Magnetic field influence and spectrum rearrangement for spatially-indirect excitons in coupled quantum wells and coupled quantum dots

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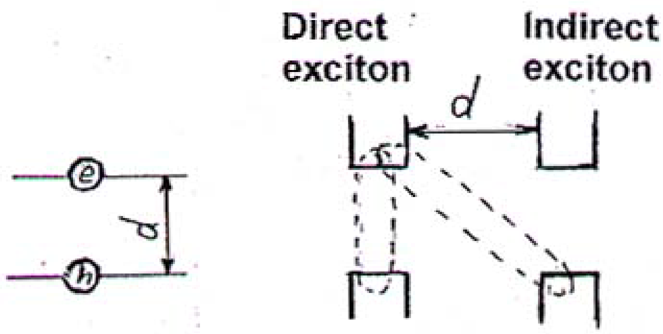
- **Energy spectra of spatially-separated excitons in coupled quantum wells in magnetic field**
  - **Weak effective magnetic field**
  - **Intermediate and strong effective magnetic field**
- **Degeneration and quasidegeneration of base energy levels**
- **Ionization of 2D spatially-separated exciton moving in transverse magnetic field**
- **Spectrum of spatially-separated electron and charged impurity in magnetic field**
- **Indirect exciton in coupled quantum dots**
- **Magnetic field influence on indirect exciton in CQDs**

SPATIALLY–SEPARATED 2D EXCITON IN CQW IN TRANSVERSE MAGNETIC FIELD



D'<< $a_0 (1 + (d/a_0)^{3/4})$ for weak magnetic field

D'<< $(a_0 r_H)^{1/2} (1+(d/a_0)^{3/4}(da_0/r_H^2)^{3/4})$ for strong magnetic field

Schrodinger equation is invariant relative to simultaneous translation e and h and gauge transformation.

C.M. $R = m_e/(m_e + m_h)r_e + m_h/(m_e + m_h)r_h)$ relative motion of e and h $r = r_e - r_h$, $M = m_e + m_h$ $\gamma = (m_h - m_e)/M$

Relative motion of e and h:

$$\left[\Delta\rho - i\gamma\omega_L\frac{\partial}{\partial\Theta} - \frac{\omega_L^2}{4}\rho^2 + \frac{1}{((\rho+\rho_0)^2 + d^2)^{1/2}} + E\right] \times$$

$$\times \Psi = 0 ,$$

$\omega_L = \omega_C/2$ is the Larmore frequency, $\rho_0 = \frac{c}{eH^2}[\mathrm{H}, \mathrm{P}]$.

$$\hat{P} = -i\hbar\nabla_e - i\hbar\nabla_h + \frac{e}{c}(\mathrm{A}_e - \mathrm{A}_h) - \frac{e}{c}[\mathrm{H}, \mathrm{r}_e - \mathrm{r}_h].$$

Operator $\hat{P}$ commutes with the Hamiltonian

We use arbitrary units:

$$E_0 = \frac{2m^* e^4}{\varepsilon^2 \hbar^2}$$ – binding energy of 2D exciton

$$r_0 = \frac{\hbar^2 \varepsilon}{2m^* e^2}$$ – radius of 2D exciton (without M.F.)

$$\omega_{c_0} = \frac{2m^* e^4}{\varepsilon^2 \hbar^3}; \quad H_0 = \frac{2(m^*)^2 e^3}{\varepsilon^2 \hbar^3}$$

$$r_H = \sqrt{\frac{1}{\omega_L}}; \quad \omega_L = \frac{\omega_c}{2}$$

Following values of parameters

$d = 1,8$; $f = 0,36$

$H = 0,26; 0,52; 0,78; 1,04; 1,3$ (in units adopted)

corresponds to experimental structures GaAs / AlGaAs with mean distance between e and h layers 120 Å

$m_e^* = 0,048\ m_e$; $m_h^* = 0,172\ m_e$

$H = 2; 4; 6; 8; 10$ Tesla

L.V. Butov et al; PRL '2001 (submitted)

Weak effective magnetic field

$\omega_L(d^2 + \rho_0^2 + 1) \ll 1$

The numerical diagonalization on 2D hydrogen-like function basis is used

$$\Phi_{mn}(\overline{r}) = C_{nm} e^{-\sqrt{|E_{0n}|}r} \cdot \left(\sqrt{|E_n|}\right)^{|m|+1/2} r^{|m|} \sum_{s=0}^{n-|m|} A_s \sqrt{|E_{0n}|}^{s} r^{s}, \quad (1)$$

where where $A_0 = 1, \quad A_s = A_{s-1} \frac{2(S+|m|-n-1)}{S(S+2|m|)} \quad S > 0; \quad C_{nm}$ **is constant coefficient .**

$$E_{0n} = -\frac{1}{4(n+1/2)^2} \quad (2)$$

where $n = S + |m| = 0, 1, 2...$

Strong effective magnetic field

$\omega_L(d^2 + \rho_0^2 + 1) >> 1$

Intermediate effective magnetic field

$\omega_L(d^2 + \rho_0^2 + 1) \sim 1$

The numerical diagonalization on magnetic function basis is used

$$f_{nm}(\rho) = e^{im\Theta} L_n^{|m|}\left(\frac{\omega_L}{2}\rho^2\right) e^{-\frac{\omega_L}{2}\frac{\rho^2}{2}} \rho^{|m|} \left(\frac{1}{\pi}\frac{n!}{(n+|m|)!}\left(\frac{\omega_L}{2}\right)^{|m|+1}\right)^{1/2} \quad (3)$$

$$E_{0nm} = 2\omega_L\left(n + \frac{|m| - \gamma m + 1}{2}\right) \quad (4)$$

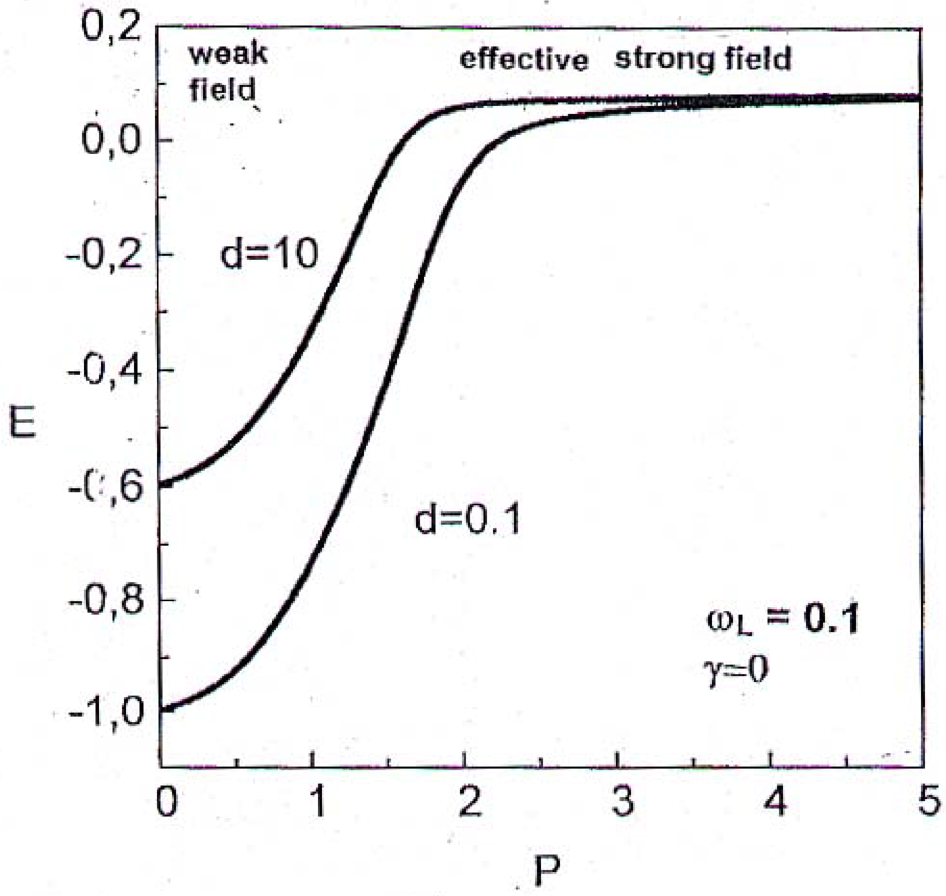
where $n = 0, 1, 2... \quad m = 0, \pm 1, \pm 2...$ **(for** $\gamma = 1$ **eigenenergy system coincides with Landau spectrum).**

For any *fixed* H effective magnetic field becomes strong with growth of distance d **and/or P.**

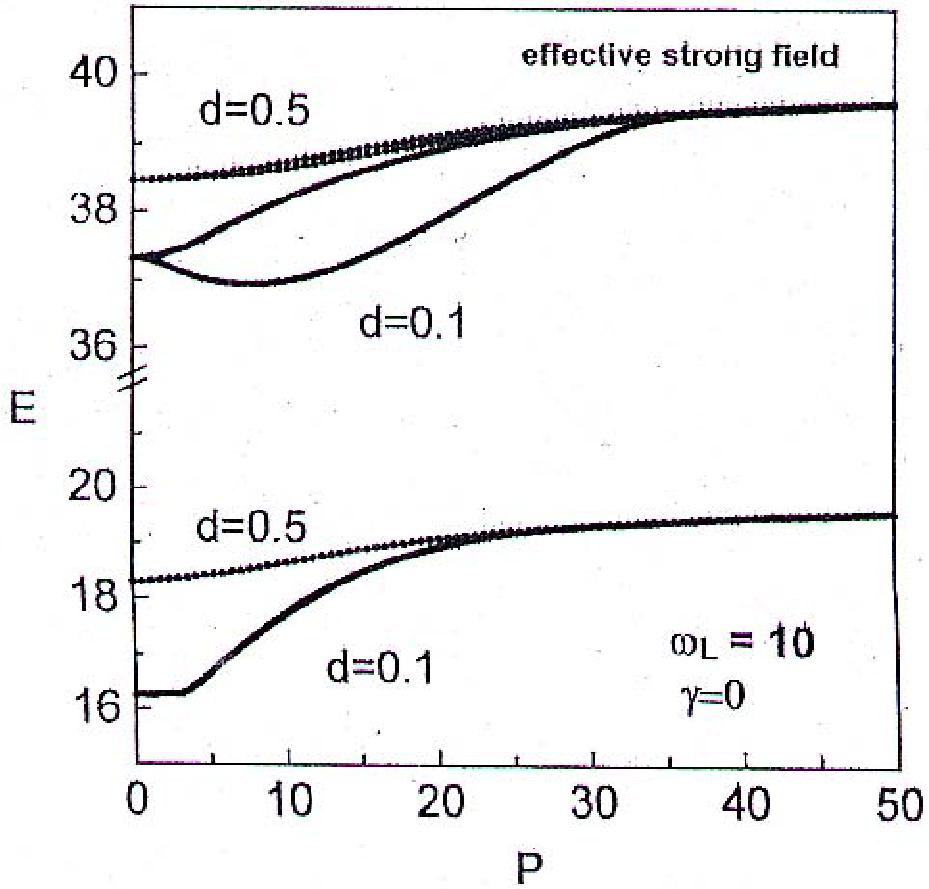
Fig.4. The dependencies of lowest energy level of indirect magnetoexciton *vs* magnetic momentum P (dispersion laws)

a) at $\omega_L = 0.1$ (weak magnetic field) Spectrum transforms from hydrogen-like to the magnetic one

b) at $\omega_L = 10$ (strong magnetic field)

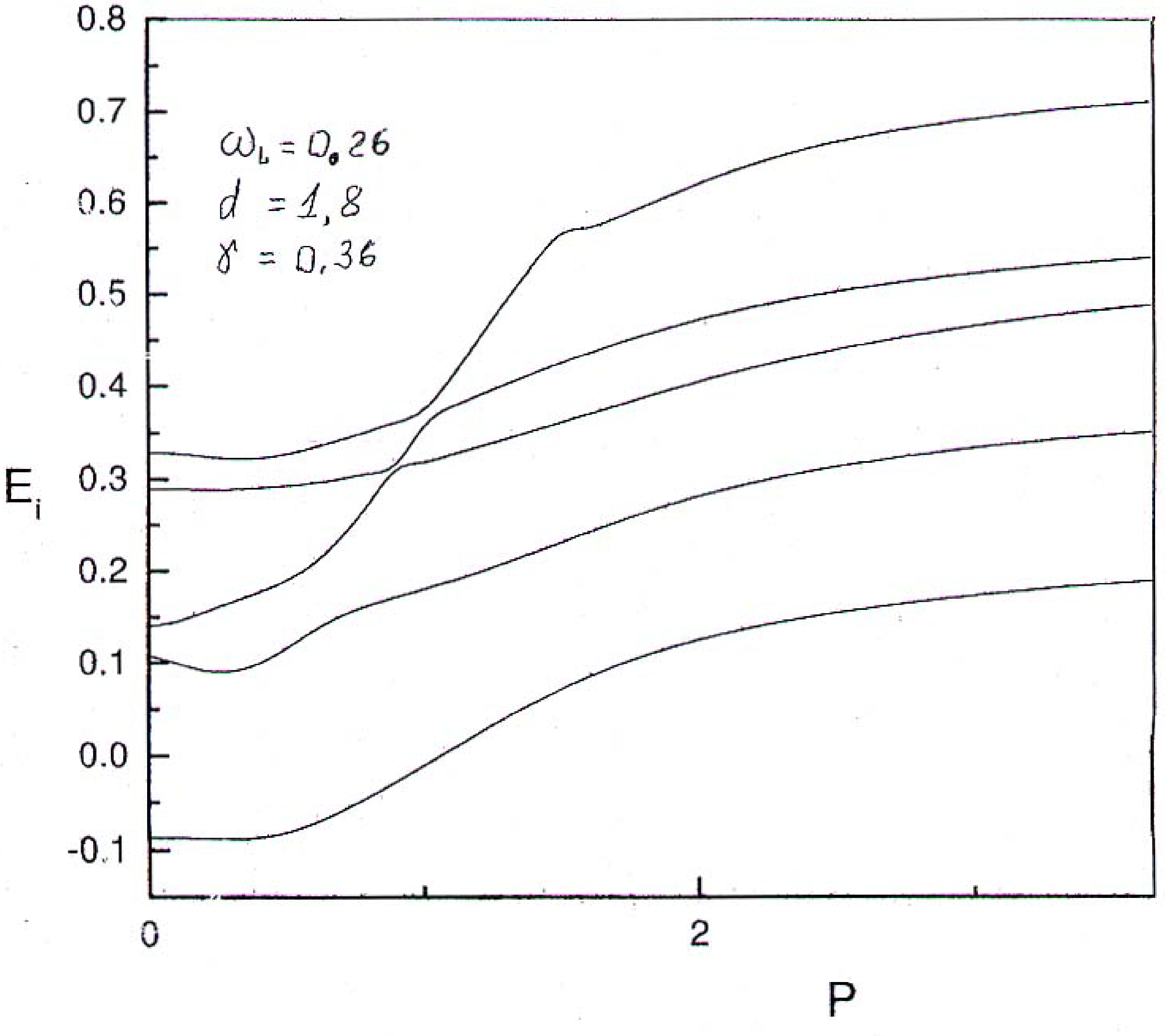


a)



b)

The dependencies of low-lying energy levels of indirect magnetoexciton *vs.* magnetic momentum P (dispersion laws) for magnetic field Larmore frequency ω_L = 0.26 (weak magnetic field) at interlayer distance d=1.8, γ=0.36.



intermediate magnetic field

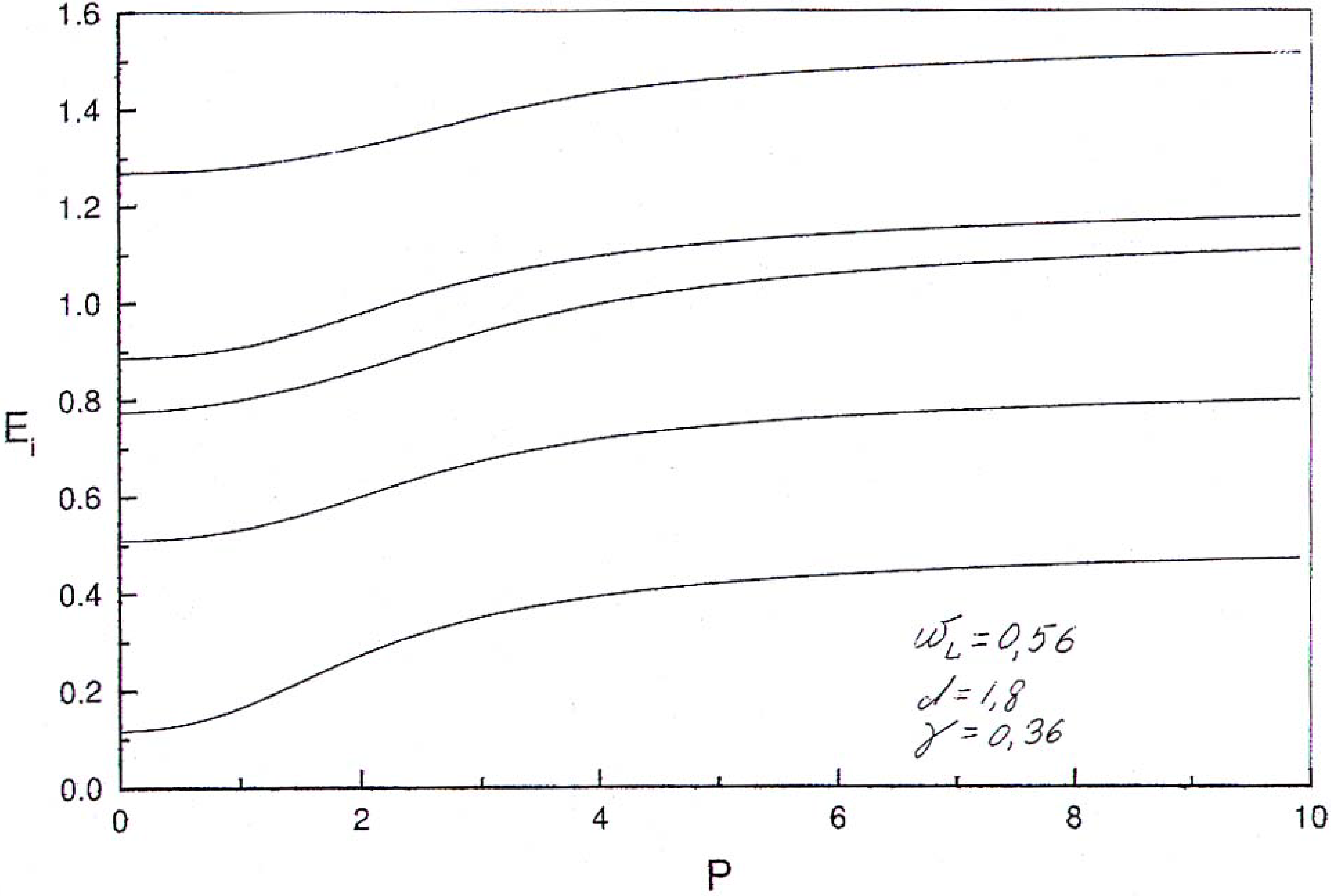
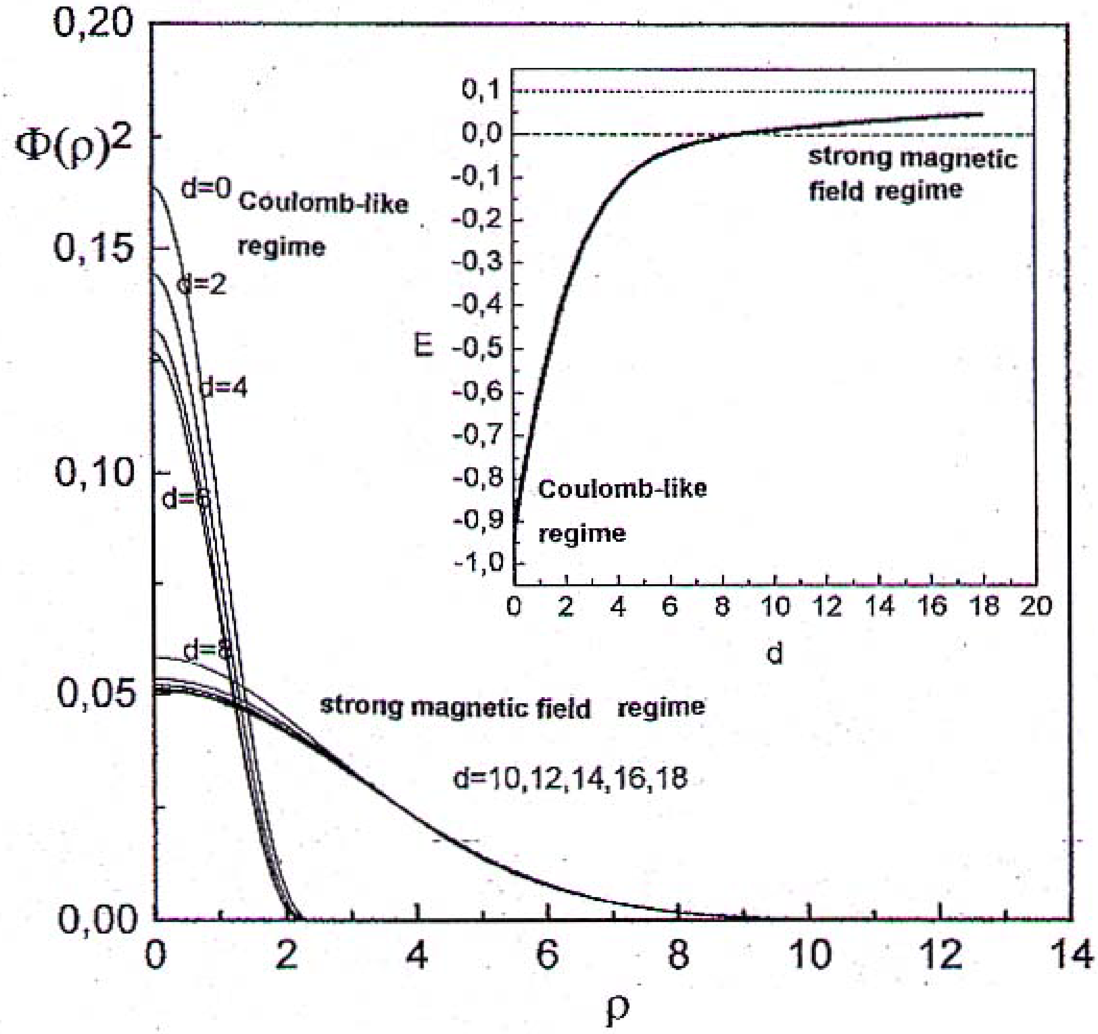
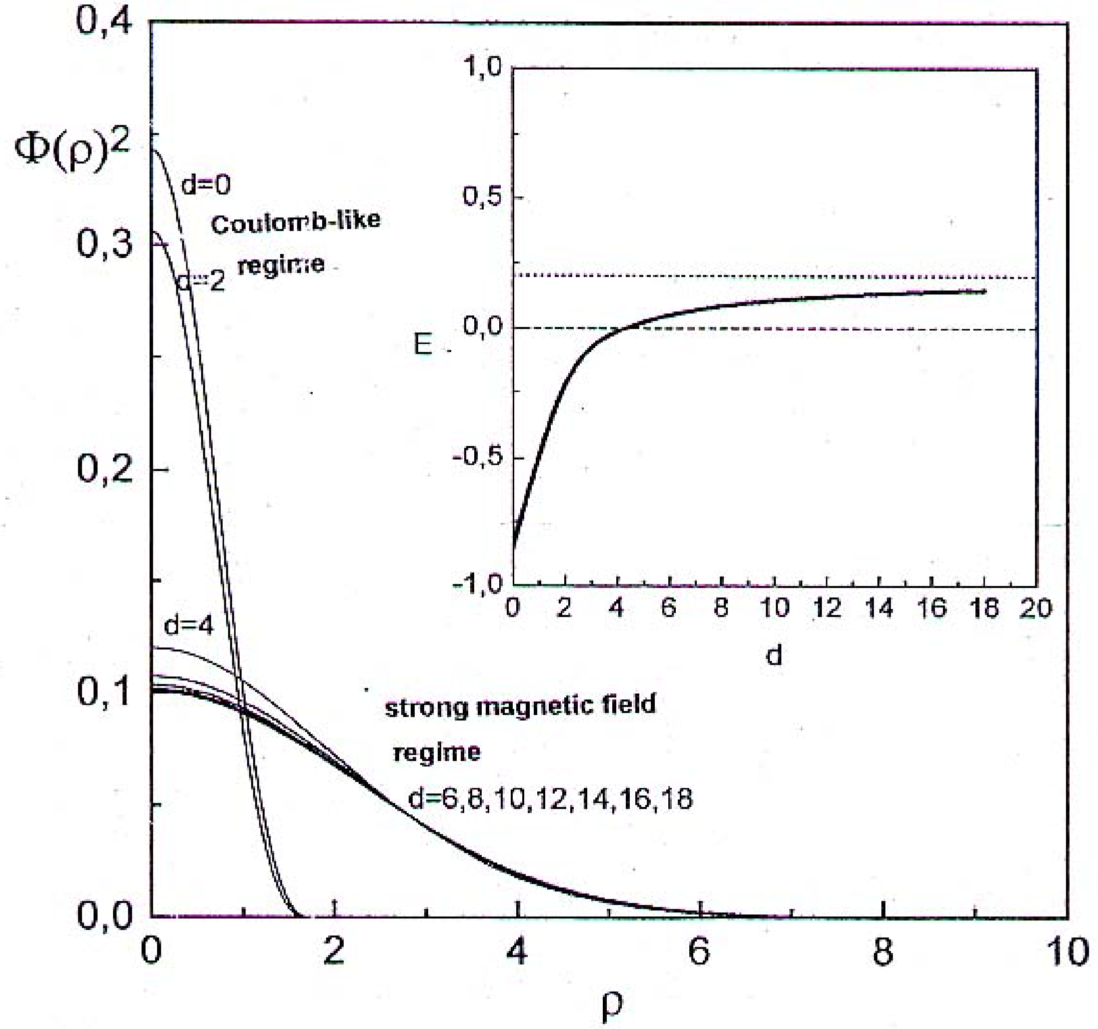


Fig.5. Transformation of ground state wave function squares $\Phi^2(\rho)$ and energy E from Coulomb-like regime to strong magnetic field regime with growth of interlayer distance d for indirect magnetoexciton
a) ω_L =0.1; b) ω_L =0.2; c) ω_L =2.

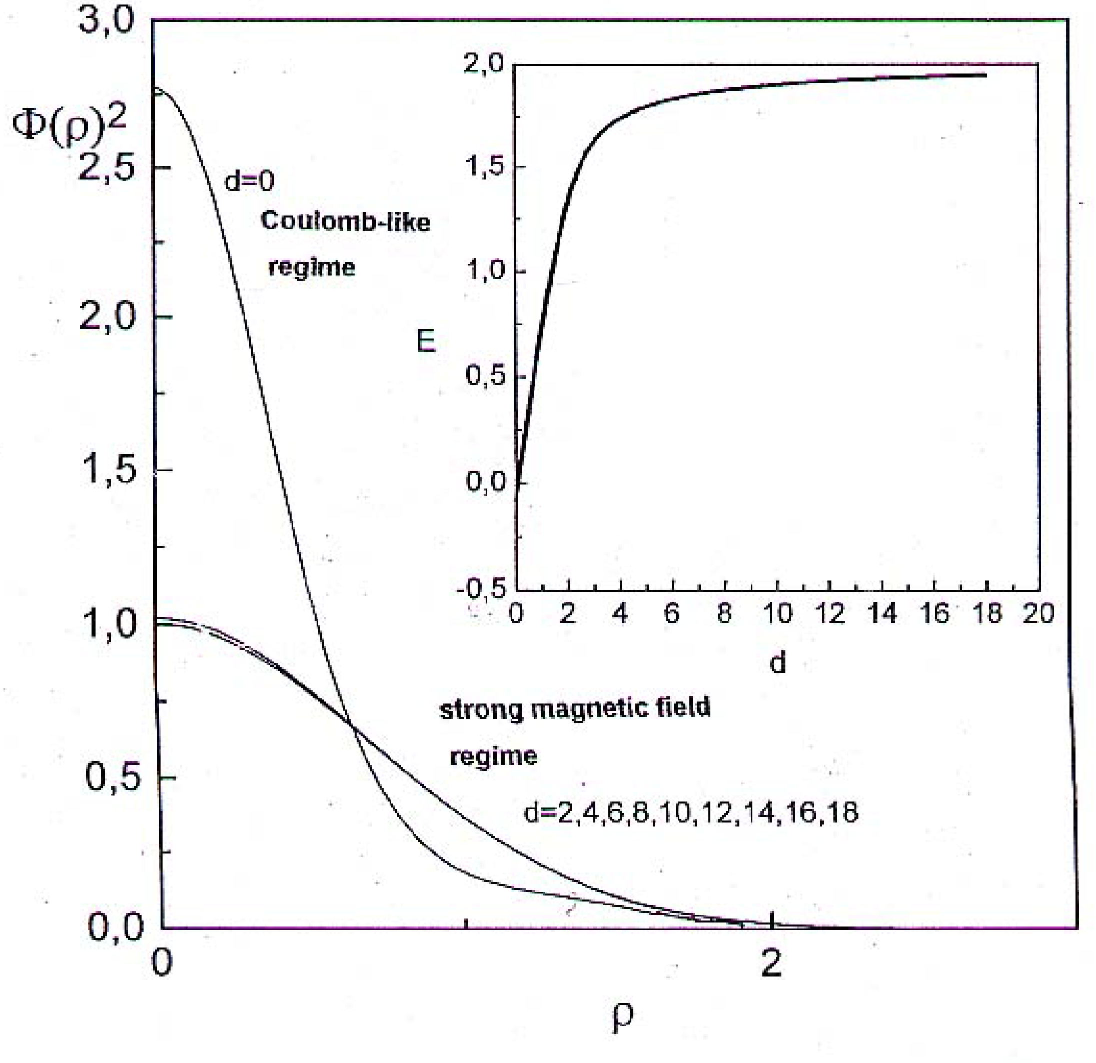


a) ω_L=0.1

Fig.5.



b) $\omega_L=0.2$



c) $\omega_L=2$

Ionization of spatially-indirect exciton moving in magnetic field

The true ionization i.e. exciton decomposition $\rho_{eh} \to 0$, $H \to 0$.

The Hamiltonian of the system has a following form:

$H = -\Delta + i2\gamma(\nabla A) + (1 - \gamma^2)P^2/4 + U$, with $U = A^2 - (1 - \gamma^2)(PA)/2 + V(r)$.

The condition of existence of the"magnetic" minimum located close by $\rho_0 = c\,[H,P]/(eH^2)$

($\rho_{min} < \rho_0$, $\rho_{min} \to \rho_0$, $H \to oo$),

Direct exciton

$P > 3\, r_H^{-2/3}$

Indirect exciton

$P > 3\, r_H^{-2/3}$

$P > (d^2 + \rho_{min}^2)/2\rho_{min}\, r_H^2$

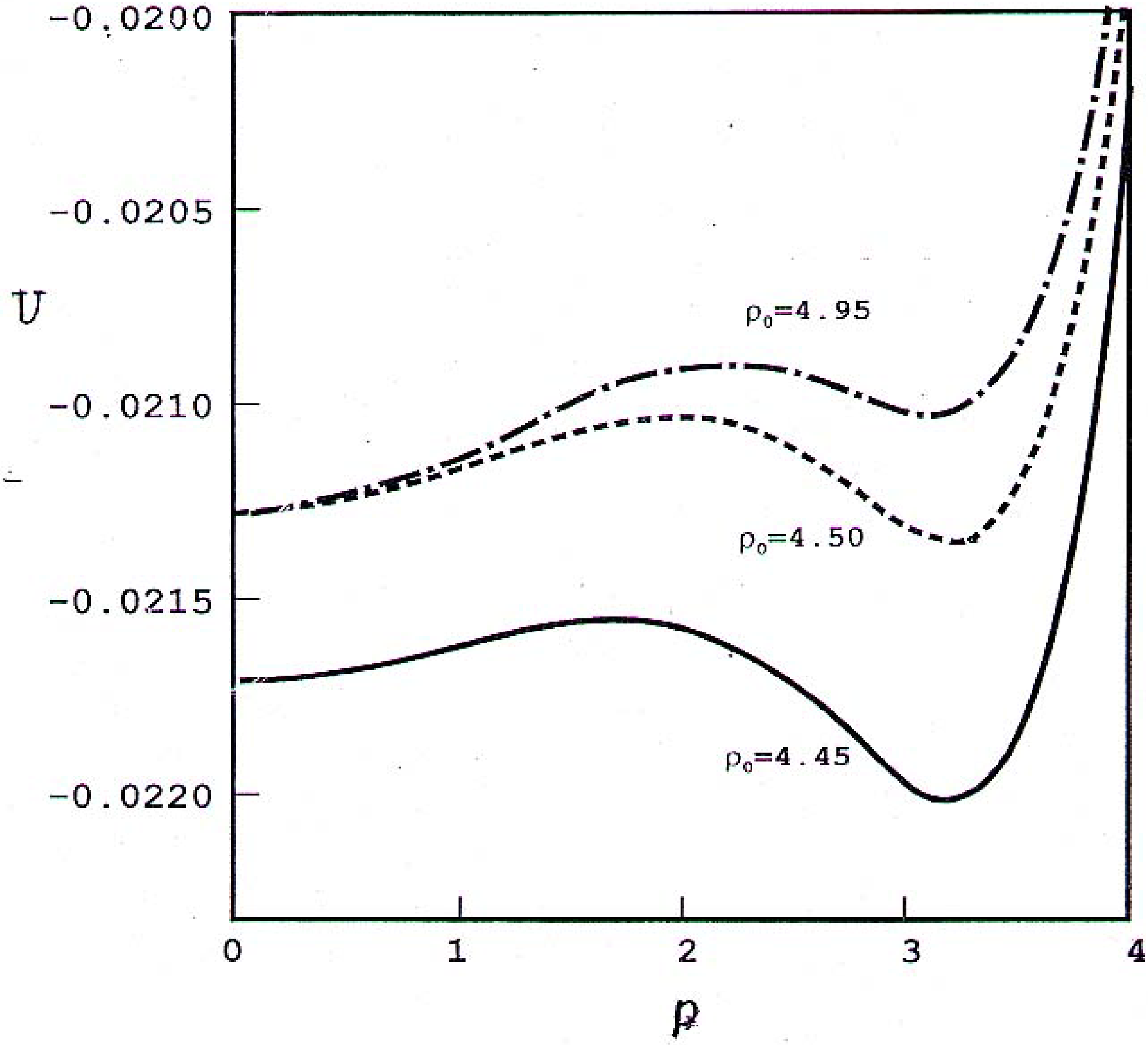
The criterion of capture in magnetic minimum

$E_C > E_M$

Direct exciton

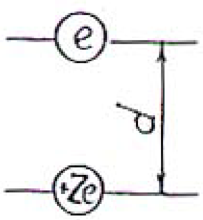
$P > 4(1 - \gamma)^{-1/2}$, $\gamma = (m^*_h - m^*_e)/(m^*_h + m^*_e)$

Tunneling term corresponding to the tunneling under the barrier at the large values of momentum is proportional to $T \sim \exp(P^2 r_H^2 / 2)$. So magnetic ionization can be observed experimentally in strong magnetic fields as $r_H < k/P$, with $k \sim 5\text{-}6$.



Effective potential $U(\rho)$ for different values of ρ_0 at $d=1.8$, $w_L' = 0.063$, $\gamma = 0$.

SPATIALLY - SEPARATED ELECTRON AND CHARGED IMPURITY IN MAGNETIC FIELD



Coulomb interaction matrix terms

$$V^{mm'}_{nn'} = 0,\ m \neq m'$$

$$V^{m}_{nn'} = -Z\left(\frac{\omega_L}{2}\right)^{1/2}\sqrt{\frac{n!n'!}{(n+|m|)!(n'+|m|)!}}\sum_{i=0}^{n}\sum_{j=0}^{n'}\frac{(-1)^{i+j}}{i!j!}\binom{n+m}{n-i}\binom{n'+|m|}{n'-j},$$

$$\times\Gamma(|m|+i+j+1)\left(\frac{d^2\omega_L}{2}\right)^{|m|+i+j+1/2}\Psi\left(|m|+i+j+1,|m|+i+j+3/2,\frac{d^2\omega_L}{2}\right),$$

The ground level n=n'=0

$$V^{m}_{nn'} = Z\left(\frac{\omega_L}{2}\right)^{1+|m|} d^{2|m|}\Psi\left(|m|,|m|+3/2,\frac{d^2\omega_L}{2}\right)$$

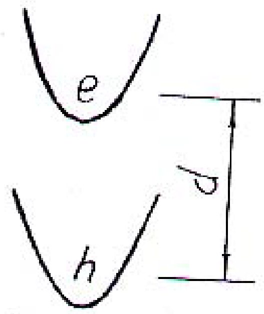
Distances d	Coulomb interaction matrix term	Fine structure (because of splitting on |m|)
Effectively large $d \gg r_H$	$V^{m}_{00} \approx -\frac{1}{d}\left[1-\frac{\lvert m\rvert+1}{d^2\omega_L}\right]$	arise up with the growth of |m| equidistantly $\left(\approx Z\frac{\lvert m\rvert}{d^3\omega_c}\right)$
Effectively small $(d \ll r_H)$	$V^{m}_{00} \approx -Z\sqrt{\frac{\omega_L}{2}}\frac{\lvert m\rvert-1/2}{\lvert m\rvert!}\left[\lvert m\rvert - 1/2 - \frac{d^2\omega_L}{4}\right]$ $\lvert m\rvert \to \infty,\ V^{\lvert m\rvert}_{00} \to -\frac{Z}{\sqrt{m}}$	thickens from down to up to unperturbed level.

Spatially-Separated Exciton in Coupled Quantum Dots

(scheme)

$U = \alpha_e r_e^2$

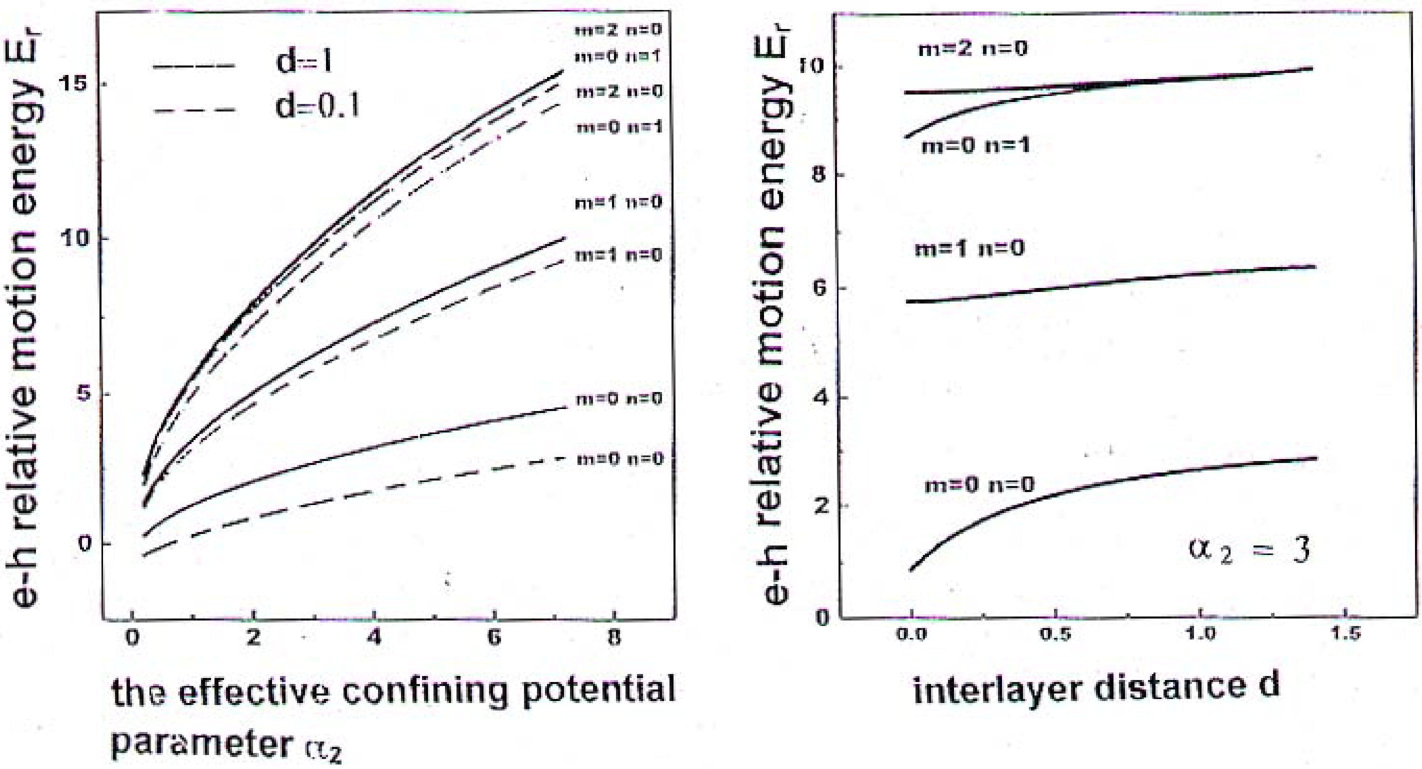
$U = \alpha_h r_h^2$



The dependencies of low-lying levels of electron-hole relative motion energy E_r

a) *versus* the effective confining potential parameter α_2

b) *versus* interlayer distance d



a) b)

Proportionality of effective masses and confining parameters of charge carriers

$$(\mu_h\alpha_e - \mu_e\alpha_h) \to 0. \quad (1)$$

where $\mu_{e,h} = \frac{m^*_{e,h}}{m^*_e + m^*_h}$ **are relative masses of electron and hole.**

$\alpha_1 = \frac{\alpha_e + \alpha_h}{\mu_e\mu_h}, \quad \alpha_2 = \mu_h^2\alpha_e + \mu_e^2\alpha_h$.

C.M.

$$E_{R_{nm}} = 4[\mu_e\mu_h(\alpha_e + \alpha_h)]^{1/2}\left(n + \frac{|m|+1}{2}\right) \quad (2)$$

$$\psi_{R_{nm}} = \left(\frac{n!}{2\pi(|m|+n)!}(\alpha_1)^{|m|+1}\right)^{1/2} R^{|m|}e^{-\sqrt{\alpha_1}R^2/2}L_n^{|m|}(\sqrt{\alpha_1}R^2)e^{im\theta}, \quad (3)$$

where $L_n^{|m|}$ **is generalized Laguerre polynomial.**

Relative motion

$$f_{nm} = \left(\frac{n!}{(|m|+n)!}\alpha_2^{|m|+1}\right)^{1/2} r^{|m|}e^{-\sqrt{\alpha_2}r^2/2}L_n^{|m|}(\sqrt{\alpha_2}r^2) \quad (4)$$

$$f_m = \sum_n C_{nm}f_{nm} \quad (5)$$

$$det\{V^m_{nn'} + \delta_{n,n'}(\varepsilon_{nm} - E_r)\} = 0 \quad (6)$$

where

$$\varepsilon_{nm} = 4\sqrt{\alpha_2}\left(n + \frac{|m|+1}{2}\right), \quad (7)$$

$$V^m_{nn'} = \left(\frac{n!n'!}{(n+|m|)!(n'+|m|)!}\right)^{1/2}\sum_{i=0}^{n}\sum_{j=0}^{n'}\frac{(-1)^{i+j}}{i!j!}\binom{n+|m|}{n-i}\binom{n'+|m|}{n'-j}$$
$$\alpha_2^{(|m|+i+j+1)/2}\Gamma(i+j+|m|+1)d^{2(i+j+|m|+1/2)}$$
$$\Psi(i+j+|m|+1, i+j+|m|+3/2; \sqrt{\alpha_2}d^2) \quad (8)$$

where Γ **is Euler gamma-function,** Ψ **is degenerated hypergeometric Tricomi function.**

Indirect Exciton in Coupled Quantum Dots in Magnetic Field

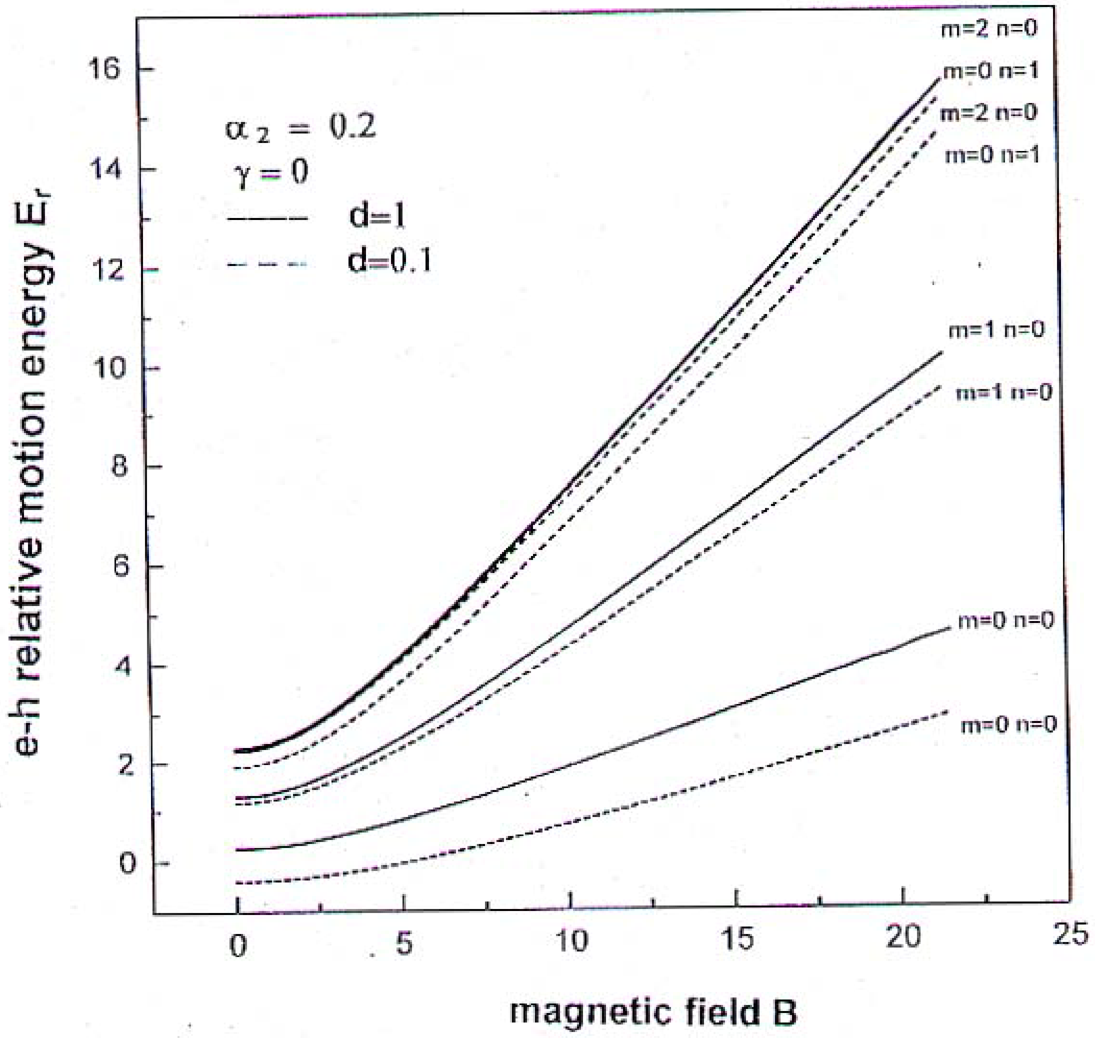
$$\hat{H}_{mf} = \hat{H}_0 + \hat{H}_m, \qquad \hat{H}_m = \frac{e}{2c}\left(\frac{\Lambda_e^2}{m_e^*} - 2\frac{i\hbar\nabla_e\Lambda_e}{m_e^*} + \frac{\Lambda_h^2}{m_h^*} + 2\frac{i\hbar\nabla_h\Lambda_h}{m_h^*}\right).$$

$$(\mu_h(\alpha_e - 1) - \mu_e(\alpha_h - 1)) \to 0$$

$$\alpha_1 \to \alpha_1' = \alpha_1 + \omega_c^2/16$$

$$\alpha_2 \to \alpha_2' = \alpha_2 + \omega_c^2/64$$

$$E_{nm}' = E_{nm} - \gamma m \omega_c/2.$$



$$\omega_c \to \infty$$

$$E_r \to 2\sqrt{\alpha_2'}(2n + |m| + 1) - \gamma\omega_c m/2$$

$$d \to \infty$$

$$E \sim 2\sqrt{\alpha_2'}(2n + |m| + 1) - \frac{\gamma\omega_c m}{2} - \frac{1}{d} + \frac{1}{4\sqrt{\alpha_2'}d^3}$$

$$d \to 0$$

$$V_{nn'}^m \to -\left(\frac{n!n'!}{(n+|m|)!(n'+|m|)!}\sqrt{\alpha_2'}\right)^{1/2} \times$$

$$\times \sum_{i=0}^{n}\sum_{j=0}^{n'} \frac{(-1)^{i+j}}{i!j!}\binom{n+|m|}{n-i}\binom{n'+|m|}{n'-j}\Gamma\left(i+j+|m|+\frac{1}{2}\right)$$

CONCLUSION

Energy spectra and wave functions of spatially-indirect magnetoexciton in CQW are obtained for wide region of control parameters - interlayer distance d and magnetic field H.
Dispersion laws E(P) at different fixed values d and H are obtained.
P - conserving at H≠0 magnetic momentum along the wells,
P is proportional to e-h separation along wells.

For *any fixed magnetic field* value *effective* magnetic field becomes strong with growth of d and/or P.
For weak fixed H exciton energy spectrum transforms with growth of P from Coulomb-like regime (weak effective magnetic field) to strong magnetic field regime.

For strong magnetic field regim spectrum consists of bands bordering to the corresponding Landau level
Energies increase with growth of H and go to the Landau levels with growth of d and P. Band widths rise *vs.* H and diminishes *vs.*d (but relative band width diminishes *vs.* H).
For energy levels corresponding excited states 'roton' minima can appear. For d≠0 with growth of effective magnetic field, 'roton' exremuma diminish and finally disappear.

The possibility of ionization of 2D spatially-separated exciton moving in transverse magnetic field is discussed. The transition from Coulomb minimum to the magnetic one is connected with tunneling. With magnetic field decreasing binding energy of exciton approaches to zero as magnetic field and exciton ionizes.

Spatially-separated electron and charged impurity in multilayer heterostructures in magnetic field are considered.
Energy spectrum of impurity state coincide with exciton energy levels at P=0 coincide (taking into account +Z e - impurity charge).

Energy spectra and wave functions are obtained for indirect magnetoexciton in CQD for wide region of controlling parameters of the problem- interlayer distance d and magnetic field H and confining potential steepness α