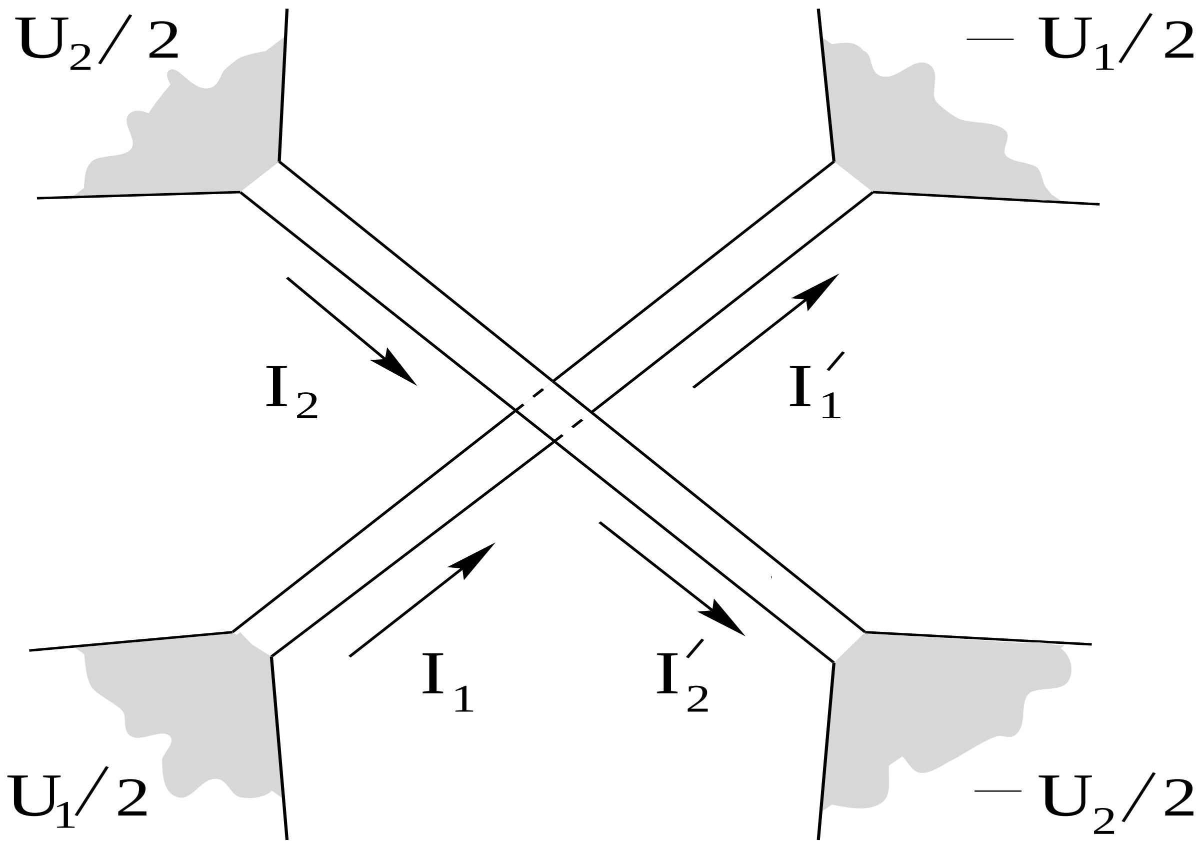
More complex situations . . .

- Crossed single-wall nanotubes (SWNTs): Coulomb drag and transmitted shot noise

- Tunneling DOS of multi-wall nanotubes (MWNTs)

Crossed Nanotubes

Komnik & Egger, PRL 80, 2881 (1998); EPJB 19, 271 (2001)



assume adiabatic coupling to leads

$\Rightarrow$ use radiative boundary conditions

Coupling?

Assume effectively point-like, not too strong coupling

Two coupling mechanisms:

1. Tunneling

   Tunneling irrelevant under RG $\Rightarrow$ can be ignored for weak tunneling

2. Electrostatic coupling

   Coulomb coupling is relevant for $g < 1/2$

$\Rightarrow$ transport strongly affected for $g < 1/2$

Model: Luttinger bosons $\varphi_{1,2}(x)$ plus local electrostatic coupling at $x = 0$

$$\begin{aligned} H &= H_0^{(1)} + H_0^{(2)} \\ &+ C \cos[\sqrt{4\pi g}\ \varphi_1(0)] \cos[\sqrt{4\pi g}\ \varphi_2(0)] \end{aligned}$$

$$H_0^{(i)} = \frac{v_F}{2} \int dx \left(\Pi_i^2 + (\partial_x \varphi_i)^2 \right)$$

Switch to new boson fields:

$$\varphi_\pm(x) = [\varphi_1(x) \pm \varphi_2(x)]/\sqrt{2}$$

$\Rightarrow$ Hamiltonian and boundary conditions completely decouple in new fields

Net result:

Two effective single-impurity problems in the $\pm$ sectors living at doubled interaction parameter $g \to 2g$ and effective applied voltage $U_\pm$:

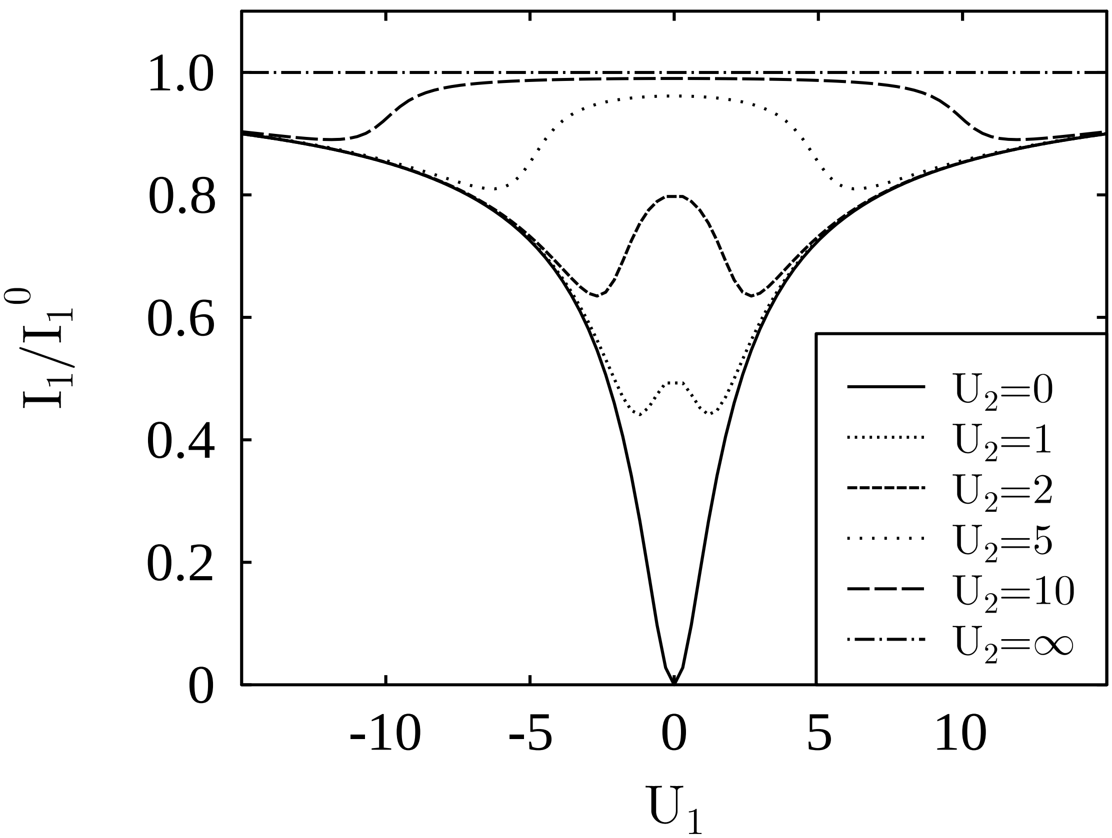
$$
\begin{aligned}
H &= H_+ + H_- \\
H_\pm &= H_0^{(\pm)} + C \cos[\sqrt{8\pi g}\; \varphi_\pm(0)] \\
U_\pm &= (U_1 \pm U_2)/\sqrt{2}
\end{aligned}
$$

$\Rightarrow$ take over exact solution of single-impurity problem
$\Rightarrow$ Dependence of current on current in other SWNT?

Coulomb drag

Coulomb drag

Characteristic dependence of current I_1 on applied cross voltage U_2, shown for $g = 1/4$:



- Perfect zero-bias anomaly for small U_1, U_2

- Dips for $U_1 = 0$ are turned into peaks for finite U_2, with minima for finite U_1

- For Fermi liquid, no singular dependence on U_2, and no U_1 dependence of conductance

- Absolute Coulomb drag

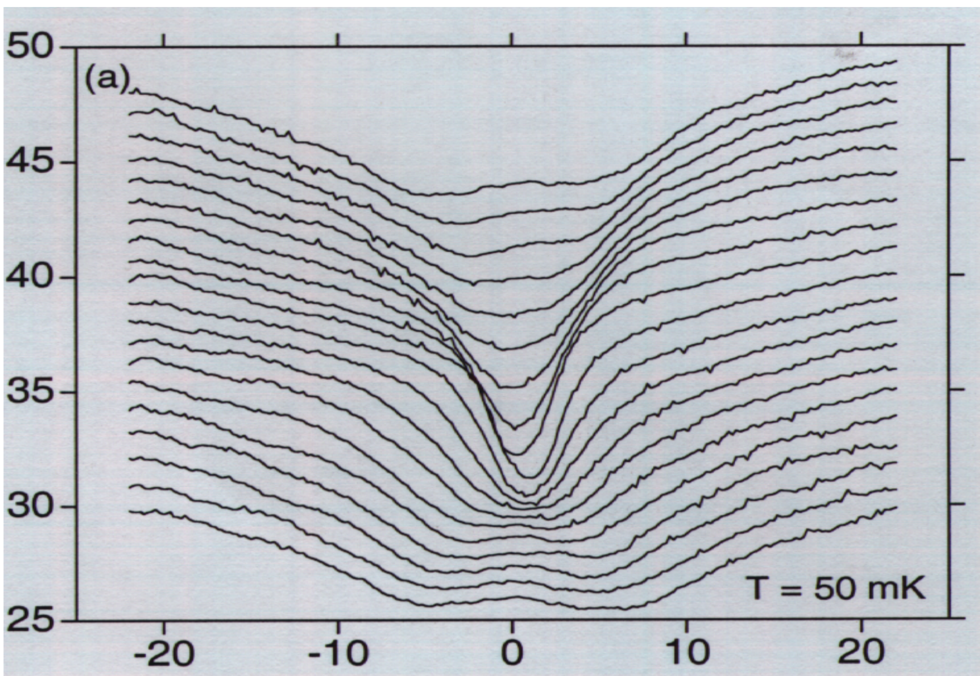
  For extended contact, the linear transconductance is universal and maximal:

$$G_t = \left.\frac{\partial I_1}{\partial U_2}\right|_{U_1=0} = \frac{e^2}{2h}$$

Experimental evidence for crossed Luttinger liquid

Kim et al., J. Phys. Soc. Jpn. 70, 1464 (2001)

I_1–U_1 characteristics, parameterized by cross voltage $U_2 = -20meV, \ldots, 20meV$



- Zero-bias anomalies for small voltage
- Conductance dip becomes a peak by increasing cross voltage

Coulomb drag shot noise

Trauzettel, Egger & Grabert, cond-mat/0109022

Shot noise at $T = 0$ gives important information beyond conductance

$$P(\omega) = \int dt e^{i\omega t} \langle \delta I(t) \delta I(0) \rangle$$

with current fluctuation δI

For weak impurity, DC shot noise of one SWNT is

$$P = 2eI_{\mathrm{BS}}(U)$$

with backscattered current I_{BS}

Now crossed nanotubes:

Imagine $U_1 = 0$ and $U_2 = U \Rightarrow$ any shot noise power $P_1 \neq 0$ must be due to applied cross voltage (Coulomb drag shot noise)

Shot noise power transmitted from one SWNT to the other?

Mapping to $\pm$ sectors goes through $\Rightarrow$ due to decoupling $H = H_+ + H_-$ correlators between φ_+ and φ_- vanish

Consequence: Perfect shot noise locking

$$P_1 = P_2 = (P_+ + P_-)/2$$

- noise $P_1 \neq 0$ due to cross voltage
- exactly equal to P_2
- requires strong interactions: $g < 1/2$
- survives thermal fluctuations

Luttinger liquid in MWNTs?

Differences to SWNTs:

- Overall energy scales are smaller
- Tunneling from the outermost into inner shells?
- Screening of the electron-electron interaction?
- Scattering potential due to inner shells?

Tunneling between shells

In bulk 3D graphite, tunneling between 2D sheets implies quantum coherence perpendicular to sheets
$\Rightarrow$ metallic behavior, band overlap

In MWNTs, this effect is strongly suppressed

- Statistically only 1/3 of all shells metallic (random helicity distribution) since inner shells are not doped

- Even if adjacent shells both metallic, momentum mismatch $k_{F,n} \neq k_{F,n+1}$ spoils inter-shell coherence

- Electron-electron interactions suppress one-electron tunneling

Modified Screening in MWNT

Egger, PRL 83, 5547 (1999)

Long-range tail of interaction still unscreened
⇒ Luttinger liquid effects would survive in a ballistic MWNT

Characteristic dependence on number $N \approx 20$ of available spin-degenerate bands (due to doping or inner shells)

$$g = \left\{ 1 + \frac{8e^2 N}{\pi \kappa \hbar v_F} \ln[L/2\pi R] \right\}^{-1/2}$$

scales to zero as $N^{-1/2}$ for large N

⇒ Luttinger parameter is strongly reduced for MWNT

Suppressed Luttinger exponents

Does reduction of g imply stronger Luttinger liquid effects in MWNTs?

End-tunneling exponent is also N-dependent:

$$\eta_{\text{end}} = \frac{(1/g) - 1}{2N} \sim N^{-1/2}$$

$\Rightarrow$ slow approach to Fermi liquid
$\Rightarrow$ end/bulk tunneling exponents are one order smaller than in SWNTs

Other exponents?

Temperature dependence of weak backscattering corrections, $\delta G \sim T^{-p}$,

$$p = \frac{1-g}{2N} \sim 1/N$$

decays much faster with N, extremly small

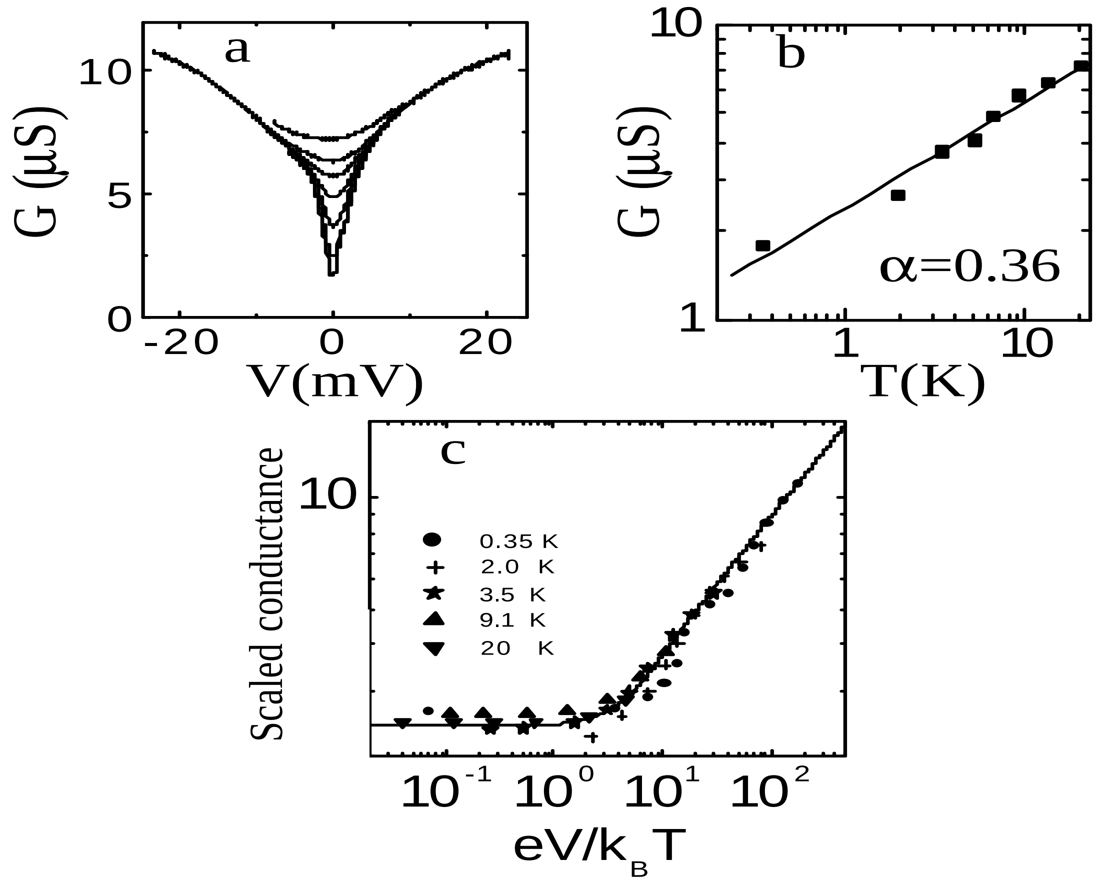
Ionic potential due to inner shells

- Inner shells never align with the outermost shell
- Quasiperiodic incommensurate inner-shell ionic potential
- Effectively diffusive behavior even for clean MWNT, mean free path $\ell \approx 10R$
- include "true" disorder: $\ell \approx 5$ to 100 nm
- MWNT is strongly interacting, weakly disordered metal on a cylinder

Tunneling density of states (TDOS)

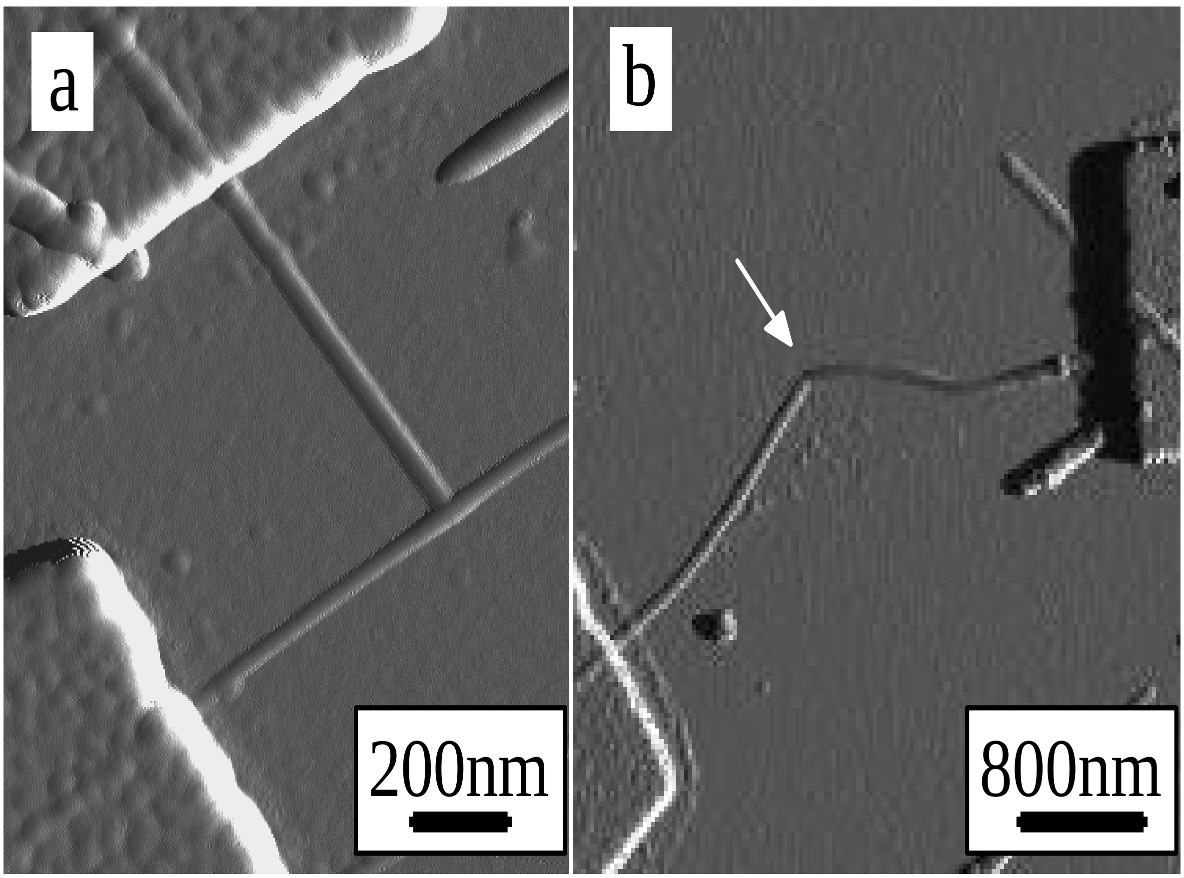
Experimental observation of TDOS as conductance through tunnel contact shows pronounced power-law zero-bias anomaly

Bachtold et al., cond-mat/0012262

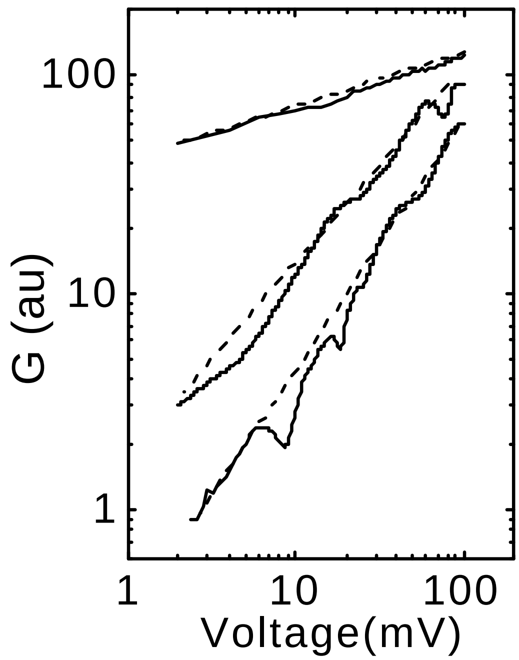


Scaling plot for $T^{-\alpha}dI/dV$ vs. eV/k_BT very similar to Luttinger liquid!

Tunneling DOS for different geometries:



Tunneling exponents are geometry-dependent again (bulk/end tunneling)



Different power law exponents for

- Gold - bulk MWNT contact: $\alpha \approx 0.3$

- End MWNT - bulk MWNT contact:

  $$\alpha + \alpha_{\text{end}} \approx 0.9$$

- End MWNT - end MWNT contact:

  $$2\alpha_{\text{end}} \approx 1.2$$

Puzzles to be explained:

- Bulk TDOS shows power law on energy scales $E < v_F/R \approx 0.1$ Volt

$$\nu(E)/\nu_0 \sim E^\alpha \, , \quad \alpha \approx 0.3$$

- End TDOS shows also power law with doubled end exponent $\alpha_{\text{end}} \approx 0.6$

- Similar experimental results by other groups

- Ballistic Luttinger liquid theory predicts much smaller exponents

- Perturbative Altshuler-Aronov-Lee (AAL) corrections also inconsistent:

$$\nu(E)/\nu_0 = 1 - \text{const.} \log[E\tau] \quad \text{(2D)}$$

$$\nu(E)/\nu_0 = 1 - \text{const.} \, E^{-1/2} \quad \text{(1D)}$$

Coulomb blockade theory

- Tunneling electron diffuses away and excites electromagnetic modes

- Assumes harmonic modes with spectral density $I(\omega)$

- Probability $P(E)$ to excite mode with energy E:

$$P(E) = \frac{1}{\pi}\text{Re} \int_0^\infty dt \exp[iEt + J(t)]$$

with phase correlation function ($T = 0$):

$$J(t) = \int_0^\infty d\omega \frac{I(\omega)}{\omega}\left(e^{-i\omega t} - 1\right)$$

"Debye-Waller" factor

Intrinsic Coulomb blockade in MWNTs

Egger & Gogolin, PRL 87, 066401 (2001)

$P(E)$ determines TDOS

$$\frac{\nu(E)}{\nu_0} = \int dE' \frac{1+\exp(-E/k_BT)}{1+\exp(-E'/k_BT)} P(E-E')$$

For constant spectral density $I(\omega) \approx I(0)$:

$\Rightarrow$ power law for TDOS with $\alpha = I(0)$

Central idea: AAL corrections reflect Coulomb blockade, but strong interactions require non-perturbative treatment

$\Rightarrow$ Coulomb blockade by intrinsic electromagnetic modes of disordered interacting system

Nonperturbative theory

- Assume $\ell < 2\pi R$ for algebraic simplicity (not essential)

- Analytical progress possible using nonlinear σ model in saddle point approximation

  Kamenev & Andreev, PRB 60, 2218 (1999)

- Approximation essentially semi-classical, good for long-ranged interactions

- Simplest case: effective short-range interaction with strength U_0

Microscopic calculation

- confirms Coulomb blockade theory (harmonic electromagnetic modes)

- yields spectral density of intrinsic electromagnetic modes ($\omega\tau \ll 1$)

$$
\begin{aligned}
I(\omega) &= \frac{U_0}{2\pi(D^* - D)}\mathrm{Re} \\
&\textstyle\sum_n \; [(-i\omega/D^* + n^2/R^2)^{-1/2} \\
&\quad -(-i\omega/D + n^2/R^2)^{-1/2}]
\end{aligned}
$$

- Field diffusion constant $D^* = D(1+\nu_0 U_0)$ and charge diffusion constant $D = v_F^2\tau/2$

- Near end: doubled "boundary" spectral density found from nonlinear σ calculation $\Leftrightarrow$ doubled boundary exponent

2D limit

Consider high energies $E > E_T$ above Thouless scale

$$E_T = \frac{D}{(2\pi R)^2}$$

(inverse diffusion time around circumference)

$\Rightarrow$ summation over n can be converted into integration

$\Rightarrow$ gives constant spectral density

$\Rightarrow$ power-law TDOS with bulk exponent

$$\alpha = \frac{R}{2\pi\nu_0 D} \ln(D^*/D)$$

tunneling into interacting 2D diffusive metal

2D logarithmic AAL correction exponentiates into power law

But: restricted to $\ell \ll R$

Low energy limit

Consider very small frequency $\omega \to 0$

$\Rightarrow$ spectral density $\sim \omega^{-1/2}$ due to $n = 0$ mode

$\Rightarrow$ Coulomb blockade equations predict pseudogap for $E \to 0$:

$$\nu(E)/\nu_0 \sim \exp(-E_0/E)$$

with

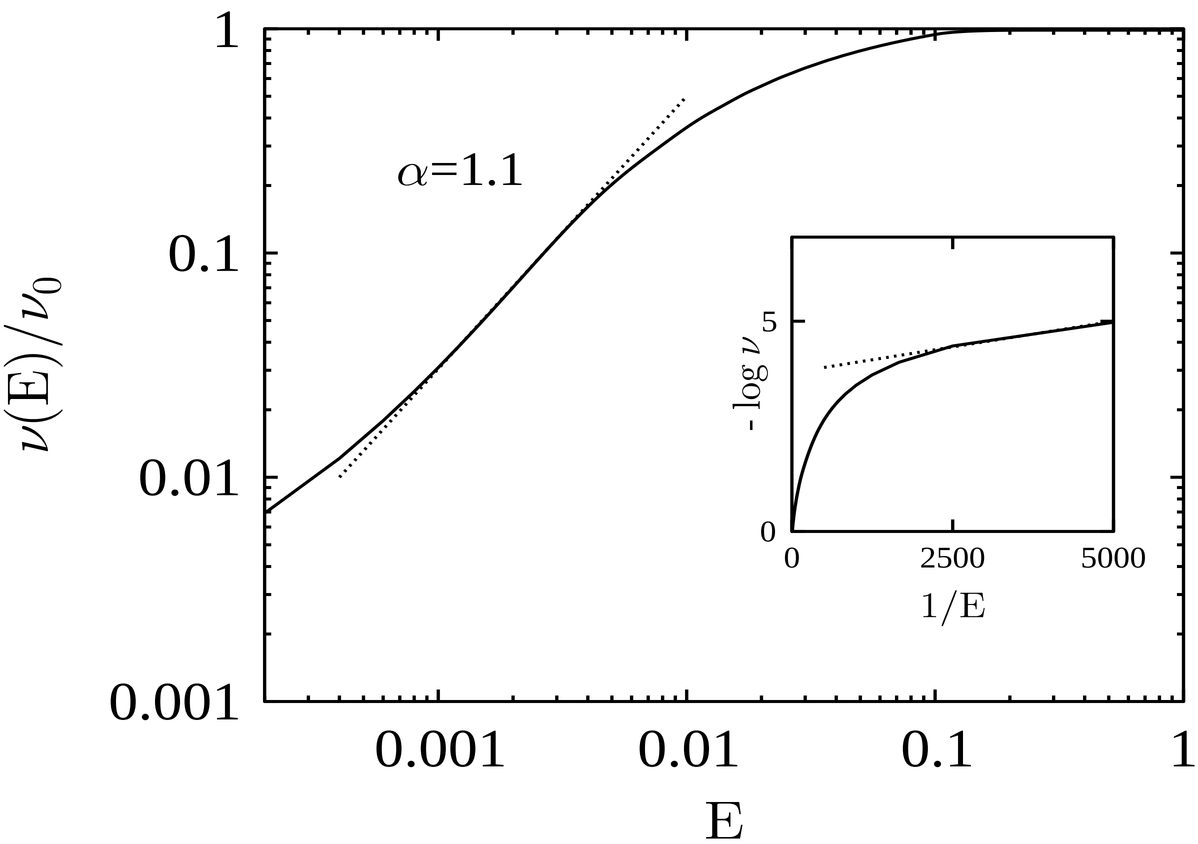
$$E_0 = U_0/8\pi\nu_0 D$$

... not yet observed ...

Numerical solution

Egger & Gogolin, cond-mat/0109336

For $\ell = 10R, \quad U_0/2\pi v_F = 1$:



- Power law well below Thouless scale

- Smaller exponent for weaker interaction

- Only small variation in α with $\ell/2\pi R$

- Pseudogap at extremely low energies

- Correct description of experiment? Check: Pseudogap

Conclusions Lecture 3

- Crossed SWNTs show spectacular consequences of correlations:

  1. Unconventional Coulomb drag

  2. Absolute Coulomb drag

  3. Perfect shot noise locking

- Power-law zero bias anomaly in MWNTs reflects intrinsic Coulomb blockade due to strong interactions in diffusive low-dimensional system

- Prediction: Pseudogap for $E \to 0$

- Open questions: Universality class of MWNTs? Manifestation of strong interactions in conductivity?