can be seen both for the ground state $N = 1 - 6$ and first excited states of the $N = 1, 2$ energy spectrum.

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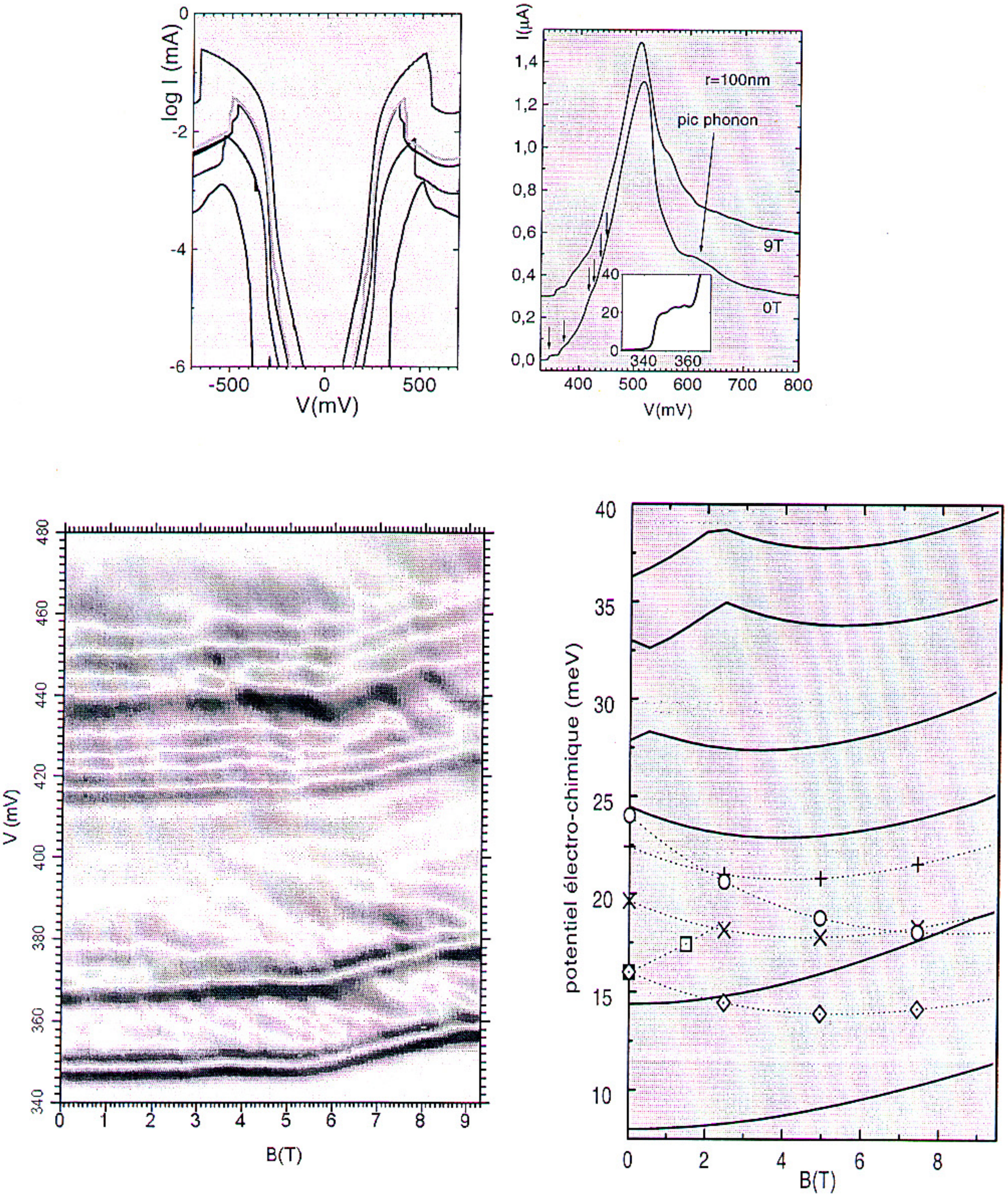
Magnetotransport experiments



B.Jouault *et al, submitted for publication*

1 Vertical Dot with few electrons: exact diagonalization

- Shell structure fixed by the confining potential

  most of the times Hartree-Fock is satisfactory except for:

- Exchange and Correlation effects at special tunings:

i) Hund's rule at even fillings

ii) Transition to higher spin states , in magnetic field B

everything can be monitored in the linear conductance

Modelisation of the QD

2D parabolic quantum dot

The Hamiltonian is given by:

$$\mathbf{H} = \sum_{i,\sigma} E_i C^{\dagger}_{i,\sigma} C_{i,\sigma} + \sum_{i,j,k,l,\sigma,\sigma'} V_{i,j,k,l} C^{\dagger}_{i,\sigma} C^{\dagger}_{j,\sigma'} C_{k,\sigma'} C_{l,\sigma} \tag{1}$$

where E_i are the Darwin-Fock single particle energy levels:

$$E_{n,m} = \hbar\omega(B)\,(2n + |m| + 1) - \frac{1}{2} m\hbar\omega_c \tag{2}$$

and V is the e-e Coulomb interaction.

Lanczos procedure to calculate the QD energy levels

Parameters : $\hbar\omega_d$= 4meV, U= 3meV (see Kouwen *et al*)

Isolated Q. DOT

parabolic confinement in 2-d : $\hbar\omega_{conf}$

Coulomb interaction $E_c \lesssim \hbar\omega_{conf}$

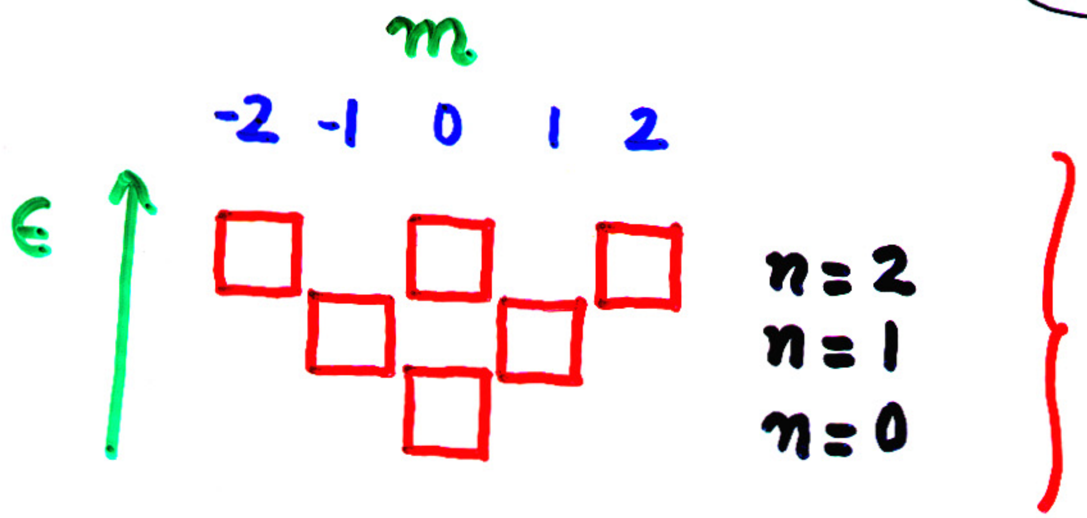
$E_c \, l_{conf} / |\underline{r} - \underline{r}'|$

Single particle levels:

$$\epsilon_{nm} = \hbar\omega_0 (n+1) - \mu_B g^* B m$$

$$\sqrt{\omega_{conf}^2 + \omega_c^2/4}$$

cyclotron frequency



2-d harmonic oscillator

States labeled by:

N	M	S	S^z
# electrons	angular momentum $\parallel z$	total spin	z component of S
	$\sum_i m_i$		$\sum_i s_i^z$

Increase B:

1

Fermi sea

Lowest M, S

S=1/2 M=1

2

Side moving to increase M

S=1/2

3

Increasing S (and M)

S=3/2

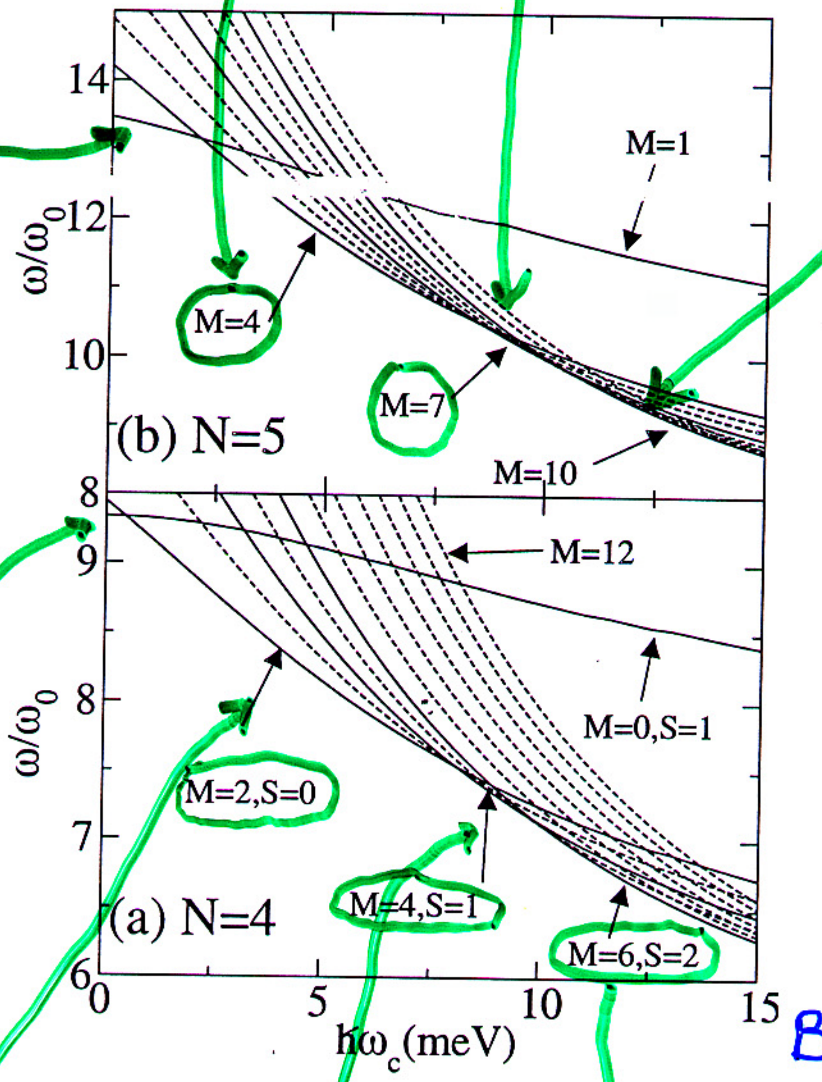
Highest S

4

S=5/2

M=10

N=5



N=4

1

Fermi Sea (Hund's rule)

M=0 S=1

FIG. 9: Energy of the $N = 4$ (a) and $N = 5$ (b) levels vs $\hbar\omega_c$ measured from the ground state energy of the $N - 1$ particle system, in units of $\hbar\omega_0$. The total angular momentum M and spin S of some of the corresponding states is reported. Dot parameters as in fig.3.

2

Side moving to increase M

M=2

S=0

3

Increasing S

M=4

S=1

Highest S

4

M=6

S=2

5

Tunneling Conductance

retarded Green's function

$$I(V) = -\frac{2e}{h}\int d\omega\, F_V(\omega)\, \Gamma(\omega)\, \mathrm{Im}\{G^r(\omega)\}$$

Fermi functions

tunneling matrix element

$$= f(\omega - \tfrac{eV}{2}) - f(\omega + \tfrac{eV}{2})$$

Weak coupling:

$$G^r(\omega) = G_0^r(\omega) + G_0^r\, \Gamma^{1/2} G^r_{dot}\, \Gamma^{1/2} G_0^r$$

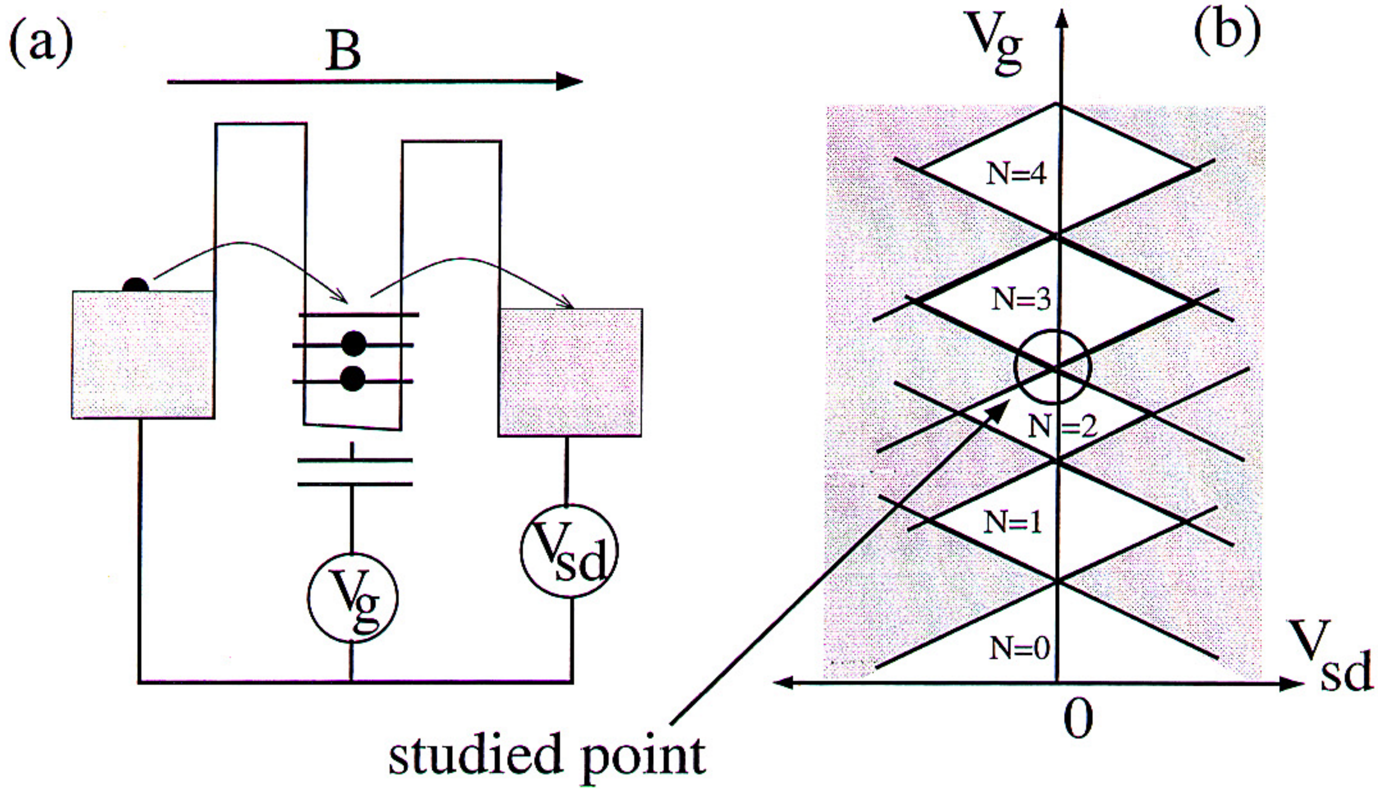
unperturbed Green's!

$$\delta I(V) = -\frac{2e}{h}\,\tilde{\Gamma}\int d\omega\, F_V(\omega)\, \mathrm{Im}\{G_0^r\, G^r_{dot}\, G_0^r\}$$

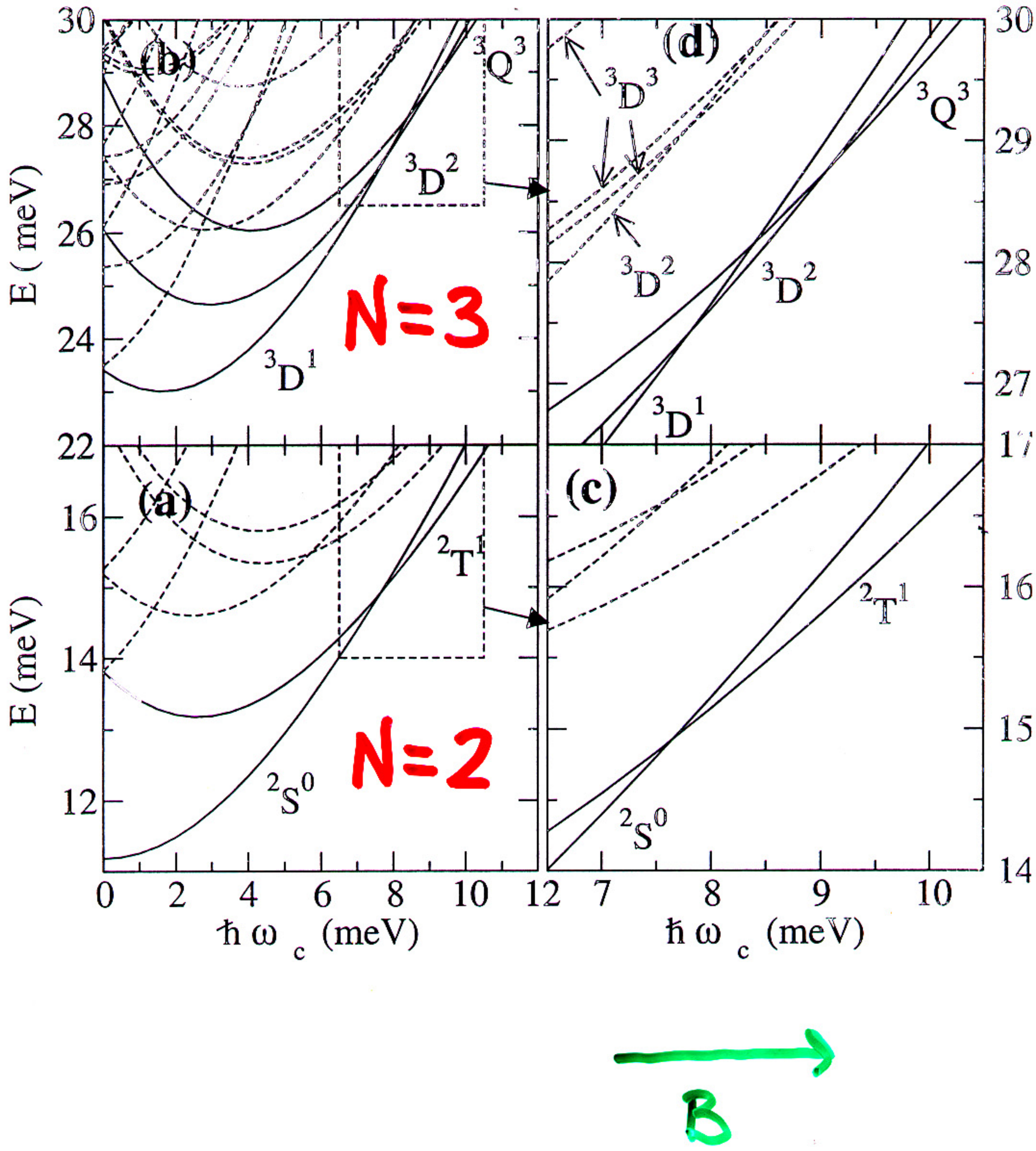
$$= -\frac{2e^2}{h}\,\tilde{\Gamma}\int d\omega\, \mathrm{Im}\{G^r_{dot}(\omega)\}\, \mathcal{F}(V,\omega)$$

peak centered at $\omega = eV$

$$-\frac{1}{\pi}\mathrm{Im}\{G^r_{dot}(\omega)\} \longrightarrow \text{dot density of states}$$



(a)
B
Vg
Vsd
studied point
(b)
Vg
N=4
N=3
N=2
N=1
N=0
Vsd
0


(b)
(d)
(a)
(c)
E (meV)
E (meV)
ħ ω c (meV)
ħ ω c (meV)
3Q3
3D3
3D2
3D1
2T1
2S0
N=3
N=2
B

Modelling the conductance

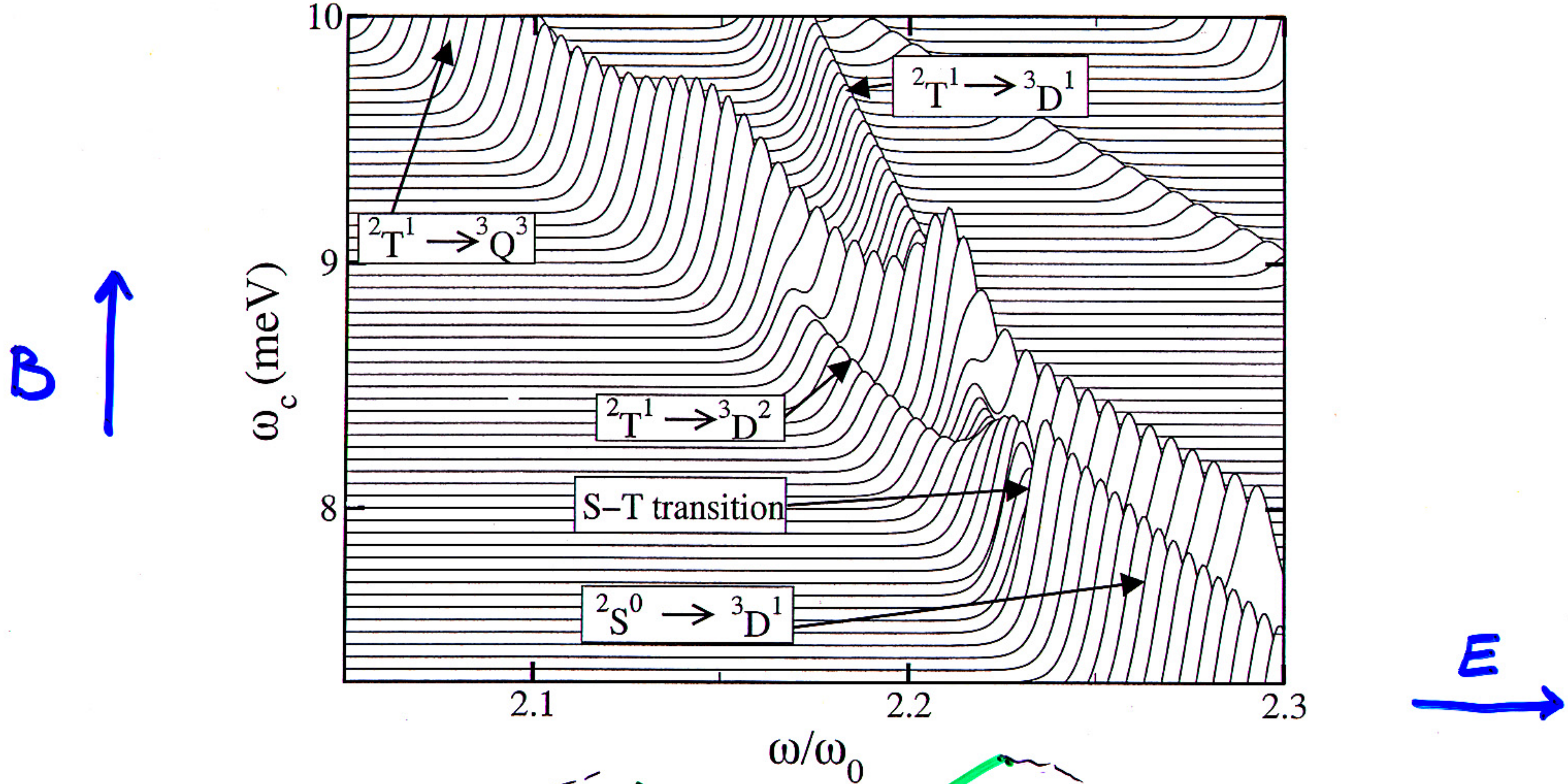
Transition rates:

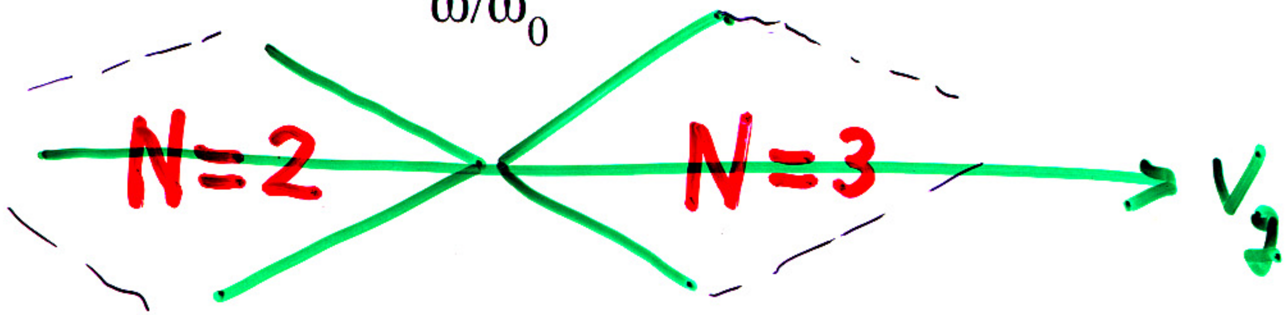
$$t_{N,0}^{N+1,\alpha}(\epsilon) = \sum_{nm} \Gamma_{nm}(\epsilon) \left| \langle N+1, \alpha \left| c_{nm\sigma}^{\dagger} \right| N, 0 \rangle \right|^2 \delta\left(\epsilon - (E_{\alpha}^{N+1} - E_0^N)\right) \tag{3}$$

where $\Gamma_{n,m}$ is the particle overlaps for the dot-contact coupling. The composant of the angular momentum in the z direction is conserved in the tunneling process due to the cylindrical symmetry. The condictance is obtained assuming that the left and right couplings to the contacts are proportional ($\Gamma_L = \lambda \Gamma_R$):

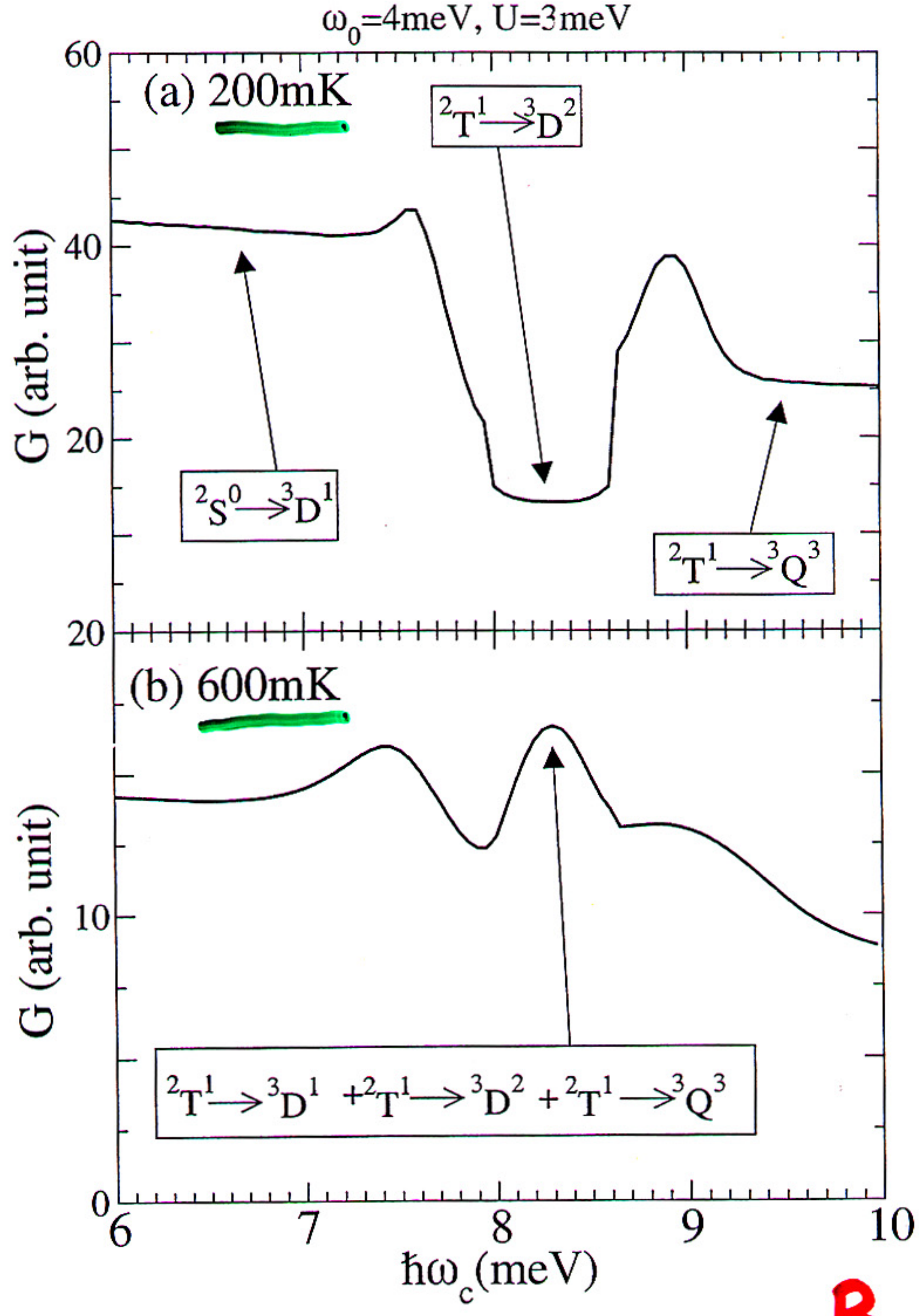
$$g(eV_g) = \left.\frac{dI}{dV}\right|_{N,0}(eV_g) = \frac{e^2}{\hbar} \frac{\lambda}{1+\lambda} \sum_{\alpha} \int_0^{\infty} d\epsilon \; t_{N,0}^{N+1,\alpha}(\epsilon) \frac{\partial f}{\partial \epsilon} \tag{4}$$

transition rates





Conductance



B

Spin [redacted] selection rule

- From the N=2 triplet to the N=3 Quartet $^2T \rightarrow {}^3Q$

$$\begin{array}{lcccc}
N=2 & & S_z=1 & S_z=0 & S_z=-1 \\
+e^- & & \uparrow\downarrow & \uparrow\downarrow & \uparrow\downarrow \\
\Downarrow & & \swarrow\ \searrow & \swarrow\ \searrow & \swarrow\ \searrow \\
N=3 & S_z=\frac{3}{2} & S_z=\frac{1}{2} & S_z=-\frac{1}{2} & S_z=-\frac{3}{2}
\end{array}$$

This gives 6 possibilities

- From the N=2 triplet to the N=3 doublet $^2T \rightarrow {}^3D$

$$\begin{array}{lccc}
N=2 & S_z=1 & S_z=0 & S_z=-1 \\
+e^- & \uparrow\downarrow & \uparrow\downarrow & \uparrow\downarrow \\
\Downarrow & \cancel{\swarrow}\ \searrow & \swarrow\ \searrow & \swarrow\ \cancel{\searrow} \\
N=3 & & S_z=\frac{1}{2} \quad S_z=-\frac{1}{2} &
\end{array}$$

missing

This gives 4 possibilities.

Possible drop in the conductance.

37 35

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