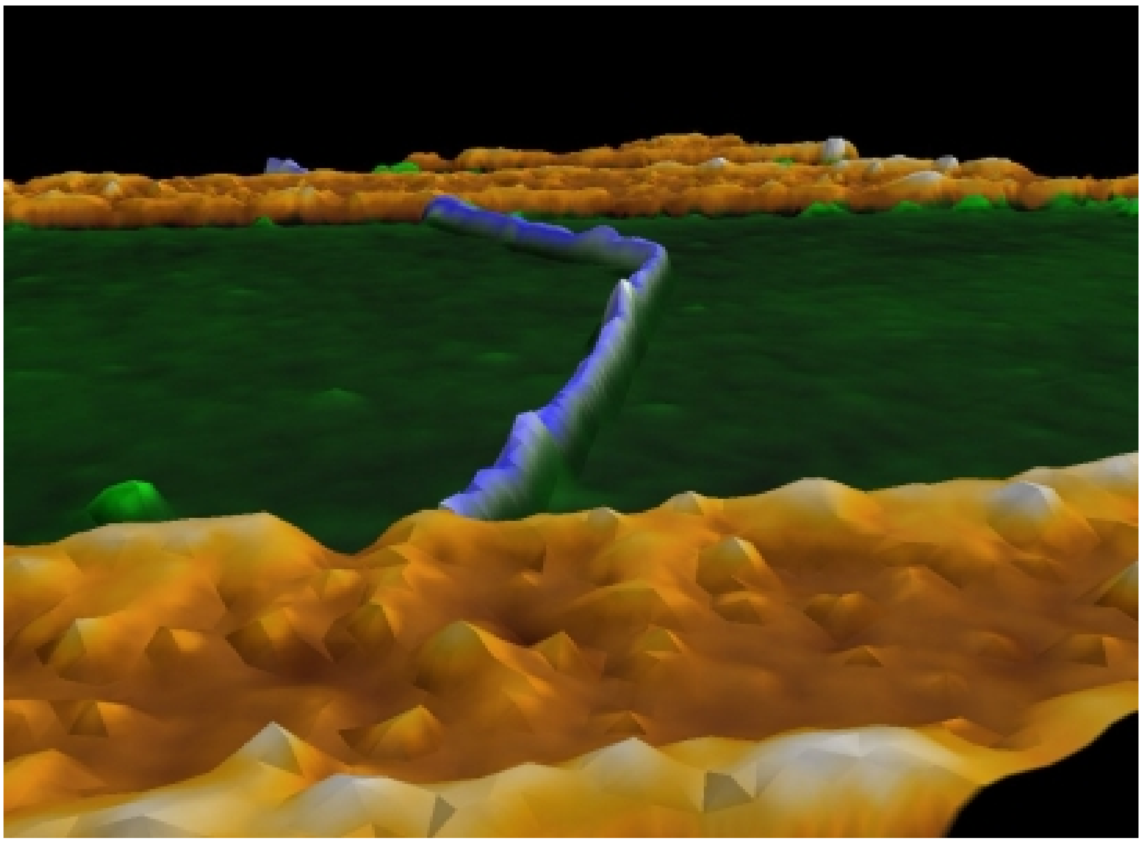
Transport in SWNTs

Typical situation of interest: SWNT in good contact to leads, with one scatterer (kink)



from: Yao et al., Nature 402, 273 (1999)

Current-voltage relation?

Theoretical description

Focus on charge sector (Luttinger phase) $\Rightarrow$ with single impurity at $x = 0$ of strength λ

$$
\begin{aligned}
H &= \frac{v_F}{2} \int dx \left(\Pi^2 + (1/g^2)(\partial_x \varphi)^2 \right) \\
&+ \lambda \cos[\sqrt{4\pi}\ \varphi(0)]
\end{aligned}
$$

Kane & Fisher, PRB 46, 15233 (1992)

Conceptual difficulty: Coupling to external leads (voltage reservoirs)? Landauer approach does not work for interacting systems!

First understand problem of injecting a charge into a Luttinger liquid ...

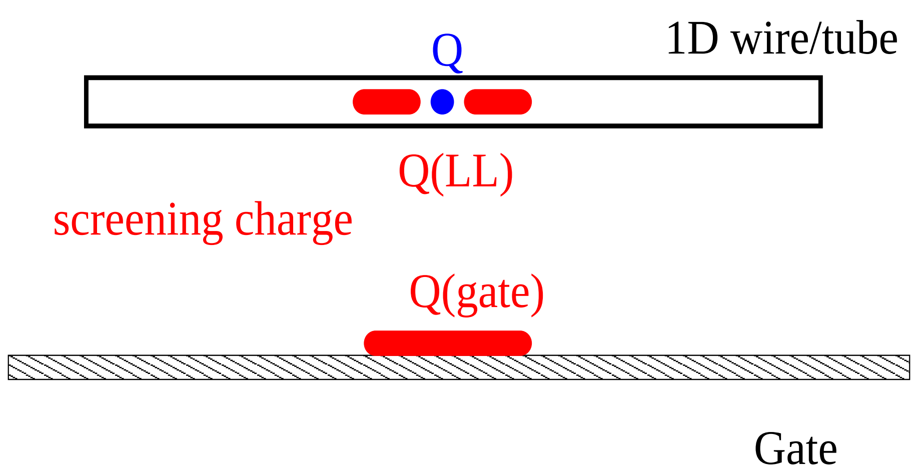
Electroneutrality

Egger & Grabert, PRL 79, 3463 (1997)

On large lengthscales, one always has

electroneutrality =
no free uncompensated charges

Inject "impurity" charge Q



$\Rightarrow$ for Luttinger liquid with screened interactions, electroneutrality holds only when induced gate charges are included

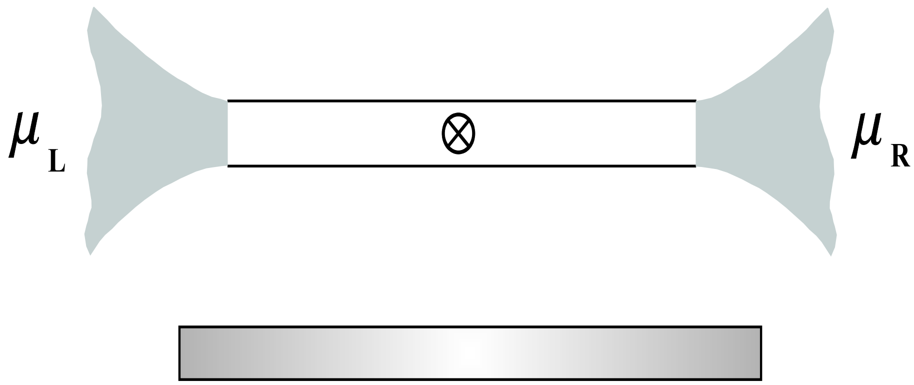
$$Q + Q_{\mathrm{LL}} + Q_{\mathrm{gate}} = 0$$

$$Q_{\mathrm{gate}} = -g^2 Q$$

$$Q_{\mathrm{LL}} = -(1 - g^2) Q$$

Without gate: True long-range interaction $\Rightarrow$ $g \to 0$ $\Rightarrow$ electroneutrality holds for quantum wire alone!

Screening



- Applied voltage $eU = \mu_L - \mu_R$

- Left/right reservoir injects “bare” density of right/left moving charges:

$$
\begin{aligned}
\rho_R^0(-L/2) &= \frac{\mu_L}{2\pi\hbar v_F} \\
\rho_L^0(L/2) &= \frac{\mu_R}{2\pi\hbar v_F}
\end{aligned}
$$

- Fraction $(1 - g^2)$ of injected charge density $\rho = \rho_R + \rho_L$ is screened off
$\Rightarrow$ charge density in Luttinger wire reduced by factor g^2

⇒ Relation between true charge density and "bare" density:

$$\rho_R(x) + \rho_L(x) = g^2[\rho_R^0(x) + \rho_L^0(x)]$$

Difference of right/left-moving densities (current) unaffected by screening:

$$\rho_R(x) - \rho_L(x) = \rho_0^R(x) - \rho_L^0(x)$$

Solving these relations for $\rho_R^0(-L/2)$ and $\rho_L^0(L/2)$ gives Sommerfeld-like boundary conditions for true charge density

Radiative boundary conditions

Egger & Grabert, PRB 59, 10761 (1998)

$$(g^{-2} \pm 1)\rho_R(\mp L/2) + (g^{-2} \mp 1)\rho_L(\mp L/2) = \pm \frac{eU}{2\pi\hbar v_F}$$

- valid for adiabatic contacts to leads, and for arbitrary correlations g & impurity strength λ
- imposed at long times (stationary non-equilibrium state) near contacts in the Luttinger wire
- voltage not just additional term in H
- preserve integrability, easy to use in bosonization

Exact solution

Full transport problem can be solved exactly for arbitrary parameters (g, U, T, λ) using integrability (even including spin)

Egger, Grabert, Koutouza, Saleur & Siano, PRL 84, 3682 (2000)

$\Rightarrow$ complete crossover from weak to strong transmission

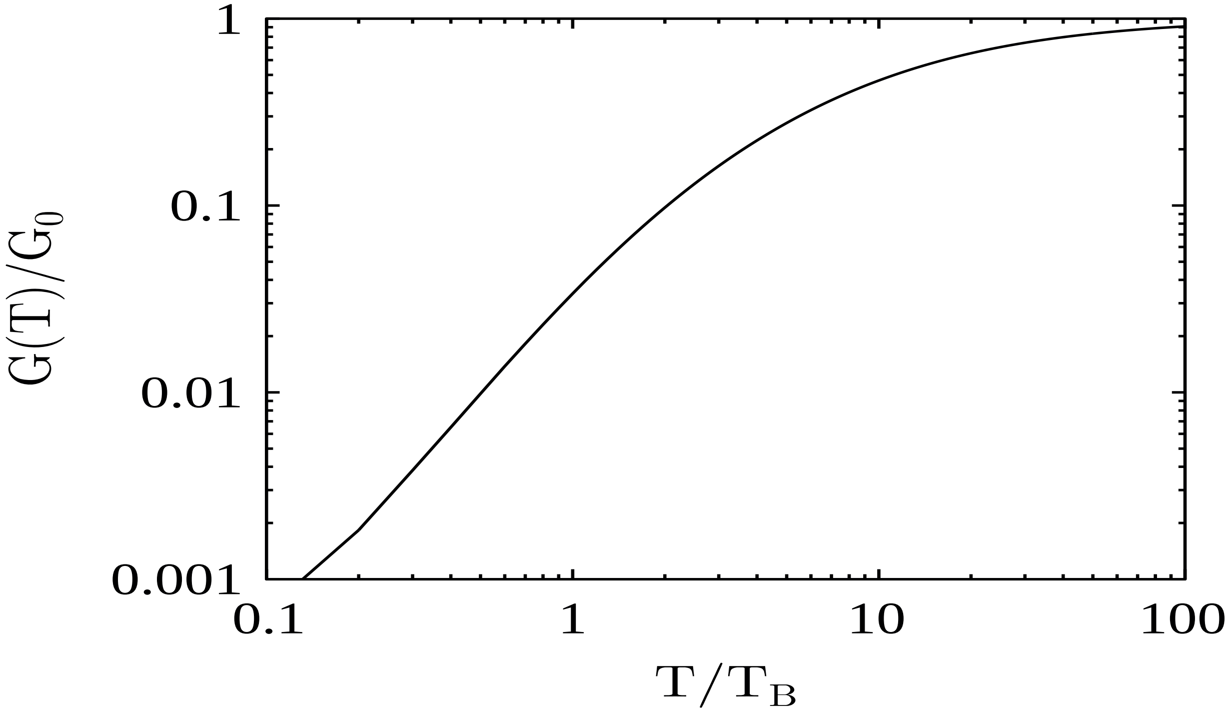
Example: Linear conductance at $g = 1/2$:

$$G(T) = \frac{e^2}{h} \quad \frac{1 - (T_B/T)\psi'(1/2 + T_B/T)}{1 + (T_B/T)\psi'(1/2 + T_B/T)}$$

with impurity scale $T_B = \pi\lambda^2/k_B D$ and trigamma function $\psi'(z)$

Linear transport

Linear conductance $(g = 1/2)$



- clear power-law scaling for low temperatures
- conductance vanishes
- only one impurity!

- On low energy scales, impurity becomes effectively very strong and cuts the SWNT into two halves

$$G(T \to 0) \sim T^{2\eta} \to 0$$

with end-tunneling exponent $\eta > 0$

- Doubled exponent in $G(T)$ since one tunnels out of one half into the other $\Rightarrow$ square of the TDOS

$$\rho(E) \sim E^{\eta}$$

Nonlinear transport

Theory predicts universal scaling functions for nonlinear conductance.

Example: Weak transmission (strong impurity) limit

$$
\begin{aligned}
& T^{-2\eta}\frac{dI}{dU} \propto \sinh(eU/2k_BT) \\
\times\ & |\Gamma(1+\eta+ieU/2\pi k_BT)|^2\{\coth(eU/2k_BT) \\
& -\frac{1}{2\pi}\,\mathrm{Im}\,\psi(1+\eta/2+ieU/2\pi k_BT)\}
\end{aligned}
$$

with digamma function $\psi(z)$

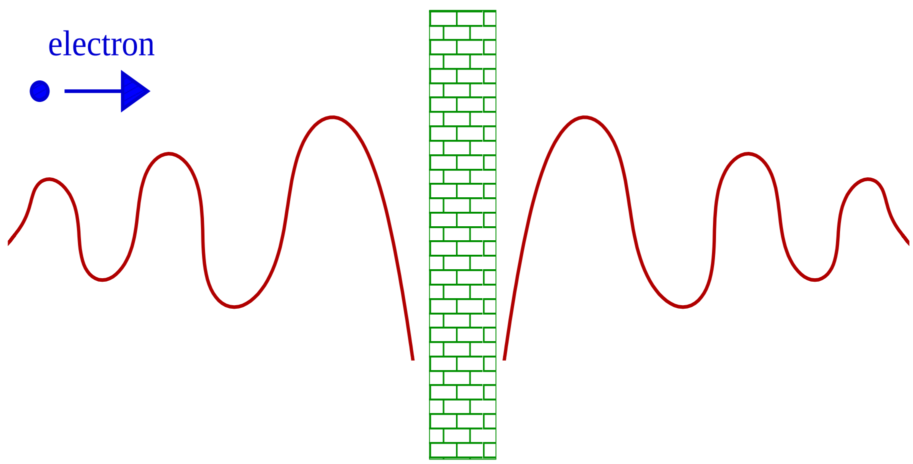
$\Rightarrow$ rhs depends only on eU/k_BT

... observed experimentally in SWNTs ...

Friedel oscillation

Matveev et al., PRL 71, 3351 (1993)

Why $G(T) \to 0$ for low temperatures (insulator!) for just one arbitrarily weak impurity?



- Impurity generates $2k_F$ oscillatory charge disturbance (Friedel oscillation)

- Incoming electron is backscattered by Hartree potential of Friedel oscillation (in addition to "bare" impurity potential)

- Energy dependence $\Leftrightarrow$ Friedel oscillation asymptotics

1D Screening

Bosonization gives

$$\delta\rho(x) \sim \cos[2k_F x - \eta_F]\langle\cos[\sqrt{4\pi}\ \varphi(x)]\rangle$$

Exact asymptotics $(x \to \infty)$

$$\delta\rho(x) \sim \cos[2k_F x - \eta_F]\ x^{-g}$$

Egger & Grabert, PRL 75, 3505 (1995)

- very slow algebraic decay with interaction-dependent exponent $g < 1$

- causes singular backscattering for low energies

- incoming electron at energy $E \to 0$ is completely reflected by Friedel oscillation

Exact Friedel oscillation at $g = 1/2$ using conformal field theory

Leclair et al., PRB 54, 13597 (1996)

$\Rightarrow$ again scaling property:

$$\delta\rho(x) = -\frac{1}{x_B}\cos[2k_F x - \eta_F] e^{x/x_B} K_0(x/x_B)$$

with $x_B \sim \lambda^{-1/(1-g)}$, Bessel function $K_0(z)$

Reproduces x^{-g} scaling for $x \gg x_B$

For $x \ll x_B$ different scaling law:

$$\delta\rho \sim \cos[2k_F x - \eta_F] \ \ln(x/x_B)$$

$\Rightarrow$ extremely slow decay

Friedel oscillation in SWNT

Egger & Gogolin, EPJB 3, 281 (1998)

Several competing Fermi momenta:

$$2q_F \ , \qquad 2(k_F \pm q_F) \ , \qquad 4q_F$$

Dominant wavelength is interaction-dependent

For $g > 0.2$ and $x \gg x_B$:

$$\delta\rho(x) \sim \cos[2q_F x - \eta_F](x/x_B)^{-(3+g)/4}$$

- same power law for $2(k_F \pm q_F)$
- for $x \ll x_B$ slower decay,

$$\delta\rho(x) \sim (x/x_B)^{-(1+g)/2}$$

For $g < 0.2$:

$$\delta\rho(x) \sim \cos[4q_F x - \eta_F](x/x_B)^{-g} \ , \quad (x \gg x_B)$$
$$\sim \cos[4q_F x - \eta_F] \ln(x/x_B) \ , \quad (x \ll x_B)$$

$\Rightarrow$ extremely slow decay

- Quite complex and rich behavior of Friedel oscillation in SWNT

- wavelength and power law depend on interaction strength

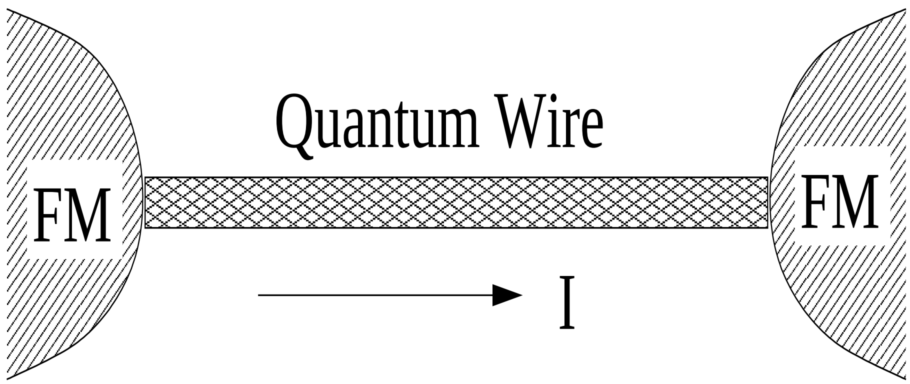
- even more complex behavior in gapped phases ($T < T_b$)

- not yet probed experimentally

Spin-dependent SWNT transport

Balents & Egger, PRL 85, 3464 (2000)

PRB 64, 035310 (2001)



Dependence of $I(V)$ on angle θ between magnetization directions $\hat{m}_{1,2}$ in the ferromagnetic reservoirs?

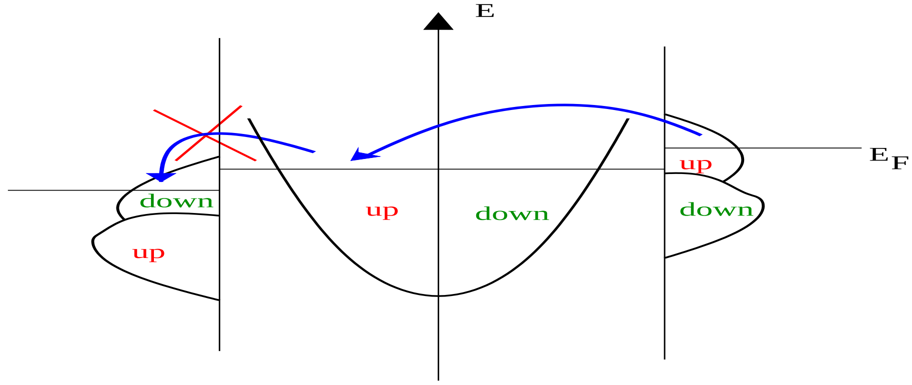
Basic assumption: spin relaxation negligible due to smallness of spin-orbit coupling
$\Rightarrow$ spin current $\vec{J} = \vec{J}_R - \vec{J}_L$ conserved

$$\vec{J}_{R/L} = \psi^\dagger_{R/L}(x)(\vec{\sigma}/2)\psi_{R/L}(x)$$

induced SWNT magnetization: $\vec{M} = \vec{J}_L + \vec{J}_R$

Spin accumulation

Consider Fermi liquid between two Stoner mean-field ferromagnets with $\theta = \pi$:



$\Rightarrow$ pile-up of $\downarrow$ spins, current suppression

For general angle θ, with polarization $P \leq 1$ and spin mixing conductance $Y \geq 1$:

$$\frac{I(\theta)}{I(0)} = 1 - P^2 \frac{\tan^2(\theta/2)}{\tan^2(\theta/2) + Y} \leq 1$$

Brataas et al., PRL 84, 2481 (2000)

$\Rightarrow$ Giant magnetoresistance effect

What happens for a Luttinger liquid?

Description of contact

(assume identical contacts)

Coupling mechanism 1: Tunneling

Spin conductance $G_{\uparrow,\downarrow}$ due to spin-dependent DOS of the FM
$\Rightarrow$ Polarization $P = (G_\uparrow - G_\downarrow)/G \leq 1$
Conductance $G = G_\uparrow + G_\downarrow$

Tunneling Hamiltonian

$$H' = F^\dagger W \Psi + \text{h.c.}$$

with spinors F, Ψ for FM and SWNT at the contact, and 2×2 tunneling matrix

$$W = \sum_{s=\uparrow,\downarrow=\pm} t_s \; (1 + s\hat{m} \cdot \vec{\sigma})/2$$

Spin conductance $G_{\uparrow,\downarrow} = (2e^2/h)|t_{\uparrow,\downarrow}|^2$

Assuming Luttinger liquid model, tunneling into nanotube is power-law suppressed (irrelevant under RG) with end-tunneling exponent

$$\eta = (g^{-1} - 1)/2 > 0$$

$\Rightarrow$ tunneling spin current $\vec{J}_T \sim (V/D)^{\eta/2}$ from lead into tube is strongly suppressed

Exact tunneling spin current:

$$\vec{J}_T = \sum_{s=\pm} (P\hat{m} + s\hat{M})\, \mathcal{I}_\eta(V - sM)$$

with $\vec{M} = \vec{J}_R + \vec{J}_L$ fixed by spin current conservation and

$$\begin{aligned} \mathcal{I}_\eta(V) &= (GV/2)(k_B T/D)^{1+\eta/2} \sinh(eV/2k_B T) \\ &\times \left| \Gamma\left(1 + \eta + \frac{ieV}{2\pi k_B T}\right)\right|^2 \end{aligned}$$

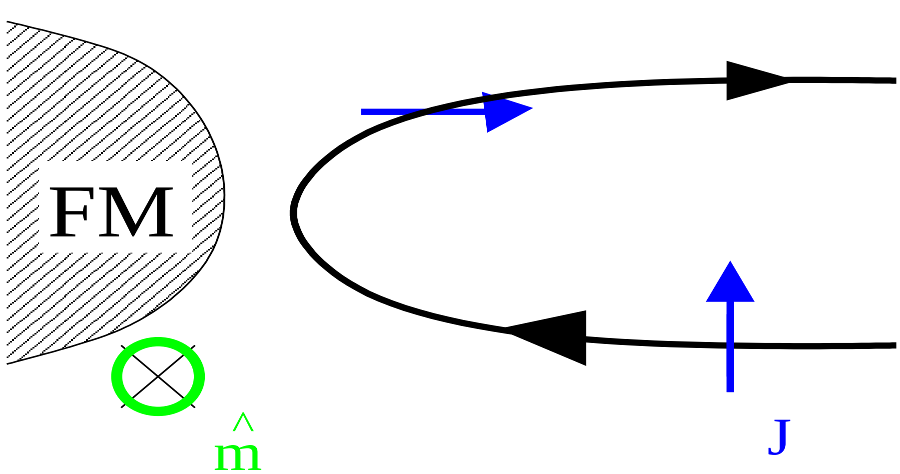
Coupling mechanism 2: Exchange

$$H'' = -K\hat{m} \cdot [\Psi^\dagger(0)(\vec{\sigma}/2)\Psi(0)]$$

With spin-charge separation, exchange acts only in the spin sector
$\Rightarrow$ Exchange current carries no power-law suppression factor, RG scaling of K not affected by interactions, but stays exactly marginal

$\Rightarrow$ Exchange drastically enhanced over tunneling due to spin-charge separation

Exchange physics: Precession of incoming (left-moving) spin current around $\hat{m}$



Without tunneling:

$$\vec{J}_R(0^+) = \mathcal{R}(\phi)\vec{J}_L(0^+)$$

$SO(3)$ rotation matrix $\mathcal{R}(\phi)$
exchange angle $\phi = K/v_F$ and axis $\hat{m}$
$\Rightarrow$ interacting analog of phase shift

With tunneling: Exact treatment of exchange implies boundary condition

$$\vec{J}_R = \mathcal{R}\vec{J}_L + \vec{J}_T$$

With $\vec{M} = \vec{J}_R + \vec{J}_L$ the injected spin current $\vec{J} = \vec{J}_R - \vec{J}_L$ at given contact is ($\phi \ll 1$)

$$\vec{J} = \phi\vec{M} \times \hat{m} + \vec{J}_T$$

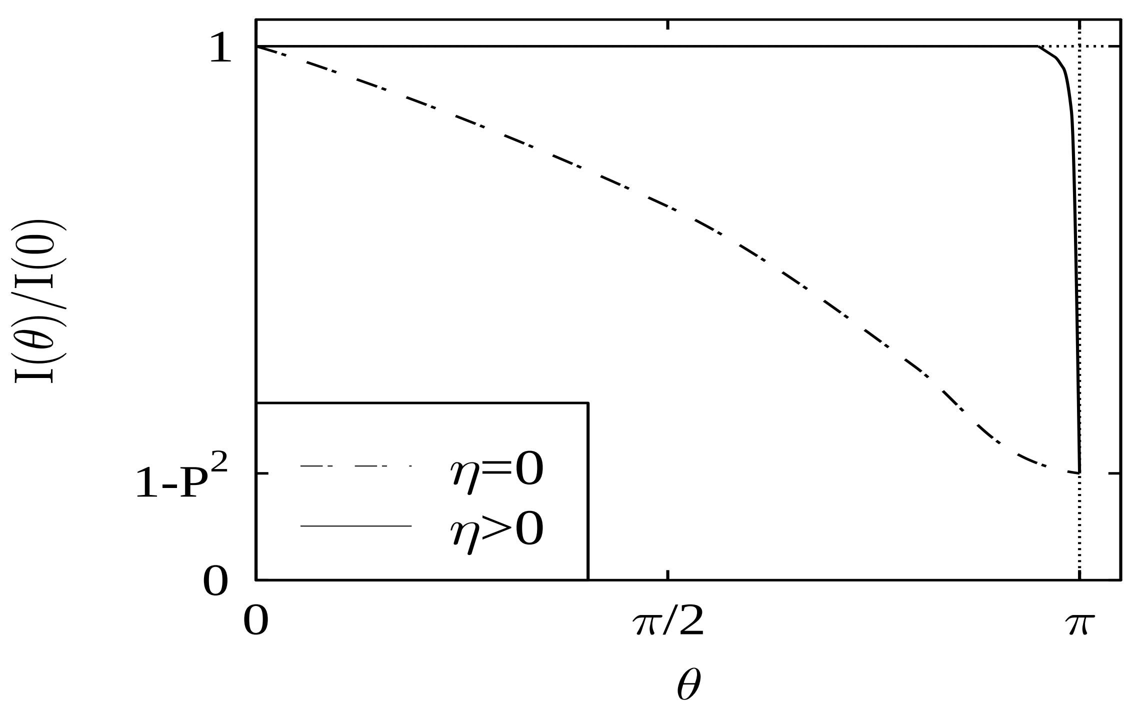
Charge and spin current conservation:
General description of arbitrary circuit of ferromagnets and SWNTs

Spin accumulation for SWNT

Current through FM-SWNT-FM device:

$$\frac{I(\theta)}{I(0)} = 1 - P^2 \frac{\tan^2(\theta/2)}{\tan^2(\theta/2) + Y}$$

$$\begin{aligned} I(0) &= (GV/2)(V/D)^{\eta/2} \\ Y &= 1 + \text{const.}\,(V/D)^{-\eta} \end{aligned}$$



Destruction of spin accumulation
due to spin-charge separation!

Exchange torques during multiple reflections scramble injected spins
$\Rightarrow$ possible observation of spin-charge separation in transport experiment

But: apparently difficult experiment
$\Rightarrow$ simpler variants are possible ...

Spin transport in magnetic field

External magnetic field $\vec{B}$ causes precession of $\vec{M}, \vec{J}$ $\Rightarrow$ simpler experimental checks of spin-charge separation for fixed antiparallel FM magnetizations $\theta = \pi$

Precession phase from left to right end:

$$\gamma = E_{\text{Zeeman}}/(\hbar v_F/L) = \mu_B B L/\hbar v_F$$

Two possibilities:

1. Fixed field strength

Take $\gamma = \pi \Rightarrow$ current as function of angle β between $\hat{B}$ and $\hat{m}$:

$$\frac{I(\beta)}{I(0)} = 1 - P^2 \frac{1}{1 + Y \tan^2 \beta}$$

$\Rightarrow$ spin accumulation destroyed for any $\beta \neq 0$

2. Fixed field orientation

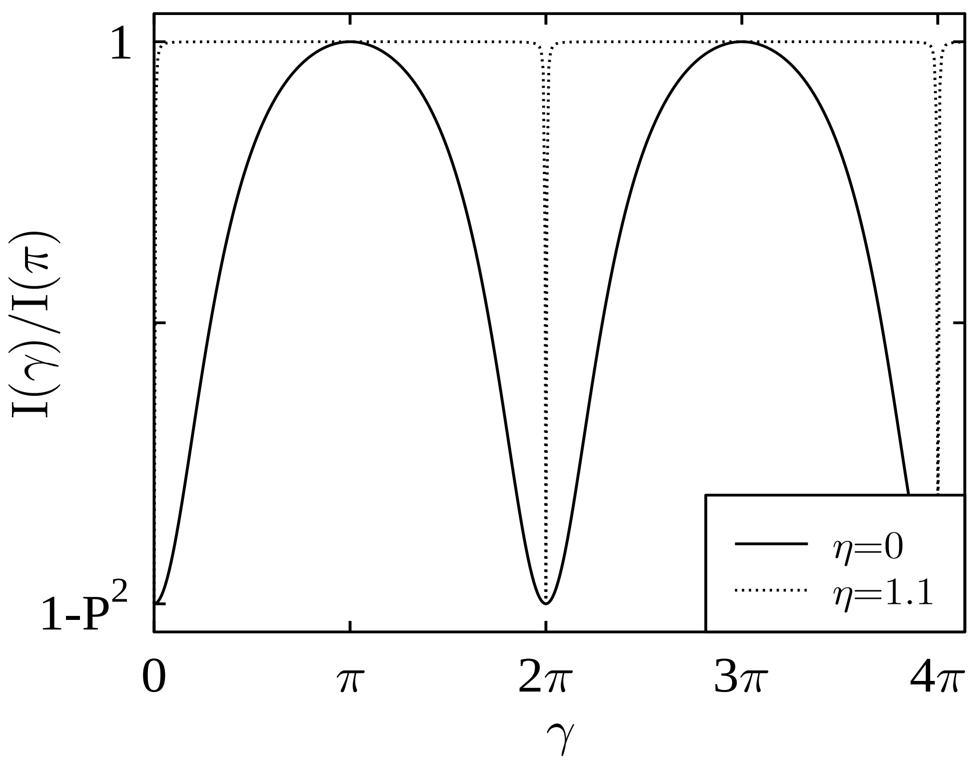
Take $\hat{B} \perp \hat{m}$ and change precession phase γ

Sharp negative magnetoresistance peaks at $\gamma = 2\pi n$ with vanishing width $\Delta\gamma \propto T^{\eta}$

$$\frac{I(\gamma)}{I(0)} = 1 - P^2 \frac{1}{1 + F(\gamma)}$$

$$F(\gamma) \propto (V/D)^{-2\eta} \frac{1 + \phi^2 \cos^{-2}(\gamma/2)}{\phi^2 + \cot^2(\gamma/2)}$$

Magnetic-field dependence of current:



Period $\Delta\gamma$ between dips in the current yields contacted tube length

Dangerous backscattering

Backscattering interaction is correction to Luttinger liquid:

$$H_b = -bv_F \int dx \vec{J}_L \cdot \vec{J}_R$$

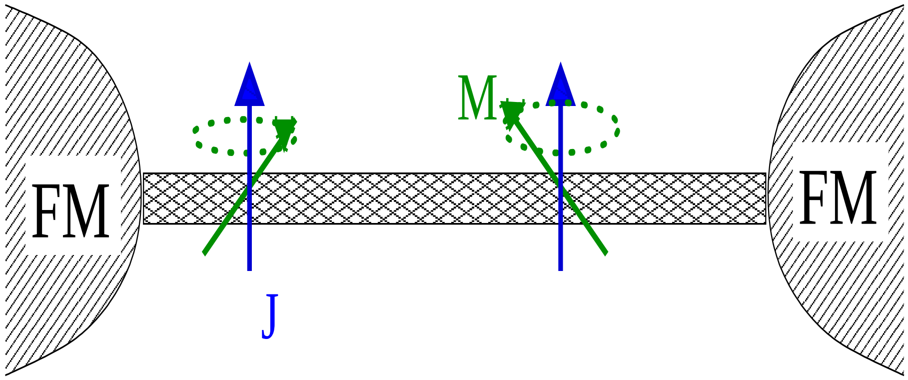
Estimate for SWNT: $b \approx (e^2/v_F)(a/R) \approx 0.1$
$\Rightarrow$ causes only exponentially small gap T_b

But: important effect in spin transport!

Bulk precession of magnetization around conserved spin current $\Rightarrow$ hydrodynamic $T = 0$ description yields ballistic steady state "Landau-Lifshitz" equation

$$v_F \partial_x \vec{M} = b\vec{M} \times \vec{J}$$

$\Rightarrow$ total precession phase $\Delta\varphi = bJL/v_F$ when going from left to right contact!



Spin current appears in the precession phase $\Delta\varphi = bJL/v_F \Rightarrow$ Charge current

$$I(b,\theta)/I(0) = 1 - P^2 X$$

obeys nonlinear self-consistency equation:

$$\sin^2(\theta/2) = \frac{YX}{1+(Y-1)X}\cos^2(\Delta\varphi/2)$$

$$[2J/PI(0)]^2 = (1-X)[1+(Y-1)X]$$

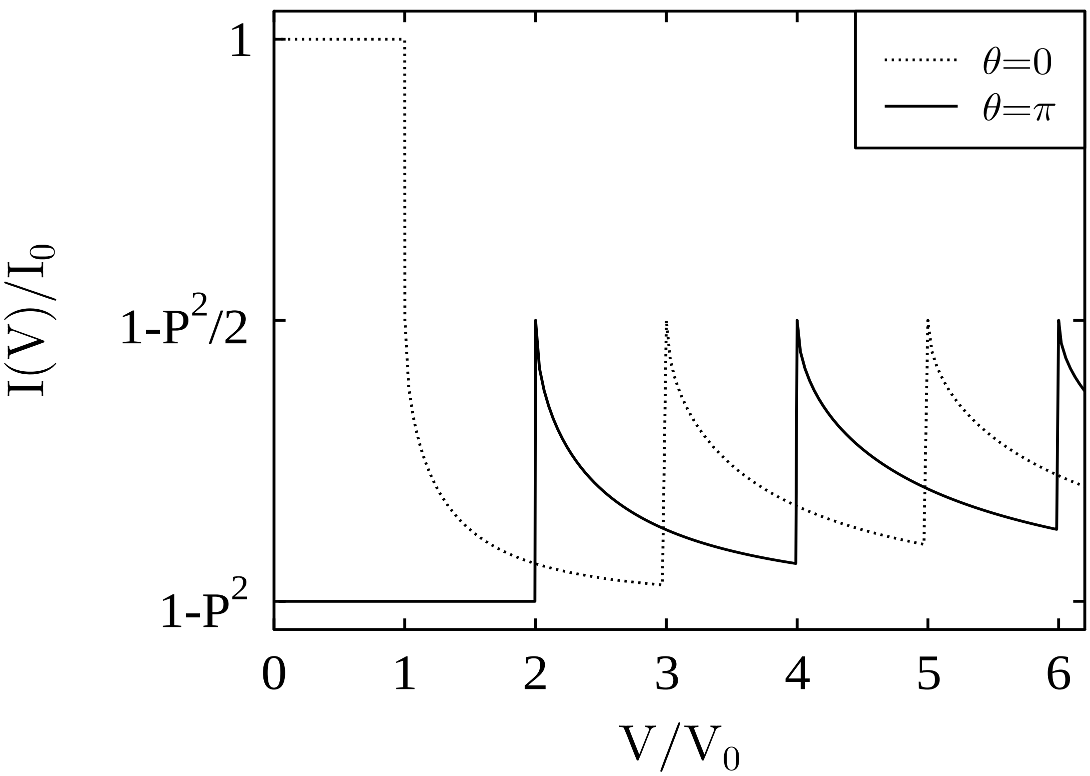
Since $Y \gg 1$, only solutions for

$$\Delta\varphi = (2n+1)\pi - \theta$$

Winding number $n = 0, 1, 2, \ldots$ counts full precession cycles of magnetization
$\Rightarrow$ Current-voltage relation follows in closed form!

Non-sinusoidal oscillations

Because of backscattering, $I-V$ characteristics shows sawtooth-like oscillations, where each oscillation period reflects a complete precession cycle:

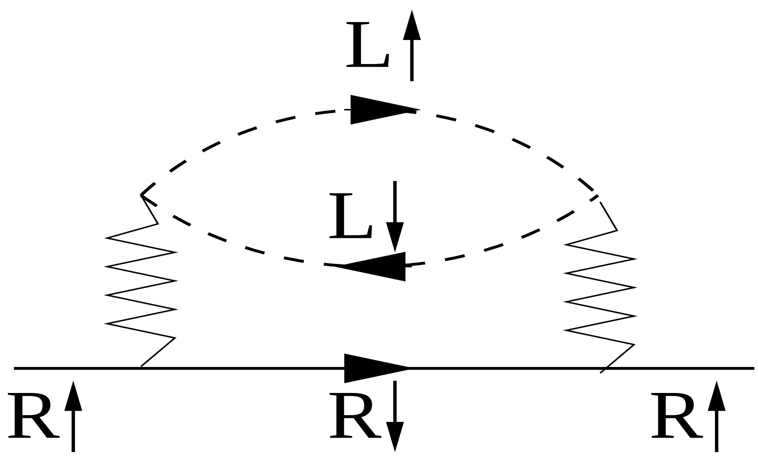


Oscillation period $2V_0$ with

$$eV_0 \approx \frac{\hbar v_F}{L}\frac{e^2/h}{G}\frac{1}{bP} \approx 0.1\,\text{eV}$$

Spin Diffusion

Backscattering scatters spin $\Rightarrow$ conserved charge current, but spin current relaxed



Fermi's golden rule: Mean free path for spin diffusion

$$\ell = \frac{1}{b^2}\frac{\hbar v_F}{k_B T} \approx 1\mu \left(\frac{500K}{T}\right)$$

Spin transport diffusive for $L > \ell$

Conclusions Lecture 2

- Charge transport for correlated 1D electron liquid in SWNT with impurity
- Coupling to voltage reservoirs via radiative boundary conditions $\Rightarrow$ exact solution of full transport problem
- Singular backscattering by Friedel oscillation
- Observation of spin-charge separation possible via spin transport
- Non-sinusoidal oscillatory current-voltage relation due to backscattering in spin transport setup